Huffman Codes

Dr. Manisha Chahal

Introduction

- Huffman Coding is a famous Greedy Algorithm.
- It is used for the lossless compression of data.
- It uses variable length encoding.
- It assigns variable length code to all the characters.
- The code length of a character depends on how frequently it occurs in the given text.
- The character which occurs most frequently gets the smallest code.
- The character which occurs least frequently gets the largest code.
- It is also known as **Huffman Encoding**.

Prefix Rule-

- Huffman Coding implements a rule known as a prefix rule.
- This is to prevent the ambiguities while decoding.
- It ensures that the code assigned to any character is not a prefix of the code assigned to any other character.

Major Steps in Huffman Coding-

- There are two major steps in Huffman Coding-
- Building a Huffman Tree from the input characters.
- Assigning code to the characters by traversing the Huffman Tree.

Huffman Tree

The steps involved in the construction of Huffman Tree are as follows-

Step-01:

- Create a leaf node for each character of the text.
- Leaf node of a character contains the occurring frequency of that character.

Step-02:

- Arrange all the nodes in increasing order of their frequency value.

Step-03:

- Considering the first two nodes having minimum frequency,
- Create a new internal node.
- The frequency of this new node is the sum of frequency of those two nodes.
- Make the first node as a left child and the other node as a right child of the newly created node.

Step-04:

- Keep repeating Step-02 and Step-03 until all the nodes form a single tree.
- The tree finally obtained is the desired Huffman Tree.

Time Complexity

- The time complexity analysis of Huffman Coding is as follows-
  - extractMin( ) is called 2 x (n-1) times if there are n nodes.
  - As extractMin( ) calls minHeapify( ), it takes O(logn) time.

- Thus, Overall time complexity of Huffman Coding becomes **O(nlogn)**.
- Here, n is the number of unique characters in the given text.

Important Formulas

- The following 2 formulas are important to solve the problems based on Huffman Coding-

$$\text{Average code length per character} = \frac{\Sigma\,(\text{frequency}_i \times \text{code length}_i)}{\Sigma\,\text{frequency}_i}$$

$$= \Sigma\,(\text{probability}_i \times \text{code length}_i)$$

Total number of bits in Huffman encoded message

= Total number of characters in the message x Average code length per character

$= \sum\,(\text{frequency}_i \times \text{Code length}_i)$

PRACTICE PROBLEM BASED ON HUFFMAN CODING-

A file contains the following characters with the frequencies as shown. If Huffman Coding is used for data compression, determine-

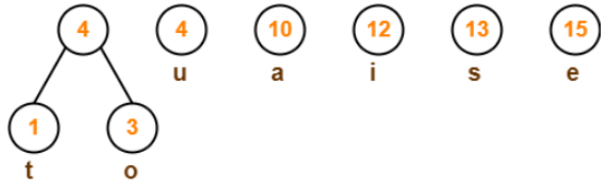
1. Huffman Code for each character
2. Average code length
3. Length of Huffman encoded message (in bits)

Characters	Frequencies
a	10
e	15
i	12
o	3
u	4
s	13
t	1

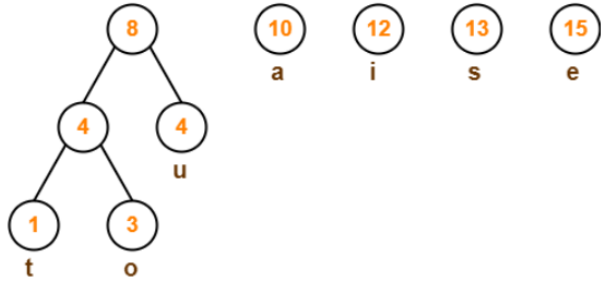
Solution

Step-01:

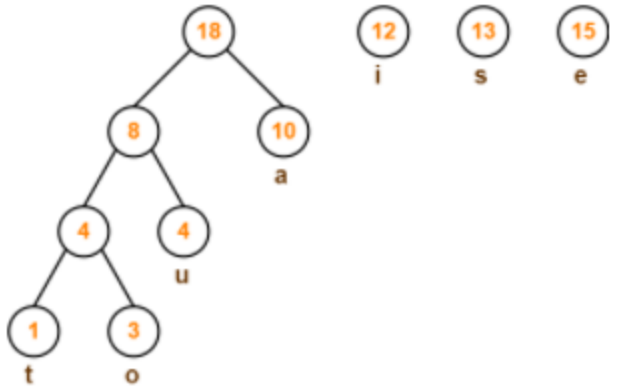
Step-02:

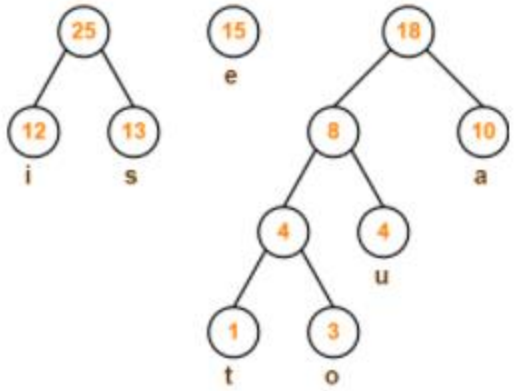


Step-03:

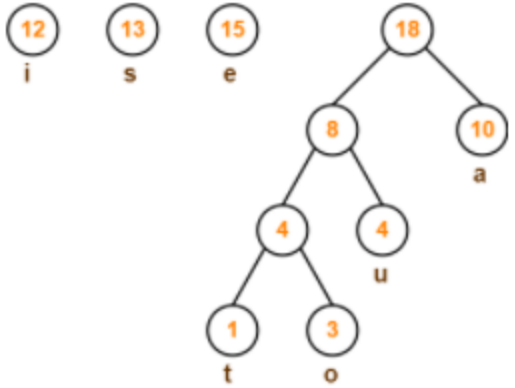


Step-04:

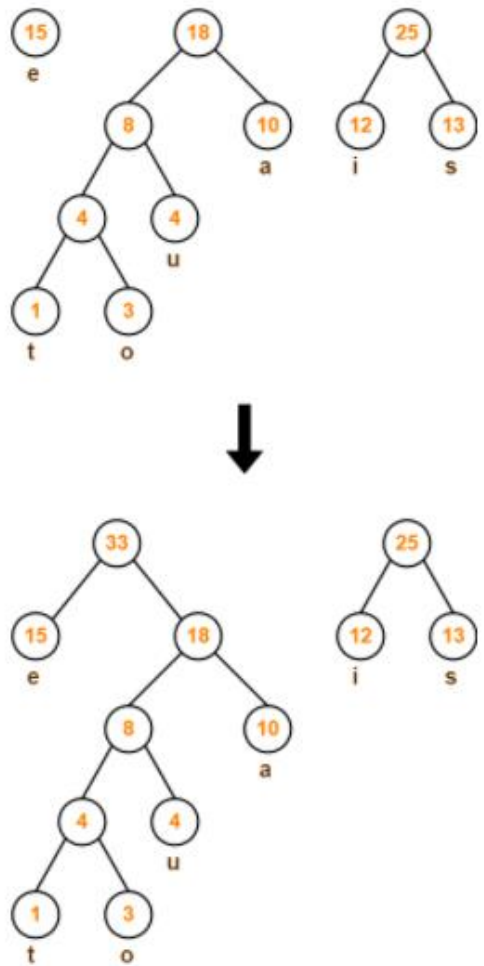




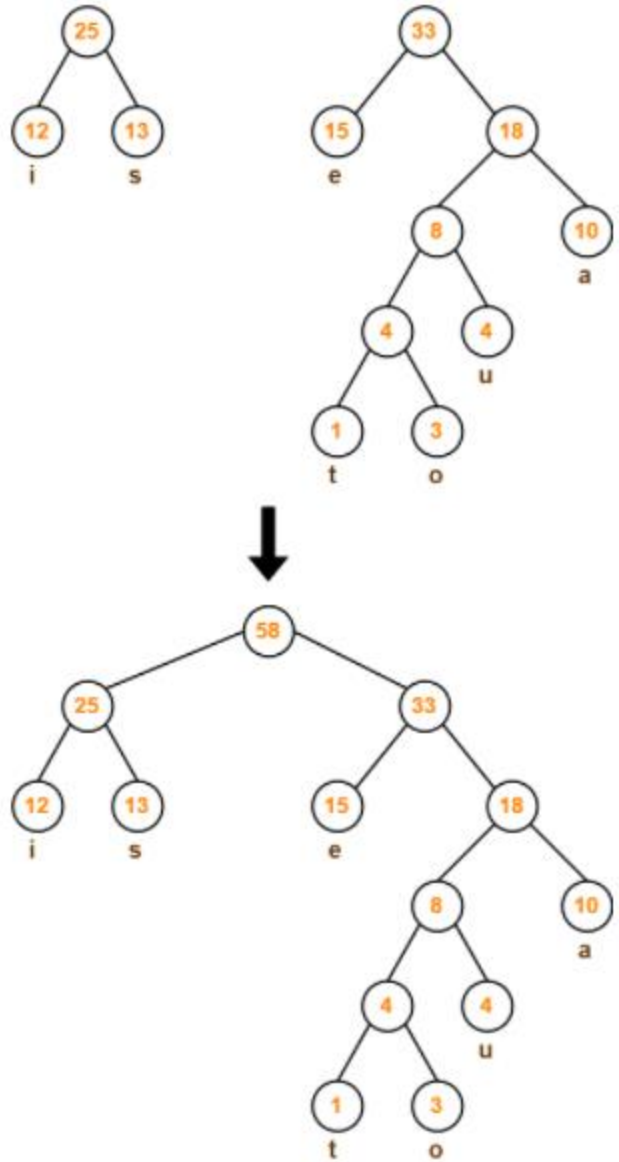
Step-05:



Step-06:

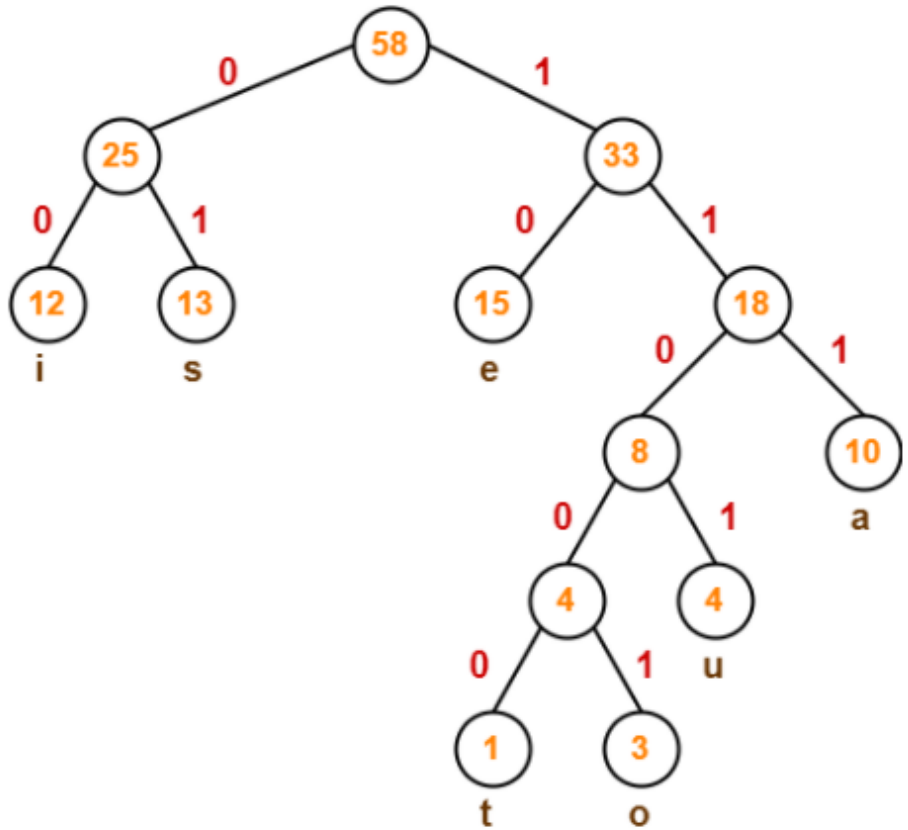


Step-07:



Huffman Tree

After assigning weight to all the edges, the modified Huffman Tree is-



Huffman Tree

1. Huffman Code For Characters-

To write Huffman Code for any character, traverse the Huffman Tree from root node to the leaf node of that character.

Following this rule, the Huffman Code for each character is-

- a = 111
- e = 10
- i = 00
- o = 11001
- u = 1101
- s = 01
- t = 11000

From here, we can observe-

- Characters occurring less frequently in the text are assigned the larger code.
- Characters occurring more frequently in the text are assigned the smaller code.

2. Average Code Length-

Using formula-01, we have-

Average code length

$= \sum (\text{frequency}_i \times \text{code length}_i) / \sum (\text{frequency}_i)$

$= \{ (10 \times 3) + (15 \times 2) + (12 \times 2) + (3 \times 5) + (4 \times 4) + (13 \times 2) + (1 \times 5) \} / (10 + 15 + 12 + 3 + 4 + 13 + 1)$

$= 2.52$

3. Length of Huffman Encoded Message-

Using formula-02, we have-

Total number of bits in Huffman encoded message

= Total number of characters in the message x Average code length per character

$= 58 \times 2.52$

$= 146.16$

$\cong$ 147 bits