EE208- CONTROL LAB

Group number: 10B

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Group members:

Harshit: 2021eeb1175

Pulkit Singh: 2021eeb1198

Objective:

1) Dynamic studies of the crane trolley system with which different Lyapunov control designs are to be incorporated.
2) To formulate the best possible Lyapunov function as a combination of the functions given to us in the problem.

Given Quantities:

We have been provided with differential equations that represent a simplified model of an overhead crane. The equations are:

$[mL + mC] . \ddot{x}_1(t) + mL . l . [\ddot{x}_3(t). \cos x_3(t) - \dot{x}_3^2(t) . \sin x_3(t)] = u(t)$

$mL . [\ddot{x}_1(t). \cos x_3(t) + l. \ddot{x}_3(t)] = -mLg . \sin x_3(t)$

Here, constants are:

mC: Mass of trolley: 10 kg.
mL: Mass of hook and load; the hook is again 10 kg, but the load can be zero to several hundred kg's, but constant for a particular crane operation.

l: Rope length; 1m or higher, but constant for a particular crane operation. g: Acceleration due to gravity, 9.8 ms−2

Input: u(t) - Force in Newtons, applied on trolley
Output: y(t) - Position of load in meters

$$\boxed{\mathbf{y(t) = x_1(t) + l*\sin(x_3(t))}}$$

States:
x_1: Position of trolley in meters.
x_2: Speed of trolley in m/s.
x_3: Rope angle in radians.
x_4: Angular speed of rope in rad/s

The given Lyapunov functions are:

Consider four different energy-based function components:

A. Proportionate to square of linear potential energy: = $K_{PEl} . (x_{1ref} - x_1)^2$

B. Proportionate to linear kinetic energy: = $K_{KEl}.x_2^2$

C. Proportionate to square of rotary potential energy: = $K_{PEr}.x_3^2$

D. Proportionate to rotary kinetic energy: = $K_{KEr}.x_4^2$

- For different control designs, Lyapunov functions can be generated by linear combinations in ones, twos, threes, or all out of "A, B, C, D".
- In Physics concepts, potential energy is proportionate to differences of distance, or angle. Function components "A" and "C" have been defined as squares of these. Kinetic energy terms are proportional to square of speeds, so these have been retained without squaring.
- "A" resembles our standard state feedback error, and can be correlated to conventional state feedback principles.

For the nonlinear system as given above –

- Create a complete four-state, single output simulation model for the crane using non-linear simulation blocks in Simulink.
- Check how each parameter can be programmed for individual dynamic studies.
- The simulation evolved should be for the complete nonlinear set of equations, and should not be confused with the linearised studies undertaken earlier.

Here, we will start analyzing the state space and look at what pattern it'll follow as we'll require the derivative ahead for analyzing the Lyapunov function ($\partial\mathbf{V}/\partial\mathbf{x}$). Where V is the function of state spaces that we have to design.

Using MATLAB, we can write the given pair of coupled nonlinear ODEs in general state-space form –

$$\dot{\boldsymbol{x}} = \begin{pmatrix} x_2 \\ \dfrac{l\, m_L \sin(x_3)\, x_4{}^2 + u + g m_L \cos(x_3)\sin(x_3)}{m_L \sin^2(x_3) + m_C} \\ x_4 \\ \dfrac{l\, m_L \cos(x_3)\sin(x_3)\, x_4{}^2 + u\cos(x_3) + g(m_C + m_L)\sin(x_3)}{(-l) \times (m_L \sin^2(x_3) + m_C)} \end{pmatrix}$$

$$y = x_1 + l\sin(x_3)$$

Apart from MATLAB, we can even perform hand calculations and substitutions in the given equations on page no. 2. By simply putting $\dot{x}1$=x2 and so on.

- Since now we have the state space system designed and the output "y" is also given we'll try to now make a patchwork of the same on MATLAB

The complete model is shown below.

• We have put the system of differential equations inside a subsystem for convenience.

• Initial conditions and input signal can be fed to the subsystem as inputs, and states obtained as outputs which can be observed with the Scope block.

• The parameter mL can be changed using the slider block, this will be required when

We wish to examine the effect of loading conditions.

• The nominal parameter values are –

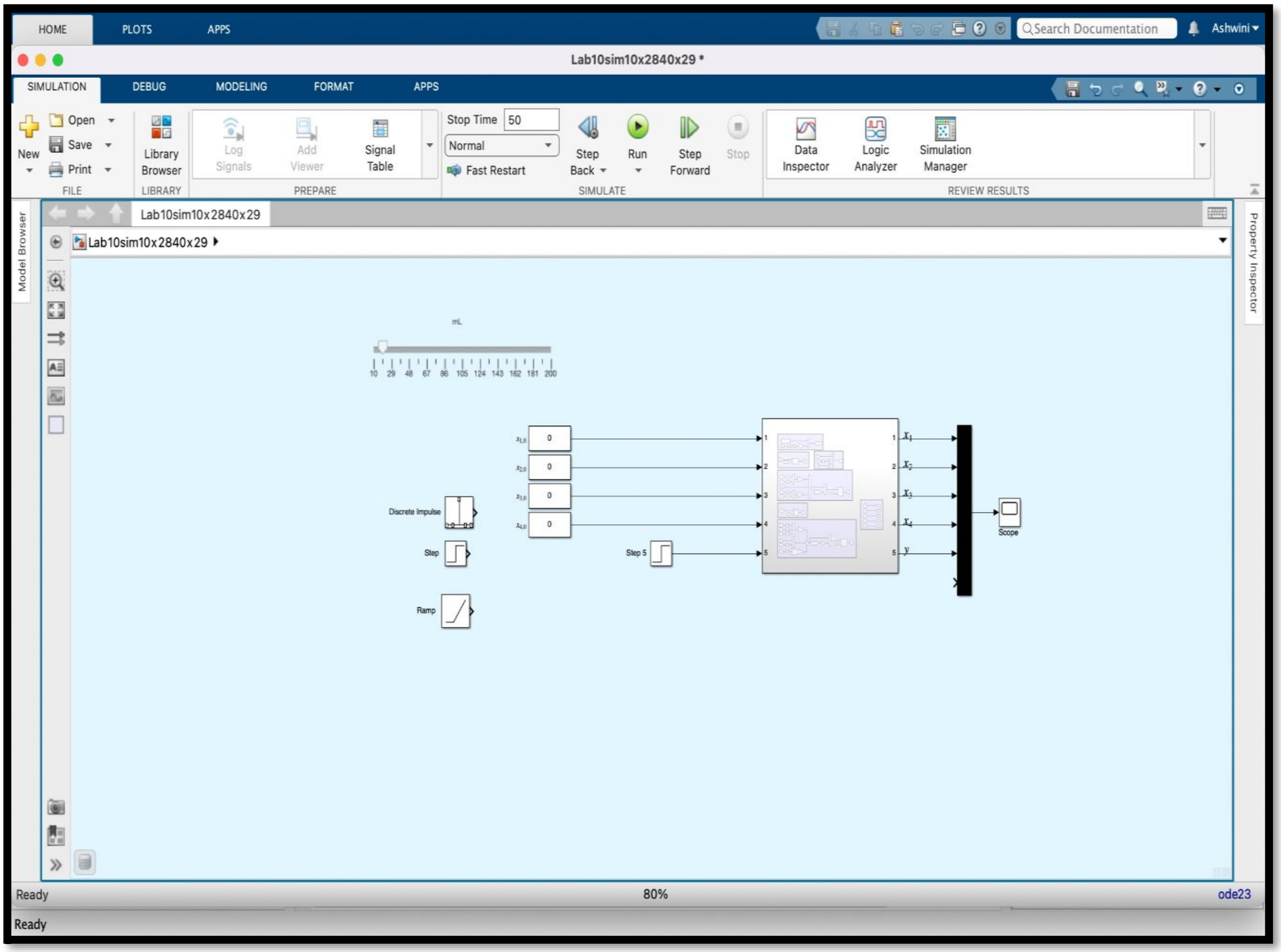
- mC = 10 kg, constant
- g = 9.81 m/s^2, constant
- l = 2 m, constant but changeable if required
- mL = 20 kg, can be changed using the slider block

• The parameters are stored inside the Model Workspace –

Name	Value	DataType	Dimensions	Complexity
L	1	double (auto)	[1 1]	real
g	9.8	double (auto)	[1 1]	real
mC	10	double (auto)	[1 1]	real
mL	20	double (auto)	[1 1]	real

• Different inputs can be given to the system as desired – impulse, step, ramp.

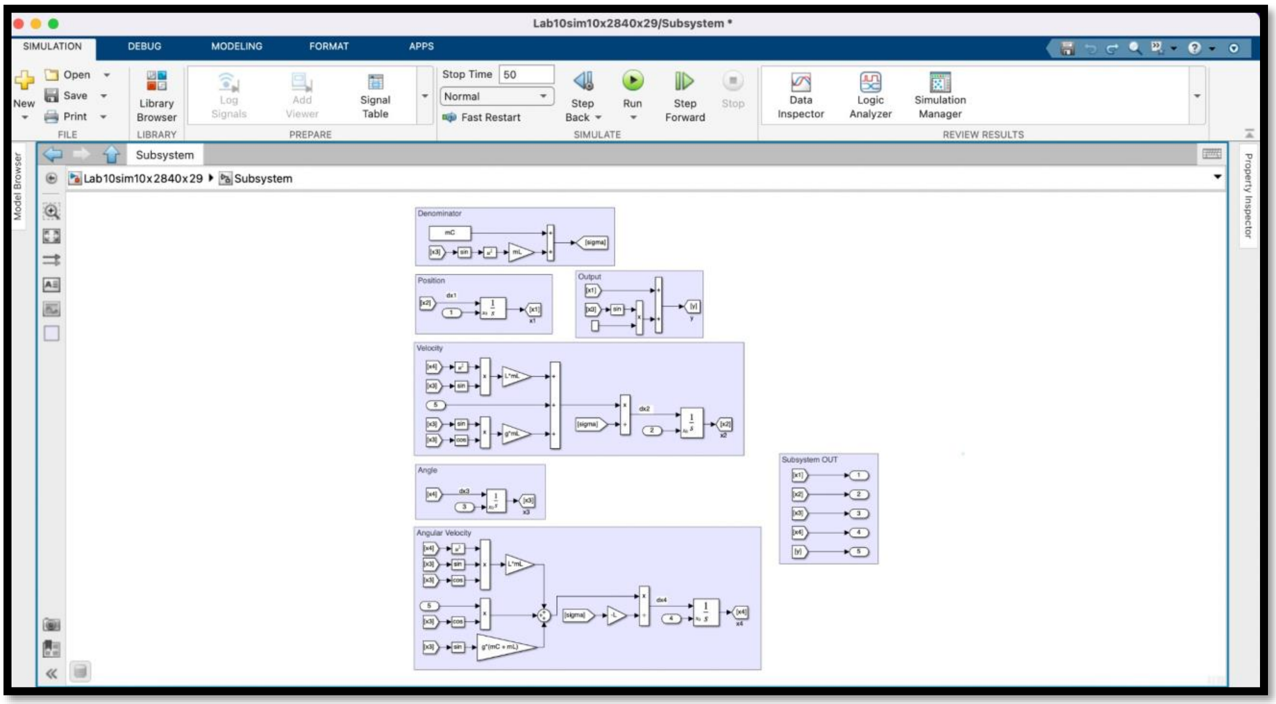
• The maximum step size is 0.0001s, and the solver used is ode23.



The model above indicates the overall system. With three inputs (note that these inputs are reference, though it was given to take only ramp and unit response as reference but we are also considering ramp response for further analysis.

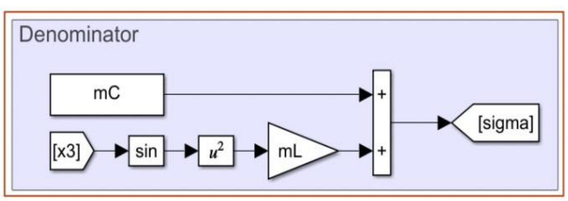
- The block in the image given above to the left of the bus operator is the "subsystem" block which contains the state space and output patchwork.

The image below is the view inside the subsystem design.



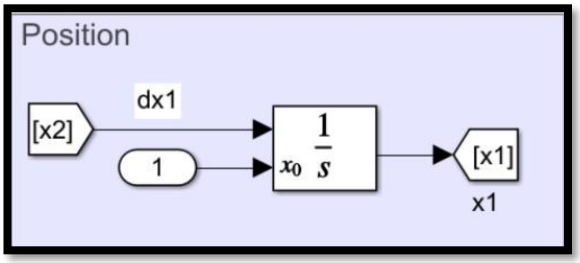
We shall take a look at each segment individually. We have used Goto-From blocks for clarity, due to the large number of terms involved in the equations.

1) Denominator



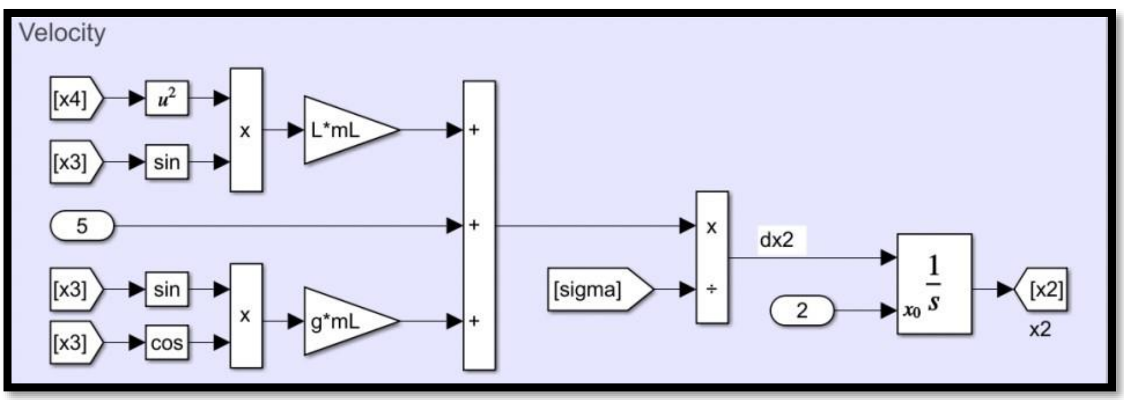
This implements the denominator term common to the 2nd and 4th rows in the ***f*** matrix.

2) **Position**



The state x1 is simply the integral of the x2, as can be seen from the f matrix. The integrator block also has a functionality to set the initial state, which is fed as an input to the subsystem.

3) **Velocity**

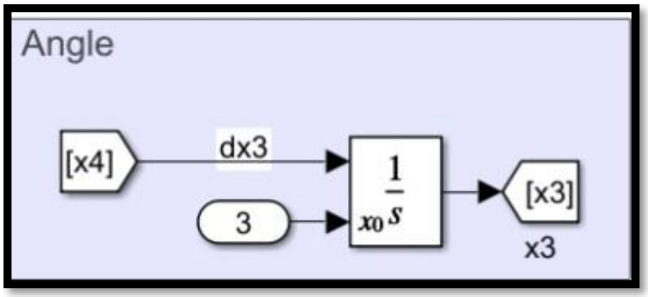


As before, we can see that the initial state (Input-2 of the subsystem to be specific) is given
as an input to the integrator. The parameters L, g, etc. are stored in the Model Workspace. The benefit of the Goto-From blocks can be seen here in making the block diagram cleaner visually.

The block diagram can be understood better as follows –

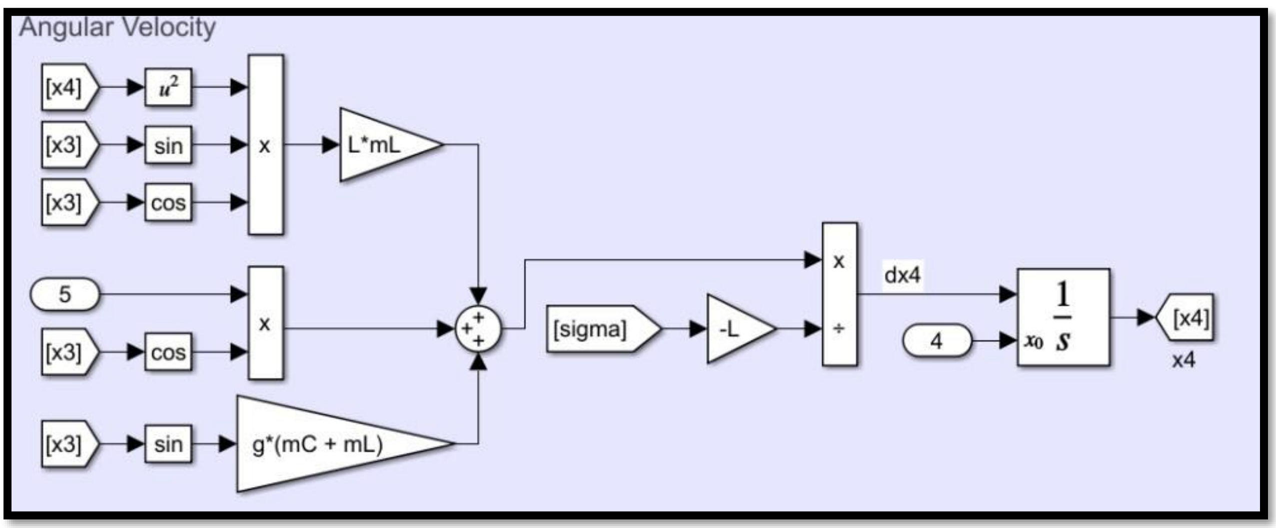
- Each of the inputs to the Sum block is a force on the Crane's body.
    - The first and third terms are contributed by the motion of the mass on the rope (basically a pendulum), such as centripetal force and weight of the Mass.
    - The second term is simply the input u, the horizontal force being applied to the crane.
- The denominator can be understood as a sort of equivalent or effective mass.
- Division gives us the acceleration of the crane, which is integrated to give the velocity.

4) Angle



This is simply the integral of angular velocity. The initial condition here is quite significant in terms of the equilibria.

5)Angular velocity



Rather than force, it is torque we focus on this time. Our explanation is only qualitative, since the expressions have already been simplified to some extent.

- The first term is the torque corresponding to centripetal force.
- The second term is torque due to the applied force.
- The third term is torque due to gravity.
- The denominator is equivalent to a moment of inertia (divided by l).
- The entire equation is already divided by l, so the description doesn't match exactly with the expressions.

Now we'll try to find out the equilibrium points of the given system. For any system to be at equilibrium, we have to have points such that it goes at a 0 state altogether.

We shall first look at the natural responses of the system at different initial conditions. There is a lot of variation in the natural responses depending on the initial state.

1) CASE 1 - [0, 0, 0, 0]

This is one of the equilibria of the system. As expected, at equilibrium the system is at rest.

2) CASE 2 – [0, 0, π, 0]

This is the second type of equilibrium this system possesses.

If we try to analyze any other cases apart from above two conditions, we'll get oscillations and therefore indicating instability due to absence of lagging in their plots. Hence for these two cases: The derivative of Lyapunov function will be equal to zero i.e. ($\partial V/\partial x=0$).
Now considering the previous Lyapunov functions A,B,C,D. We'll try to find their derivatives separately.

Following are the **derivatives** for individual Energy profiles with respect to time:

$\boldsymbol{A} = -2\, K_{PE}\,(x_{1ref} - x_1)\,\dot{x}_1$

$\boldsymbol{B} = 2\, K_{KE}\, x_2\, \dot{x}_2$

$\boldsymbol{C} = 2\, K_{PE}\, x_3\, \dot{x}_3$

$\boldsymbol{D} = 2\, K_{KE}\, x_4\, \dot{x}_4$

Lyapunov's Second Method is an extension of two fundamental physical ideas to the theory of nonlinear systems of ODEs: A conservative physical system's state is stable only if its potential energy has a local minimum at that point. Except for the equilibrium point, the function must be positive everywhere. It should be zero at equilibrium. The fundamental idea is to create a feedback control regulation that makes the derivative of a given Lyapunov function candidate negative definite or negative semi-definite. If a Lyapunov function with a negative definite derivative for all exists in the vicinity of an autonomous system's zero solution, the system's equilibrium point is asymptotically stable. A negative semidefinite matrix is a Hermitian matrix all of whose eigenvalues are nonpositive.

Best Lyapunov's Selection Criterion:

- In a neighbourhood of the zero solution of the system, the Lyapunov function should have a negative definite derivative for all, then the equilibrium point of the system is asymptotically stable.
- However, if the Lyapunov's derivative is negative semidefinite, then asymptotic stability cannot be concluded from Lyapunov's method.

Briefs on Project Idea Approach:

1) We have already determined the equilibrium points for the system. We'll have to make combinations such that its corresponding Lyapunov functions come out to be 0 at the point of equilibria.

2) We observed that our states were showing oscillatory nature at points where the system was not at equilibrium. So while solving this problem, our major challenge was to make this crane system work stable without oscillations at almost all points (increasing the points of stability). Thus we tried to make the system show a lagging nature. That's our base approach towards designing the combinations as well: minimizing the oscillations.
3) This implies that once we stabilized the system, we worked on evaluating the best Lyapunov function from the linear combinations of the functions given and determined the values of the k- constants given to us in the expressions.

We'll now describe the Lyapunov functions of A,B,C,D individually and try to analyze the possible values of K-constants (a range of values). Refer to the image below for the same

PULKIT SINGH 2021eeb1198
HARSHIT 2021eeb1175

we know, A: $K^{L}_{PE}(x_{1,ref} - x_1)^2 \Rightarrow \dot{A} = -2K^{L}_{PE}(x_{1,ref} - x_1)\cdot\dot{x}_1$

If.

$$\boxed{\begin{array}{l} x_1 > x_{1,ref} \Rightarrow K^{L}_{PE} < 0 \\ x_{1,ref} > x_1 \Rightarrow K^{l}_{PE} > 0 \end{array}}$$

for lyapunov stability, $\frac{dV}{dx} < 0$

$B = K^{L}_{KE}\cdot x_2^2 \Rightarrow \dot{B} = 2K^{l}_{KE}\, x_2\cdot\dot{x}_2$

Here, $x_2 \geq 0$ always since its velocity

$K^{l}_{KE} < 0$.

$\dot{x}_2 = \frac{u}{m_c}$ at equilibrium which is > 0

∴ $\dot{x}_2$ demonstrates that

∴ $\boxed{K^{L}_{KE} < 0}$ for lyapunov stability

Then, 'C' becomes,

C: $K^{r}_{PE}\cdot x_3^2 \Rightarrow \dot{C} = +2K^{r}_{PE}\cdot x_3\cdot\dot{x}_3$

$\dot{C} = 2K^{r}_{PE}\, x_3\cdot x_4$.

x_3 & $x_4 > 0 \Rightarrow K^{r}_{PE} < 0$ for lyapunov stability

$$D : K^{r}_{KE} \cdot x_4^{2} \qquad \dot{D} = 2K^{r}_{KE}\, x_4 \cdot \dot{x}_4$$

$$\hat{x}_4 = \frac{-u}{l\, m_C}$$

$$\therefore \hat{D} = 2K^{r}_{KE} \cdot x_4 \cdot \left[\frac{-u}{l\, m_C}\right]$$

For lyapunov function,

$$\boxed{K^{r}_{KE} > 0}$$

Nowdealing with references: we first put **step response**

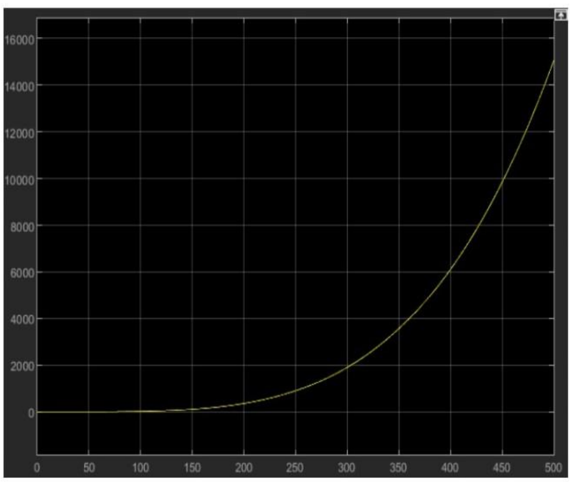


Fig-1: Variance of Linear Potential Energy

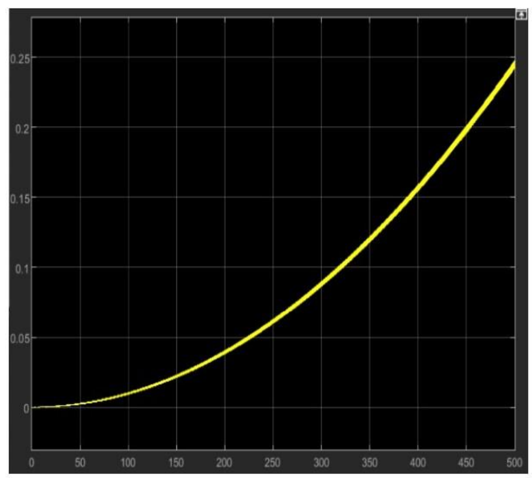


Fig-2: Variance of Linear Kinetic Energy

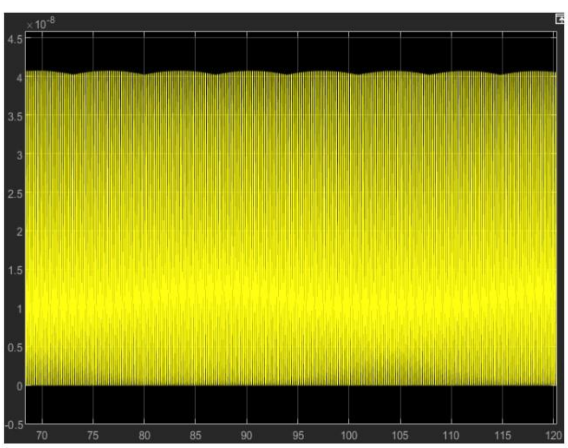


Fig-3: Variance of Rotary Potential Energy

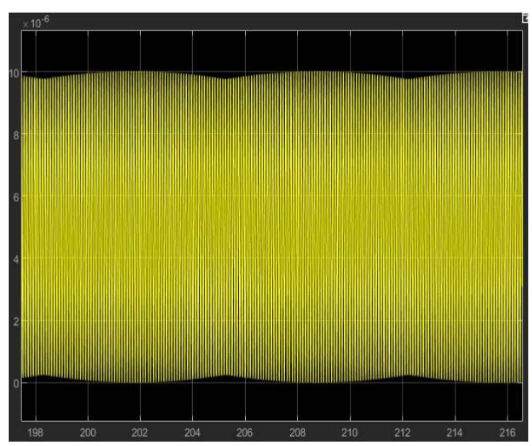


Fig-4: Variance of Rotary Kinetic Energy

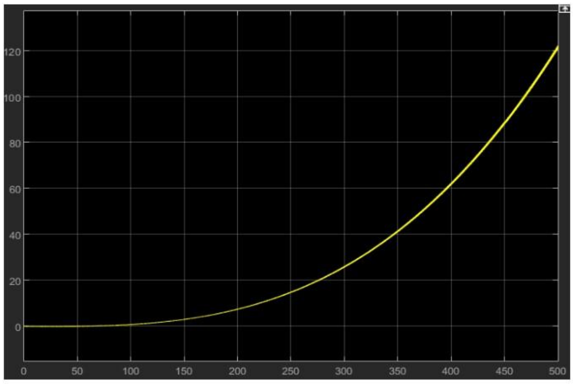


Fig-5: Variance of Derivative of Linear Potential Energy

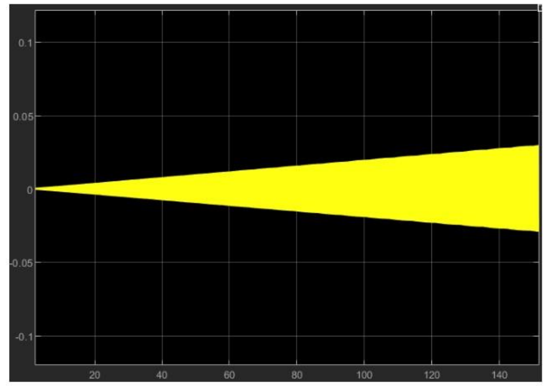


Fig-6: Variance of Derivative of Linear Kinetic Energy

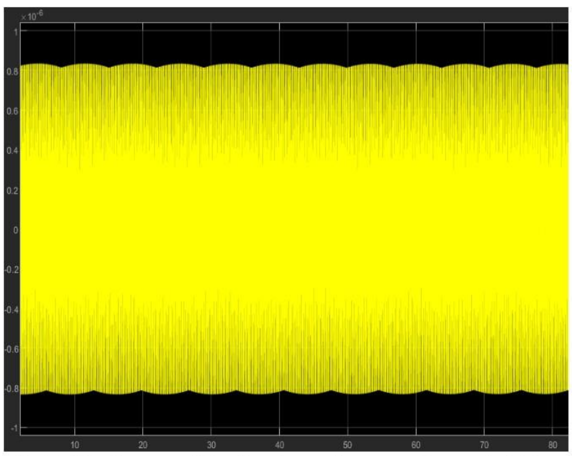


Fig-7: Variance of Derivative of Rotary Potential Energy

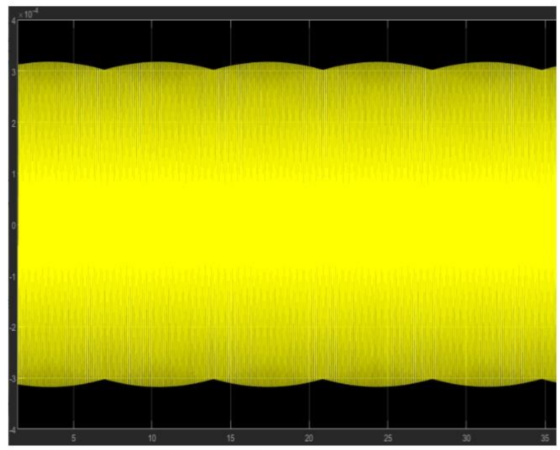


Fig-8: Variance of Derivative of Rotary Kinetic Energy

Referring: K_{PE1}=K1, $K_{PE\ R}$= K2, $K_{KE\ R}$=K3, K_{KE1}=K4

Since the Variance of Derivative of Linear Potential Energy is always positive and increasing monotonically, K_{PE1} must be negative. Magnitude of Derivative of Rotary Potential Energy and Rotary Kinetic Energy is negligible (of order e (-6)), though they show an oscillatory nature about the x axis. Linear Kinetic energy shows increasing magnitude of the oscillations symmetrically to the x axis.

Since we require negative definite function, we can have:

- $K_{PE1}<0$
- $K_{PE\ R}$, $K_{KE\ R}$, $K_{KE1} > 0$
- These proportional will render us with negative overall derivative functions and hence asymptotically stable.

Lyapunov Function (A): - $1 * (x_{1ref} - x_1)^2 + x_2^2 + x_3^2 + x_4^2$

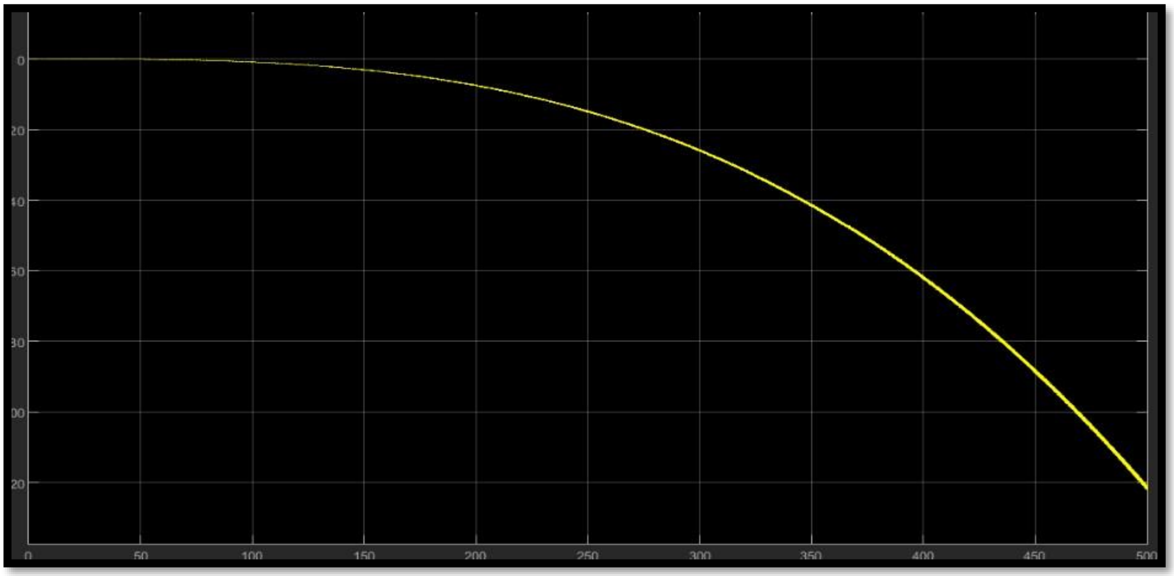


Fig-9: The derivative of the designed Lyapunov function (semi definitely negative)

Ramp Response:

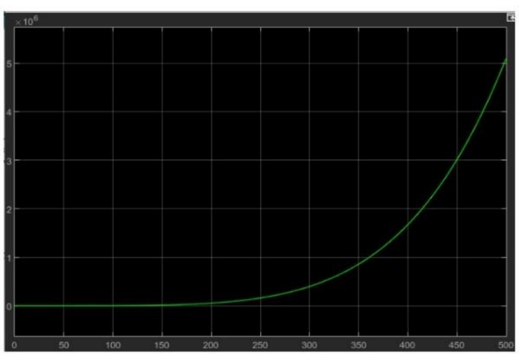


Fig-10: Variance of Derivative of Linear Potential Energy

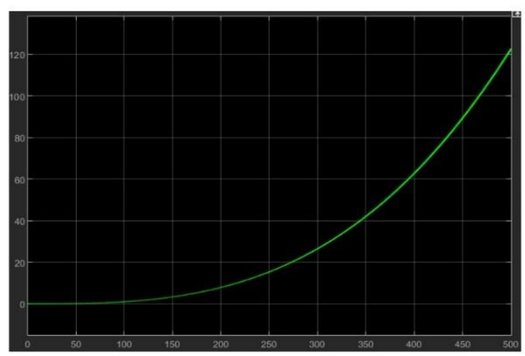


Fig-11: Variance of Derivative of Linear Kinetic Energy

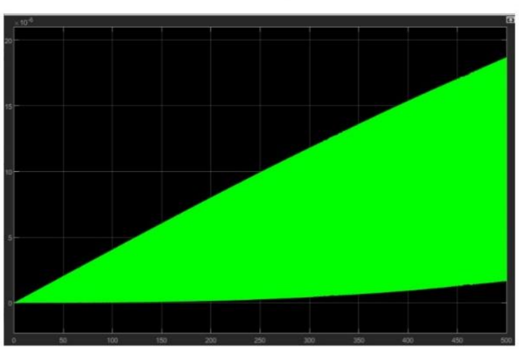


Fig-12: Variance of Derivative of Rotary Potential Energy

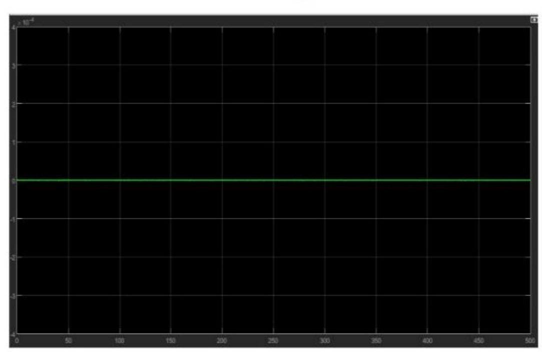


Fig-13: Variance of Derivative of Rotary Kinetic Energy

The derivative of the Linear Potential Energy and Linear kinetic energy shows a monotonous increasing curve; hence they are liable of negative proportional value for designing the Lyapunov. The derivative of the rotary potential energy shows an expected oscillatory behavior of increasing magnitude with a monotonous increase in the equilibrium position. Whereas the derivative of the rotary kinetic energy shows an oscillatory symmetric behavior about x axis of negligible magnitude of the power of e (-8). The desired Lyapunov function should have the following proportion constant:

- $K_{PE1}, K_{PER} < 0$
- $K_{KER}, K_{KE1} > 0$
- $K_{PE1} = (-0.00001) * K_{PE1}$ (we prefer a small value of k to reduce its effect)

Lyapunov Function (B): - $0.00001*(x_{1ref} - x_1)^2 + x_2^2 - x_3^2 + x_4^2$

- Since rotary potential energy also shows increasing magnitude though of small magnitude has been assigned with the negative proportional.

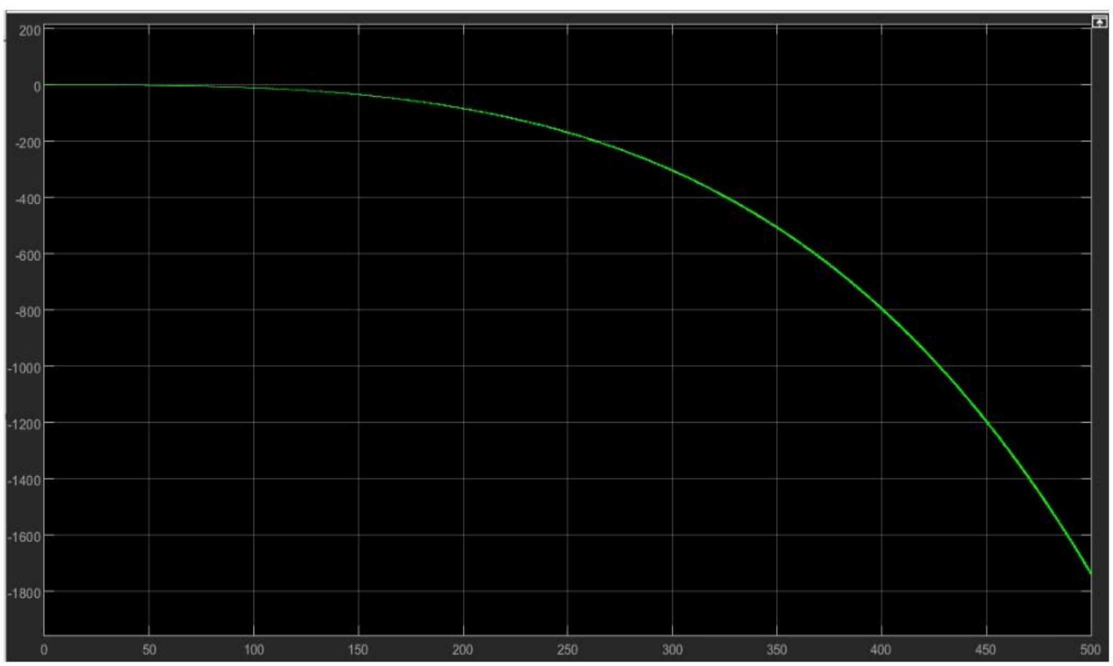


Fig-14: The derivative of the designed Lyapunov function (definitely negative)

Impulse:

<u>Taking the derivative of energy profiles:</u>

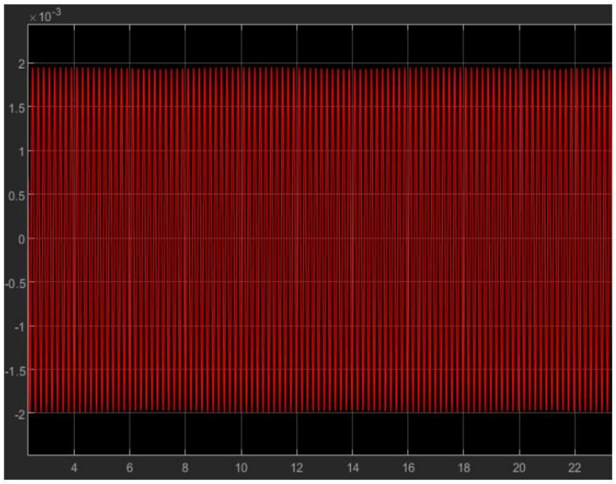


Fig-15: Variance of Derivative of Linear Potential Energy

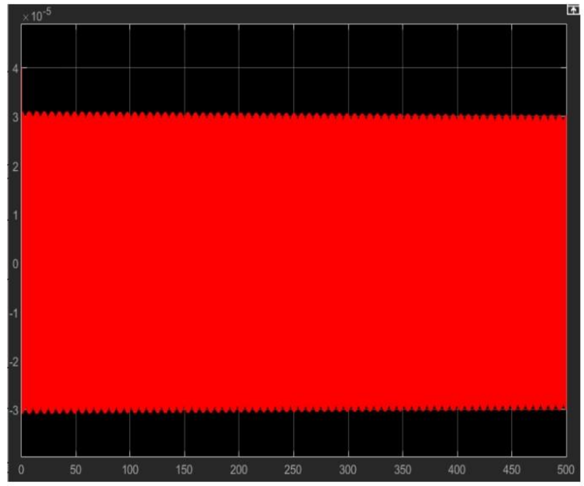


Fig-16: Variance of Derivative of Linear Kinetic Energy

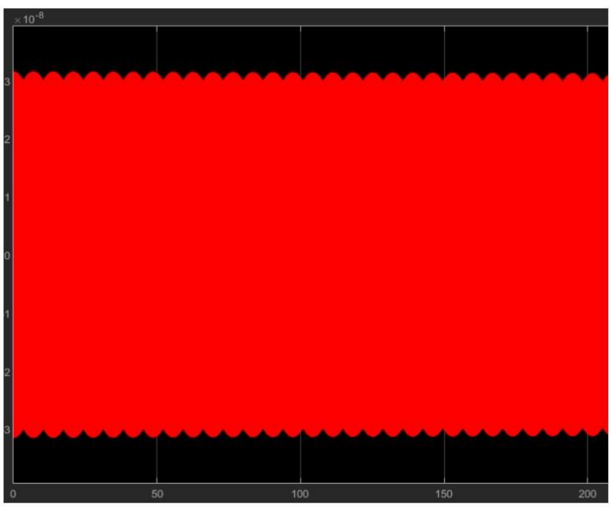


Fig-17: Variance of Derivative of Rotary Potential Energy

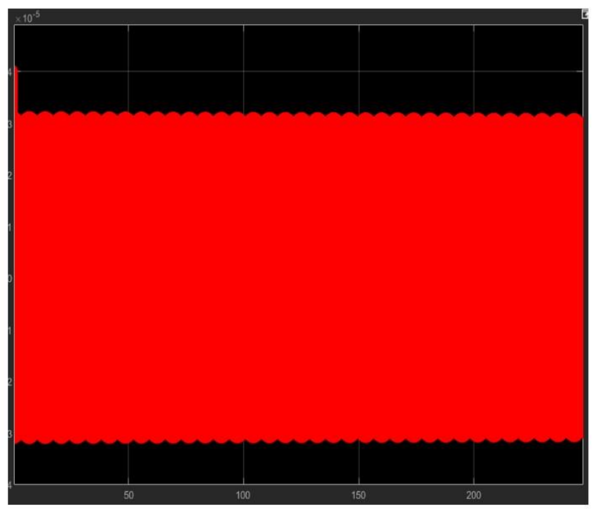


Fig-18: Variance of Derivative of Rotary Kinetic Energy

The impulsive response shows oscillatory derivative functions, which makes the design of the suitable Lyapunov function difficult to generate through simple algebraic operations. The desired Lyapunov function should have the following proportion constant:

- $K_{KE1}, K_{PER} < 0$
- $K_{KER}, K_{PE1} > 0$
- $K_{KE1} = (-1000) * K_{KE1}$
- $K_{PER} = (-100) * K_{PER}$

Lyapunov Function (C): $(x_{1ref} - x_1)^2 - 1000*x_2^2 - 100*x_3^2 + x_4^2$

The basis of the choice was to eliminate the oscillations and produce a somewhat constant line by n superimposing the scaled equal magnitudes.

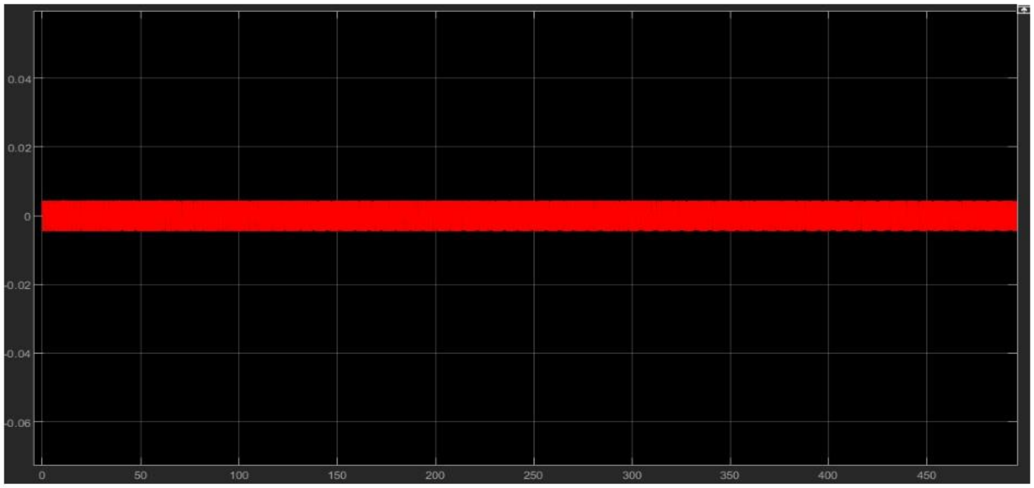
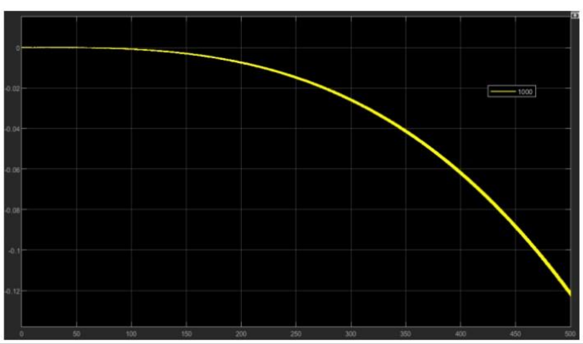


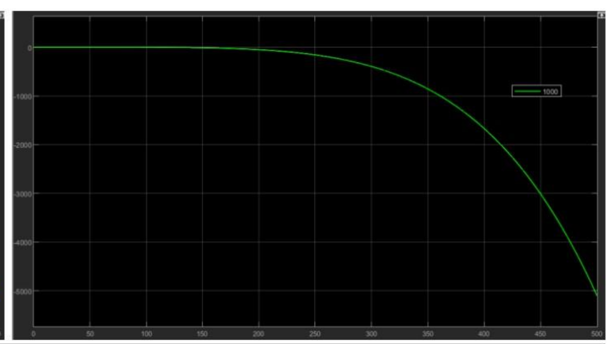
Fig-19: Lyapunov Response

The obtained response doesn't satisfy the criteria of negative definiteness and does not meet the expectation, though the magnitude has decreased significantly.

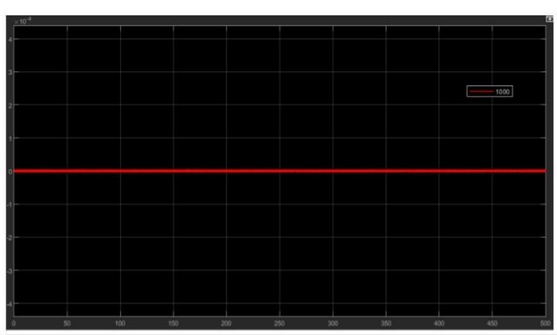
Best Designed Lyapunov function:



For step input



For Ramp Input



For Impulse Input

Now in the context to the handwritten calculation of Lyapunov derivatives we did earlier, we'll design the function "E" where the k-constants that are analyzed by previous combinations are considered such that the Lyapunov derivative graph shows minimum oscillations and tend towards a lagging nature with negative gradient assuring more stability.

- The designed Lyapunov function shows desired results especially for step and ramp input, also for impulse and the customised input, the system is stable.
- The desired Lyapunov function should have the following proportion constant:
  - K_{KE1}, $K_{KE\,R}$, $K_{PE1} < 0$
  - $K_{PER} > 0$
  - K_{PE1}= -1; K_{KE1}= -0.01
  - K_{PER}= 0.02; K_{PE1}=-0.05

Lyapunov Function (E): - $(x_{1ref} - x_1)^2 - 0.01*x_2^2 + 0.02*.x_3^2 - 0.05.x_4^2$

In case of variable load mass,
It could be well observed that with increase in load mass the Lyapunov function tends to destabilize in each case. Though none of the derivatives crossed the x axis until 10,000 kgs of load mass, a gradual upward increase is observed.

Conclusion:

Five different Lyapunov functions were obtained after the system underwent analysis for a variety of inputs. Simulink was used to construct the Lyapunov model, and its modifications were noted as well as the impact they had on the load mass.

- In order to demonstrate the optimal reaction to the particular applied input, four Lyapunov functions were initially created for the distinct inputs.
- A general Lyapunov function was created that can maintain stability regardless of the inputs used.