EE 208-CONTROL ENGINEERING LAB

1nd SEMESTER OF AY 2023-2024

Lab Report of Experiment 9

(Impact of gain and sampling time on gain and phase margins)

Pulkit Singh (2021EEB1150)

Harshit (2021EEB1175)

Objective of the Experiment

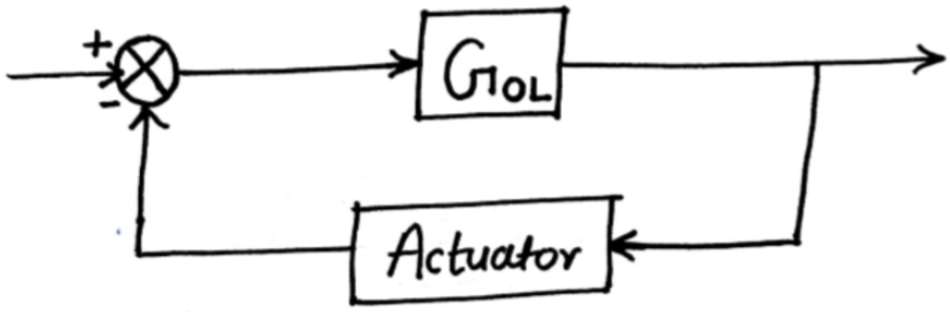
To study the variation in gain margins and phase margins of a system with changes in feedback gain and sampling time.

Given Data

1. Furnace Transfer Function $G_{OL}(z) = 10^{-5} * (4.95z + 4.901)/(z^2 - 1.97z + 0.9704)$.
2. Actuator Transfer Function $A(z) = \frac{0.9515}{(z-0.9048)}$
3. Sampling Time = 0.01s.

Block Diagram

The Block Diagram of the given system is as shown below.



Transfer Function of Complete System

The open loop transfer function of the system along with actuator (without feedback) is as follows-

$$G_{OL}(z) = 10^{-5} * \frac{4.711z+4.644}{z^3-2.785z^2+2.735z-0.8781}$$

After taking the negative feedback with a positive integer (K) gain, the closed loop transfer function will look like-

$$G_{CL}(z) = \frac{10^{-5}*(4.7117z + 4.664)}{z^3-2.875z^2+z(2.753+4.711*10^{-5}k)+4.66*10^{-5}-0.8781}$$

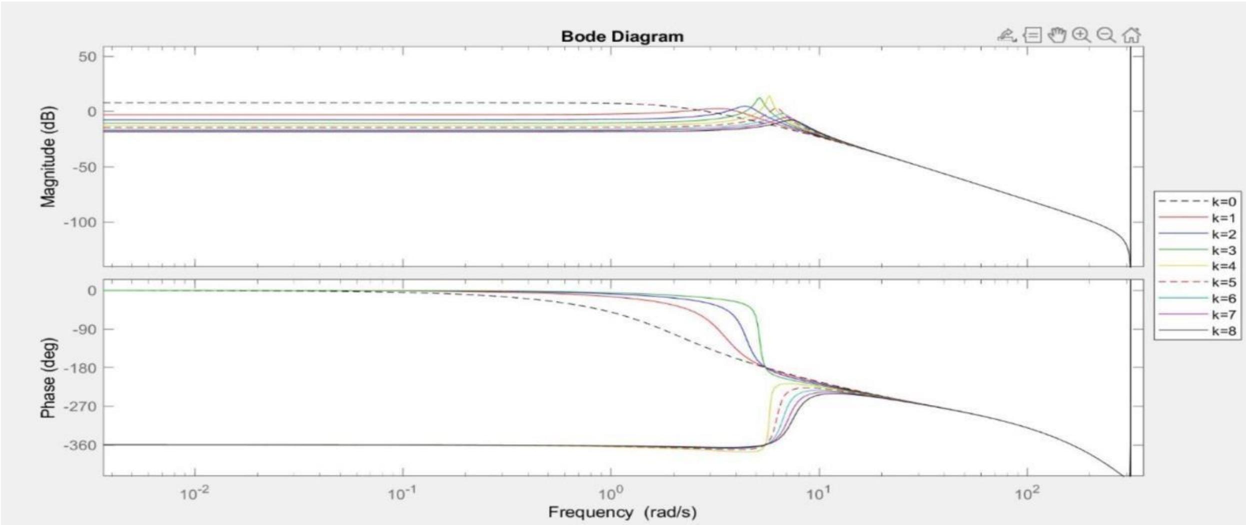
Approach

We approached the problem given by creating multiple arrays to store the required information like sampling time, different gain values, gain margins etc. Then we iterated through different values of feedback gain and stored the required result in respective array. Then we kept feedback gain constant and varied the sampling time and observed the results obtained.

For stability , we check the gain margins and phase margins to know if the system is stable, unstable or marginally stable. We, then confirm our results from the pole-zero plots .

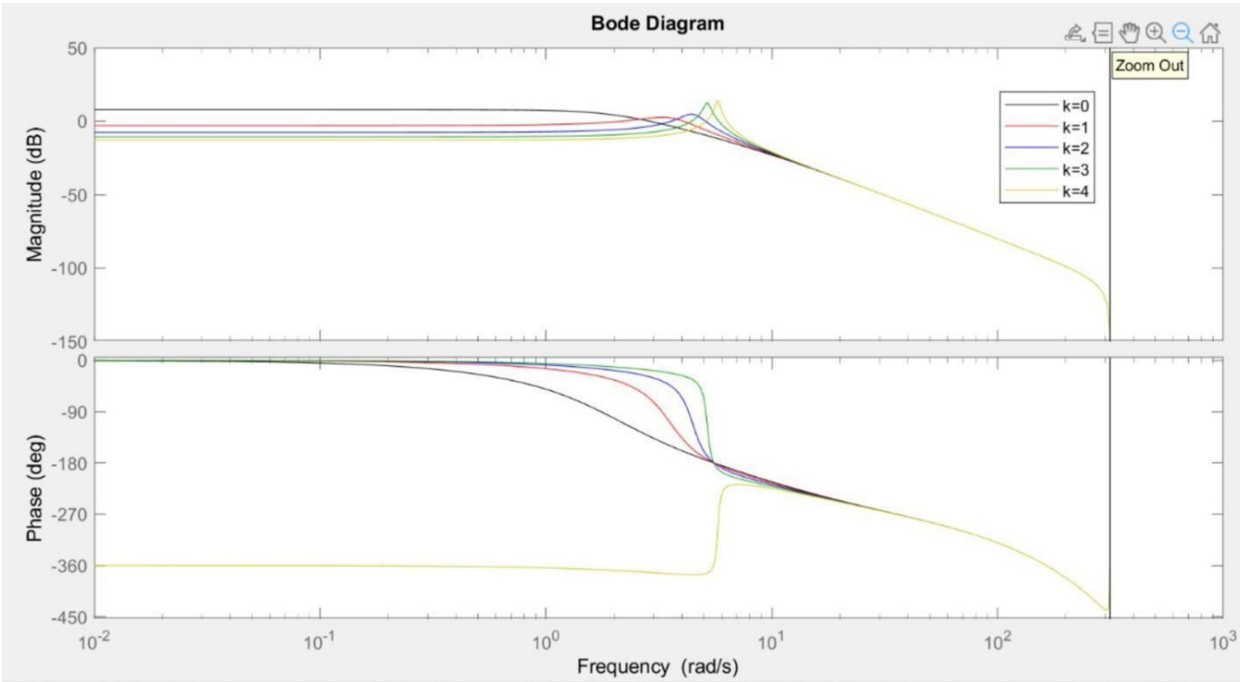
Observations

Varying the feedback gain:



BODE PLOT FOR DIFFERENT K

From the above plot, it is evident that the Gain Margin is infinity for K>4. So, for these K, the system is unstable. Bode Plot for K<4 are given below.



BODE PLOT FOR K<4.

[

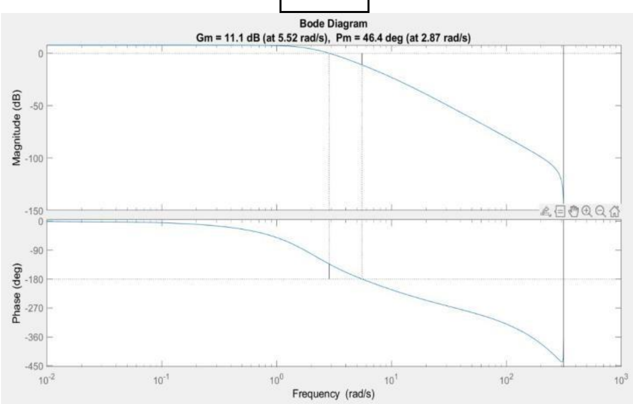
Gain Margins and Phase Margins:

The Gain Margins and Phase Margins for different values of feedback gain (K) can be seen in the table below:

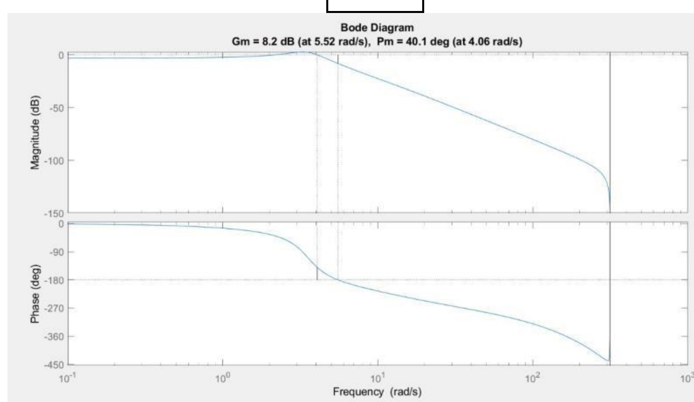
FEEDBACK GAIN (K)	GAIN MARGIN (dB)	PHASE MARGIN (deg)	GCO (rad/s)	PCO (rad/s)
0	11.1	46.35	5.52	2.87
1	8.2	40.13	5.52	4.05
2	3.92	15.86	5.52	5.07
3	-4.88	-12.18	5.52	5.80
4	Inf	-41.73	Nan	6.31
5	Inf	-77.59	Nan	6.60
6	Inf	Inf	Nan	Nan
7	Inf	Inf	Nan	Nan

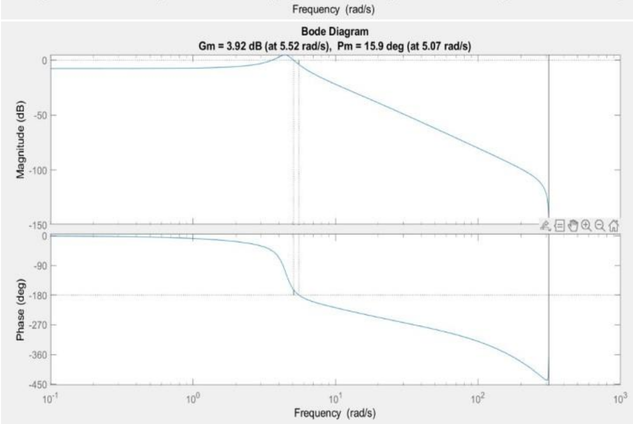
The above values can be confirmed from the Bode Plots for K=0, 1, 2, 3, 4 as shown along:

K=0

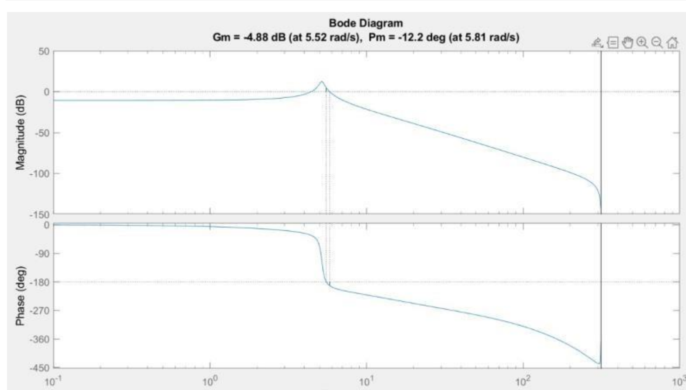


K=1





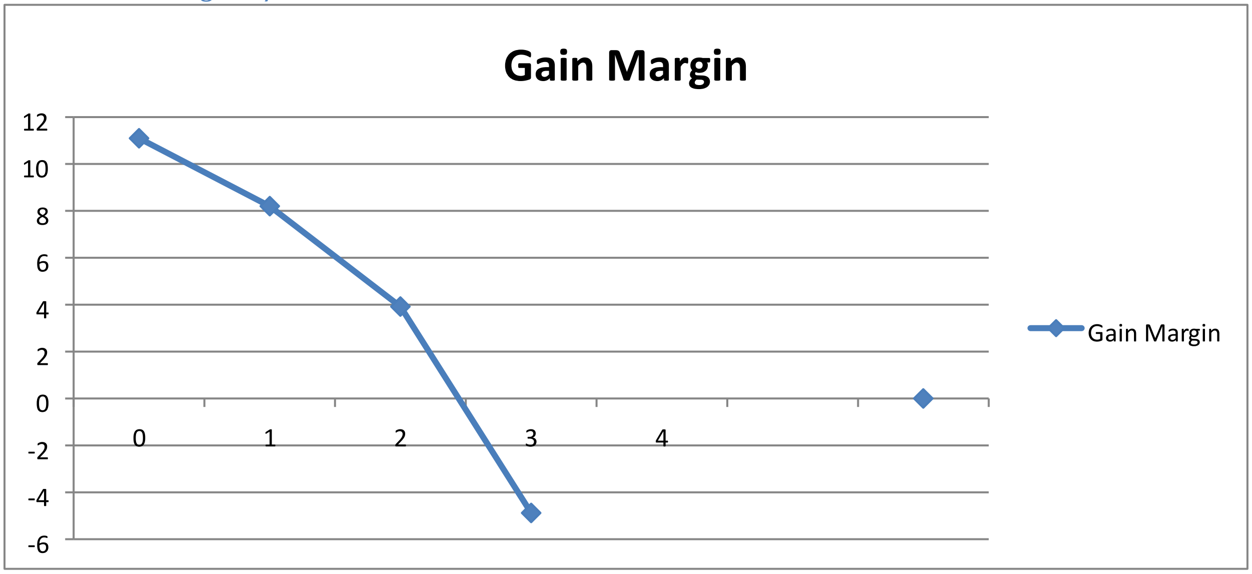
K=2



K=3

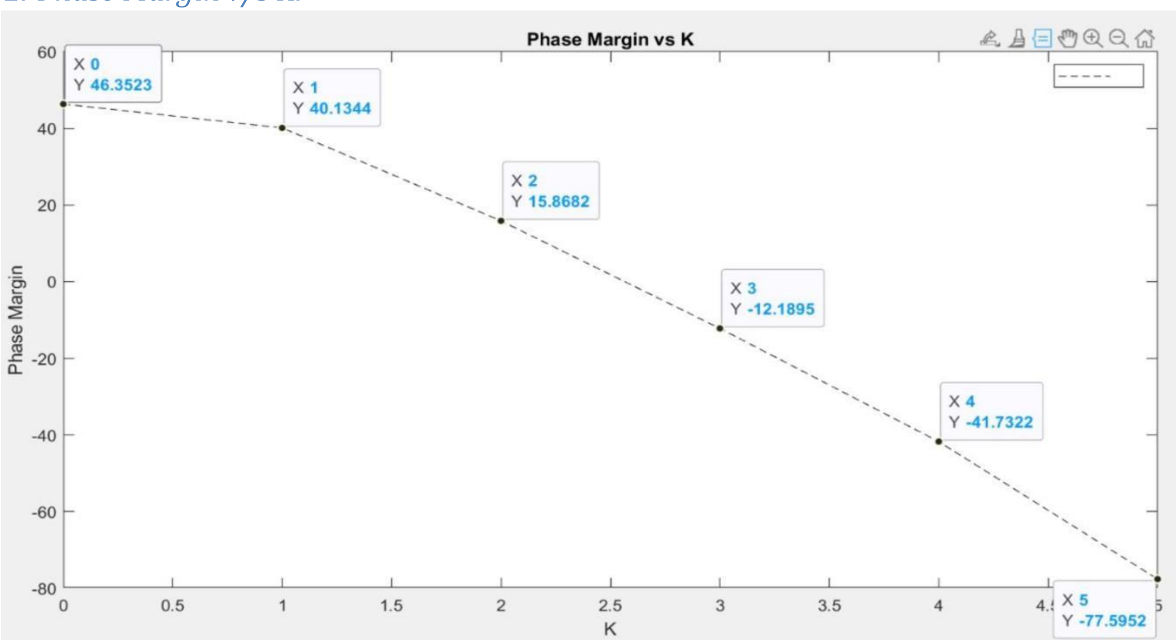
Plots for variation of GM, PM, GCO and PCO with K:

1. Gain Margin v/s K:



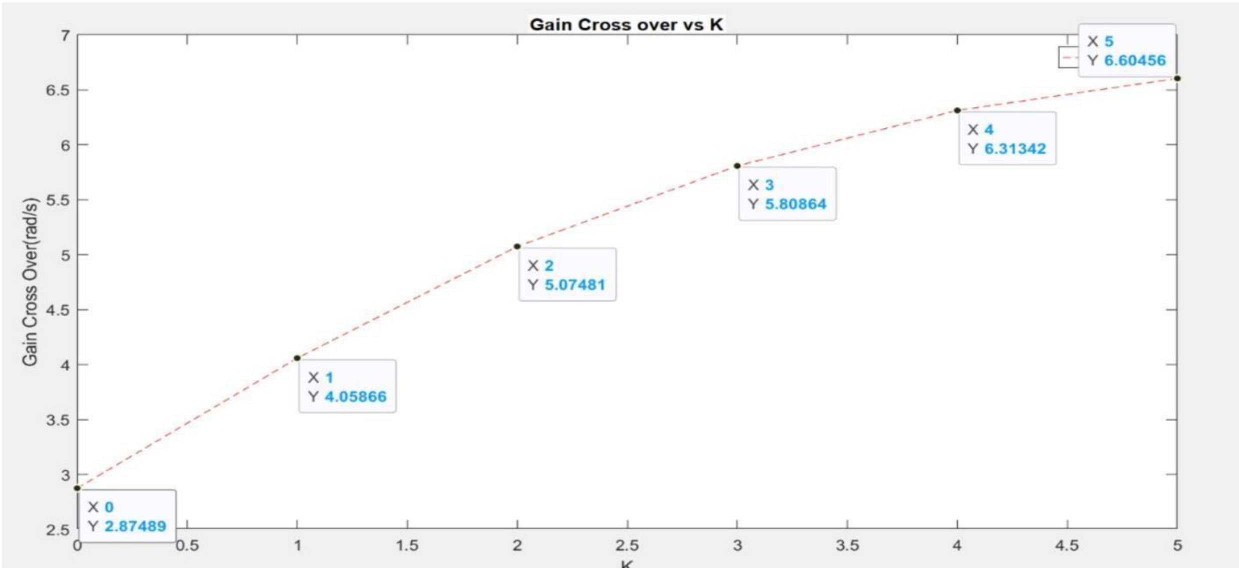
- For K>3, the Gain Margin approaches to infinity.
- Maximum GM is 11.1 dB and decreases further as K increases.

2. Phase Margin v/s K:



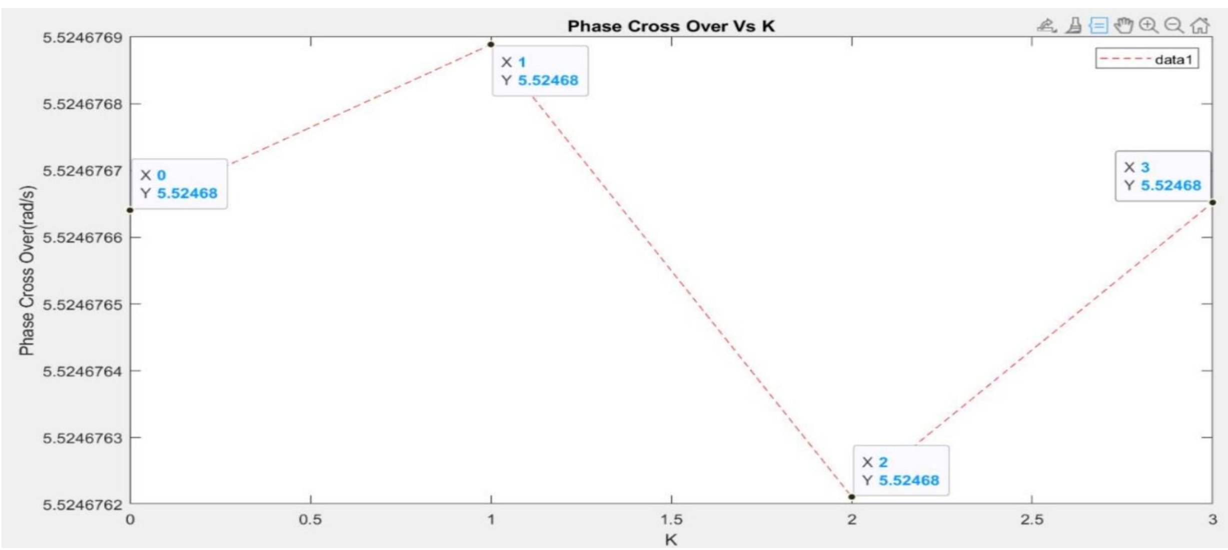
- With increase in K, the Phase Margin decreases.
- PM approaches to infinity beyond K=5.

3. GCO v/s K:



- GCO increases as K increases.

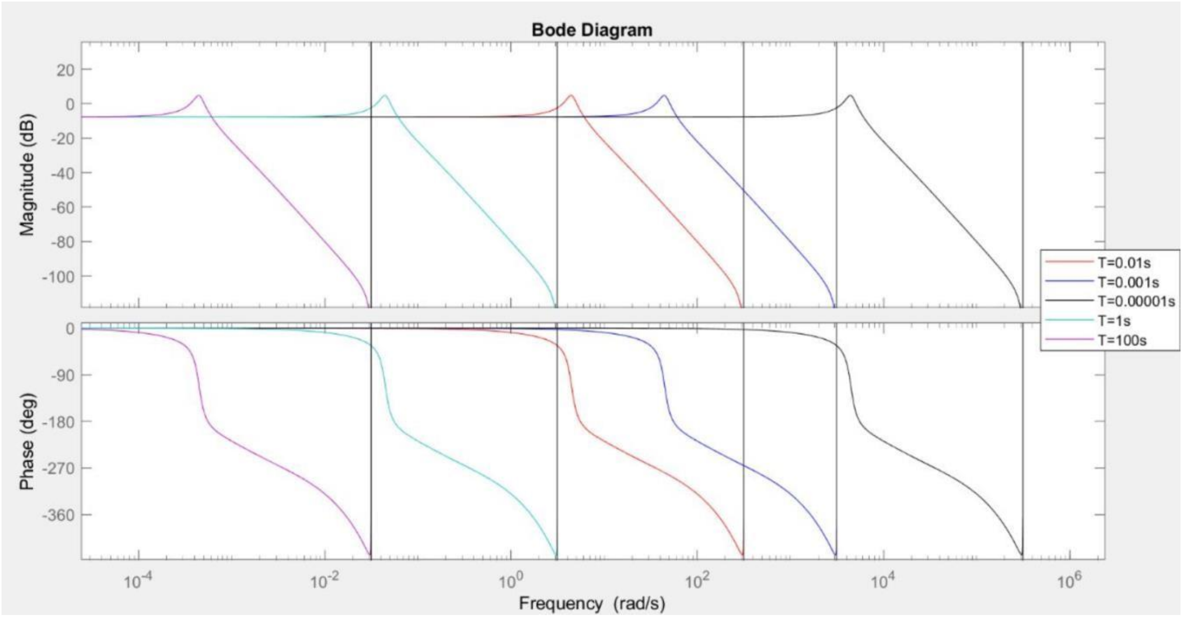
4. PCO v/s K:



- PCO don't change when we change K.

Variations with Sampling Time:

Bode Plot for different sampling time is obtained as follows:

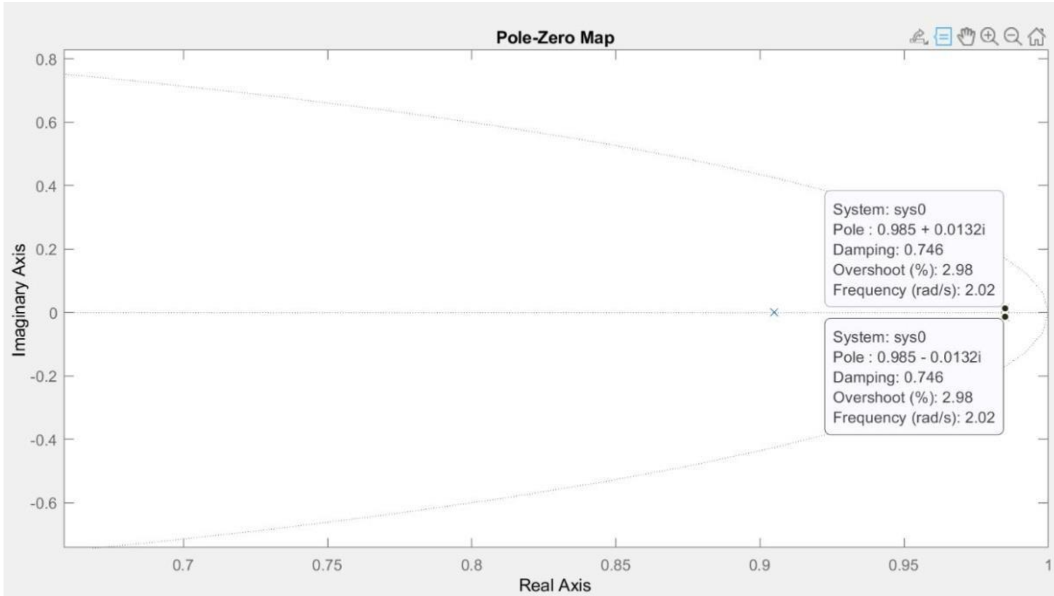


- From the bode plot for multiple sampling times, it is evident that the GM and PM don't change with change in sampling time but only translates GCO and PCO.
- So, it does not affect the stability of the system.

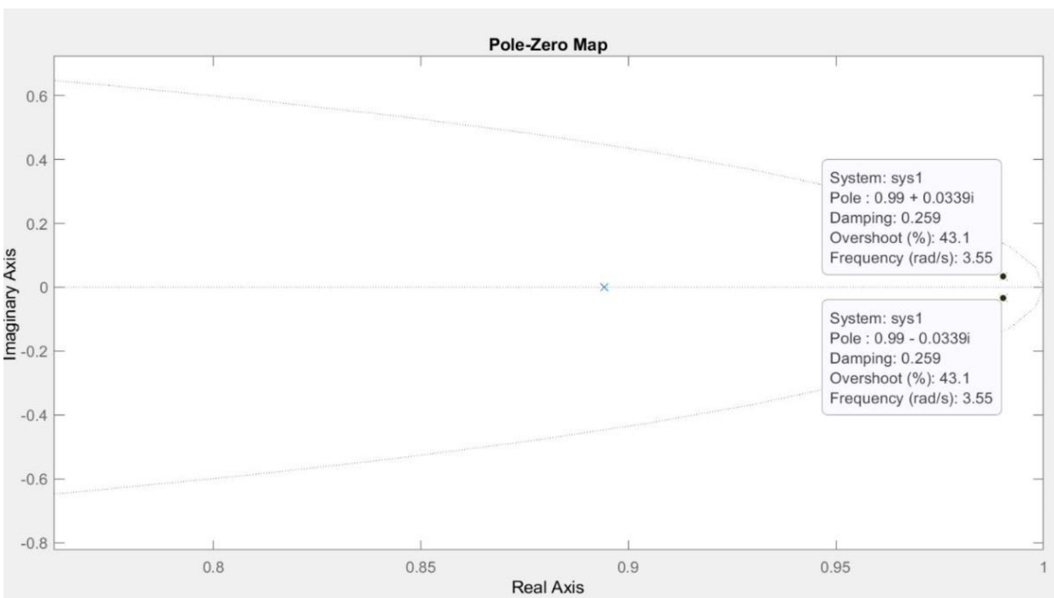
Discussions

Effect of K:

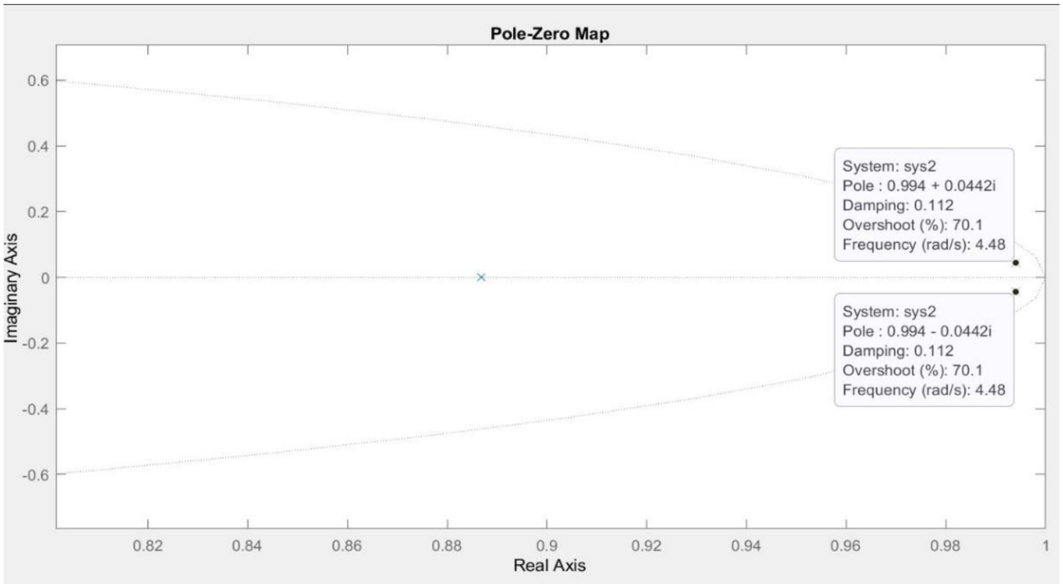
1. We saw effect of changing K on the system through various plots shown in the previous section.
2. We found that the Gain Margin becomes infinity for K>=4. Therefore the system becomes unstable for K>=4.
3. To confirm the stability for K = 0, 1, 2, 3 we will use pole zero plots for given K and check if the poles are inside the unit circle. The pole zero plots can be found below.


Pole-Zero Map
Imaginary Axis
Real Axis
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.7
0.75
0.8
0.85
0.9
0.95
1
System: sys0
Pole : 0.985 + 0.0132i
Damping: 0.746
Overshoot (%): 2.98
Frequency (rad/s): 2.02
System: sys0
Pole : 0.985 - 0.0132i
Damping: 0.746
Overshoot (%): 2.98
Frequency (rad/s): 2.02

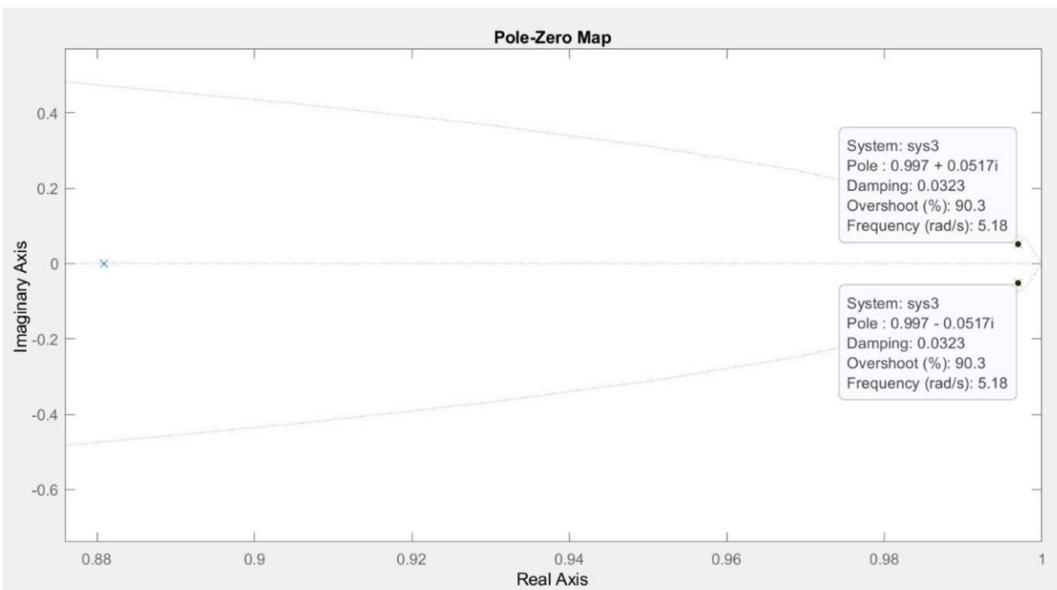
K=0


Pole-Zero Map
Imaginary Axis
Real Axis
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0.8
0.85
0.9
0.95
1
System: sys1
Pole : 0.99 + 0.0339i
Damping: 0.259
Overshoot (%): 43.1
Frequency (rad/s): 3.55
System: sys1
Pole : 0.99 - 0.0339i
Damping: 0.259
Overshoot (%): 43.1
Frequency (rad/s): 3.55

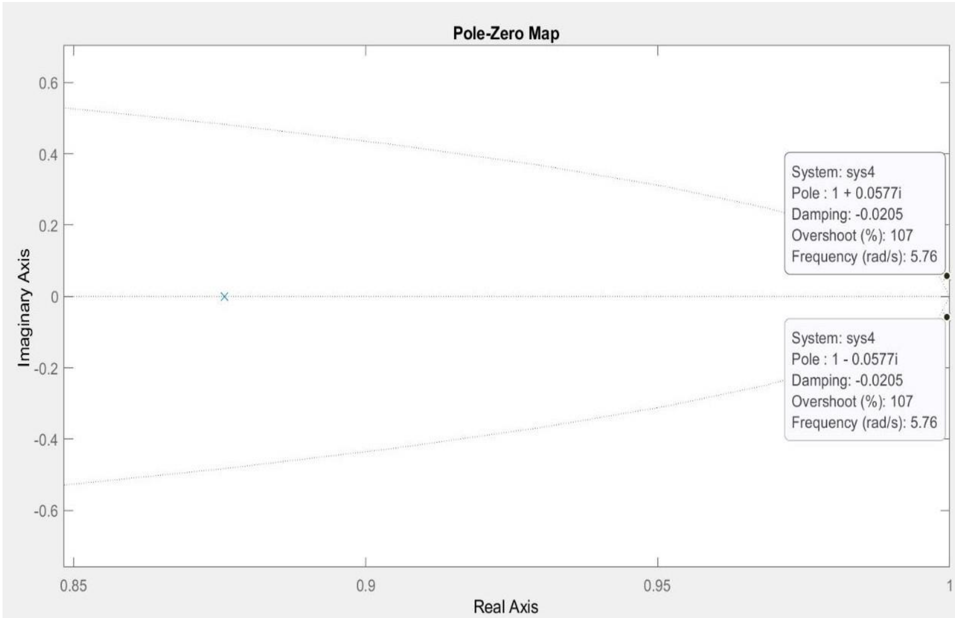
K=1


Pole-Zero Map
System: sys2
Pole : 0.994 + 0.0442i
Damping: 0.112
Overshoot (%): 70.1
Frequency (rad/s): 4.48
System: sys2
Pole : 0.994 - 0.0442i
Damping: 0.112
Overshoot (%): 70.1
Frequency (rad/s): 4.48
Imaginary Axis
Real Axis
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.82
0.84
0.86
0.88
0.9
0.92
0.94
0.96
0.98
1

K=2


Pole-Zero Map
System: sys3
Pole : 0.997 + 0.0517i
Damping: 0.0323
Overshoot (%): 90.3
Frequency (rad/s): 5.18
System: sys3
Pole : 0.997 - 0.0517i
Damping: 0.0323
Overshoot (%): 90.3
Frequency (rad/s): 5.18
Imaginary Axis
Real Axis
0.4
0.2
0
-0.2
-0.4
-0.6
0.88
0.9
0.92
0.94
0.96
0.98
1

K=3

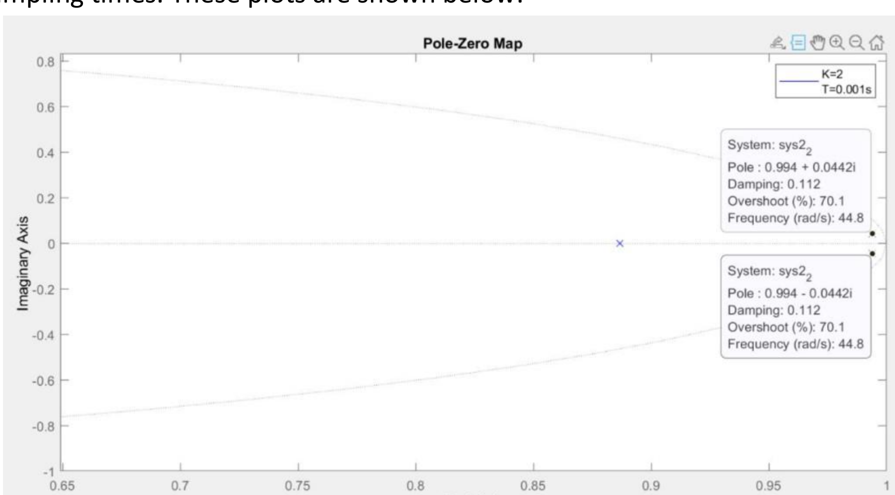


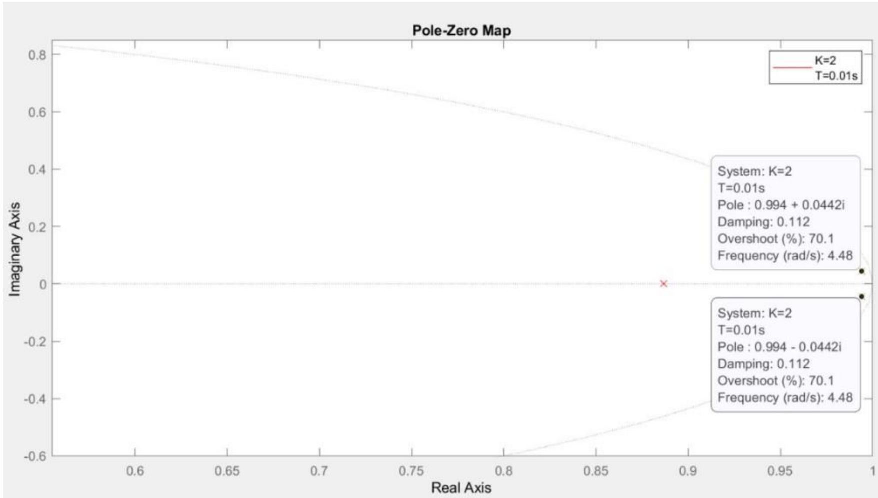
K=4 (UNSTABLE)

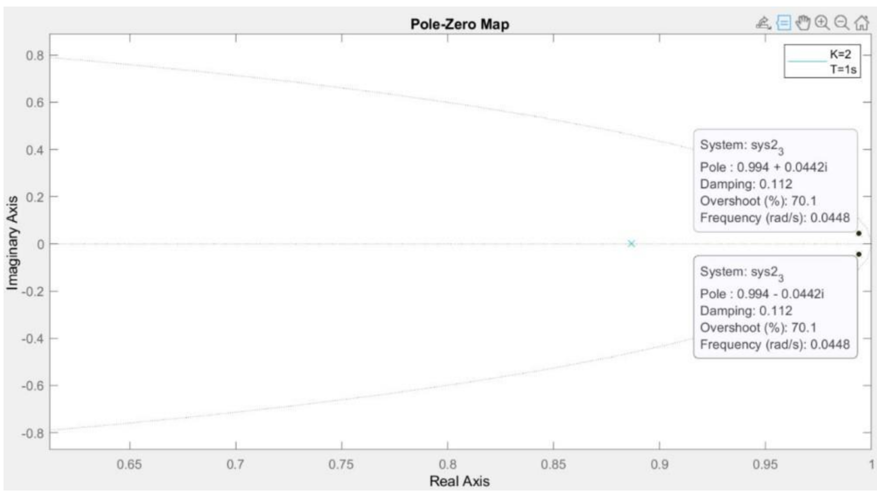
4. From all the above plots, we can see that the poles lie in unit circle for all K = 0, 1, 2, 3 which means that the considered systems are stable.
5. Thus we can summarize that:
    The closed loop transfer function of the given furnace with actuator is stable for K=0,1,2,3 and unstable for K>3.

Effect of Sampling Time:

1. We saw the effect of changing sampling time on the system through the plots in previous section.
2. We found that the GM and PM are not affected when we change the sampling time.
3. The effect of sampling time can be seen in time domain analysis.
4. Therefore, we can say that changing sampling time does not affect the stability of the system.
5. The above said can also be confirmed from pole-zero plots of the transfer function for different sampling times. These plots are shown below:







6. From the plots above, it is clearly evident that for different sampling time the poles remain at same position and inside unit circle showing that the system is stable for different sampling time.

Conclusions

In this experiment we worked with the digital transfer function of a furnace model. We studied the variation of feedback gains and the effect on system stability and frequency domain stability margins. We also studied the effect of changing sampling time. Our findings are summarized as follows:

1. Feedback gain of K=0,1,2,3 provide a stable closed loop system. The corresponding GM, PM, GCO, PCO have been found and plotted in previous sections.
2. Changing sampling time does not affect the gain and phase margins, and does not show any effect on the stability of the system.

Matlab Script

```
%BOde plot of each K
G=tf([4.95 4.901],[1 -1.97 0.9704],0.01);
G=(1E-5)*G;
T=tf([0.9516],[1 -0.9048],0.01);
C=T*G;
sys0=feedback(C,0);
sys1=feedback(C,1);
sys2=feedback(C,2);
sys3=feedback(C,3);
sys4=feedback(C,4);
sys5=feedback(C,5);
sys6=feedback(C,6);
sys7=feedback(C,7);
sys8=feedback(C,8);
bode(sys0,"k",sys1,"r",sys2,"b",sys3,"g",sys4,"y",sys5,"r--",sys6,"c",sys7,"m",sys8,"k");
disp(C);
```

To plot Bode plot for different K

```
G=tf([4.95 4.901],[1 -1.97 0.9704],0.01);
G=(1E-5)*G;
T=tf([0.9516],[1 -0.9048],0.01);
C=T*G;
M=zeros(1, 9);
W=zeros(1, 9);
X=zeros(1, 9);
Y=zeros(1, 9);
Z=zeros(1, 9);
for i=0:1:8
    sys=feedback(C,i);
    [GM,PM,w_c,w_180]=margin(sys);
    W(i+1)=PM;
    X(i+1)=GM;
    Y(i+1)=w_c;
    Z(i+1)=w_180;
    M(i+1)=i;
end
disp(M);
disp(W);
disp(X);
disp(Y);
disp(Z);
```

To obtain GM, PM, GCO, PCO for different K

EDITOR PUBLISH VIEW

```
hold off;
G=tf([4.95 4.901],[1 -1.97 0.9704],0.01);
G=(1E-5)*G;
T=tf([0.9516],[1 -0.9048],0.01);
C=T*G;
sys2_1=feedback(C,2);
bode(sys2_1,"r");
hold on;


G=tf([4.95 4.901],[1 -1.97 0.9704],0.001);
G=(1E-5)*G;
T=tf([0.9516],[1 -0.9048],0.001);
C=T*G;
sys2_2=feedback(C,2);
bode(sys2_2,"b");
hold on;

G=tf([4.95 4.901],[1 -1.97 0.9704],0.00001);
G=(1E-5)*G;
T=tf([0.9516],[1 -0.9048],0.00001);
C=T*G;
sys2_5=feedback(C,2);
bode(sys2_5,"k");
hold on;



G=tf([4.95 4.901],[1 -1.97 0.9704],1);
G=(1E-5)*G;
T=tf([0.9516],[1 -0.9048],1);
C=T*G;
sys2_3=feedback(C,2);
bode(sys2_3,"c");
hold on;


G=tf([4.95 4.901],[1 -1.97 0.9704],100);
G=(1E-5)*G;
T=tf([0.9516],[1 -0.9048],100);
C=T*G;
sys2_4=feedback(C,2);
bode(sys2_4,"m");
hold on;
```

For bode plot when considering different sampling times