# Array

# Array ADT

`float marks[10];`

- The simplest but useful data structure.
- Assign single name to a homogeneous collection of instances of one abstract data type.
  - All array elements are of same type, so that a pre–defined equal amount of memory is allocated to each one of them.
- Individual elements in the collection have an associated index value that depends on array dimension.

# Contd…

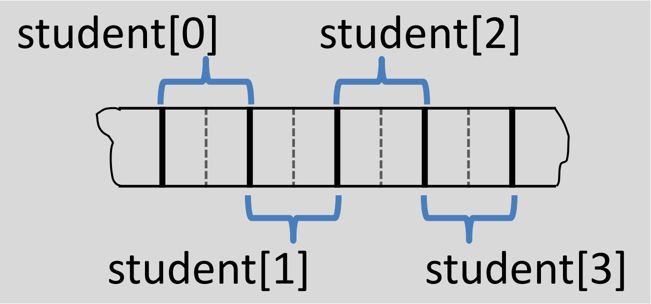
- One-dimensional and two-dimensional arrays are commonly used.
- Multi–dimensional arrays can also be defined.

- Usage:
  - Used frequently to store relatively permanent collections of data.
  - Not suitable if the size of the structure or the data in the structure are constantly changing.

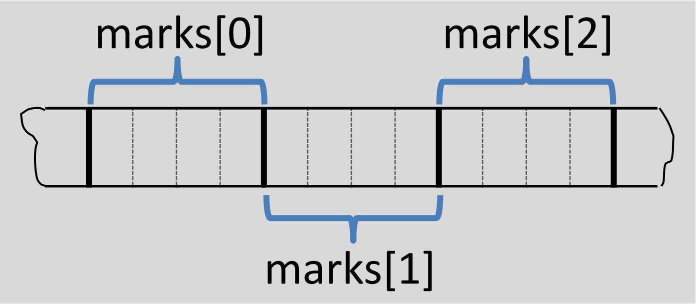
# Memory Storage

# Memory Storage – One Dimensional Array

```
int student[4];
```




```
float marks[3];
```



# Memory Storage – Two Dimensional Array

```
int marks[3][5];
```

- Can be visualized in the form of a matrix as

| | Col 0 | Col 1 | Col 2 | Col 3 | Col 4 |
|---|---|---|---|---|---|
| Row 0 | marks[0][0] | marks[0][1] | marks[0][2] | marks[0][3] | marks[0][4] |
| Row 1 | marks[1][0] | marks[1][1] | marks[1][2] | marks[1][3] | marks[1][4] |
| Row 2 | marks[2][0] | marks[2][1] | marks[2][2] | marks[2][3] | marks[2][4] |

# Contd…

- Row-major order

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (0,0) | (0,1) | (0,2) | (0,3) | (0,4) | (1,0) | (1,1) | (1,2) | (1,3) | (1,4) | (2,0) | (2,1) | (2,2) | (2,3) | (2,4) |

Row0 Row1 Row2

- Column-major order

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (0,0) | (1,0) | (2,0) | (0,1) | (1,1) | (2,1) | (0,2) | (1,2) | (2,2) | (0,3) | (1,3) | (2,3) | (0,4) | (1,4) | (2,4) |

Col0 Col1 Col2 Col3 Col4

# Array Address Computation

# 1D array – address calculation

- Let A be a one dimensional array.
- Formula to compute the address of the $I^{th}$ element of an array (A[I]) is:

**Address of A[I] = B + W * ( I – LB )**

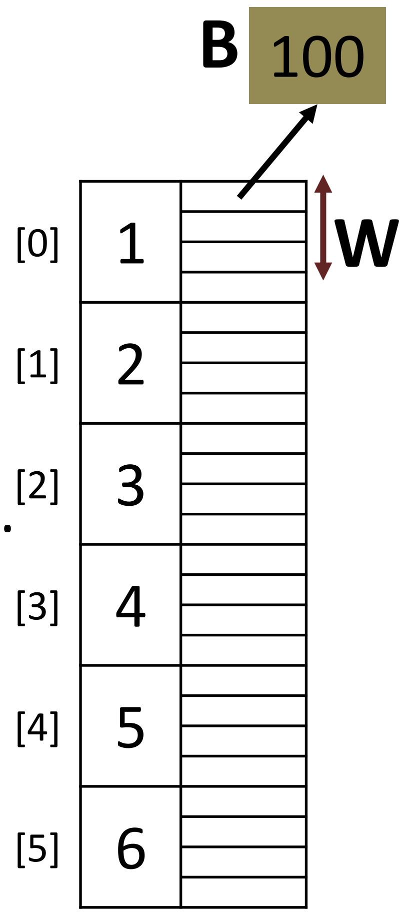
where,

**B** = Base address/address of first element, i.e. A[LB].

**W** = Number of bytes used to store a single array element.

**I** = Subscript of element whose address is to be found.

**LB** = Lower limit / Lower Bound of subscript, if not specified assume 0 (zero).



# 1D array – address calculation

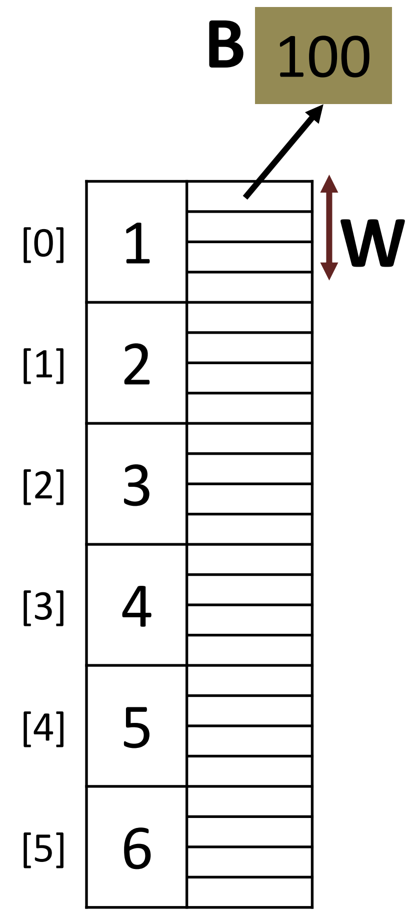
- Let A be a one dimensional array.
- Formula to compute the address of the $I^{th}$ element of an array (A[I]) is:

**Address of A[I] = B + W * ( I – LB )**

Given:

**B = 100, W = 4, and LB = 0**

**A[0] = 100 + 4 * (0 – 0) = 100**



# 1D array – address calculation

- Let A be a one dimensional array.
- Formula to compute the address of the $I^{th}$ element of an array (A[I]) is:

**Address of A[I] = B + W * ( I – LB )**

Given:

**B = 100, W = 4, and LB = 0**

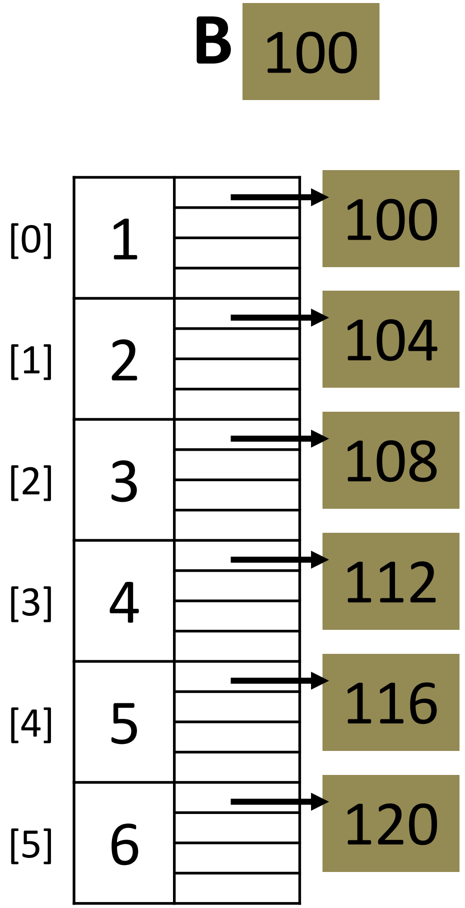
**A[1] = 100 + 4 * (1 – 0) = 104**

**A[2] = 100 + 4 * (2 – 0) = 108**

**A[3] = 100 + 4 * (3 – 0) = 112**

**A[4] = 100 + 4 * (4 – 0) = 116**

**A[5] = 100 + 4 * (5 – 0) = 120**



# Example – 1

- Similarly, for a character array where a single character uses 1 byte of storage.
- If the base address is 1200 then,

**Address of A[I] = B + W * ( I – LB )**

Address of A[0] = 1200 + 1 * (0 – 0) = 1200

Address of A[1] = 1200 + 1 * (1 – 0) = 1201

…

Address of A[10] = 1200 + 1 * (10 – 0) = 1210

# Example – 2

- If **LB** = 5, **Loc(A[LB])** = 1200, and **W** = 4.
- Find **Loc(A[8])**.

**Address of A[I] = B + W * ( I – LB )**

$$
\begin{aligned}
\text{Loc}(A[8]) &= \text{Loc}(A[5]) + 4 * (8 - 5) \\
&= 1200 + 4 * 3 \\
&= 1200 + 12 \\
&= 1212
\end{aligned}
$$

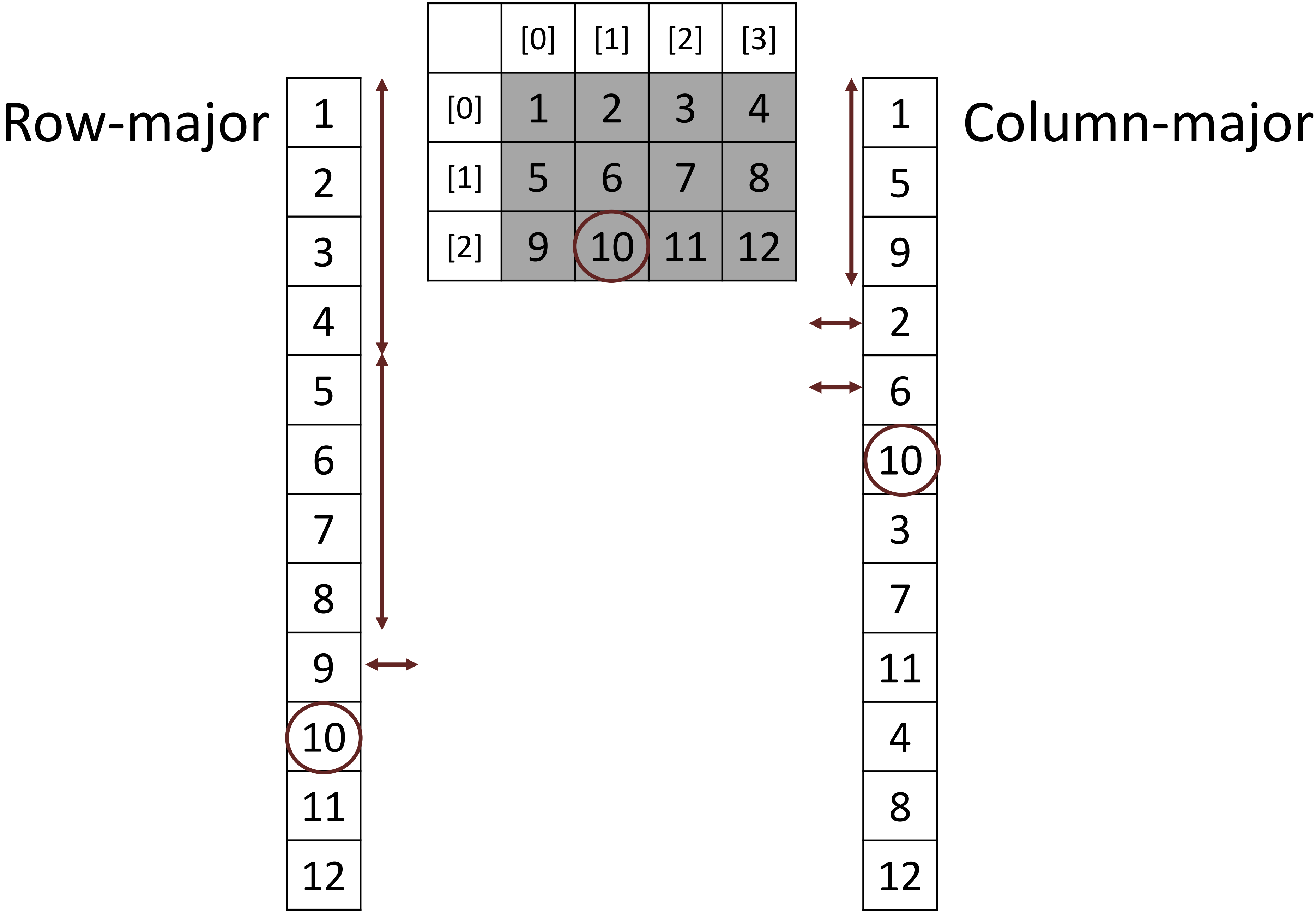
# Example – 3

- Base address of an array **B[1300…..1900]** is **1020** and size of each element is 2 bytes in the memory. Find the address of **B[1700]**.

**Address of A[I] = B + W * ( I – LB )**

- Given: **B** = 1020, **W** = 2, **I** = 1700, **LB** = 1300

**Address of B[1700]** = 1020 + 2 * (1700 – 1300)
= 1020 + 2 * 400
= 1020 + 800
= 1820



Row-major
Column-major
[0]
[1]
[2]
[3]
1 2 3 4
5 6 7 8
9 10 11 12
1 2 3 4 5 6 7 8 9 10 11 12
1 5 9 2 6 10 3 7 11 4 8 12

# 2D Array – Address Calculation

- If **A** be a two dimensional array with **M** rows and **N** columns. We can compute the address of an element at **I**$^{th}$ row and **J**$^{th}$ column of an array (**A[I][J]**).

  **B** = Base address/address of first element, i.e. A[LBR][LBC]
  **I** = Row subscript of element whose address is to be found
  **J** = Column subscript of element whose address is to be found
  **W** = Number of bytes used to store a single array element
  **LBR** = Lower limit of row/start row index of matrix, if not given assume 0
  **LBC** = Lower limit of column/start column index of matrix, if not given assume 0
  **N** = Number of column of the given matrix
  **M** = Number of row of the given matrix

Row-major

| |
|---|
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 1 | 2 | 3 | 4 |
| [1] | 5 | 6 | 7 | 8 |
| [2] | 9 | 10 | 11 | 12 |

**M = 3**

**N = 4**

**Address of A[2][1] =**

**B + W * (4 * (2 – 0) + (1 – 0))**

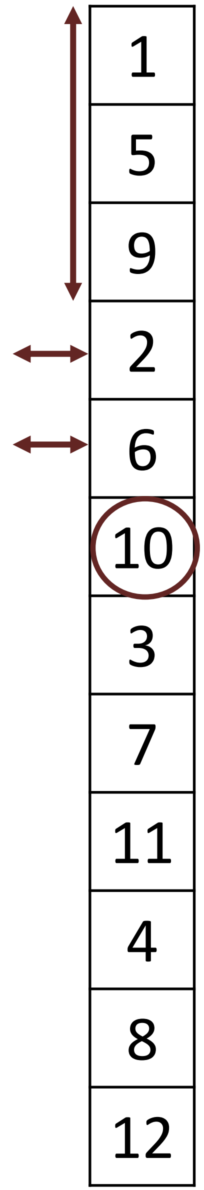
**Address of A[I][J] =**

**B + W * ( N * ( I – LBR ) + ( J – LBC ))**

**M = 3**

**N = 4**

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 1 | 2 | 3 | 4 |
| [1] | 5 | 6 | 7 | 8 |
| [2] | 9 | 10 | 11 | 12 |




Column-major

**Address of A [2][1] =**

**B + W * ((2 – 0) + 3 * (1 – 0))**

**Address of A [I][J] =**

**B + W * ((I – LBR) + M * (J – LBC))**

# Contd…

- Row Major

**Address of A[I][J] = B + W * ( N * ( I – LBR ) + ( J – LBC ))**

- Column Major

**Address of A [I][J] = B + W * (( I – LBR ) + M * ( J – LBC ))**

- Note: **A[LBR…UBR, LBC…UBC]**

**M = (UBR – LBR) + 1**

**N = (UBC – LBC) + 1**

# Example – 4

- Suppose elements of array **A[5][5]** occupies **4** bytes, and the address of the first element is **49**. Find the address of the element **A[4][3]** when the storage is row major.

**Address of A[I][J] = B + W * ( N * ( I – LBR ) + ( J – LBC ))**

- Given: **B** = 49, **W** = 4, **M** = 5, **N** = 5, **I** = 4, **J** = 3, **LBR** = 0, **LBC** = 0.

**Address of A[4][3]** = 49 + 4 * (5 * (4 – 0) + (3 - 0))

= 49 + 4 * (23)

= 49 + 92

= 141

# Example – 5

- An array **X [-15…10, 15…40]** requires **one** byte of storage. If beginning location is **1500** determine the location of **X [0][20]** in column major.

**Address of A[I][J] = B + W * [ ( I – LBR ) + M * ( J – LBC ) ]**

- Number or rows (**M**) = **(UBR – LBR) + 1** = [10 – (- 15)] +1 = 26
- Given: **B** = 1500, **W** = 1, **I** = 0, **J** = 20, **LBR** = -15, **LBC** = 15, **M** = 26

**Address of X[0][20]** = 1500 + 1 * [(0 – (-15)) + 26 * (20 – 15)]

= 1500 + 1 * [15 + 26 * 5]

= 1500 + 1 * [145]

= 1645

# Example – 6

- A two-dimensional array defined as **A [-4 … 6] [-2 … 12]** requires **2 bytes** of storage for each element. If the array is stored in row major order form with the address **A[4][8]** as **4142**. Compute the address of **A[0][0].**

  **Address of A[I][J] = B + W (N ( I – LBR ) + ( J – LBC ))**

- **Given:**

  **W = 2, LBR = –4, LBC = –2**

  **#rows = M = 6 + 4 + 1 = 11 #columns = N = 12 + 2 + 1 = 15**

  **Address of A[4][8] = 4142**

- **Address of A[4][8] = B + 2 (15 (4 – (–4)) + (8 – (–2)))**

  4142 = B + 2 (15 (4 + 4) + (8 + 2)) = B + 2 (15 (8) + 10) = B + 2 (120 + 10)

  4142 = B + 260

  **Thus, B = 4142 – 260 = 3882**

- **Now, Address of A[0][0] = 3882 + 2 (15 (0 – (–4)) + (0 – (–2)))**

  = 3882 + 2 (15(4) + 2) = 3882 + 2 (62)

  = 3882 + 124

  = 4006

# Array Basic Operations

# Operations on Linear Data Structures

- Traversal
- Search – Linear and Binary.
- Insertion
- Deletion
- Sorting – Different algorithms are there.
- Merging – During the discussion of Merge Sort.

# TRAVERSAL

Processing each element in the array.

# Example – Print all the array elements.

**Algorithm** arrayTraverse(A,n)

**Input:** An array **A** containing **n** integers.

**Output:** All the elements in **A** get printed.

1. for i = 0 to n-1 do
2.         Print A[i]

```
int arrayTraverse(int arr[], int n)
{
   for (int i = 0; i < n; i++)
        cout << "\n" << arr[i];
}
```

# Example – Find minimum element in the array.

**Algorithm** arrayMinElement(A,n)

**Input:** An array **A** containing **n** integers.

**Output:** The minimum element in **A**.

```
min = 0
for i = 1 to n-1 do
    if A[min] > A[i]
        min = i
return A[min]
```

```c
int arrayMinElement(int arr[], int n)
{  int min = 0;
   for (int i = 1; i < n; i++)
   {   if (arr[i] < arr[min])
           min = i;
   }
   return arr[min];
}
```

# Search

Find the location of the element with a given value.

# Linear Search

- Used if the array is unsorted.
- Example:

Search 7 in the following array

i → 0 → 1 → 2 → 3 → 4 → 5 → 6

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| a[] | 10 | 5 | 1 | 6 | 2 | 9 | 7 | 8 | 3 | 4 |

Found at index 6

Search 11 in the following array

i → 0 → 1 → 2 → 3 → 4 → 5 → 6 → 7 → 8 → 9 → 10

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| a[] | 10 | 5 | 1 | 6 | 2 | 9 | 7 | 8 | 3 | 4 |

Not found

# Contd…

**Algorithm** linearSearch(A,n,num)

**Input:** An array **A** containing **n** integers and number **num** to be searched.

**Output:** Index of **num** if found, otherwise -1.

```
for i = 0 to n-1 do
    if A[i] == num
        return i
return -1
```

```
int linearSearch(int a[], int n, int num)
{  for (int i = 0; i < n; i++)
       if (a[i] == num)
            return i;
     return -1;
}
```

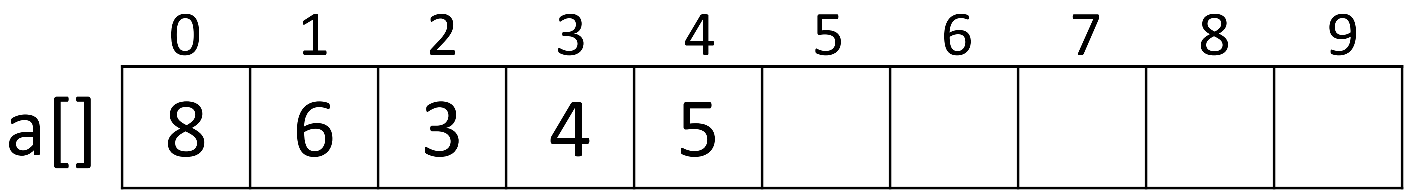
# Insertion

Insert an element in the array

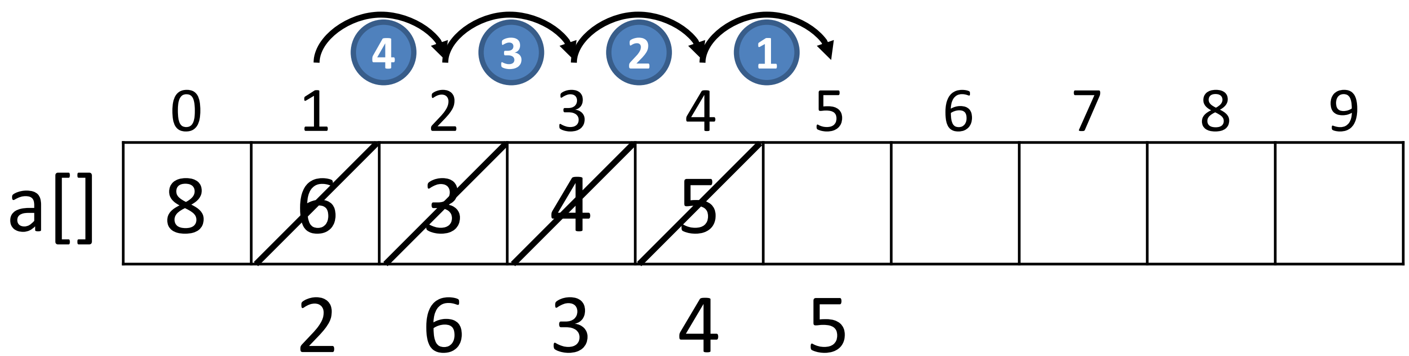
# Deletion

Delete an element from the array

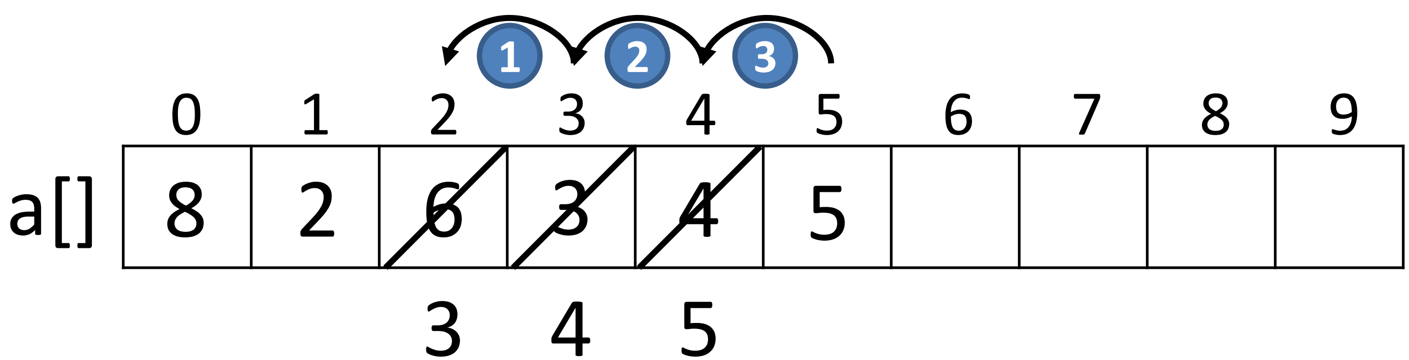
# Insertion and Deletion




- Insert 2 at index 1




- Delete the value at index 2



# Algorithm – Insertion

**Algorithm** insertElement(A,n,num,indx)

**Input:** An array **A** containing **n** integers and the number **num** to be inserted at index **indx**.

**Output:** Successful insertion of **num** at **indx**.

1. for i = n – 1 to indx do
2. A[i + 1] = A[i]
3. A[indx] = num
4. n = n + 1

```
void insert(int a[], int num, int pos)
{   for(int i = n-1; i >= pos; i--)
            a[i+1] = a[i];
        a[pos] = num;
         n++;
}
```

# Algorithm – Deletion

**Algorithm** deleteElement(A,n,indx)
**Input:** An array **A** containing **n** integers and the index **indx** whose value is to be deleted.
**Output:** Deleted value stored initially at **indx**.

1. temp = A[indx]
2. for i = indx to n – 2 do
3.     A[i] = A[i + 1]
4. n = n – 1
5. return temp

```
int deleteElement(int a[], int pos)
{   int temp = a[pos];
    for(int i = pos; i <= n-2; i++)
        a[i] = a[i+1];
    n--;
    return temp;
}
```

# Handling Special Matrices

# Special Matrices

- Square – same number of rows and columns.
- Some special forms of square matrices are
  - Diagonal: $M(i,j) = 0$ for $i \neq j$
  - Tridiagonal: $M(i,j) = 0$ for $|i-j| > 1$
  - Lower triangular: $M(i,j) = 0$ for $i >= j$
  - Upper triangular: $M(i,j) = 0$ for $i <= j$
  - Symmetric $M(i,j) = M(j,i)$ for all $i$ and $j$

# Contd…

M(i,j) = 0 for i ≠ j

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

Diagonal

M(i,j) = 0 for |i-j| > 1

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 3 & 1 & 3 & 0 \\ 0 & 5 & 2 & 7 \\ 0 & 0 & 9 & 0 \end{bmatrix}$$

Tri-Diagonal

M(i,j) = 0 for i >= j

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 4 & 2 & 7 & 0 \end{bmatrix}$$

Lower Triangular

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Upper Triangular

M(i,j) = 0 for i <= j

$$\begin{bmatrix} 2 & 4 & 6 & 0 \\ 4 & 1 & 9 & 5 \\ 6 & 9 & 4 & 7 \\ 0 & 5 & 7 & 0 \end{bmatrix}$$

Symmetric

M(i,j) = M(j,i) for all i and j

# Contd…

- Why are we interested in these "special" matrices?
  - We can provide more efficient implementations for specific special matrices.
  - Rather than having a space complexity of $O(n^2)$, we can find an implementation that is $O(n)$.
  - We need to be clever about the "store" and "retrieve" operations to reduce time.

# Diagonal Matrix

- Naive way to represent n x n diagonal matrix
  - <datatype> d[n][n]
  - d[i][j] for D(i,j)
  - Requires $n^2$ x sizeof(<datatype>) bytes of memory.

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

- Better way
  - <datatype> d[n]
  - d[i] for D(i,j) where i = j
    0 for D(i,j) where i ≠ j
  - Requires n x sizeof(<datatype>) bytes of memory.

# Example

```
#include<stdio.h>
#define MAX 4
int main()
{  int i,j, a[MAX];
   printf("\nEnter elements (row major):\n");
   for(i = 0; i < MAX; i++)
     scanf("%d",&a[i]);
   printf("\nThe matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i==j)
              printf("%d ", a[i]);
           else
              printf("0 ");   }
      printf("\n");         }
   return 0;   }
```

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| a[]: | 2 | 1 | 4 | 6 |

# Contd...

```
#include<stdio.h>
#define MAX 4
int main()
{  int i,j, a[MAX];
   printf("\nEnter elements (row major):\n");
   for(i = 0; i < MAX; i++)
    scanf("%d",&a[i]);
   printf("\nThe matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i==j)
             printf("%d ", a[i]);
           else
             printf("0 ");   }
      printf("\n");         }
   return 0;   }
```

a[]:

| 0 | 1 | 2 | 3 |
|---|---|---|---|
| 2 | 1 | 4 | 6 |

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 2 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 2 | 0 | 0 | 4 | 0 |
| 3 | 0 | 0 | 0 | 6 |

# Tridiagonal Matrix

- Nonzero elements lie on one of three diagonals:
  - main diagonal: i = j
  - diagonal below main diagonal: i = j+1
  - diagonal above main diagonal: i = j-1
- Total elements are 3n – 2: <datatype> d[3n-2]
- Mappings
  - by row [2,1,3,1,3,5,2,7,9,0]
  - by column [2,3,1,1,5,3,2,9,7,0]
  - by diagonal [3,5,9,2,1,2,0,1,3,7]

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 3 & 1 & 3 & 0 \\ 0 & 5 & 2 & 7 \\ 0 & 0 & 9 & 0 \end{bmatrix}$$

# Example

```
#include<stdio.h>
#define MAX 4
int main()
{  int i,j,k=0, size = 3*MAX-2, a[size];
   printf("\nEnter elements (row major):\n");
   for(i = 0; i < size; i++)
     scanf("%d",&a[i]);
   printf("\nThe matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i-j == -1 || i-j == 0 || i-j == 1)
              {  printf("%d ", a[k]);  k++;  }
             else
                 printf("0 ");              }
      printf("\n");        }
   return 0;   }
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| a[]: | 2 | 1 | 3 | 1 | 3 | 5 | 2 | 7 | 9 | 0 |

# Example

```c
#include<stdio.h>
#define MAX 4
int main()
{  int i,j,k=0, size = 3*MAX-2, a[size];
   printf("\nEnter elements (row major):\n");
   for(i = 0; i < size; i++)
    scanf("%d",&a[i]);
   printf("\nThe matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
     {    if(i-j == -1 || i-j == 0 || i-j == 1)
          {  printf("%d ", a[k]);  k++;  }
          else
             printf("0 ");              }
     printf("\n");        }
   return 0;   }
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| a[]: | 2 | 1 | 3 | 1 | 3 | 5 | 2 | 7 | 9 | 0 |

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 2 | 1 | 0 | 0 |
| 1 | 3 | 1 | 3 | 0 |
| 2 | 0 | 5 | 2 | 7 |
| 3 | 0 | 0 | 9 | 0 |

# Triangular Matrix

- Nonzero elements lie in the upper triangular or lower triangular region.
- Total elements are 1 + 2 + … + n = n(n+1)/2:

  <datatype> d[(n(n+1)/2)]
- Mappings
  - by row

    [2,5,1,0,3,1,4,2,7,0]
  - by column

    [2,5,0,4,1,3,2,1,7,0]

    [2,5,3,1,1,2,0,4,7,0]

$$\begin{bmatrix} 2 & 5 & 1 & 0 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Upper Triangular

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 4 & 2 & 7 & 0 \end{bmatrix}$$

Lower Triangular

# Example

```
#include<stdio.h>
#define MAX 4
int main()
{  int i,j,k=0, size = (MAX*(MAX+1))/2, a[size];
   printf("\nEnter elements (row major):\n");
   for(i = 0; i < size; i++)
     scanf("%d",&a[i]);
   printf("\nThe upper triangular matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i <= j)
             {  printf("%d ", a[k]);  k++;  }
           else
             printf("0 ");       }
      printf("\n");            }
   k = 0;
   printf("\nThe lower triangular matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i >= j)
             {  printf("%d ", a[k]);  k++;  }
           else
             printf("0 ");       }
      printf("\n");            }
   return 0;     }
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| a[]: | 2 | 5 | 1 | 0 | 3 | 1 | 4 | 2 | 7 | 0 |

# Example

```c
#include<stdio.h>
#define MAX 4
int main()
{  int i,j,k=0, size = (MAX*(MAX+1))/2, a[size];
   printf("\nEnter elements (row major):\n");
   for(i = 0; i < size; i++)
     scanf("%d",&a[i]);
   printf("\nThe upper triangular matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i <= j)
             {  printf("%d ", a[k]);  k++;  }
           else
             printf("0 ");      }
      printf("\n");          }
   k = 0;
   printf("\nThe lower triangular matrix is...\n");
   for(i = 0; i < MAX; i++)
    {  for(j = 0; j < MAX; j++)
       {    if(i >= j)
              {  printf("%d ", a[k]);  k++;  }
            else
              printf("0 ");      }
       printf("\n");          }
    return 0;     }
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| a[]: | 2 | 5 | 1 | 0 | 3 | 1 | 4 | 2 | 7 | 0 |

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 2 | 5 | 1 | 0 |
| 1 | 0 | 3 | 1 | 4 |
| 2 | 0 | 0 | 2 | 7 |
| 3 | 0 | 0 | 0 | 0 |

# Example

```
#include<stdio.h>
#define MAX 4
int main()
{  int i,j,k=0, size = (MAX*(MAX+1))/2, a[size];
   printf("\nEnter elements (row major):\n");
   for(i = 0; i < size; i++)
     scanf("%d",&a[i]);
   printf("\nThe upper triangular matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i <= j)
             {  printf("%d ", a[k]);  k++;  }
           else
             printf("0 ");       }
      printf("\n");            }
   k = 0;
   printf("\nThe lower triangular matrix is...\n");
   for(i = 0; i < MAX; i++)
   {  for(j = 0; j < MAX; j++)
      {    if(i >= j)
             {  printf("%d ", a[k]);  k++;  }
           else
             printf("0 ");       }
      printf("\n");            }
   return 0;     }
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| a[]: | 2 | 5 | 1 | 0 | 3 | 1 | 4 | 2 | 7 | 0 |

| | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 2 | 0 | 0 | 0 |
| 1 | 5 | 1 | 0 | 0 |
| 2 | 0 | 3 | 1 | 0 |
| 3 | 4 | 2 | 7 | 0 |

# Symmetric Matrix

- An n x n matrix can be represented using 1-D array of size n(n+1)/2 by storing either the lower or upper triangle of the matrix.

$$\begin{bmatrix} 2 & 4 & 6 & 0 \\ 4 & 1 & 9 & 5 \\ 6 & 9 & 4 & 7 \\ 0 & 5 & 7 & 0 \end{bmatrix}$$

- Use one of the methods for a triangular matrix.
- The elements that are not explicitly stored may be computed from those that are stored.

# Sparse Matrix

- A matrix is sparse if many of its elements are zero.
- A matrix that is not sparse is dense.
- Two possible representations
  - Array (also known as triplet)
  - Linked list

$$\begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 9 \\ 0 & 5 & 4 & 0 \end{bmatrix}$$

# Array representation

| | [0] | [1] | [2] | [3] | [4] | [5] |
|---|---|---|---|---|---|---|
| [0] | 15 | 0 | 0 | 22 | 0 | -15 |
| [1] | 0 | 11 | 3 | 0 | 0 | 0 |
| [2] | 0 | 0 | 0 | -6 | 0 | 0 |
| [3] | 0 | 0 | 0 | 0 | 0 | 0 |
| [4] | 91 | 0 | 0 | 0 | 0 | 0 |
| [5] | 0 | 0 | 28 | 0 | 0 | 0 |

| Row | Col | Value |
|---|---|---|
| 6 | 6 | 8 |
| 0 | 0 | 15 |
| 0 | 3 | 22 |
| 0 | 5 | -15 |
| 1 | 1 | 11 |
| 1 | 2 | 3 |
| 2 | 3 | -6 |
| 4 | 0 | 91 |
| 5 | 2 | 28 |

# Operations

- Transpose
- Addition
- Multiplication

# Transpose

| | [0] | [1] | [2] | [3] | [4] | [5] |
|---|---|---|---|---|---|---|
| [0] | 15 | 0 | 0 | 22 | 0 | -15 |
| [1] | 0 | 11 | 3 | 0 | 0 | 0 |
| [2] | 0 | 0 | 0 | -6 | 0 | 0 |
| [3] | 0 | 0 | 0 | 0 | 0 | 0 |
| [4] | 91 | 0 | 0 | 0 | 0 | 0 |
| [5] | 0 | 0 | 28 | 0 | 0 | 0 |

| | [0] | [1] | [2] | [3] | [4] | [5] |
|---|---|---|---|---|---|---|
| [0] | 15 | 0 | 0 | 0 | 91 | 0 |
| [1] | 0 | 11 | 0 | 0 | 0 | 0 |
| [2] | 0 | 3 | 0 | 0 | 0 | 28 |
| [3] | 22 | 0 | -6 | 0 | 0 | 0 |
| [4] | 0 | 0 | 0 | 0 | 0 | 0 |
| [5] | -15 | 0 | 0 | 0 | 0 | 0 |

| Row | Col | Value |
|---|---|---|
| 6 | 6 | 8 |
| 0 | 0 | 15 |
| 0 | 3 | 22 |
| 0 | 5 | -15 |
| 1 | 1 | 11 |
| 1 | 2 | 3 |
| 2 | 3 | -6 |
| 4 | 0 | 91 |
| 5 | 2 | 28 |

Original

| Row | Col | Value |
|---|---|---|
| 6 | 6 | 8 |
| 0 | 0 | 15 |
| 3 | 0 | 22 |
| 5 | 0 | -15 |
| 1 | 1 | 11 |
| 2 | 1 | 3 |
| 3 | 2 | -6 |
| 0 | 4 | 91 |
| 2 | 5 | 28 |

Column Major

| Row | Col | Value |
|---|---|---|
| 6 | 6 | 8 |
| 0 | 0 | 15 |
| 0 | 4 | 91 |
| 1 | 1 | 11 |
| 2 | 1 | 3 |
| 2 | 5 | 28 |
| 3 | 0 | 22 |
| 3 | 2 | -6 |
| 5 | 0 | -15 |

Row Major

# Addition

| | [0] | [1] | [2] | [3] | [4] | [5] |
|---|---|---|---|---|---|---|
| [0] | 15 | 0 | 0 | 22 | 0 | -15 |
| [1] | 0 | 11 | 3 | 0 | 0 | 0 |
| [2] | 0 | 0 | 0 | -6 | 0 | 0 |
| [3] | 0 | 0 | 0 | 0 | 0 | 0 |
| [4] | 91 | 0 | 0 | 0 | 0 | 0 |
| [5] | 0 | 0 | 28 | 0 | 0 | 0 |

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| | [0] | [1] | [2] | [3] | [4] | [5] |
|---|---|---|---|---|---|---|
| [0] | 15 | 0 | 0 | 0 | 91 | 0 |
| [1] | 0 | 11 | 0 | 0 | 0 | 0 |
| [2] | 0 | 3 | 0 | 0 | 0 | 28 |
| [3] | 22 | 0 | -6 | 0 | 0 | 0 |
| [4] | 0 | 0 | 0 | 0 | 0 | 0 |
| [5] | -15 | 0 | 0 | 0 | 0 | 0 |

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| | [0] | [1] | [2] | [3] | [4] | [5] |
|---|---|---|---|---|---|---|
| [0] | 30 | 0 | 0 | 22 | 91 | -15 |
| [1] | 0 | 22 | 3 | 0 | 0 | 0 |
| [2] | 0 | 3 | 0 | -6 | 0 | 28 |
| [3] | 22 | 0 | -6 | 0 | 0 | 0 |
| [4] | 91 | 0 | 0 | 0 | 0 | 0 |
| [5] | -15 | 0 | 28 | 0 | 0 | 0 |

# Addition

Counter = 14

| Row | Col | Value |
|---|---|---|
| 6 | 6 | 8 |
| 0 | 0 | 15 |
| 0 | 3 | 22 |
| 0 | 5 | -15 |
| 1 | 1 | 11 |
| 1 | 2 | 3 |
| 2 | 3 | -6 |
| 4 | 0 | 91 |
| 5 | 2 | 28 |

| Row | Col | Value |
|---|---|---|
| 6 | 6 | 8 |
| 0 | 0 | 15 |
| 0 | 4 | 91 |
| 1 | 1 | 11 |
| 2 | 1 | 3 |
| 2 | 5 | 28 |
| 3 | 0 | 22 |
| 3 | 2 | -6 |
| 5 | 0 | -15 |

| Row | Col | Value |
|---|---|---|
| 6 | 6 | 14 |
| 0 | 0 | 30 |
| 0 | 3 | 22 |
| 0 | 4 | 91 |
| 0 | 5 | -15 |
| 1 | 1 | 22 |
| 1 | 2 | 3 |
| 2 | 1 | 3 |
| 2 | 3 | -6 |
| 2 | 5 | 28 |
| 3 | 0 | 22 |
| 3 | 2 | -6 |
| 4 | 0 | 91 |
| 5 | 0 | -15 |
| 5 | 2 | 28 |

# Multiplication

- Compute A x B
- First take transpose of B.
- Multiply only if the corresponding elements are present and add them for each position in the resultant matrix.

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | | | | |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

=

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | | | |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

×

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

=

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | | |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

×

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 45 | 0 |
| [3] | 0 | 0 | 120 | 276 |

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 0 | 20 |
| [1] | 0 | 0 | 0 | 25 |
| [2] | 8 | 0 | 9 | 0 |
| [3] | 0 | 23 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | | | | |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 45 | 0 |
| [3] | 0 | 0 | 120 | 276 |

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 0 | 20 |
| [1] | 0 | 0 | 0 | 25 |
| [2] | 8 | 0 | 9 | 0 |
| [3] | 0 | 23 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | | | |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 45 | 0 |
| [3] | 0 | 0 | 120 | 276 |

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 0 | 20 |
| [1] | 0 | 0 | 0 | 25 |
| [2] | 8 | 0 | 9 | 0 |
| [3] | 0 | 23 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | | |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

$$
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 10 & 0 & 12 \\
[1] & 0 & 0 & 0 & 0 \\
[2] & 0 & 0 & 5 & 0 \\
[3] & 15 & 12 & 0 & 0
\end{array}
\times
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 0 & 8 & 0 \\
[1] & 0 & 0 & 0 & 23 \\
[2] & 0 & 0 & 9 & 0 \\
[3] & 20 & 25 & 0 & 0
\end{array}
=
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 240 & 300 & 0 & 230 \\
[1] & 0 & 0 & 0 & 0 \\
[2] & 0 & 0 & 45 & 0 \\
[3] & 0 & 0 & 120 & 276
\end{array}
$$

$$
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 10 & 0 & 12 \\
[1] & 0 & 0 & 0 & 0 \\
[2] & 0 & 0 & 5 & 0 \\
[3] & 15 & 12 & 0 & 0
\end{array}
\times
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 0 & 0 & 20 \\
[1] & 0 & 0 & 0 & 25 \\
[2] & 8 & 0 & 9 & 0 \\
[3] & 0 & 23 & 0 & 0
\end{array}
=
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 240 & 300 & 0 & \\
[1] & & & & \\
[2] & & & & \\
[3] & & & &
\end{array}
$$

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 45 | 0 |
| [3] | 0 | 0 | 120 | 276 |

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

×

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 0 | 20 |
| [1] | 0 | 0 | 0 | 25 |
| [2] | 8 | 0 | 9 | 0 |
| [3] | 0 | 23 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

×

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 8 | 0 |
| [1] | 0 | 0 | 0 | 23 |
| [2] | 0 | 0 | 9 | 0 |
| [3] | 20 | 25 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 45 | 0 |
| [3] | 0 | 0 | 120 | 276 |

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 10 | 0 | 12 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 5 | 0 |
| [3] | 15 | 12 | 0 | 0 |

×

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 0 | 0 | 0 | 20 |
| [1] | 0 | 0 | 0 | 25 |
| [2] | 8 | 0 | 9 | 0 |
| [3] | 0 | 23 | 0 | 0 |

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| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | | | |
| [2] | | | | |
| [3] | | | | |

# Multiplication

$$
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 10 & 0 & 12 \\
[1] & 0 & 0 & 0 & 0 \\
[2] & 0 & 0 & 5 & 0 \\
[3] & 15 & 12 & 0 & 0
\end{array}
\times
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 0 & 8 & 0 \\
[1] & 0 & 0 & 0 & 23 \\
[2] & 0 & 0 & 9 & 0 \\
[3] & 20 & 25 & 0 & 0
\end{array}
=
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 240 & 300 & 0 & 230 \\
[1] & 0 & 0 & 0 & 0 \\
[2] & 0 & 0 & 45 & 0 \\
[3] & 0 & 0 & 120 & 276
\end{array}
$$

$$
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 10 & 0 & 12 \\
[1] & 0 & 0 & 0 & 0 \\
[2] & 0 & 0 & 5 & 0 \\
[3] & 15 & 12 & 0 & 0
\end{array}
\times
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 0 & 0 & 0 & 20 \\
[1] & 0 & 0 & 0 & 25 \\
[2] & 8 & 0 & 9 & 0 \\
[3] & 0 & 23 & 0 & 0
\end{array}
=
\begin{array}{c|cccc}
 & [0] & [1] & [2] & [3] \\
\hline
[0] & 240 & 300 & 0 & 230 \\
[1] & 0 & 0 & 0 & 0 \\
[2] & 0 & 0 & 45 & 0 \\
[3] & 0 & 0 & 120 & 276
\end{array}
$$

# Multiplication

Counter = 0

| Row | Col | Value |
|---|---|---|
| 4 | 4 | 5 |
| 0 | 1 | 10 |
| 0 | 3 | 12 |
| 2 | 2 | 5 |
| 3 | 0 | 15 |
| 3 | 1 | 12 |

| Row | Col | Value |
|---|---|---|
| 4 | 4 | 5 |
| 0 | 3 | 20 |
| 1 | 3 | 25 |
| 2 | 0 | 8 |
| 2 | 2 | 9 |
| 3 | 1 | 23 |

| Row | Col | Value |
|---|---|---|
| 4 | 4 | 5 |
| 0 | 2 | 8 |
| 1 | 3 | 23 |
| 2 | 2 | 9 |
| 3 | 0 | 20 |
| 3 | 1 | 25 |

| Row | Col | Value |
|---|---|---|
| 4 | 4 | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

# Multiplication

Counter = 6

| Row | Col | Value |
|---|---|---|
| 4 | 4 | 5 |
| 0 | 1 | 10 |
| 0 | 3 | 12 |
| 2 | 2 | 5 |
| 3 | 0 | 15 |
| 3 | 1 | 12 |

| Row | Col | Value |
|---|---|---|
| 4 | 4 | 5 |
| 0 | 3 | 20 |
| 1 | 3 | 25 |
| 2 | 0 | 8 |
| 2 | 2 | 9 |
| 3 | 1 | 23 |

| Row | Col | Value |
|---|---|---|
| 4 | 4 | 6 |
| 0 | 0 | 240 |
| 0 | 1 | 300 |
| 0 | 3 | 230 |
| 2 | 2 | 45 |
| 3 | 2 | 120 |
| 3 | 3 | 276 |

| | [0] | [1] | [2] | [3] |
|---|---|---|---|---|
| [0] | 240 | 300 | 0 | 230 |
| [1] | 0 | 0 | 0 | 0 |
| [2] | 0 | 0 | 45 | 0 |
| [3] | 0 | 0 | 120 | 276 |