Objectives

- Introduction to Fractional Knapsack problem
- Greedy Strategies to solve fractional knapsack problem
- Optimal solution approach
- Practical example of fractional knapsack problem
- Algorithm and its complexity

Problem statement:

Consider there are n objects and each object is having a weight w and contributes to profit p . There is a knapsack, a bag having capacity W .

Objective is to fill the knapsack in such a way that the profit shall be maximum. That is:

$$\max . \sum_{i=1}^{n} p_i x_i \quad and \sum_{i=1}^{n} w_i x_i \leq W$$

Three different strategies to solve fractional knapsack problem:

Strategy 1:

Items are arranged by their profit values. Here an item with maximum profit is selected first.

Strategy 2:

Items are arranged by weights and an item with minimum weight is selected first.

Strategy 3:

1. Calculate the ratio (value/wt.) for each item.
2. Sort the item based on the ratio.
3. Take the item with highest ratio and add them until we cannot add the next item as whole.
4. At the end add the next item as much (fraction) as we can.

Example:

objects	1	2	3	4	5	6	7
value	5	10	15	7	8	9	4
Wt.	1	3	5	4	1	3	2

Weight of knapsack is: 15 units

Strategy 1: Select the item first which is having maximum profit.

Objects	profit	Wt.	Remaining wt.
3	15	5	15-5=10
2	10	3	10-3=7
6	9	3	7-3=4
5	8	1	4-1=3
4	7*(3/4)=5.25	3	3-3=0

Total profit: = 47.25 units

Strategy 2: Select the item first which is having minimum weight.

Objects	profit	Wt.	Remaining wt.
1	5	1	15-1=14
5	8	1	13
7	4	2	11
2	10	3	8
6	9	3	5
4	7	4	1
3	15*(1/5)=3	1	0

Total profit: = 46 units

Strategy 3: select the item according to the profit by weight ratio.

objects	1	2	3	4	5	6	7
profit	5	10	15	7	8	9	4
Wt.	1	3	5	4	1	3	2
p/w	5	3.3	3	1.75	8	3	2

Weight of knapsack is: 15 units

Objects	profit	Wt.	Remaining wt.
5	8	1	15-1=14
1	5	1	13
2	10	3	10
3	15	5	5
6	9	3	2
7	4	2	0

Total profit: = 51 units

Assignment 1:

objects	1	2	3	4	5	6	7
value	12	5	15	7	6	18	5
Wt.	2	3	5	7	2	6	1

Weight of knapsack is: 20 units.

Assignment 2:

objects	1	2	3	4	5	6	7
value	5	10	15	7	8	9	4
Wt.	1	3	5	4	1	3	2

Weight of knapsack is: 15 units

Assignment 3:

objects	1	2	3
profit	60	100	120
Wt.	10	20	30

Weight of knapsack is: 50 units

Fractional knapsack problem using greedy approach:

Time Complexity: $O(n \log n)$---------- (verify?)