LECTURE 9

TYPES OF ERRORS

COMBINATION OF QUANTITIES WITH LIMITING ERRORS

UNCERTAINTY OR ERROR ANALYSIS

- All measurements have some degree of uncertainty that may come from different sources
- The process of evaluating the uncertainty associated with a measurement is often called uncertainty or error analysis

Measurement = (best estimate ± uncertainty)units

LIMITING ERROR

LIMITING ERROR

The limits of the deviations from the specified value $\pm\delta A$

- RELATIVE LIMITING ERROR

Ratio of the error to the nominal value of the measured quantity

$$\frac{\delta A}{A_s}$$

Accuracy is often reported quantitatively by using relative error.

TYPES OF ERRORS

- GROSS ERRORS

Arise due to human mistakes in reading instruments and recording results

  - Great care should be taken while taking the readings
  - Better not to rely on a single reading

- SYSTEMATIC ERRORS

✓ Instrumental

✓ Environmental

✓ Observational

- RANDOM ERRORS

Caused due to disturbances about which we are unaware

TYPES OF ERRORS(CONTD.)

- Instrumental errors

These errors arise due to three main reasons :

- Due to inherent shortcomings in the instrument

Inherent in the instruments due to their mechanical structure

Can be reduced by adopting following methods

(i) Correction factors should be applied after determining the instrumental errors.

(ii) The instrument may be re-calibrated carefully

TYPES OF ERRORS(CONTD.)

- Due to misuse of the instruments

Caused due to fault of operator (like failure to adjust zero, poor initial adjustments etc.)

- Due to loading effects of instruments

Improper use of instrument for measurement

For example, a well calibrated voltmeter may give a misleading voltage reading when connected across a high resistance circuit . The same voltmeter when connected in a low resistance circuit, may give a more dependable reading .

- Environmental errors

Caused due to conditions prevailing around the instrument. These may be effects of temperature pressure, humidity, dust, vibrations or of external magnetic or electrostatic fields.

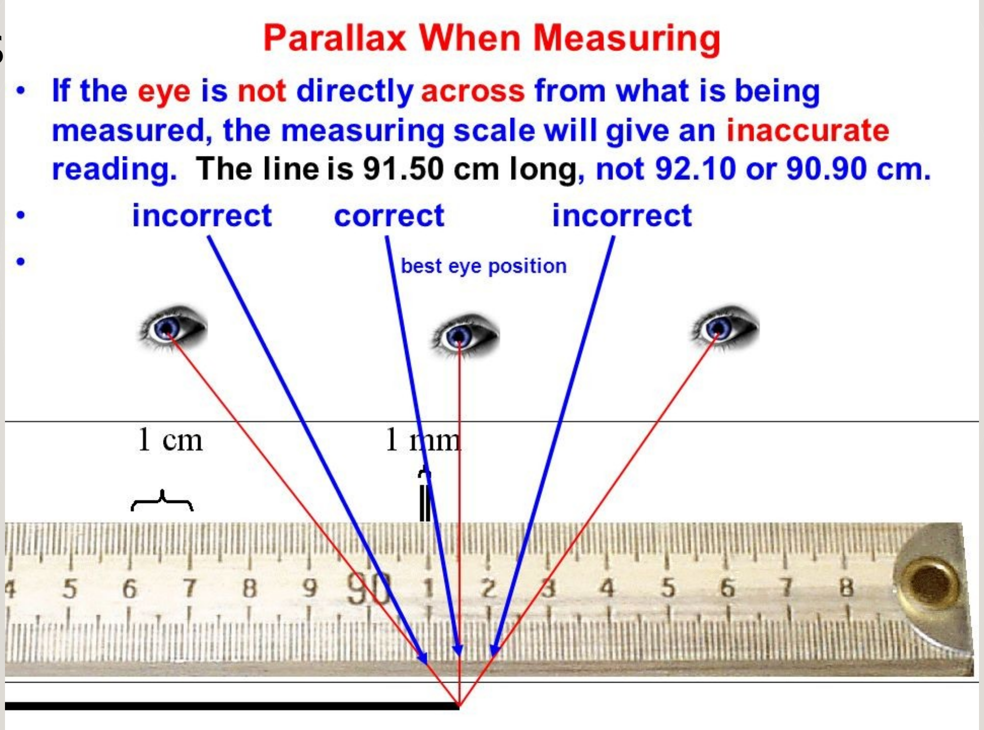
Can be reduced by adopting following methods

1. Arrangements are made to keep the conditions constant to the possible extent.

2. Using equipment which is immune to these effects. For example, if we use resistance materials having very low temperature coefficient of resistance, variations in resistance with temperature can be minimized

3. Employing techniques which eliminate the effects of these disturbances. For eg., to reduce the effects of humidity or dust on instrument readings, hermetically seal the equipment

Observational errors



COMBINATION OF QUANTITIES WITH LIMITING ERRORS

- Sum of two quantities
- Difference of two quantities
- Product of two quantities
- Quotient of two quantities
- Power of a factor

SUM OF TWO QUANTITIES

$$X = x_1 + x_2$$

$$\frac{dX}{X} = \frac{d\left(x_1 + x_2\right)}{X} = \frac{dx_1}{X} + \frac{dx_2}{X}$$

$$\frac{dX}{X} = \frac{x_1}{X}\frac{dx_1}{x_1} + \frac{x_2}{X}\frac{dx_2}{x_2}$$

$$\frac{\delta X}{X} = \pm\left[\frac{x_1}{X}\frac{\delta x_1}{x_1} + \frac{x_2}{X}\frac{\delta x_2}{x_2}\right]$$

DIFFERENCE OF TWO QUANTITIES

$$X = x_1 - x_2$$

$$\frac{dX}{X} = \frac{d\left(x_1 - x_2\right)}{X} = \frac{dx_1}{X} - \frac{dx_2}{X}$$

$$\frac{dX}{X} = \frac{x_1}{X}\frac{dx_1}{x_1} - \frac{x_2}{X}\frac{dx_2}{x_2}$$

$$\frac{\delta X}{X} = \pm\left[\frac{x_1}{X}\frac{\delta x_1}{x_1} + \frac{x_2}{X}\frac{\delta x_2}{x_2}\right]$$

PRODUCT OF TWO QUANTITIES

$$X = x_1 x_2$$

$$\log_e X = \log_e x_1 + \log_e x_2$$

Differentiating w.r.t. X

$$\frac{1}{X} = \frac{1}{x_1}\frac{dx_1}{dX} + \frac{1}{x_2}\frac{dx_2}{dX}$$

$$\frac{dX}{X} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2}$$

$$\frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}\right)$$

QUOTIENT OF TWO QUANTITIES

$$X = \frac{x_1}{x_2}$$

$$\log_e X = \log_e x_1 - \log_e x_2$$

$$\frac{dX}{X} = \frac{dx_1}{x_1} - \frac{dx_2}{x_2}$$

$$\frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}\right)$$

POWER OF A FACTOR

$$X = x_1^n$$

$$\log_e X = n \log_e x_1$$

Differentiating w.r.t. X

$$\frac{1}{X} = n \frac{1}{x_1} \frac{dx_1}{dX}$$

$$\frac{dX}{X} = n \frac{dx_1}{x_1}$$

$$\frac{\delta X}{X} = \pm n \frac{\delta x_1}{x_1}$$

EXAMPLE

- The resistance of a circuit is found by measuring current flowing and the power fed into the circuit. Find the limiting error in the measurement of resistance when the limiting errors in the measurement of power and current are respectively ±1.5% and ±1.0% .

Soln. Resistance $R = \frac{P}{I^2} = PI^{-2}$

$$\frac{\delta R}{R} = \pm\left(\frac{\delta P}{P} + 2\frac{\delta I}{I}\right) = \pm(1.5 + 2 \times 1.0)$$

$$= \pm 3.5\,\%$$