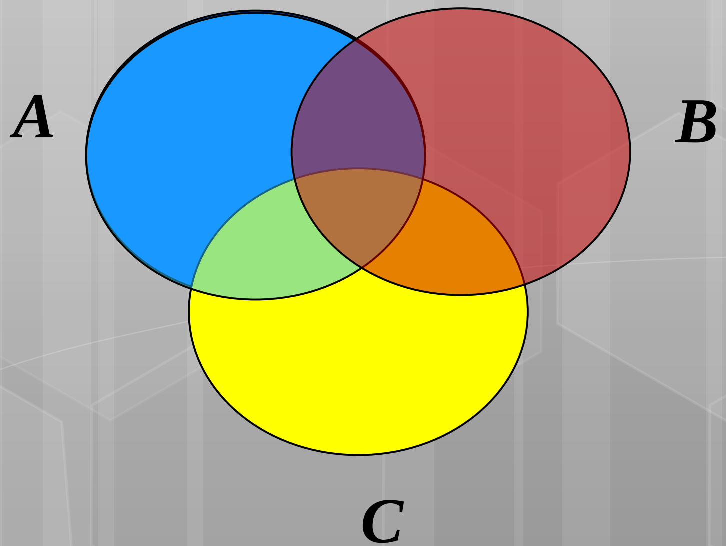

Set Theory

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Defining Sets

Definition: A **set** is an unordered collection of objects.

The objects in a set are called the **elements** or **members** of the set S, and we say S **contains** its elements.

The notation $a \in A$ denotes that a is an element of the set A.
If a is not a member of A, write $a \notin A$

We can define a set by directly listing all its elements.

e.g. S = {2, 3, 5, 7, 11, 13, 17, 19},

S = {CSC1130, CSC2110, ERG2020, MAT2510}

After we define a set, the set is a single mathematical object, and it can be an element of another set.

e.g. S = {{1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}}

Defining Sets by Properties

It is inconvenient, and sometimes impossible, to define a set by listing all its elements.

Alternatively, we can define by a set by describing the *properties* that its elements should satisfy.

We use the notation $\{x \in A \mid P(x)\}$

to define the set as the set of elements, x, in *A* such that x satisfies property *P*.

e.g. $\{x \mid x \text{ is a prime number and } x < 1000\}$

$$\{x \mid x \text{ is a real number and } -2 < x < 5\}$$

Example of Sets

Well known sets:

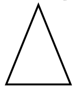
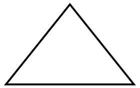
- the set of all real numbers, $\mathbb{R}$
- the set of all complex numbers, $\mathbb{C}$
- the set of all integers, $\mathbb{Z}$
- the set of all positive integers $\mathbb{Z}^+$

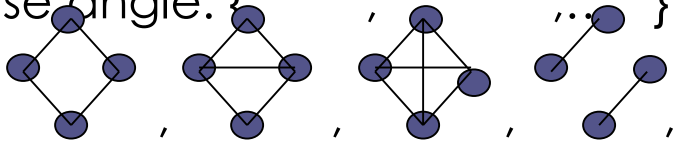
Other examples:

- empty set, $\emptyset = \{\}$ the set with no elements.

The set of all polynomials with degree at most three: $\{1, x, x^2, x^3, 2x+3x^2, \ldots\}$.

The set of all n-bit strings: {000…0, 000…1, …, 111…1}

The set of all triangles without an obtuse angle: {  ,  ,… }

The set of all graphs with four nodes: {  ,…}

Membership

Order, number of occurrence is not important.

e.g. {a,b,c} = {c,b,a} = {a,a,b,c,b}

The most basic question in set theory is whether an element is in a set.

$x \in A$ x is an element of A

x is in A

$x \notin A$ x is not an element of A

x is not in A

e.g.

Recall that Z is the set of all integers. So $7 \in Z$ and $2/3 \notin Z$.

Let P be the set of all prime numbers. Then $97 \in P$ and $321 \notin P$

Let Q be the set of all rational numbers. Then $0.5 \in Q$ and $\sqrt{2} \notin Q$

(will prove later)

Numerical Sets (Well-Defined)

- Set of even numbers:

  {…, -4, -2, 0, 2, 4, …}
- Set of odd numbers:

  {…, -3, -1, 1, 3, …}
- Set of prime numbers:

  {2, 3, 5, 7, 11, 13, 17, …}
- Positive multiples of 3 that are less than 10:

  {3, 6, 9}

Numerical Sets (Not Well-Defined)

- There can also be sets of numbers that have no common property, they are just defined that way. For example:

- {2, 3, 6, 828, 3839, 8827}
- {4, 5, 6, 10, 21}
- {2, 949, 48282, 428859, 119484}
- {111, 8888, 001922, 98373773}

Examples of Sets (Well-defined)

- A set of even numbers between 1 and 15

  A= {2, 4, 6, 8, 10, 12,14}

- B set of multiple of 5 between 8 and 28

  B= {10, 15, 20, 25}

- **Note:** A set may be denoted by a capital letter as shown in above examples.

Your Task☺

- **Which of the following are well-defined sets?**

1. All the colors in the rainbow.
2. All the points that lie on a straight line.
3. All the honest members in the family.
4. All the consonants of the English alphabet.
5. All the tall boys of the school.
6. All the hardworking teachers in a school.
7. All the prime numbers less than 100.
8. All the letters in the word GEOMETRY.

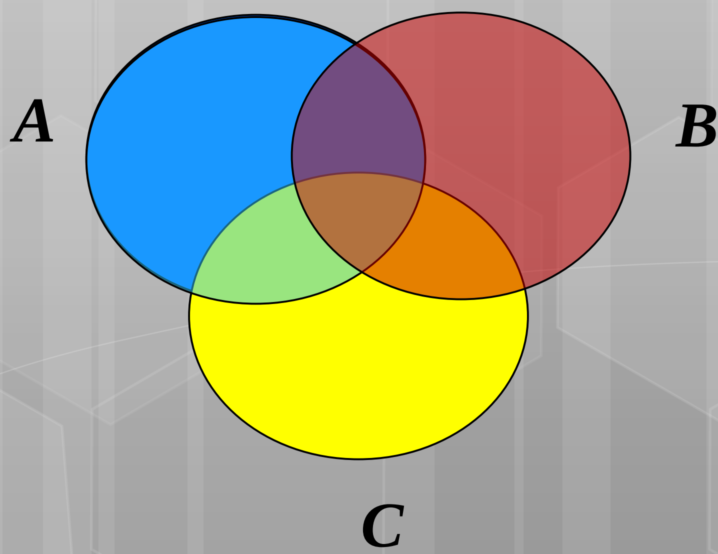
Your Task☺

- **Which of the following are well-defined sets?**

1. All the colors in the rainbow.
2. All the points that lie on a straight line.
3. All the honest members in the family.
4. All the consonants of the English alphabet.
5. All the tall boys of the school.
6. All the hardworking teachers in a school.
7. All the prime numbers less than 100.
8. All the letters in the word GEOMETRY.

Answers: 1, 2, 4, 7, 8 are well-defined.


Thank you!



Set Theory

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Cardinality of Sets

- The cardinality of a set S, denoted $|S|$, is the number of elements in S. If the set has an infinite number of elements, then its cardinality is ∞.
- The cardinality of a set A is denoted by $|A|$.
- 1: If $A = \phi$, then $|A| = 0$.
- 2: If A has exactly n elements, then
- $|A| = n$.
- Note that n is a nonnegative number.
- 3: If A is an infinite set, then $|A| = \infty$.

Size of a Set

In this course we mostly focus on finite sets.

Definition: The **size** of a set S, denoted by **|S|**, is defined as the number of elements contained in S.

e.g. if S = {2, 3, 5, 7, 11, 13, 17, 19}, then |S|=8.

if S = {CSC1130, CSC2110, ERG2020, MAT2510}, then |S|=4.

if S = {{1,2}, {1,3}, {1,4}, {2,3}, {2,4}, {3,4}}, then |S|=6.

Subsets

- A set A is a subset of a set B, if all the elements of A are contained in/members of the larger set B.
- set A is a subset of B if and only if every element of A is also an element of B.
- We use the notation $A \subseteq B$ to indicate that A is a subset of the set B.
- The empty set({ } or ϕ) is a subset of every set.

Subsets

- **Example 1:**
- If, B = {3, 5, 6, 8, 9, 10, 11, 13}
- And, A = {5, 11, 13}
- Then, A is a subset of B.
- A subset of this is {5, 11, 13}. Another subset is {3, 5} or even another is {3}, etc.
- But {1, 6} is not a subset, since it has an element (1) which is not in the parent set.
- That is, $A \subseteq B$ (where $\subseteq$ means 'is a subset of').
- A is a subset of B if and only if every element of A is in B.

Subsets

- **Example 2:**
- If, $F = \{1, 2, 3\}$
- And, $G = \{1, 2\}$
- Then, G is a subset of F.

Not a Subsets Example

- If, A = {1, 2, 3}
- And, B = {1, 2, 3, 4, 5}
- Then, B is not a subset of A.
- That is, B $\not\subseteq$ A (Where $\not\subseteq$ means ‘not is a subset of’)

- **Note:** Every set is a subset of itself. The empty set is a subset of every set.

Your Task☺

- A = {3, 9}, B = {5, 9, 1, 3}, $A \subseteq B$?
- **Answer:** Yes
- A = {3, 3, 3, 9}, B = {5, 9, 1, 3}, $A \subseteq B$?
- **Answer:** Yes
- A = {1, 2, 3}, B = {2, 3, 4}, $A \subseteq B$?
- **Answer:** No

Number of Subsets

- If, $M = \{a, b, c\}$
- Then, the subsets of M are:
- $\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{\}$
- Therefore, the number of subsets, $S = 8$
- And the formula, $S = 2^n$
- Where, S is the number of sets And, n is the number of elements of the set.
- Is the formula used to calculate the number of subsets of a given set.
- So from above, $M = \{a, b, c\}$
- $S = 2^n$, $2^3 = 2\times 2 \times 2 = 8$

Proper Subset

- A is a **proper** subset of B if and only if every element in A is also in B, and there exists **at least one element** in B that is **not** in A.
- **Example 1:**
- A = {1, 2, 3} is a subset of B = {1, 2, 3}, but is not a proper subset of {1, 2, 3}.
- **Example 2:**
- A = {1, 2, 3} is a proper subset of B = {1, 2, 3, 4} because the element 4 is not in the first set.
- Hence $A \subset B$

Subset and Proper Subset

Difference Between Subset and Proper Subset

- If every member of one set is also a member of a second set, then the first set is said to be a subset of the second set. Usually, it turns out that the first set is smaller than the second, but not always. The definition of "subset" allows the possibility that the first set is the same as (equal to) the second set. But a "proper subset" must be smaller than the second set.
- The set {2,3,5,7} is a subset of {2,3,5,7}.
- The set {2,3,5,7} is NOT a proper subset of {2,3,5,7}.
- The set {2,3,5} is a proper subset of {2,3,5,7}.

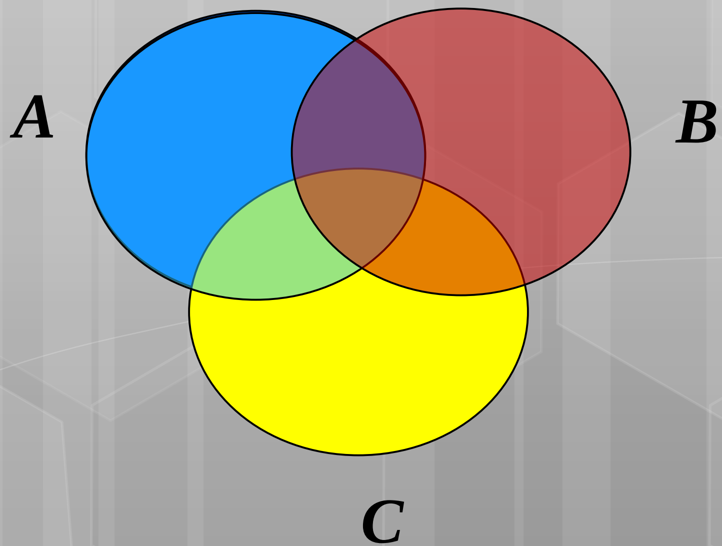
Your Task☺

- **Determine whether each of the following statements is true or false.**
  - $x \in \{x\}$
  - $\{x\} \subseteq \{x\}$
  - $\{x\} \in \{x\}$
  - $\{x\} \in \{\{x\}\}$
  - $\varnothing \subseteq \{x\}$
  - $\varnothing \in \{x\}$

Solution

- **Determine whether each of the following statements is true or false.**
  - $x \in \{x\}$ TRUE
    - (Because x is the member of the singleton set { x })
  - $\{x\} \subseteq \{x\}$ TRUE
    - (Because Every set is the subset of itself. Note that every Set has necessarily two subsets $\varnothing$ and the Set itself, these two subset are known as Improper subsets and any other subset is called Proper Subset)
  - $\{x\} \in \{x\}$ FALSE
    - (Because { x} is not the member of {x}) Similarly other
  - $\{x\} \in \{\{x\}\}$ TRUE
  - $\varnothing \subseteq \{x\}$ TRUE
  - $\varnothing \in \{x\}$ FALSE


Thank you!



Set Theory

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Representation of a Set

- Sets can be represented in two ways :

- **Roster or Tabular Form**
- **Descriptive Form**
- **Set Builder Notation**

Tabular Form

- **Tabular Form:**
  - Listing all the elements of a set, separated by commas and enclosed within braces or curly brackets{}.

- **Examples:**
  - In the following examples we write the sets in Tabular Form.
  - A = {1, 2, 3, 4, 5} is the set of first five **Natural Numbers**.
  - B = {2, 4, 6, 8, …, 50} is the set of **Even numbers** up to 50
  - C = {1, 3, 5, 7, 9, …} is the set of **positive odd numbers.**

Descriptive Form

- **Descriptive Form:**
  - Stating in words the elements of a set.
- **Examples**:
  - Now we will write the same examples which we write in Tabular Form ,in the Descriptive Form.
  - A = set of first five Natural Numbers.(is the Descriptive Form)
  - B = set of positive even integers less or equal to fifty. (is the Descriptive Form)
  - C = {1, 3, 5, 7, 9, ...} (is the Tabular Form)
  - C = set of positive odd integers. (is the Descriptive Form)

Set Builder Form

- **Set Builder Form:**
  - Writing in symbolic form the common characteristics shared by all the elements of the set.

- **Examples:**
  - Now we will write the same examples which we write in Tabular as well as Descriptive Form ,in Set Builder Form .
  - $A = \{x \in N \mid x <= 5\}$ (is the Set Builder Form)
  - $B = \{x \in E \mid 0 < x <= 50\}$ (is the Set Builder Form)
  - $C = \{x \in O \mid 0 < x \}$ (is the Set Builder Form)

Your Task☺

- **Write the following sets in the set builder form.**

- (a) A = {2, 4, 6, 8}
- (b) B = {3, 9, 27, 81}
- (c) C = {1, 4, 9, 16, 25}
- (d) D = {1, 3, 5, ……}
- (e) E = {a, e, i, o, u}

Answers of the Previous Questions

- **Write the following sets in the set builder form.**

- (a) $\{x : x \text{ is even and } x \leq 8$
- (b) $\{x : x = 3n, n \in N, n \leq 4\}$
- (c) $\{x : x = n^2, n \leq 5, n \in N\}$
- (d) $\{x : x \text{ is odd}\}$
- (e) $\{x : x = \text{Vowels in English alphabets}\}$

Your Task☺

- **Write the following sets in the roster form.**

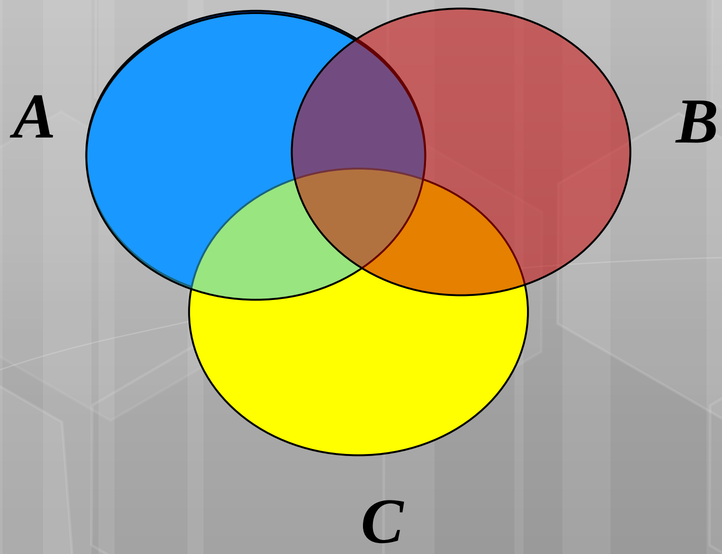
- (a) $A = \{x : x \in W, x \leq 5\}$
- (b) B = {The set all even numbers less than 12)
- (c) $C = \{x : x \text{ is divisible by } 12\}$
- (d) D = {The set of first seven natural numbers}
- (e) E = {The set of whole numbers less than 5}

Answers of the Previous Questions

- **Write the following sets in the roster form.**

- (a) {0, 1, 2, 3, 4, 5}
- (b) {2, 4, 6, 8. 10}
- (c) {12, 24, 36, ...}
- (d) {1, 2, 3, 4, 5, 6, 7}
- (e) {0, 1, 2, 3, 4}


Thank you!



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Finite and infinite Sets

- **Finite Set:** A finite set is one in which it is possible to list and count all the members of the set.
- **Example: 1,** D = {days of week}
- D = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
- So, n(D) = 7 which is countable, so it is finite set.
- **Example: 2,** A = {1, 2, 3, 4, 5}
- The set 'A' has 5 elements and so it is finite set.
- **Example: 3,** F = {-3, -2, -1, 0, 1, 2, 3}
- The set 'F' has countable number of elements so it is also a finite set.

Finite and infinite Sets

- **Infinite Set:** An infinite set is one in which it is not possible to list and count all the members of the set.
- **Example: 1**
- E = {even numbers greater than 9}
- E = {10, 12, 14, 16...}
- Here n(E) = infinite
- **Example: 2**
- G = { whole numbers greater than 2000 }
- G = { 2001, 2002, 2003, 2004, ...}
- Here n(G) = infinite

Your Task☺

- **Classify the following as finite and infinite sets.**

a. A = {x : x ∈ N and x is even}
b. B = {x : x ∈ N and x is composite}
c. C = {x : x ∈ N and $3x - 2 = 0$}
d. D = {x : x ∈ N and $x^2 = 9$ }
e. E = {The set of numbers which are multiple of 3}
f. F = {The set of letters in English alphabets}
g. G = {The set of persons living in a house}
h. H = {x : x ∈ P, P is a number}
i. I = {The set of fractions with numerator 3}

Answer of the Previous Questions

a. Infinite
b. Infinite
c. Finite
d. Finite
e. Infinite
f. Finite
g. Finite
h. Infinite
i. Infinite

Empty Set / Null Set

- An empty set is a set which has no members.
- **Example:**
- If, H = {the number of dinosaurs on earth}
- Then, H is an empty set.
- That is, H = {}
- **Note:** An empty set is denoted by the symbol {} or ϕ.
- **Note** the subtlety in $\phi \neq \{\phi\}$
  - The left-hand side is the empty set
  - The right hand-side is a singleton set, and a set containing a set

Singleton Set or Unit Set

- A singleton is a set that contains exactly one element.
- Sometimes, it is known as unit set.
- The singleton containing only the element a can be written {a}.
- Note that ϕ is empty set and $\{\phi\}$ is not empty set but it is a singleton set.

- Singleton set or unit set contains only
- one element. A singleton set is denoted
- by {s}.
- **Example** : $S = \{x \mid x \in N, 7 < x < 9\}$

Your Task☺

- **Identify the following as null set or singleton set.**

a. $A = \{x : x \in N, 1 < x < 2\}$

b. P = {Point of intersection of two lines}

c. C = {x : x is an even prime number greater than 2}

d. Q = {x : x is an even prime number}

e. $E = \{x : x^2 = 9$, x is even$\}$

f. $B = \{0\}$

g. D = {The set of largest 1 digit number}

h. F = {The set of triangles having 4 sides}

i. H = {The set of even numbers not divisible by 2}

Answers:

a. Null
b. Singleton
c. Null
d. Singleton
e. Null
f. Singleton
g. Singleton
h. Null
i. Null

Equal and Equivalent Sets

- **Equal Set:** Two sets are equal if they both have the same members.
- **Example 1:** if $A = \{1, 2, 3\}$
- And $B = \{1, 2, 3\}$
- Then $A = B$, that is both sets are equal.
- **Example 2:** if $C = \{1, 2, 5\}$
- And $D = \{5, 1, 2\}$
- Then $C = D$, that is both sets are equal.
- **Note:** The order in which the members of a set are written does not matter.

Equal and Equivalent Sets

- **Equivalent Set:** Two sets are equivalent if they have the same number of elements.
- **Example 1:** if F = {2, 4, 6, 8, 10}
- And G = {10, 20, 30, 40, 50}
- Then n(F)=n(G), that is, sets F and G are equivalent.

- **Example 2:** if A = {1, 2, 3}
- And B = {a, b, c}
- Then n(A)=n(B), that is, sets A and B are equivalent.

Equal and Equivalent Sets

- **Note:** When each member of a set matches one and only one member of the other set, there is a 1-1 correspondence between the two sets.
- For Example:

$$\begin{array}{ccc} 1 & 2 & 3 \\ \updownarrow & \updownarrow & \updownarrow \\ a & b & c \end{array}$$

- Sets that cannot be paired in a 1-1 correspondence are called **non-equivalents sets**.

Your Task☺

- **Which of the following pairs of sets are equivalent or equal?**

a. $A = \{x : x \in N, x \leq 6\}$

$B = \{x : x \in W, 1 \leq x \leq 6\}$

b. P = {The set of letters in word "plane"}

Q = {The set of letters in word "plain"}

c. X = {The set of color in the rainbow}

Y = {The set of days in a week}

d. $M = \{4, 8, 12, 16\}$

$N = \{8, 12, 4, 16\}$

Answers:

a. Equal Sets
b. Equivalent Sets
c. Equivalent Sets
d. Equal Sets

One More Task☺

- **Is this is equal set?**
- {2,3,5,7} , {2,2,3,5,3,7}

✓ Equal

- And-----------------:
- {2,3,5,7} , {2,3}

✓ Not Equal

Note that:

- Equal sets are always equivalent.
- Equivalent sets may not be equal

Examples

- Let A be the set of odd positive integers less than 10. Then $|A|=5$.
- Let S be the set of letters in the English alphabet. Then $|S|=26$.
- Let P be the set of infinite numbers.

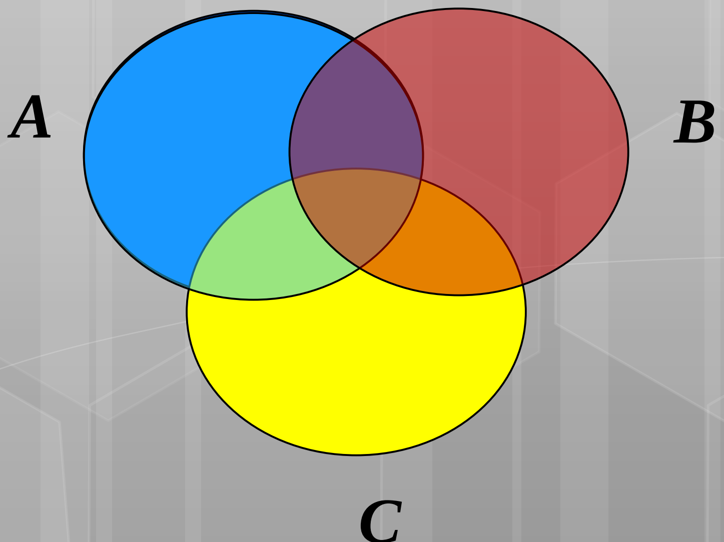
  Then $|P|=\infty$.

  - The cardinality of the empty set is $|\varnothing|=0$
  - The sets $\mathcal{N}, \mathcal{Z}, \mathcal{Q}, \mathcal{R}$ are all infinite

Cardinality of Sets

- A = {Mercedes, BMW, Porsche}, $|A| = 3$
- B = { x: x is an odd number divisible by 2}, $|B| = 0$
- $C = \{1, \{2, 3\}, \{4, 5\}, 6\}$ $|C| = 4$
- D = {x: x is a counting number <10}, $|D| = 9$
- $E = \emptyset$ $|E| = 0$
- F = {Letter in the words BANANA}, $|F| = 3$
- $G = \{ x \in \mathbf{N} \mid x \leq 7000 \}$, $|G| = 7000$


Thank you!



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Power Sets

- A Power Set is a set of all the subsets of a set.
- The power set of S is denoted by P(S).
- **Notation:**
- The number of members of a set is often written as |S|, so we can write:

$$|P(S)| = 2^n$$

Power Sets

- **A= { a, b, }**

- The power set of A is $2^4 = 16$
- P(A)={}, {a}, {b}, {a, b}, {c}, {a, c}, {b, c}, {a, b, c}, {d}, {a, d}, {b, d}, {a, b, d}, {c, d}, {a, c, d}, {b, c, d}, {a, b, c, d}.

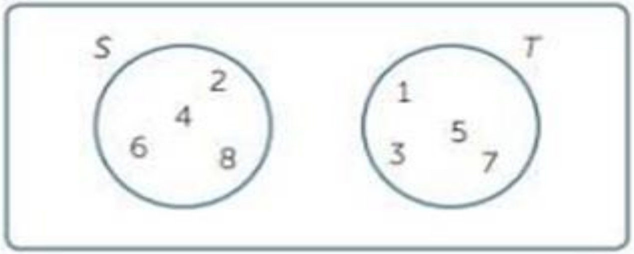
- **B={1, 2, 3}**

- The power set of B is $2^3 = 8$
- P(B)={}, {1}, {2}, {1, 2}, {3}, {1, 3}, {2, 3}, {1, 2, 3}

Disjoint Sets

- **Disjoint sets:**
- Two sets are called disjoint if they have no elements in common.

- **For Example:**
- The sets S = {2, 4, 6, 8} and T = {1, 3, 5, 7} are disjoint.



Disjoint Sets

- Another way to define disjoint sets is to say that their intersection is the empty set,
- Two sets A and B are disjoint if $A \cap B = \{\ \}$.
- In the example above,
- $S \cap T = \{\ \}$ because no number lies in both sets.
- The overlapping region of two circles represents the intersection of the two sets.
- When two sets are disjoint, we can draw the two circles without any overlap.

Your Task ☺

- **Which of the following sets are disjoint or overlapping:**

a. A = {The set of boys in the school}
   B = {The set of girls in the school}

b. P = {The set of letters in the English alphabets}
   Q = {The set of vowels in the English alphabets}

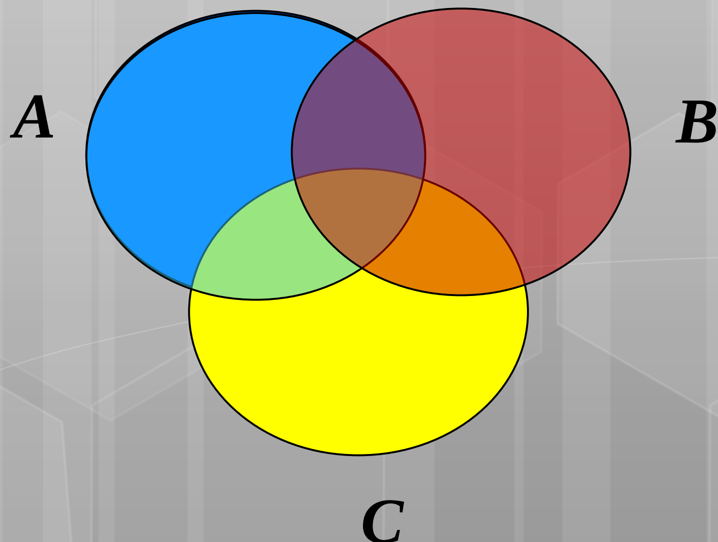
c. $X = \{x : x \text{ is an odd number}, x < 9\}$
   $Y = \{x : x \text{ is an even number}, x < 10\}$

d. $E = \{9, 99, 999\}$
   $F = \{1, 10, 100\}$

Answers:

a. Disjoint Sets
b. Overlapping Sets
c. Disjoint Sets
d. Disjoint Sets


Thank you!



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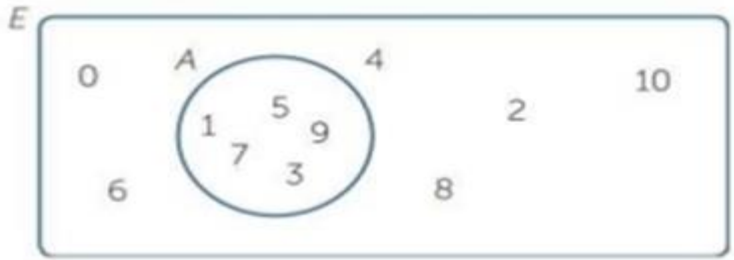
Venn Diagrams

- Diagrams make mathematics easier because they help us to see the whole situation at a glance. The English mathematician John Venn (1834–1923) began using diagrams to represent sets. His diagrams are now called Venn diagrams.
- In most problems involving sets, it is convenient to choose a larger set that contains all of the elements in all of the sets being considered. This larger set is called the universal set, and is usually given the symbol E. In a Venn diagram, the universal set is generally drawn as a large rectangle, and then other sets are represented by circles within this rectangle.

Venn Diagrams

- In the Venn diagram below, the universal set is $E = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$, and each of these numbers has been placed somewhere within the rectangle.



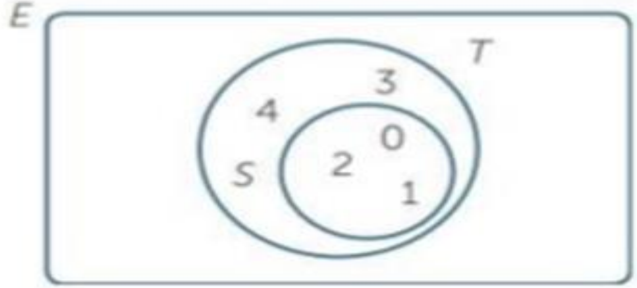
- The region inside the circle represents the set A of odd whole numbers between 0 and 10. Thus we place the numbers 1, 3, 5, 7 and 9 inside the circle, because $A = \{ 1, 3, 5, 7, 9 \}$.Outside the circle we place the other numbers 0, 2, 4, 6, 8 and 10 that are in E but not in A

Venn Diagrams

- For example, if V = { vowels }, we could choose the universal set as E = { letters of the alphabet } and all the letters of the alphabet would then need to be placed somewhere within the rectangle, as shown below.

Representing Subsets on a Venn diagram

- When we know that *S* is a subset of *T*, we place the circle representing *S* inside the circle representing *T*. For example, let $S = \{ 0, 1, 2 \}$, and $T = \{ 0, 1, 2, 3, 4 \}$. Then *S* is a subset of *T*, as illustrated in the Venn diagram below.



- Make sure that 5, 6, 7, 8, 9 and 10 are placed outside both circles

Cartesian Product

Definition: The Cartesian Product of two sets A and B, denoted by A $\times$ B is the set of ordered pairs (a,b) where a $\in$ A and b $\in$ B .

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Example:

$A = \{a,b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

Definition: A subset R of the Cartesian product $A \times B$ is called a *relation* from the set A to the set B.

Cartesian Product

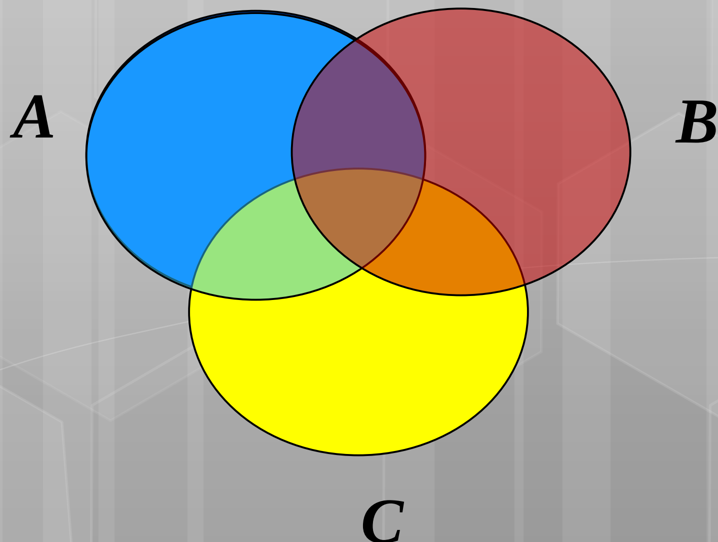
Definition: The Cartesian products of the sets $A_1, A_2, \ldots\ldots, A_n$, denoted by $A_1 \times A_2 \times \ldots\ldots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots\ldots, a_n)$ where a_i belongs to A_i for $i = 1, \ldots n$.

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } i = 1, 2, \ldots n\}$$

Example: What is $A \times B \times C$ where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

Solution: $A \times B \times C$ **=** {(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)}


Thank you!



Set Theory

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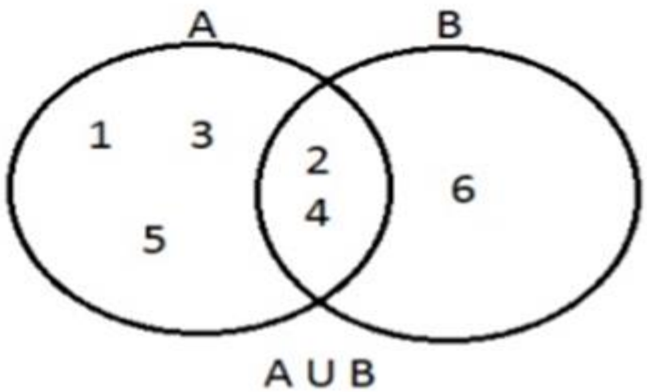
Union of Sets

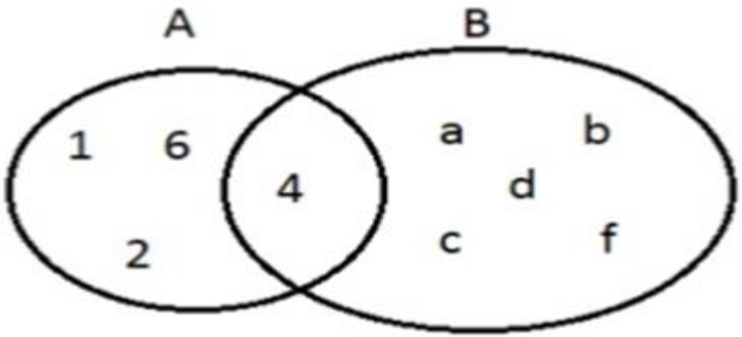
- Combining all the elements of any two sets is called the Union of those sets.
- Union of two sets A and B is obtained by combining all the members of the sets and is represented as A U B
- **Examples of Union of Sets**
- If A = {1, 2, 3, 4, 5} and
- B = {2, 4, 6},
- Then the union of these sets is A U B = {1, 2, 3, 4, 5, 6}

Union of Sets



Examples of Union of Sets

- A = {1, 2, 4, 6} and B = {4, a, b, c, d, f}
- Then the union of these sets is A U B = {1, 2, 4, 6, a, b, c, d, f}



Union of Sets

- **Examples of Union of Sets**
- A = {x / x is a number bigger than 4 and smaller than 8}
- B = {x / x is a positive number smaller than 7}

- A = {5, 6, 7} and B = {1, 2, 3, 4, 5, 6}
- A U B = {1, 2, 3, 4, 5, 6, 7}
- Or A U B = {x / x is a number bigger than 0 and smaller than 8}

Union of Sets

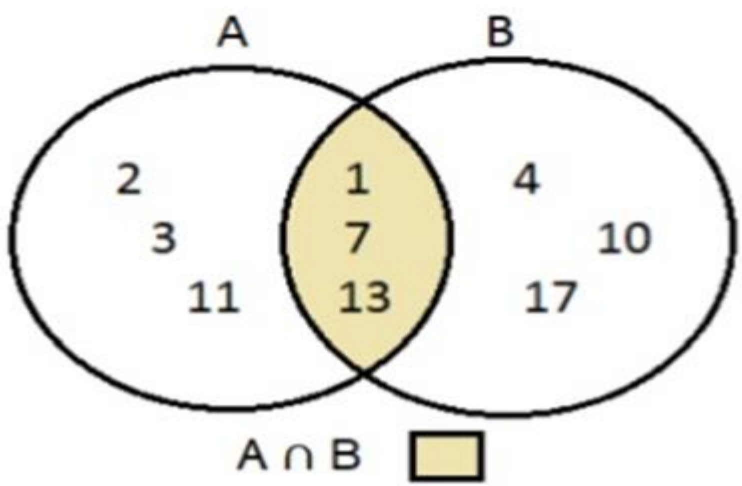
- **Examples of Union of Sets**
- A = {#, %, $}
- B = { }
- Then, A ∪ B = {#, %, $}

- **Examples of Union of Sets**
- N = {-5, -4, 0, 6, 8} and O = {-4, 0, 8, 9}
- Then, N ∪ O = {-5, -4, 0, 6, 8, 9}

Intersection of Sets

- Intersection of Sets is defined as the grouping up of the common elements of two or more sets.
- It is denoted by the symbol ∩
- **Example of Intersection of Sets**
- When Set A = {1, 2, 3, 7, 11, 13} and Set B = {1, 4, 7, 10, 13, 17},
- A ∩ B is all the common elements of the set A and B.
- Therefore, A ∩ B = {1, 7, 13}.
- This can be shown by using Venn diagram as:

Intersection of Sets

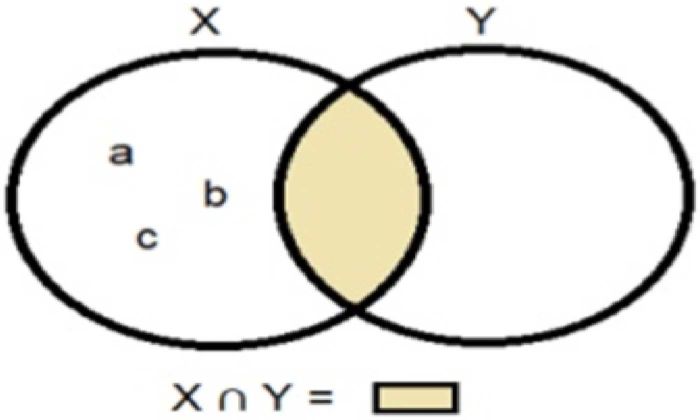


Intersection of Sets

- **Example of Intersection of Sets**
- If A = {a, b, c, d} and B = {1, a, 2, b}.
- A n B is all the common elements of the set A and B.
- Therefore, $A \cap B = \{a, b\}$.

More Example

- If X = {a, b, c} and Y = {ϕ}. Find intersection of two given sets X and Y.
- **Solution:**
- $X \cap Y = \{\ \}$



$X \cap Y = \{\ \}$ or ϕ (an empty set). Non empty sets which have no members in common are called "**disjoint sets**".

Your Task☺

- If set A = {4, 6, 8, 10, 12}, set B = {3, 6, 9, 12, 15, 18} and set C = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

- (i) Find the intersection of sets A and B.
- (ii) Find the intersection of two set B and C.
- (iii) Find the intersection of the given sets A and C.

Solution

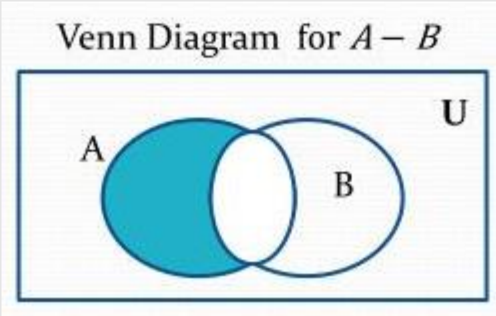
- (i) Intersection of sets A and B is $A \cap B$
- Set of all the elements which are common to both set A and set B is {6, 12}.

- (ii) Intersection of two set B and C is $B \cap C$
- Set of all the elements which are common to both set B and set C is {3, 6, 9}.

- (iii) Intersection of the given sets A and C is $A \cap C$
- Set of all the elements which are common to both set A and set C is {4, 6, 8, 10}.

Difference of Sets

- The difference set of any two sets A and B. is the set of the members of set A which is not the members of set B.
- **Example of Difference of Sets**
- A = {0, 1, 2, 3}
- B = {2, 3}
- The difference set is {0, 1}.



- We can write it as A – B or A \ B. We say: 'A difference B'.
- B – A or B \ A = { }

Your Task☺

- A = {1, 2, 3} and B = {4, 5, 6}.

- Find the difference between the two sets:
- (i) A and B
- (ii) B and A

Solution

- The two sets are disjoint as they do not have any elements in common.
- (i) $A - B = \{1, 2, 3\} = A$
- (ii) $B - A = \{4, 5, 6\} = B$

More Task☺

- **Given three sets P, Q and R such that:**
- P = {x : x is a natural number between 10 and 16},
- Q = {y : y is a even number between 8 and 20} and
- R = {7, 9, 11, 14, 18, 20}
-
- (i) Find the difference of two sets P and Q
- (ii) Find Q - R
- (iii) Find R - P
- (iv) Find Q – P

Solution

- According to the given statements:
- P = {11, 12, 13, 14, 15}
- Q = {10, 12, 14, 16, 18}
- R = {7, 9, 11, 14, 18, 20}
- (i) P – Q = {Those elements of set P which are not in set Q}
  = {11, 13, 15}
- (ii) Q – R = {Those elements of set Q not belonging to set R}
  = {10, 12, 16}
- (iii) R – P = {Those elements of set R which are not in set P}
  = {7, 9, 18, 20}
- (iv) Q – P = {Those elements of set Q not belonging to set P}
  = {10, 16, 18}

Universal Set

- The universal set is the set of all elements that are considered in a specific theory. We'll note the universal set with U.
- We'll choose as universal set: U = {6,7,8,9,15,16,17,18,20,21}.
- We have to determine the sets:
- M = {x / x are the multiple of 3}
- N = {x / x are the multiple of 5}
- The elements of M and N have to be chosen from the universal set U.
- To determine M, we'll identify the multiples of 3 from U: {6,9,15,18,21}
- M = {6,9,15,18,21}
- To determine N, we'll identify the multiples of 5 from U: {15,20}.
- N = {15,20}

Complement of a Set

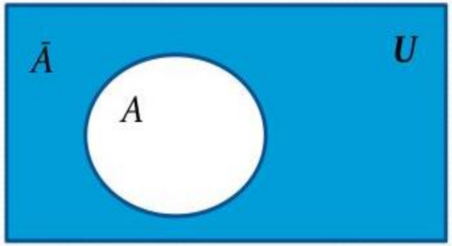
- If a set A is a subset of a given Universal Se U, Then the difference U – A or U \ A id the complement of A.
- We write U – A or U \ A = A′. We say A complement.
- **Example:**
- If, U = {3, 5, 7, 9, 11, 13, 15, 17, 19}
- And, A = {5, 11, 17, 19}
- Then, U – A = A′ = {3, 7, 9, 13, 15}

Complement of a Set

- Where, A′ is "the complement of A".
- The union of A and A′ is the Universal set.
- $U = A \cup A' = \{5, 11, 17, 19\} \cup \{3, 7, 9, 13, 15\}$
- $U = A \cup A' = \{3, 5, 7, 9, 11, 13, 15, 17, 19\}$

- The intersection of A and A′ is an empty set.
- $A \cap A' = \{\ \}$ or ϕ

Venn Diagram for Complement

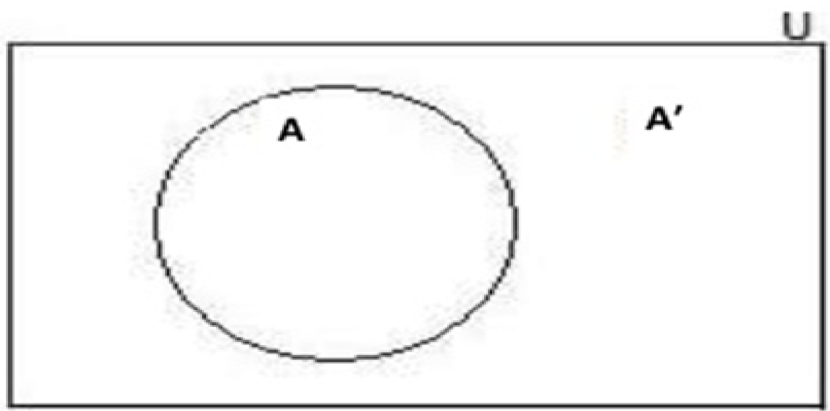


Your Task ☺

- **Find the complement of A in U**
- A = { x / x is a number bigger than 4 and smaller than 8}
- U = { x / x is a positive number smaller than 7}
- A = { 5, 6, 7} and U = { 1, 2, 3, 4, 5, 6}
- A′ = { 1, 2, 3, 4}
- Or A′ = { x / x is a number bigger than 1 and smaller than 5 }

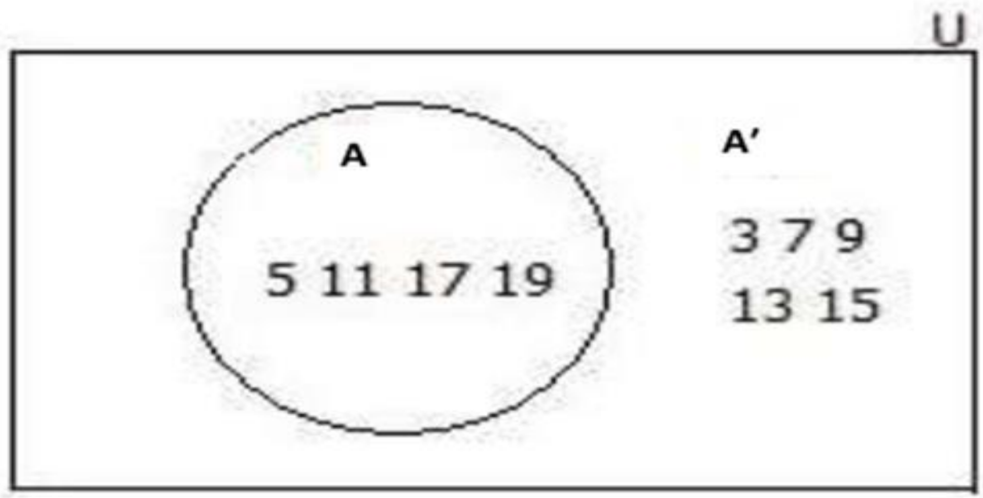
Example- Using Venn diagram

- Sets are represented by a drawing called a Venn diagram, in which a rectangle is used to represent a universal set, U, and circles inside the rectangle, used to represent subsets.



Example- Using Venn diagram

- Using the previous above, below is a Venn diagram showing A'.



Note Some Points:

- The Complement of a universal set is an empty set.
- **For Example:**

$$U = \{1, 2, 3, 4\}$$

$$U' = \phi$$

- The Complement of an empty set is a universal set.
- **For Example:**

$$U = \{1, 2, 3, 4\}$$

$$A = \{\ \}$$

Then $A' = \{1, 2, 3, 4\}$ or we simply say U.

Note Some Points:

- The set and its complement are disjoint sets.
- **For Example:**

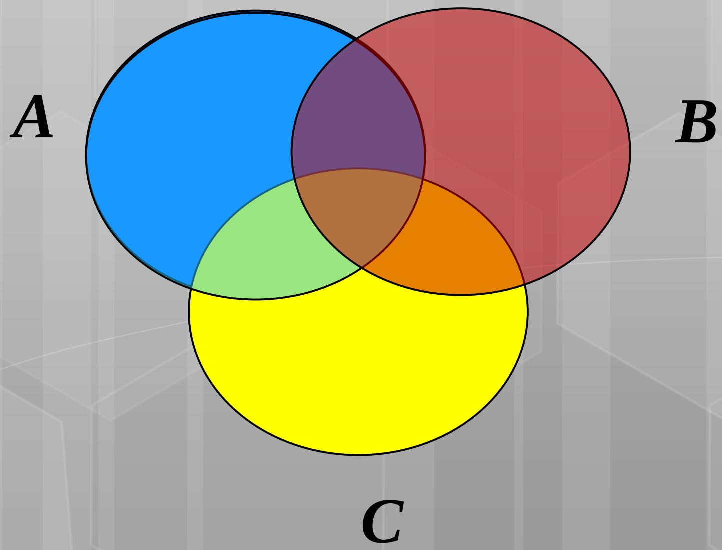
$$U = \{1, 2, 3, 4\}$$
$$A = \{1, 2\}$$
$$A' = \{3, 4\}$$

- And $A \cap A' = \{\ \}$ or ϕ

Set identities

$A \cup \varnothing = A$ $A \cap U = A$	Identity Law	$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination law
$A \cup A = A$ $A \cap A = A$	Idempotent Law	$(A^c)^c = A$	Complementation Law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Law	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$	De Morgan's Law
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Law	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive Law
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Law	$A \cup A^c = U$ $A \cap A^c = \varnothing$	Complement Law


Thank you!



Set Theory

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Multi-set

Definition: These are unordered collection of elements where an element can occur as a member more than once. The notation $\{m_1.a_1\ ,\ m_2.a_2\ \ldots\ldots\ m_r.a_r\ \}$ denotes the multi-set with element a_1 occurring m_1 times, element a_2 occurring m_2 times and so on. The numbers m_i , *i = {1, 2,.....r} are called multiplicities of the element* a_i , *i = {1, 2,.....r}*

Multi-set of prime factors of a number n

$$120 = 2^3 3^1 5^1$$

which gives the multiset {2, 2, 2, 3, 5}.

Operations on Multi-set

1. Union:

For example, if $A = \{2, 3, 4, 4\}$, $B = \{1, 4, 3, 3\}$
then $A \cup B = \{1, 2, 3, 3, 4, 4\}$.

2. Intersection:

For example, if $A = \{3, 3, 3, 4, 4\}$, $B = \{1, 4, 3, 3\}$
then $A \cap B = \{3, 3, 4\}$.

Operations on Multi-set

3. Addition/Sum/Merge:

For example, if A = {1, 1, 2, 2, 4, 4, 4}, B = {1, 2, 3, 3}
then A + B = {1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4}.

4. Difference:

For example, if A = {3, 3, 3, 4, 4}, B = {1, 4, 3, 3}
then A - B = {3, 4}.

Exercise: Suppose that A is the multiset that has as its elements the types of computer equipment needed by one department of a university and the multiplicities are the number of pieces of each type needed, and B is the analogous multiset for a second department of the university. For instance, A could be the multiset {107 · personal computers, 44 · routers, 6 · servers} and B could be the multiset {14 · personal computers, 6 · routers, 2 · mainframes}.

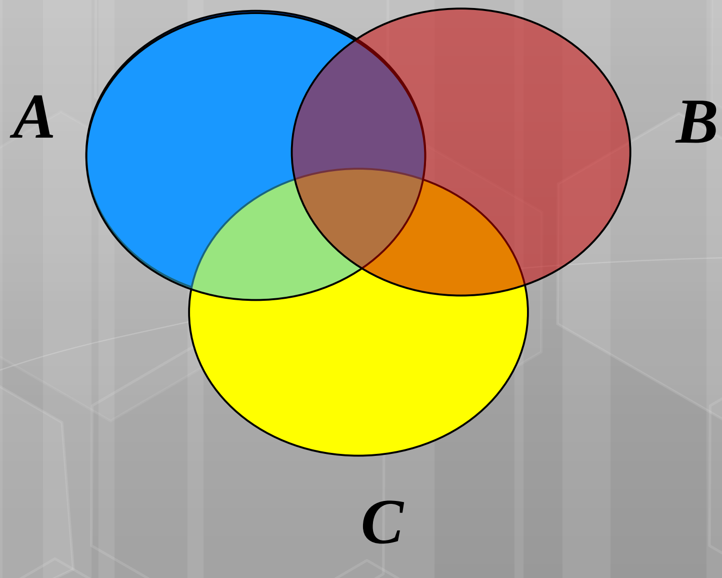
a) What combination of A and B represents the equipment the university should buy assuming both departments use the same equipment?

b) What combination of A and B represents the equipment that will be used by both departments if both departments use the same equipment?

c) What combination of A and B represents the equipment that the second department uses, but the first department does not, if both departments use the same equipment?

d) What combination of A and B represents the equipment that the university should purchase if the departments do not share equipment?


Thank you!



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Fuzzy Set Operation

Given X to be the universe of discourse and Ã and Ḃ to be fuzzy sets with $\mu_A(x)$ and $\mu_B(x)$ are their respective membership function, the fuzzy set operations are as follows:

Union:

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$$

Intersection:

$$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

Complement:

$$\mu_A(x) = 1 - \mu_A(x)$$

Fuzzy Set Operation (Continue)

Example:

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$ $B = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$

Union:

$$A \cup B = \{(x_1,0.8),(x_2,0.7),(x_3,1)\}$$

Because

$$\begin{aligned}\mu_{A \cup B}(x_1) &= \max(\mu_A(x_1), \mu_B(x_1)) \\ &= \max(0.5,0.8) \\ &= 0.8\end{aligned}$$

$$\mu_{A \cup B}(x_2) = 0.7 \text{ and } \mu_{A \cup B}(x_3) = 1$$

Fuzzy Set Operation (Continue)

Example:

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$ $B = \{(x_1,0.8),(x_2,0.2),(x_3,1)\}$

Intersection:

$$A \cap B = \{(x_1,0.5),(x_2,0.2),(x_3,0)\}$$

Because

$$\mu_{A\cap B}(x_1) = \min\ (\mu_A(x_1),\ \mu_B(x_1))$$
$$= \max(0.5,0.8)$$
$$= 0.5$$

$$\mu_{A\cap B}(x_2) = 0.2 \quad \text{and}\ \mu_{A\cap B}(x_3) = 0$$

Fuzzy Set Operation (Continue)

Example:

$A = \{(x_1,0.5),(x_2,0.7),(x_3,0)\}$

Complement:

$$A^c = \{(x_1,0.5),(x_2,0.3),(x_3,1)\}$$

Because

$$\mu_A(x_1) = 1 - \mu_A(x_1)$$
$$= 1 - 0.5$$
$$= 0.5$$

$$\mu_A(x_2) = 0.3 \quad \text{and} \quad \mu_A(x_3) = 1$$

Consider the fuzzy set A

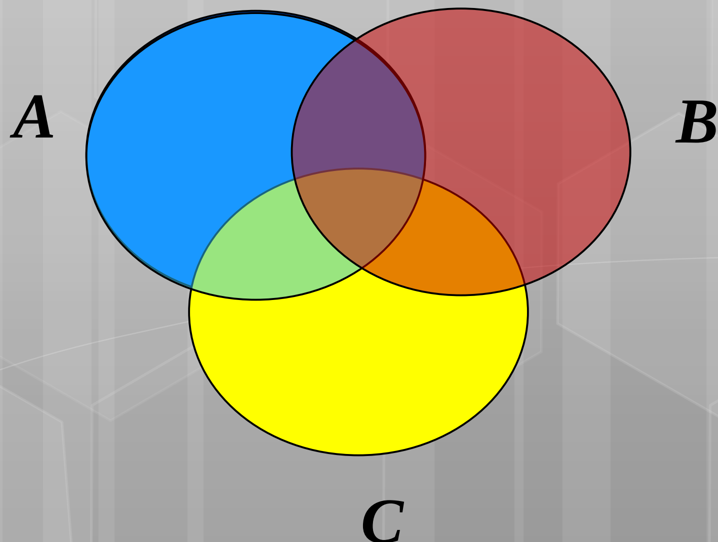
$A = \{(1, 0), (2, 0), (3, 0.2), (4, 0.5), (5, 0.8), (6, 1)\}$

Ans:

$\text{Card}(A) = |A| = 0 + 0 + 0.2 + 0.5 + 0.8 + 1 = 2.5$

$\text{Relcard}(A) = ||A|| = \dfrac{2.5}{6} \approx 0.417$


Thank you!



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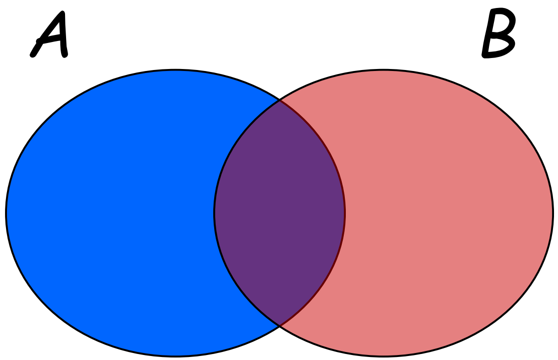
Inclusion-Exclusion (2 sets)

For two arbitrary sets *A* and *B*

$$|A \cup B| = |A| + |B| - |A \cap B|$$



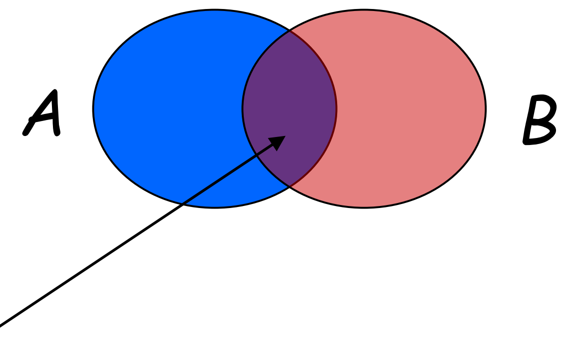
Inclusion-Exclusion (2 sets)

Let S be the set of integers from 1 through 1000 that are multiples of 3 or multiples of 5.

Let A be the set of integers from 1 to 1000 that are multiples of 3.

Let B be the set of integers from 1 to 1000 that are multiples of 5.

It is clear that S is the union of A and B, but notice that A and B are not disjoint.



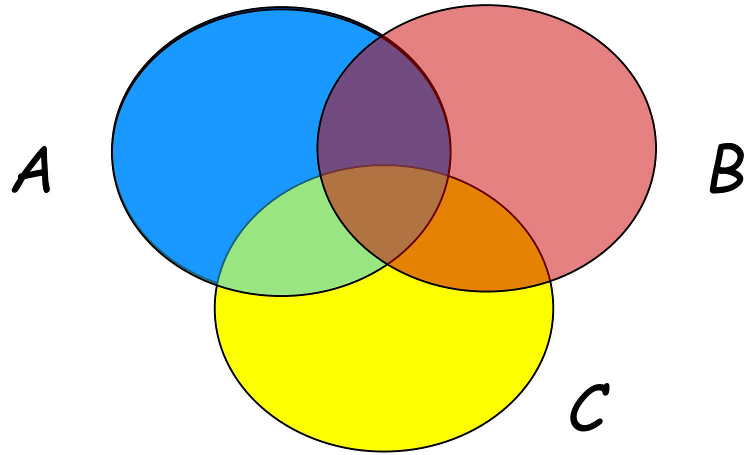
$|A| = 1000/3 = 333$ $|B| = 1000/5 = 200$

$A \cap B$ is the set of integers that are multiples of 15, and so $|A \cap B| = 1000/15 = 66$

So, by the inclusion-exclusion principle, we have $|S| = |A| + |B| - |A \cap B| = 467$.

Inclusion-Exclusion (3 sets)

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$



Inclusion-Exclusion (3 sets)

From a total of 50 students:

$|A|$ → 30 know Java

$|B|$ → 18 know C++

$|C|$ → 26 know C#

$|A \cap B|$ → 9 know both Java and C++

$|A \cap C|$ → 16 know both Java and C#

$|B \cap C|$ → 8 know both C++ and C#

$|A \cup B \cup C|$ → 47 know at least one language.

How many know none?

How many know all? → $|A \cap B \cap C|$

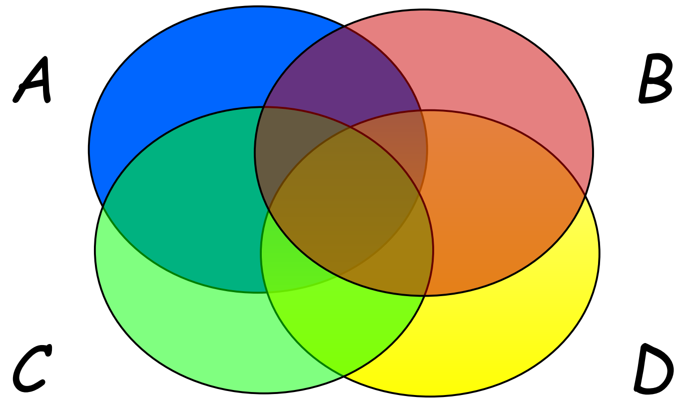
$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$47 = 30 + 18 + 26 - 9 - 16 - 8 + |A \cap B \cap C|$$

$$|A \cap B \cap C| = 6$$

Inclusion-Exclusion (4 sets)

$$
\begin{aligned}
|A \cup B \cup C \cup D| = {} & |A| + |B| + |C| + |D| \\
& - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| \\
& + |A \cap B \cap C| + |A \cap B \cap D| + |A \cap C \cap D| + |B \cap C \cap D| \\
& - |A \cap B \cap C \cap D|
\end{aligned}
$$

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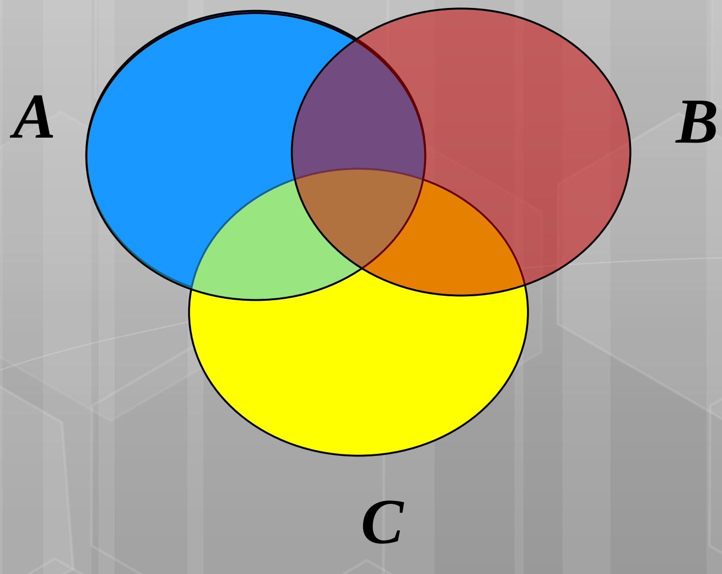

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Partitioning of a Set

Partition of a set, say *S*, is a collection of *n* disjoint subsets, say $P_1, P_2, \ldots P_n$ that satisfies the following three conditions −

- P_i does not contain the empty set.

$$[P_i \neq \{\emptyset\}\ for\ all\ 0 < i \leq n]$$

- The union of the subsets must equal the entire original set.

$$[P_1 \cup P_2 \cup \cdots \cup P_n = S]$$

- The intersection of any two distinct sets is empty.

$$[P_a \cap P_b = \{\emptyset\},\ for\ a \neq b\ where\ n \geq a,\ b \geq 0]$$

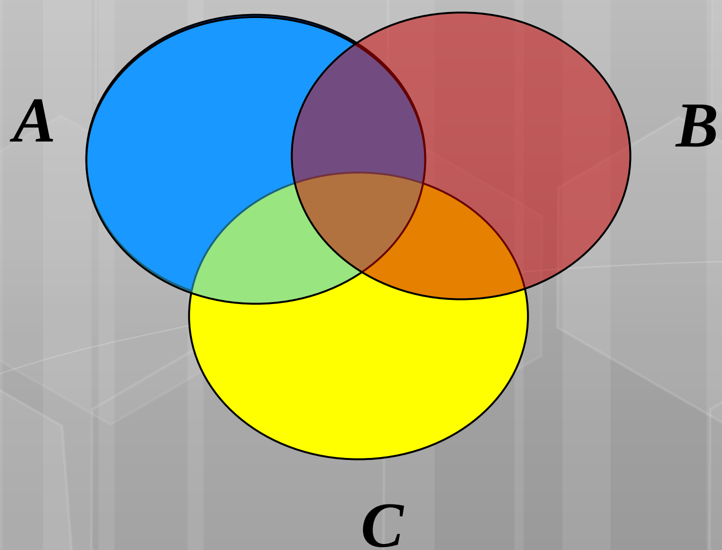
Covering on Set A

It is defined as a set on non-empty subsets Ai, whose union leads to the original set A and which are need not be pairwise disjoint. Here are the two conditions that are to be satisfied:

$$\bigcup_{i \in n} A_i = A$$

$$A_i \cap A_j \neq \phi \; for\ each \, (i, j) \in n \, ; i \neq j$$


Thank you!



Set Theory

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Set identities

$A \cup \varnothing = A$ $A \cap U = A$	Identity Law	$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination law
$A \cup A = A$ $A \cap A = A$	Idempotent Law	$(A^c)^c = A$	Complementation Law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Law	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$	De Morgan's Law
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Law	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive Law
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Law	$A \cup A^c = U$ $A \cap A^c = \varnothing$	Complement Law

Proving Set Identities

Different ways to prove set identities:

1. Prove that each set (side of the identity) is a subset of the other.
2. Use set builder notation and propositional logic.
3. Membership Tables: Verify that elements in the same combination of sets always either belong or do not belong to the same side of the identity. Use 1 to indicate it is in the set and a 0 to indicate that it is not.

Proof of Second De Morgan Law

Example: Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Solution: We prove this identity by showing that:

1) $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ and

2) $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

Proof of Second De Morgan Law

These steps show that: $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$

$x \in \overline{A \cap B}$	by assumption
$x \notin A \cap B$	defn. of complement
$\neg((x \in A) \wedge (x \in B))$	defn. of intersection
$\neg(x \in A) \vee \neg(x \in B)$	1st De Morgan Law for Prop Logic
$x \notin A \vee x \notin B$	defn. of negation
$x \in \overline{A} \vee x \in \overline{B}$	defn. of complement
$x \in \overline{A} \cup \overline{B}$	defn. of union

Proof of Second De Morgan Law

These steps show that: $\overline{A} \cup \overline{B} \subseteq \overline{A \cap B}$

$$
\begin{array}{ll}
x \in \overline{A} \cup \overline{B} & \text{by assumption} \\
(x \in \overline{A}) \vee (x \in \overline{B}) & \text{defn. of union} \\
(x \notin A) \vee (x \notin B) & \text{defn. of complement} \\
\neg(x \in A) \vee \neg(x \in B) & \text{defn. of negation} \\
\neg((x \in A) \wedge (x \in B)) & \text{by 1st De Morgan Law for Prop Logic} \\
\neg(x \in A \cap B) & \text{defn. of intersection} \\
x \in \overline{A \cap B} & \text{defn. of complement}
\end{array}
$$

Set-Builder Notation: Second De Morgan Law

$$
\begin{array}{rcll}
\overline{A \cap B} & = & \{x | x \notin A \cap B\} & \text{by defn. of complement} \\
 & = & \{x | \neg(x \in (A \cap B))\} & \text{by defn. of does not belong symbol} \\
 & = & \{x | \neg(x \in A \wedge x \in B\} & \text{by defn. of intersection} \\
 & = & \{x | \neg(x \in A) \vee \neg(x \in B)\} & \text{by 1st De Morgan law} \\
 & & & \text{for Prop Logic} \\
 & = & \{x | x \notin A \vee x \notin B\} & \text{by defn. of not belong symbol} \\
 & = & \{x | x \in \overline{A} \vee x \in \overline{B}\} & \text{by defn. of complement} \\
 & = & \{x | x \in \overline{A} \cup \overline{B}\} & \text{by defn. of union} \\
 & = & \overline{A} \cup \overline{B} & \text{by meaning of notation}
\end{array}
$$

Membership Table

Example: Construct a membership table to show that the distributive law holds.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Solution:

A	B	C	$B \cap C$	$A \cup (B \cap C)$	$A \cup B$	$A \cup C$	$(A \cup B) \cap (A \cup C)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0


Thank you!