Introduction to Functions(I)


Activity One

Function definition

- Let A and B be nonempty sets.
- A *function* f from A to B is an assignment of exactly one element of B to each element of A.
- We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.
- If f is a function from A to B, we write
  - $f : A \rightarrow B$.

Domain and Range

- If f is a function from A to B, we say that A is the *domain* of f and B is the *codomain* of f.
- If $f(a) = b$, we say that b is the *image* of a and a is a *preimage* of b.
- The *range*, or *image*, of f is the set of all images of elements of A.
- Also, if f is a function from A to B, we say that f *maps* A to B.

Equal functions

- Two functions **f** and **g** are **equal** when
- they have the same domain **or** domain of f = domain of g
- have the same co-domain **or** co-domain of f = co-domain of g
- Map each element of their common domain to the same element in their common co-domain.

  or

  f(x) = g(x) for every x belonging to their common domain.

f A={1,2} and B={3,6} and two functions $f : A \rightarrow B$ and $g : A \rightarrow B$ are defined respectively as :
$f(x) = x^2 + 2 \text{ and } g(x) = 3x$
Find whether $f = g$

Summary

- Concept of Functions
- Domain and Range of functions
- Equal Functions

Introduction to Functions(II)

Real valued/Integer valued functions

- A function is called **real-valued** if its codomain is the set of real numbers.
- A function is called **integer-valued** if its codomain is the set of integers.
- Two real-valued functions or two integer-valued functions with the same domain can be added, as well as multiplied.

Function addition/multiplication

- Let f_1 and f_2 be functions from A to $\mathbf{R}$. Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to $\mathbf{R}$ defined for all $x \in A$ by
- $(f_1 + f_2)(x) = f_1(x) + f_2(x)$,
- $(f_1 f_2)(x) = f_1(x)\, f_2(x)$.

Example

Question: Let f_1 and f_2 be functions from **R** to **R** such that

$f_1(x) = x^2$ and $f_2(x) = x - x^2$.

What are the functions $f_1 + f_2$ and $f_1 f_2$?

Answer:

Image of a subset

- Let f be a function from A to B and let S be a subset of A.
- The *image* of S under the function f is the subset of B that consists of the images of the elements of S.
- The image of S is denoted by $f(S)$ where
- $f(S) = \{t \mid \exists s \in S\ (t = f(s))\}$.

Example

- Question: Let $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4\}$ with $f(a) = 2$, $f(b) = 1$, $f(c) = 4$, $f(d) = 1$, and $f(e) = 1$. Consider subset $S = \{b, c, d\}$.

  What is image of S?

  Answer:

Summary

- Concept of Functions
- Domain and Range of functions
- Equal Functions
- Function addition/multiplication
- Image of a subset

Types of Function

Types of Function

- A function can be of three types:

1. One-to-One function (Injective function)
2. Onto function (Surjective function)
3. One-to-One correspondence (Bijective function)

One-to-One function

- A function f is said to be *one-to-one*, or an *injunction*, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.
- A function is said to be *injective* if it is one-to-one.

Example 1

- Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, and $f(d) = 3$ is one-to-one.

Example 2

- Determine whether the function $f(x) = x^2$ from the set of integers to the set of integers is one-to-one.

Onto function

- A function f from A to B is called *onto*, or a *surjection*, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.
- A function f is called *surjective* if it is onto.

Example 1

- Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3\}$ defined by $f(a) = 3$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f an onto function?

Example 2

- Is the function $f(x) = x^2$ from the set of integers to the set of integers onto?

One-to-One correspondence

- The function f is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto.
- A function f is called *bijective* if it is one to one and onto.

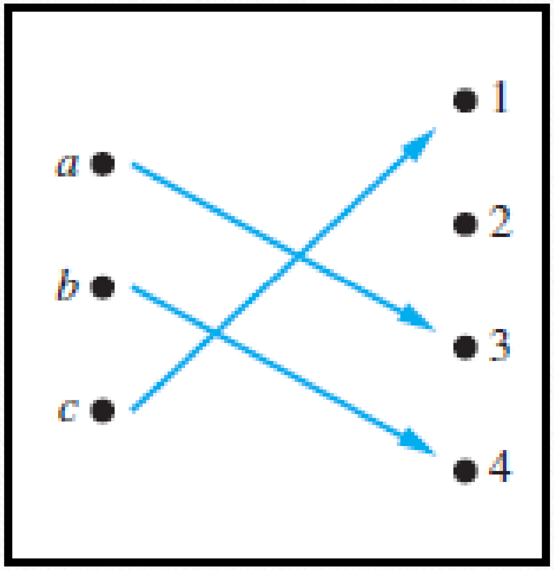
Example 1

- Let f be the function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4\}$ with $f(a) = 4$, $f(b) = 2$, $f(c) = 1$, and $f(d) = 3$. Is f a bijection?

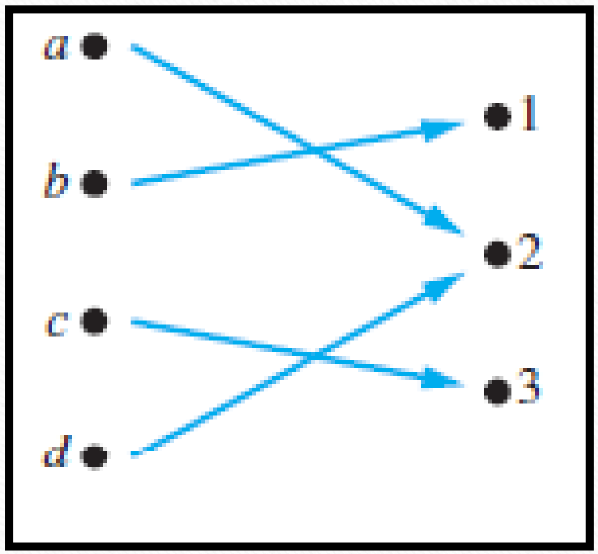
Example 2

- Is the function $f(x) = x^2$ from the set of integers to the set of integers bijective?

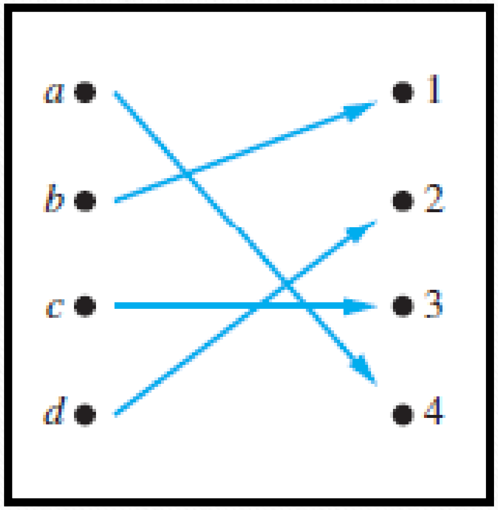
Examples of Different Types of Correspondences



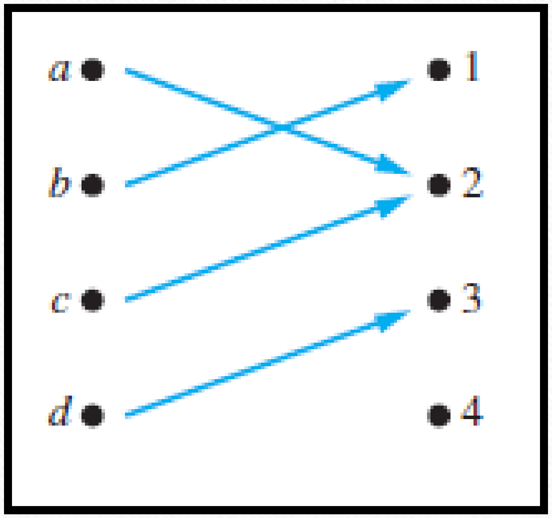
Examples of Different Types of Correspondences



Examples of Different Types of Correspondences



Examples of Different Types of Correspondences



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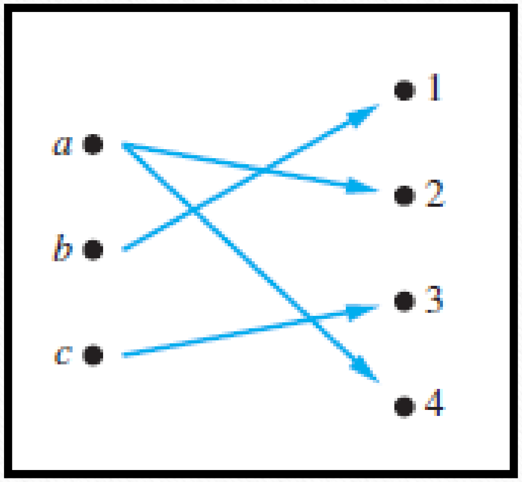

Inverse Function and Compositions of Functions

Inverse function

- Let f be a one-to-one correspondence from the set A to the set B.
- The *inverse function* of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$.
- The inverse function of f is denoted by f^{-1}.
- $f^{-1}(b) = a$ when $f(a) = b$.
- A one-to-one correspondence is called **invertible** as we can define an inverse of it.
- A function is **not invertible** if it is not a one-to-one correspondence.

Function f^{-1} Is the Inverse of Function f

Example 1

- Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f invertible, and if it is, what is its inverse?

Example 2

- Let f be the function from **R** to **R** with $f(x) = x^2$. Is f invertible?

Example 3

- Find the inverse function of $F(x) = x^3+1$.

Example 4

- Find the inverse function of $F(x) = \dfrac{x-3}{2}$.

Example 5

- Find the inverse function of $F(x) = \dfrac{3x+2}{4x-1}$.

Example 6

- Let f is a function from R to R given by $f(x) = x^2+1$. Find $f^{-1}(-5)$.

Composition of functions

- Let g be a function from the set A to the set B and let f be a function from the set B to the set C.
- The *composition* of the functions f and g, denoted for all $a \in A$ by $f \circ g$, is defined by
  - $(f \circ g)(a) = f(g(a))$.

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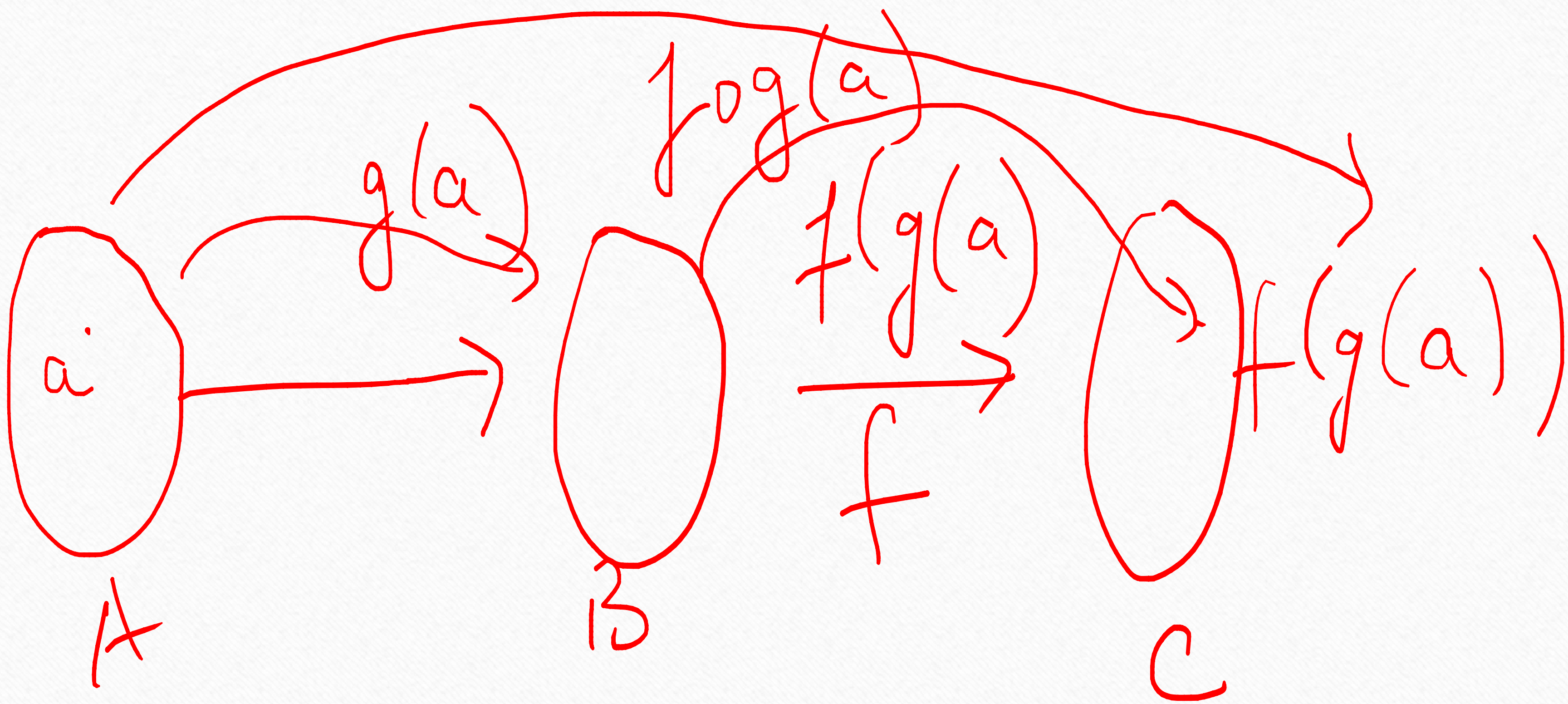


Example 1

- Let g be the function from the set $\{a, b, c\}$ to itself such that $g(a) = b$, $g(b) = c$, and $g(c) = a$. Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 3$, $f(b) = 2$, and $f(c) = 1$. What is the composition of f and g, and what is the composition of g and f ?

Example 2

- Let f and g be functions from the set of integers to the set of integers defined by $f(x) = 2x+3$ and $g(x) = 3x+2$. What is the composition of f and g? What is the composition of g and f?

Floor and Ceil Function

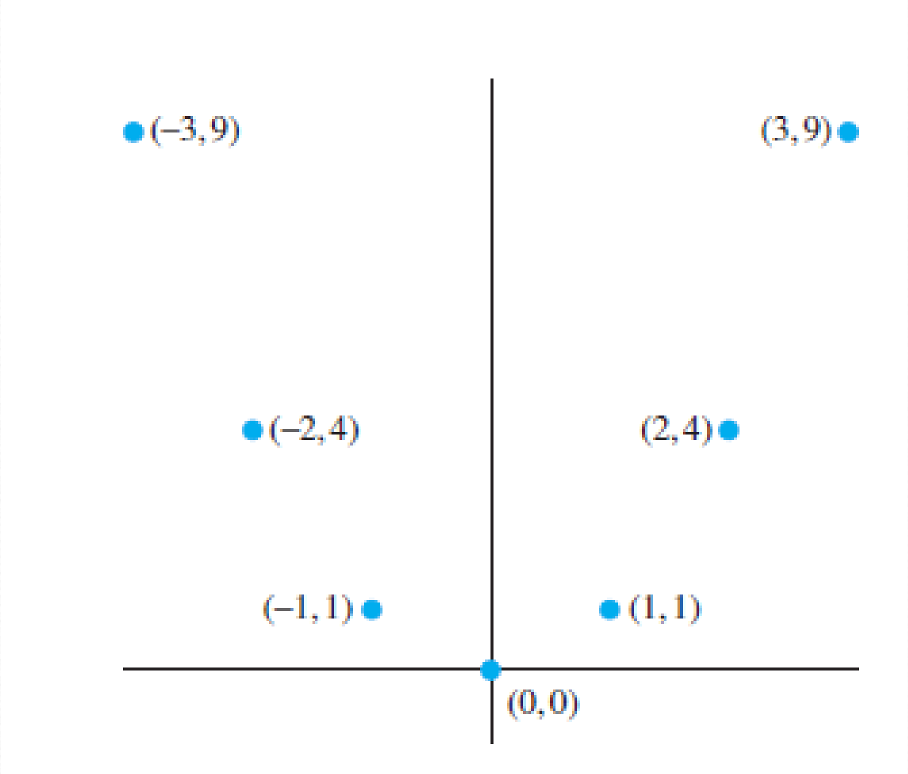
Graphs of Functions

- Let f be a function from the set A to the set B.
- The *graph* of the function f is the set of ordered pairs $\{(a, b) \mid a \in A$ and $f(a) = b\}$.

Example

- Display the graph of the function $f(x) = x^2$ from the set of integers to the set of integers.

The graph of f is the set of ordered pairs of the form $(x, f(x)) = (x, x^2)$, where x is an integer

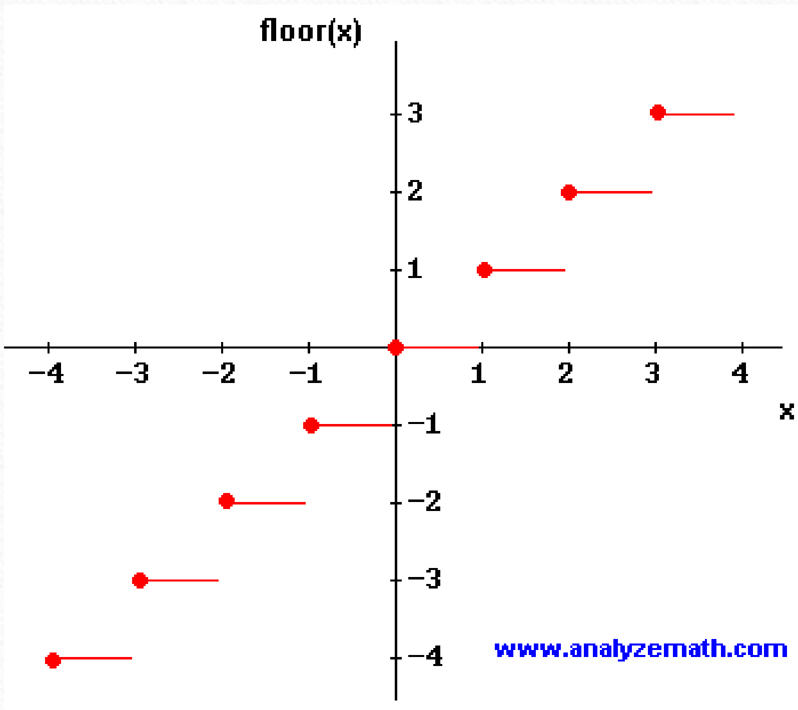


The Graph of $f(x) = x^2$ from Z to Z.

Floor function

- The ***floor function*** assigns to the real number x the largest integer that is less than or equal to x.
- The value of the floor function at x is denoted by $\lfloor x \rfloor$.

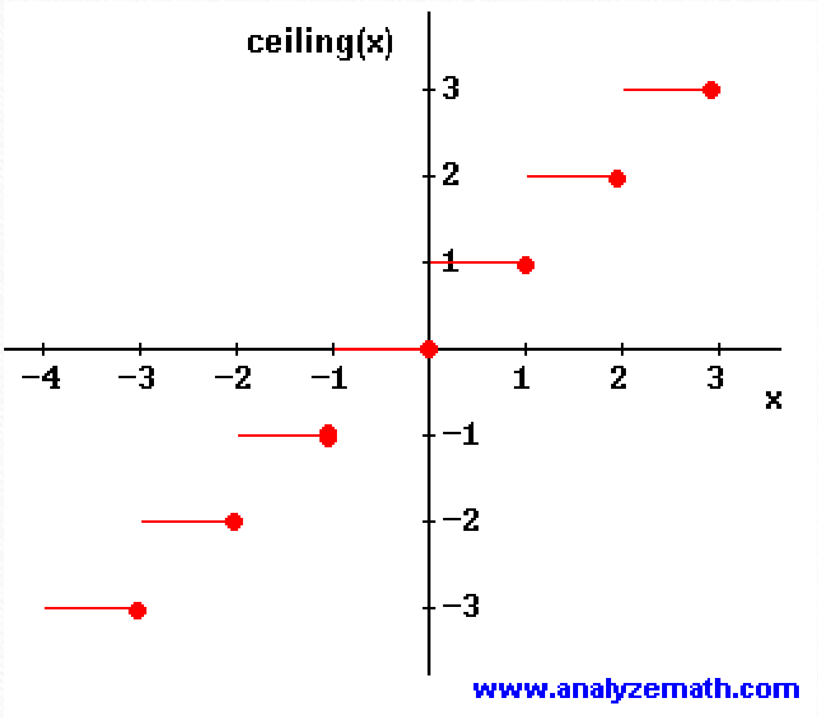
Graph of floor function



Ceil function

- The ***ceiling function*** assigns to the real number x the smallest integer that is greater than or equal to x.
- The value of the ceiling function at x is denoted by $\lceil x \rceil$.

Graph of ceil function



Example 1

- Data stored on a computer disk or transmitted over a data network are usually represented as a string of bytes. Each byte is made up of 8 bits. How many bytes are required to encode 100 bits of data?

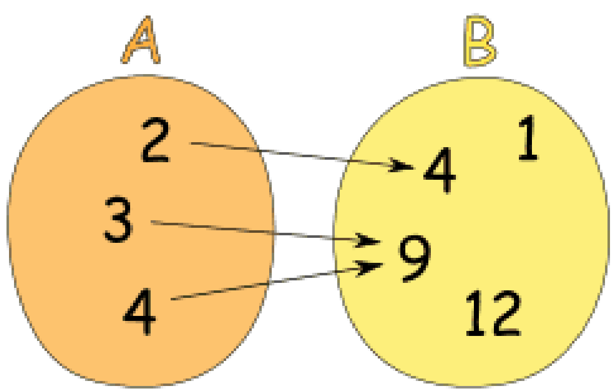
Example 2

- In asynchronous transfer mode, data are organized into cells of 53 bytes. How many ATM cells can be transmitted in 1 minute over a connection that transmits data at the rate of 500 kilobits per second?

Thank You

Practice on Basics of Functions

Question 1

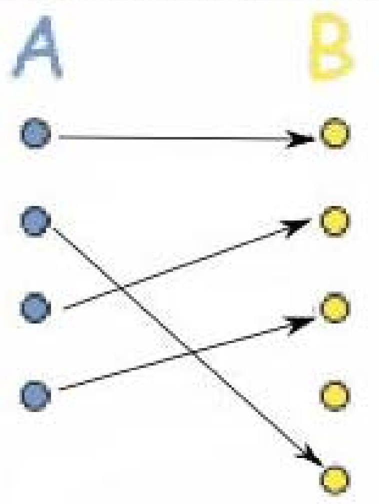


For the function illustrated above, what is the range?

Question 2

What is the domain for the function $f(x) = \dfrac{(x-2)(x-4)}{(x-1)(x-3)}$?

Question 3



The function from set A to set B is

Question 4

Which of the following functions is NOT injective (one-to-one)?

A $f(x) = x^3 + 4$ from R to R

B $f(x) = x^3 + 4$ from N to N

C $f(x) = x^2 + 4$ from R to R

D $f(x) = x^2 + 4$ from N to N

Question 5

If X = Floor(X) = Ceil(X) then :

a) X is a fractional number

b) X is a Integer

c) X is less than 1

d) none of the mentioned

Question 6

Suppose that $f(x) = 3x - 8$

a) Is f^{-1} a function?

b) Find the inverse function of f.

c) Compute $f(f^{-1}(7))$ and $f^{-1}(f(7))$

Thank You