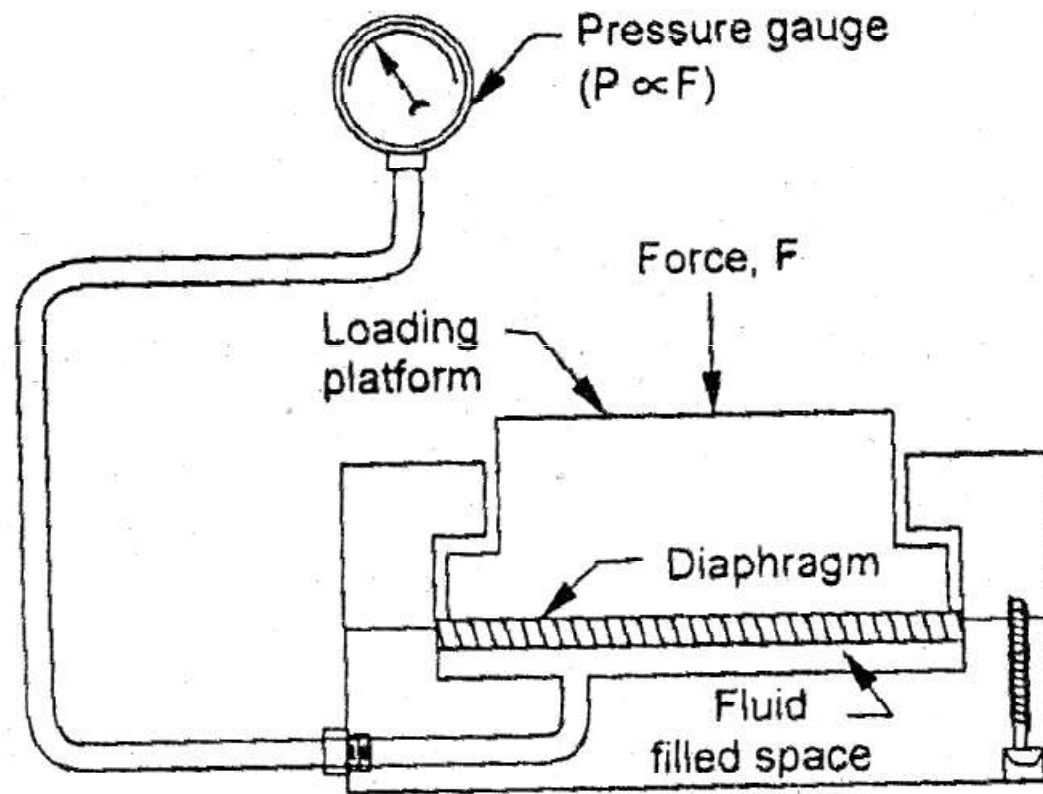


# Force Measurement

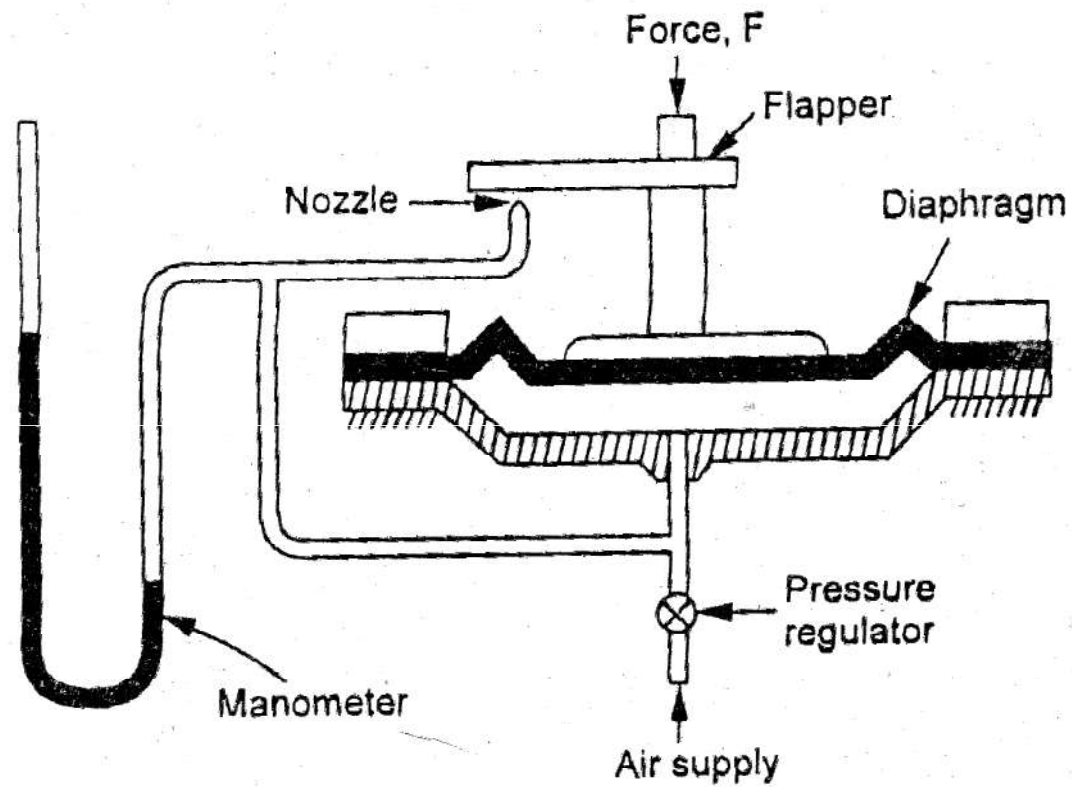
# Hydraulic Load Cell



# Hydraulic Load Cell

- **Force** is impressed upon **diaphragm** which deflects & transmit it to liquid
- The liquid in a space has preload pressure of the order of 2 bar.
- Application of force increases the liquid pressure. It equals force magnitude divided by effective area of diaphragm.
- The pressure is transmitted to pressure gauge calibrated in force units.
- This system has good dynamic response.
- Used to measure loads up to 2.5 MN with accuracy of the order of 0.1 % of full scale ; resolution is about 0.02%.

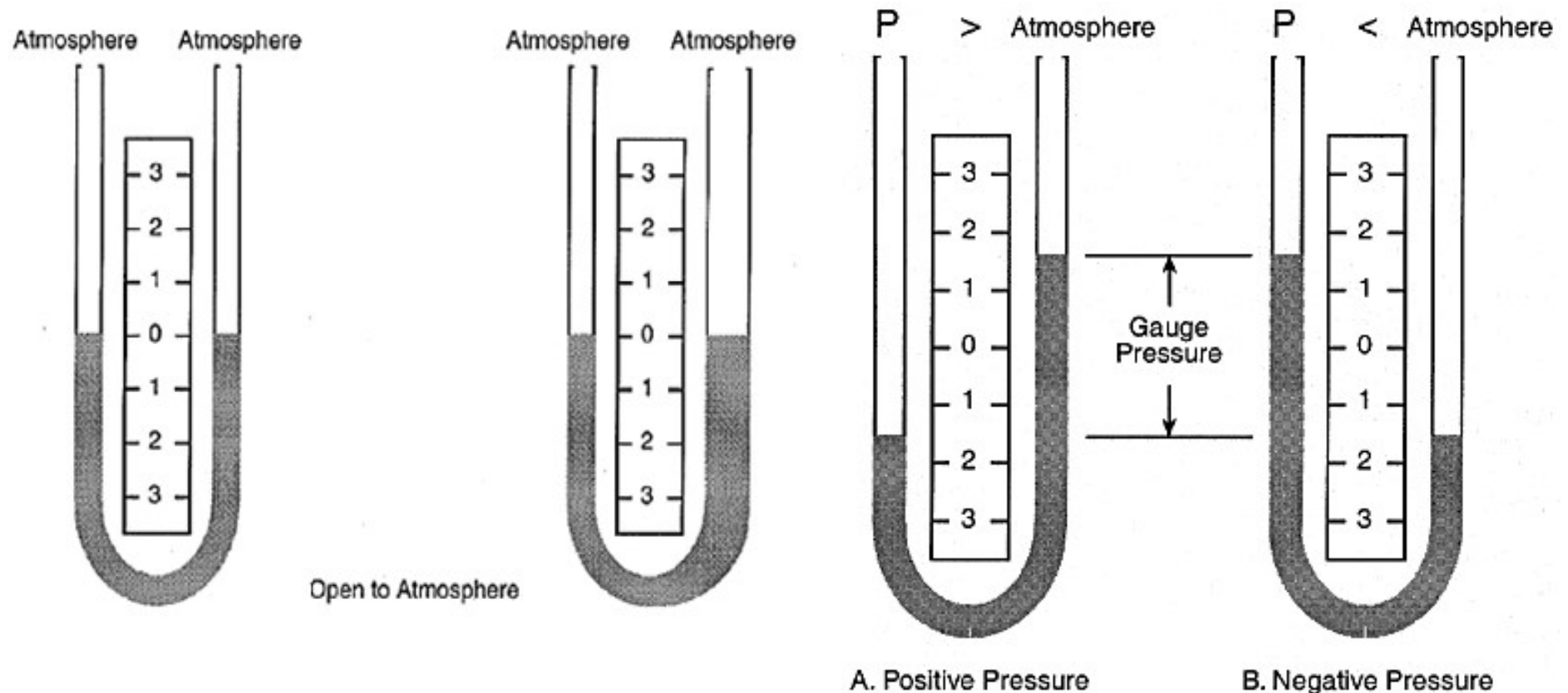
# Pneumatic Load Cell



# Pneumatic Load Cell

- A downward force is balanced by an upward force of air pressure against the effective area of a diaphragm.
- Application of force causes flapper to come closer to nozzle, and the diaphragm to deflect downwards.
- Nozzle opening is nearly shut-off and this results into an increased back pressure in the system.
- For constant applied force system attains equilibrium at a specific nozzle opening and a corresponding pressure is indicated by height of mercury column .
- Can measure loads upto 250 kN with an accuracy of 0.5 % of full scale

# Variations on the U-Tube Manometer



# U – Tube Manometer

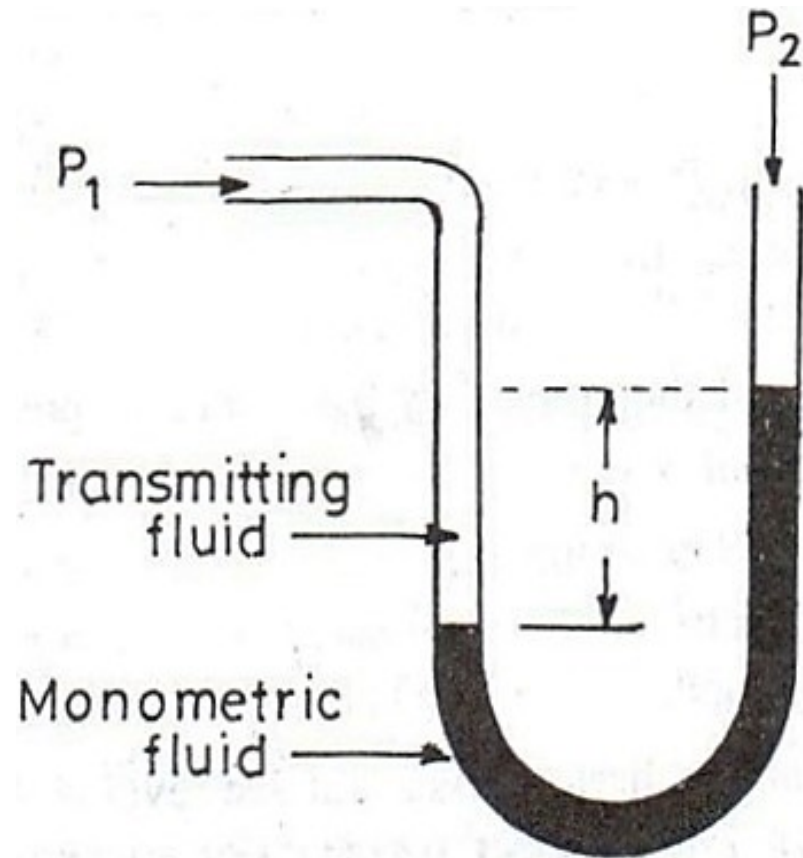
- Filled with Manometric fluid whose specific gravity is known
- Pressure balance equation is,

$$P_1 + gh\rho_f = P_2 + gh\rho_m$$

$$\Delta P = gh (\rho_m - \rho_f)$$

Where,

$g$  is gravitational force ( $9.81 \text{ m/s}^2$ ),  
 $\rho_m$  and  $\rho_f$  are densities of  
manometric fluid and transmitting  
fluid in  $\text{kg/m}^3$



- ▶ If pressure due to transmitting fluid can be neglected, difference in pressure,

$$\Delta P = P_1 - P_2 = gh \rho_m$$

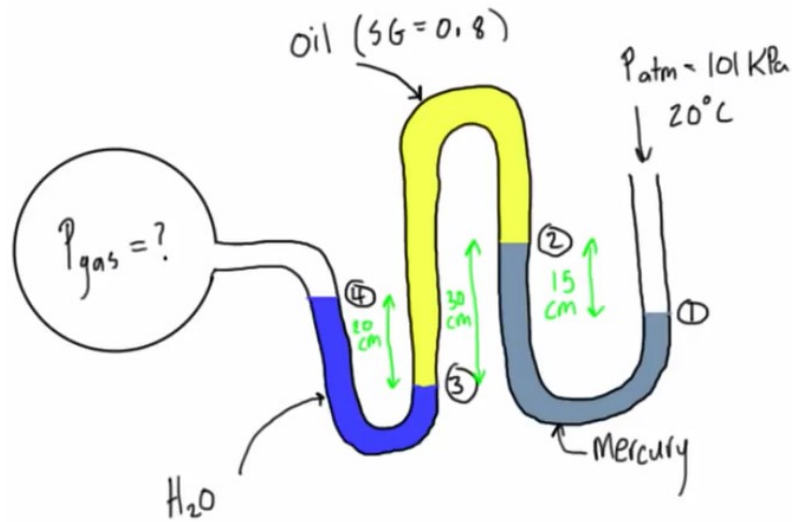
- ▶ Pressure sensitivity is:  $S = \frac{\Delta h}{\Delta P} = \frac{1}{g \rho_m}$

- ▶ If  $P_1$  is pressure being measured, U-tube manometer can be used for measurement of:

- ✓ Absolute pressure, if  $P_2 = 0$ ,
- ✓ Gauge pressure if  $P_2 =$  atmospheric pressure,
- ✓ Differential pressure, if  $P_2$  is the second pressure to be compared with  $P_1$



# U Tube Manometer



$$\begin{aligned}
 P_2 &= P_1 - \rho g h \\
 &= 101,000 \text{ Pa} - (13,580 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (0.15 \text{ m}) \\
 &= 101,000 \text{ Pa} - 19.9 \text{ Pa} \\
 P_2 &= 81,100 \text{ Pa}
 \end{aligned}$$

$$\begin{aligned}
 P_3 &= P_2 + \rho g h \\
 &= 81,000 \text{ Pa} + (798.4 \frac{\text{kg}}{\text{m}^3}) (9.81 \frac{\text{m}}{\text{s}^2}) (0.3 \text{ m}) \\
 &= 81,000 + 2,300 \text{ Pa} \\
 &= 83.4 \text{ kPa}
 \end{aligned}$$

$0.8(\rho_{H_2O}) \rho_{oil}$   
 $0.8(998 \frac{\text{kg}}{\text{m}^3}) = 798.4 \frac{\text{kg}}{\text{m}^3}$

$$\begin{aligned}
 P_4 &= P_{gas} = P_3 - \rho g h \\
 &= 83,400 \text{ Pa} - (998)(9.81)(0.2) \\
 &= 83,400 - 1,958 \rightarrow
 \end{aligned}$$

$$P_{gas} = 81,442 \text{ Pa}$$

## Slide 23

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**s1**

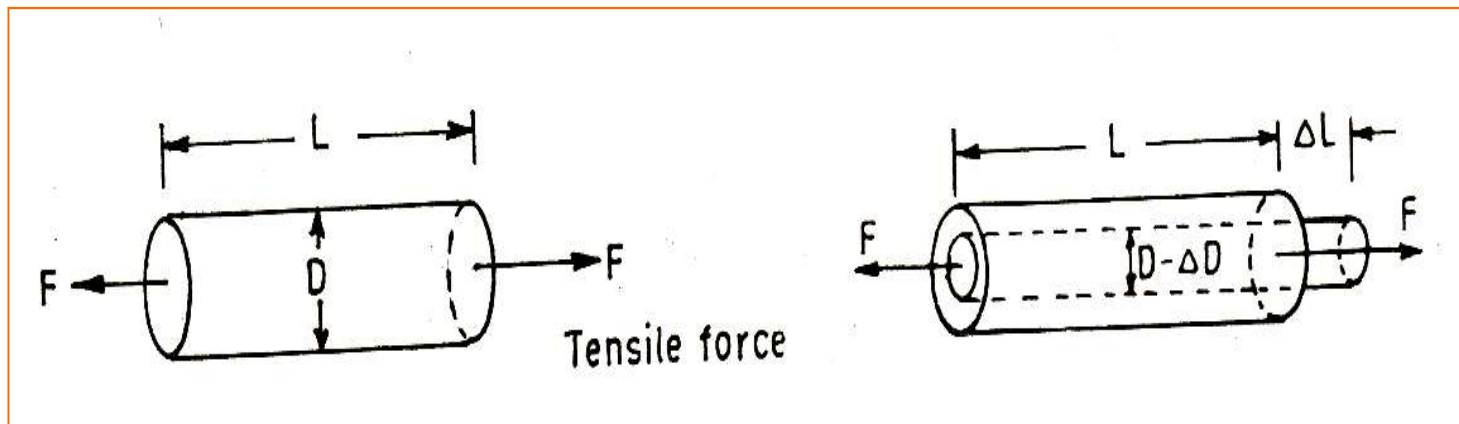
saaurabh\_bhardwaj, 01-02-2016

# Strain Gauge Load Cell

- Uses change in resistance to sense strain produced by a force
- If a metal conductor is stretched/compressed its resistance changes as both length and diameter of conductor changes.
- Gauge of circular wire with length( $l$ ), area( $A$ ), diameter( $D$ ) before strain. Let tensile stress ' $s$ ' be applied causing length to increase and area to decrease.
- When strained there are changes in its dimensions. Let  $\Delta L$  is change in length,  $\Delta A$  is change in area,  $\Delta D$  is change in diameter and  $\Delta R$  is change in resistance

# Strain Gauge Load Cell

- Uses change in resistance to sense strain produced by a force
- If a metal conductor is stretched/compressed its resistance changes as both length and diameter of conductor changes.



# Strain Gauge Load Cell

- Resistance of unstrained gauge is

$$R = \frac{\rho L}{A}$$

To find how  $\Delta R$  depends upon the material physical quantities, the expression for  $R$  is differentiated with respect to stress  $s$

$$\frac{dR}{ds} = \frac{\rho}{A} \frac{\partial L}{\partial s} - \frac{\rho L}{A^2} \frac{\partial A}{\partial s} + \frac{L}{A} \frac{\partial \rho}{\partial s}$$

- Dividing both side by resistance  $R = \rho L / A$ , we get

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} - \frac{1}{A} \frac{\partial A}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s}$$

- Per unit change in resistance is due to the factors like per unit change in length =  $\Delta L/L$ , area =  $\Delta A/A$  and resistivity =  $\Delta \rho/\rho$

# Strain Gauge Load Cell

$$A = \frac{\pi}{4} D^2 \therefore \frac{\partial A}{\partial s} = 2 \cdot \frac{\pi}{4} D \cdot \frac{\partial D}{\partial s}$$

$$\frac{1}{A} \frac{dA}{ds} = \frac{(2\pi/4)D}{(\pi/4)D^2} \frac{\partial D}{\partial s} = \frac{2}{D} \frac{\partial D}{\partial s}$$

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} - \frac{2}{D} \frac{\partial D}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s}$$

Also we know that Poisson's ratio  $\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = - \frac{\partial D / D}{\partial L / L}$

$$\partial D / D = -\nu \times \partial L / L$$

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} + \nu \frac{2}{L} \frac{\partial L}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s}$$

# Strain Gauge Load Cell

- For small variations, relationship can be written as

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\nu \frac{\Delta L}{L} + \frac{\Delta \rho}{\rho}$$

- The gauges factor is defined as the ratio of per unit change in resistance to per unit change in length gauge factor

$$G_f = \frac{\Delta R / R}{\Delta L / L} \quad G_f = \frac{\Delta R / R}{\Delta L / L} = 1 + 2\nu + \frac{\Delta \rho / \rho}{\Delta L / L}$$

$$G_f = 1 + 2\nu + \frac{\Delta \rho / \rho}{\epsilon}$$

$$\epsilon = \text{strain} = \frac{\Delta L}{L}$$

- If the change in the value of resistivity of a material when strained is neglected, the gauge factor is given by

$$G_f = 1 + 2\nu$$

# Strain Gauge Load Cell

## **Quarter Bridge:**

Only one strain gauge is used and other 3 elements are fixed resistors.

## **Half Bridge:**

Two of the bridge elements are strain gauges and the other two are fixed resistors.

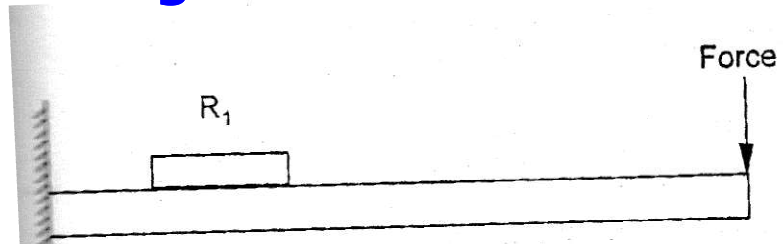
## **Full Bridge:**

All the four elements are strain gauges.



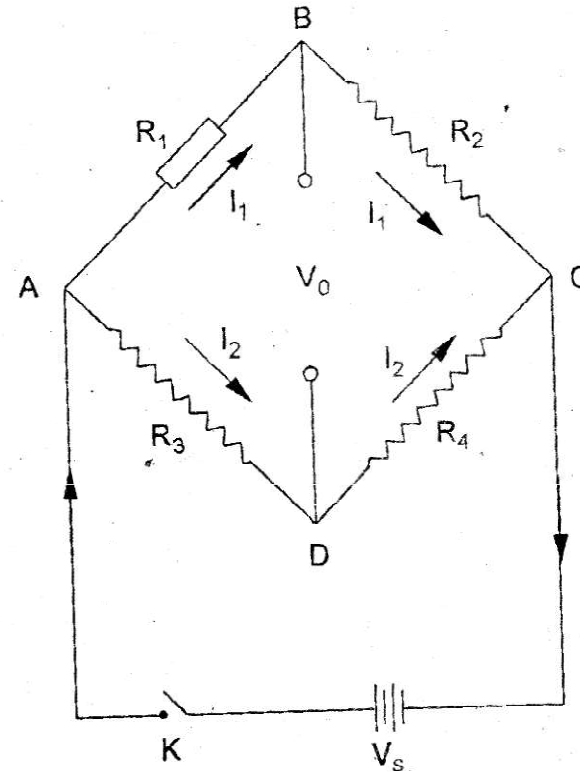
# Strain Gauge Load Cell

## Quarter Bridge:



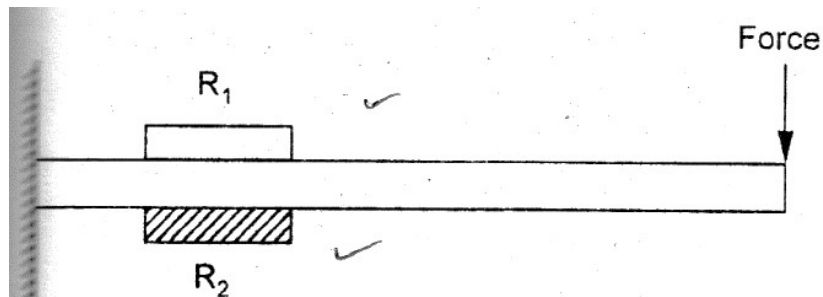
Strain gauge  $R_1$  is under tension  
 $R_2$ ,  $R_3$  and  $R_4$  are fixed resistors

$$dV_o = \frac{V_s}{4} \frac{dR}{R}$$



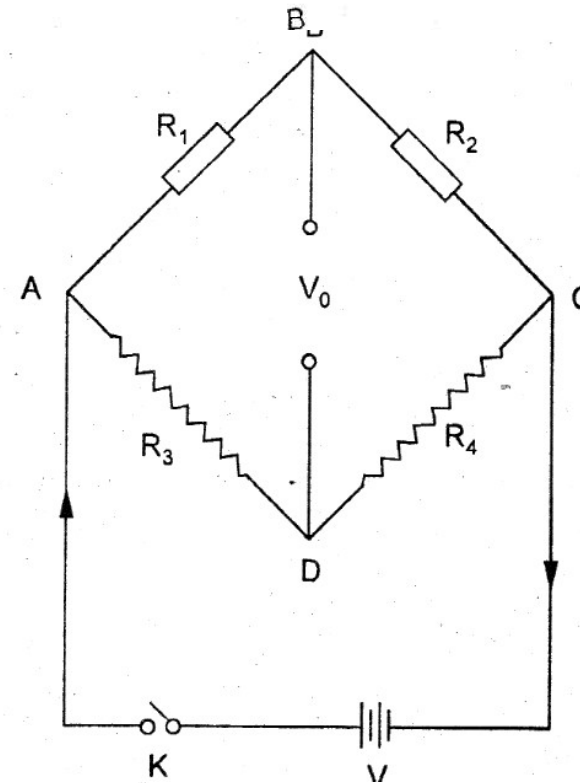
# Strain Gauge Load Cell

## Half Bridge:



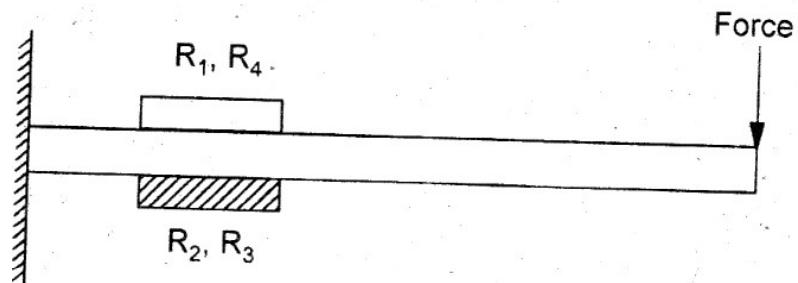
Strain gauge  $R_1$  is under tension  
Strain gauge  $R_2$  is under compression  
 $R_3$  and  $R_4$  are fixed resistors

$$dV_o = \frac{V_s}{2} \frac{dR}{R}$$



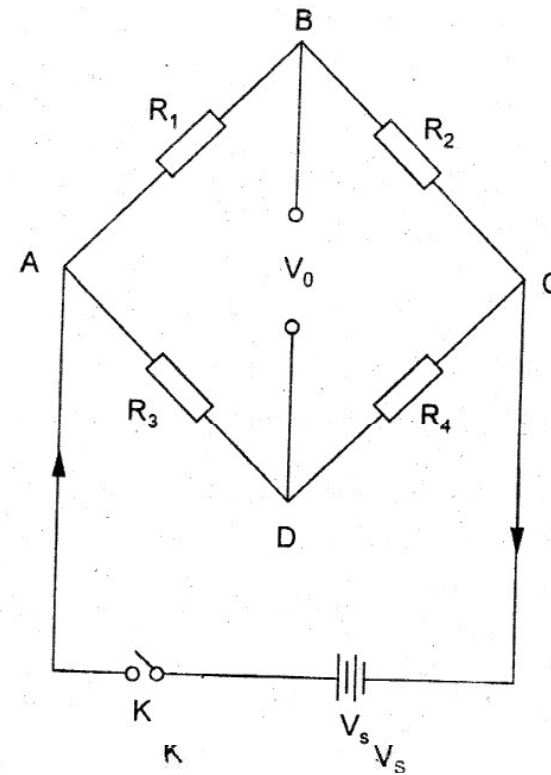
# Strain Gauge Load Cell

## Full Bridge:



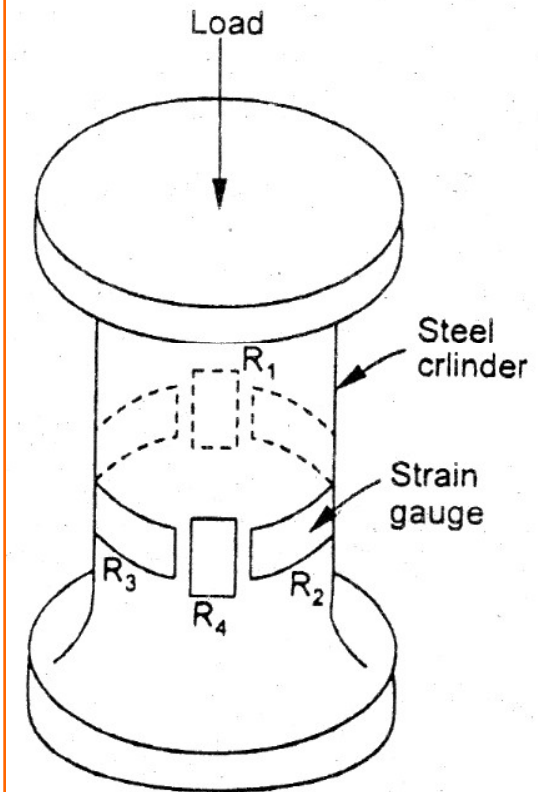
Gauge  $R_1$  and  $R_4$  is under tension  
Gauge  $R_2$  and  $R_3$  is under compression

$$dV_o = V_s \frac{dR}{R}$$



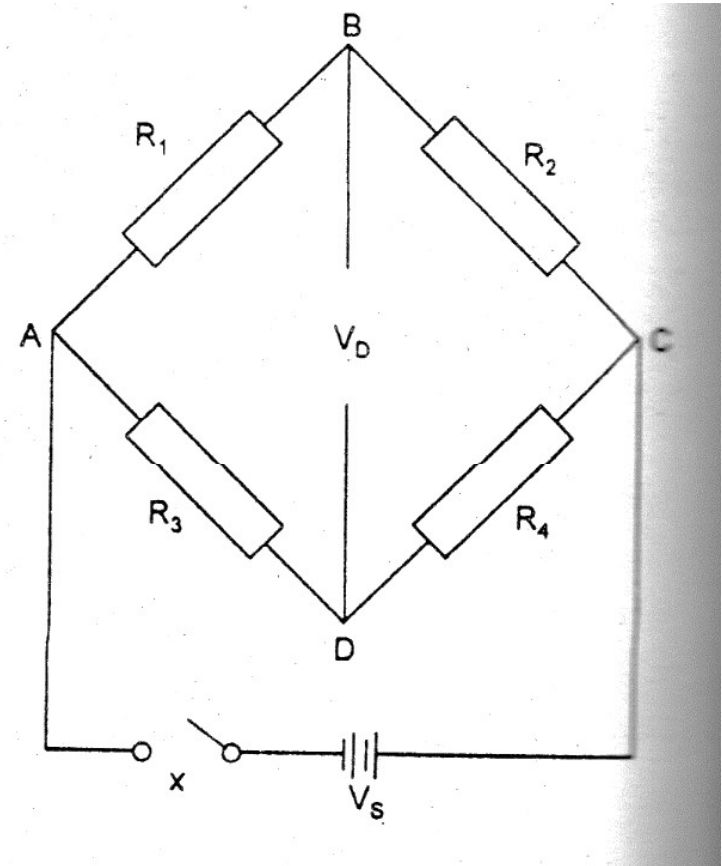
# Strain Gauge Load Cell

- The Gauges  $R_1$  and  $R_4$  are along the direction of applied load
- $R_2$  and  $R_3$  are attached at right angles to gauges  $R_1$  and  $R_4$
- These 4 gauges are connected to the 4 limbs of wheat stone bridge



# Strain Gauge Load Cell

- When a compressive load is applied the vertical gauges ( $R_1$  and  $R_4$ ) undergo compression and so decrease in resistance.
- Gauges  $R_2$  and  $R_3$  undergo tension and so increase in resistance.
- In Poisson's arrangement changes in resistance are related to each other by poisson's ratio



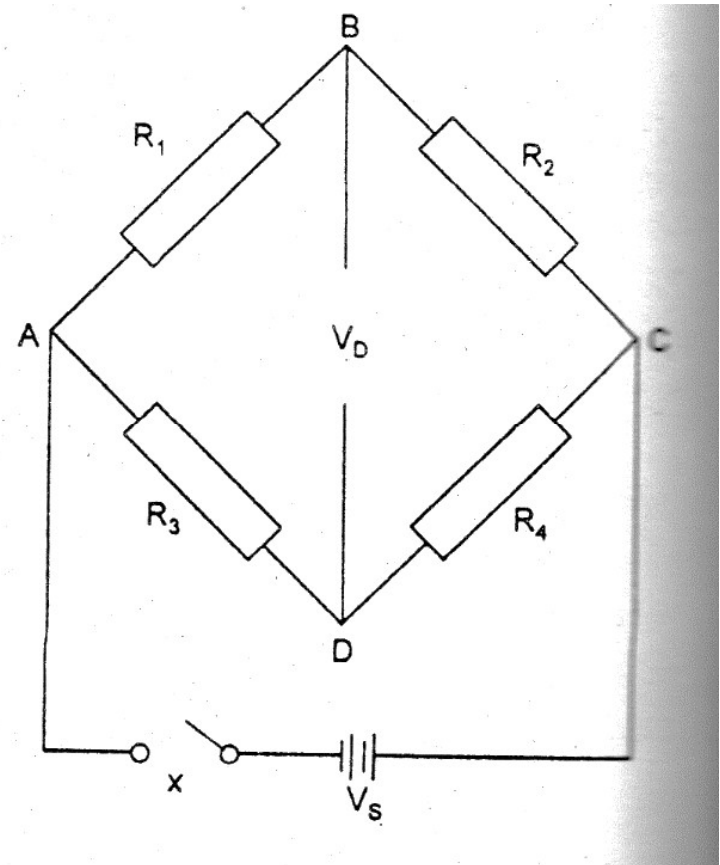
# Strain Gauge Load Cell

- When strained ,

$$R_1 \text{ and } R_4 = R - dR$$

$$R_2 \text{ and } R_3 = R + \mu dR$$

$$\text{Changed output voltage} = 2(1+\mu)\left(\frac{V_s}{4} \frac{dR}{R}\right)$$



# Speed Measurement

# Things that Turn - Examples

Tire on a car or bike

Buckets on a waterwheel

Teeth on a gear

Horse on a Merry-Go-Round

Fins on a fan or a windmill

Earth on its axis





# Linear & Angular Velocity - Examples

Film on a projector or tape on a videotape

Turntable in a microwave oven

Blade on a lawnmower

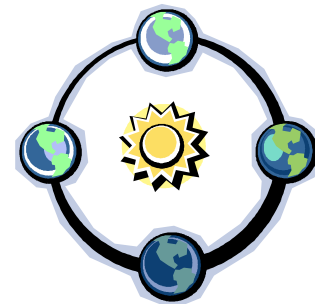
Earth around the sun

Seat on a Ferris wheel

Rope around a pulley

A record on an old record player

Drum/Barrel in a clothes dryer



# Linear & Angular Velocity - Examples

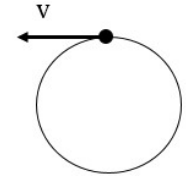
- Linear Velocity is distance/time:

$$v = \frac{s}{t}$$

- Angular Velocity is turn/time:

$$\omega = \frac{\theta}{t}$$

# Linear/Tangential Velocity



- Objects moving in a circle also have a rotational or angular velocity, which is the rate angular position changes.
- Rotational velocity is measured in degrees/sec, rotations/minute (rpm), *etc.*
- Common symbol,  $\omega$

## Rotational/Angular Velocity

- ▶ Objects moving in a circle still have a linear velocity = distance/time.
- ▶ This is often called tangential velocity, since direction of the linear velocity is tangent to the circle

# Measurement of Linear Velocity

- ▶ Velocity is first derivative of displacement
- ▶ Linear velocity utilize
  - ✓ Electromagnetic transducers
  - ✓ Seismic transducers
  - ✓ Digital transducers
  - ✓ Transducers utilizing Doppler effect

# Electromagnetic Transducers

- ▶ Here, voltage produced in a coil on account of change in flux linkages resulting from change in reluctance

- ▶ O/P voltage from coil is  $e_o = \frac{d\Phi}{dt}$

- ▶ Flux  $\Phi = \frac{Ni}{R}$

$$e_o = \frac{d}{dt} \left( \frac{Ni}{R} \right) = N \frac{d}{dt} \left( \frac{i}{R} \right) = \frac{N}{R} \frac{di}{dt} - \frac{Ni}{R^2} \frac{dR}{dt}$$

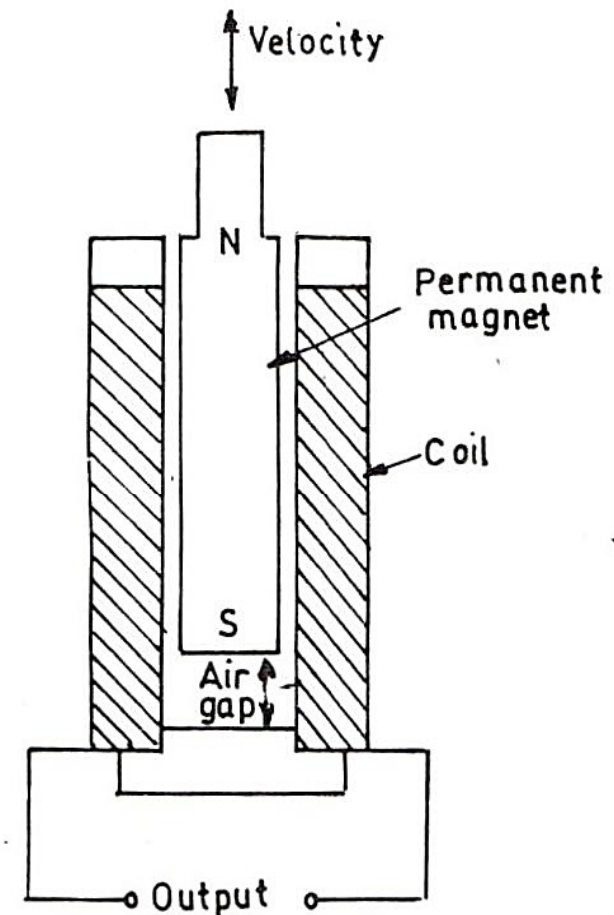
- ▶ Suppose current ( $i$ ) is constant, O/P voltage

$$e_o = \frac{Ni}{R^2} \frac{dR}{dt} = K \frac{dR}{dt}$$

- ▶ Electromagnetic transducers classification: **Reluctance varies directly as length of air gap &  $e_o$  is  $\propto$  to rate of change of length of air gap, hence v**
- Moving magnet type
- Moving coil type

# Moving Magnet Type

- ▶ Constant mmf  $Ni$ , can be provided by a solenoid of  $N$  turns and carrying a constant current
- ▶ Sensing element is rod i.e., rigidly coupled to the device whose velocity is being measured
- ▶ This rod is Permanent Magnet (PM)
- ▶ Motion of magnet induces a voltage in coil and amplitude of voltage is directly proportional to the velocity
- ▶ Reluctance varies directly as length of air gap and O/P voltage  $\propto$  rate of change of length of air gap, hence linear velocity



# Moving Magnet Type

- ▶ For a coil placed in magnetic field, the voltage generated is

$$e_0 = BANv = Kv$$

where B = flux density, A = area of coil, N = number of turns of coil, v = relative velocity of magnet w.r.t coil

- ▶ Polarity of the O/P determines direction of motion

- ▶ **Advantages**

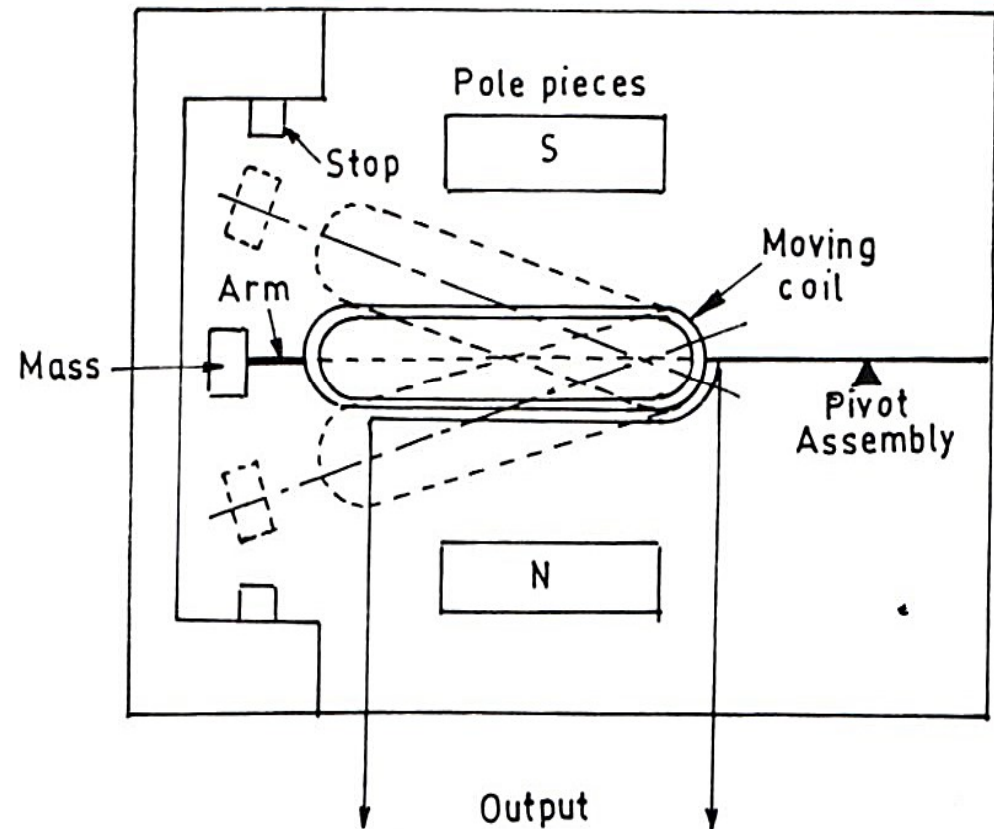
- ✓ Negligible maintenance (no mech. contact)
- ✓ O/P is linearly proportional to velocity
- ✓ Inexpensive

- ▶ **Disadvantages**

- ✓ Performance is adversely affected by stray magnetic fields, *resulting noise*
- ✓ Frequency response is limited
- ✓ Not very useful for measurement of vibrations

# Moving Coil Type

- ▶ Operates through the action of a coil moving in a magnetic field
- ▶ Voltage is generated in the coil which is  $\propto$  to velocity of coil
- ▶ Velocity transducer uses a pivoted arm on which a coil is mounted
- ▶ Velocity to be measured is applied to the pivoted arm and the coil moves in the PM field



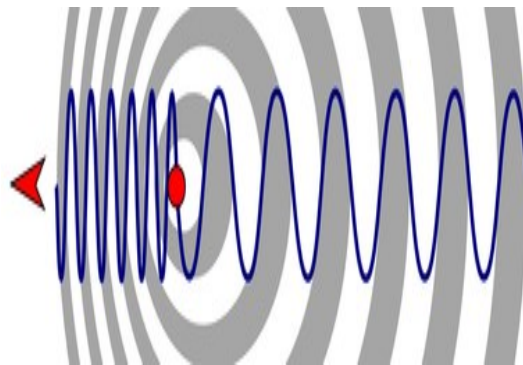


# Moving Coil Type

- ▶ Voltage is generated on account of motion of the coil in the magnetic field
- ▶ O/P voltage is proportional to velocity
- ▶ Used for measurement of velocities developed in
  - linear
  - sinusoidal or random
- ▶ **Advantages**
  - ✓ More satisfactory arrangement as the system now forms a closed magnetic circuit with a constant air gap
  - ✓ Whole device is contained in antimagnetic case which reduces the effects of stray magnetic fields

# Doppler Effect Transducers

- ▶ If a narrow beam or ultrasonic beam is aimed at an object, the beam will be reflected back to the source
- ▶ If the object is moving, frequency of signals received back differs from that of the transmitted signal
- ▶ Difference of frequencies is a measure of velocity of moving object
- ▶ Received frequency is higher than transmitted frequency if the object is travelling towards receiver and lower if the object is travelling away



# Doppler Effect Transducers

- In physics (waves in a medium), relationship between observed frequency  $f$  and emitted frequency  $f_0$  is given by:

$$f = \left( \frac{v + v_r}{v + v_s} \right) f_0$$

where

$v$  is velocity of waves in the medium

$v_s$  is velocity of the source relative to the medium

$v_r$  is velocity of the receiver relative to the medium

Observed frequency

$$f = \left( 1 - \frac{v_{s,r}}{c} \right) f_0$$

Change in frequency

$$\Delta f = -\frac{v_{s,r}}{c} f_0 = -\frac{v_{s,r}}{\lambda_0}$$