TIME RESPONSE ANALYSIS OF FIRST ORDER SYSTEM

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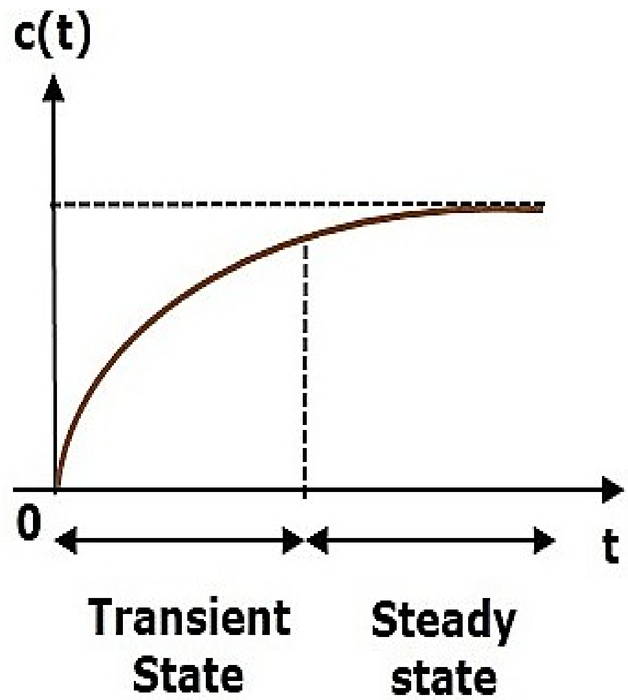
TIME RESPONSE

- The time response of a system is the output (response) which is function of the time, when input (excitation) is applied.
- Time response of a control system consists of two parts

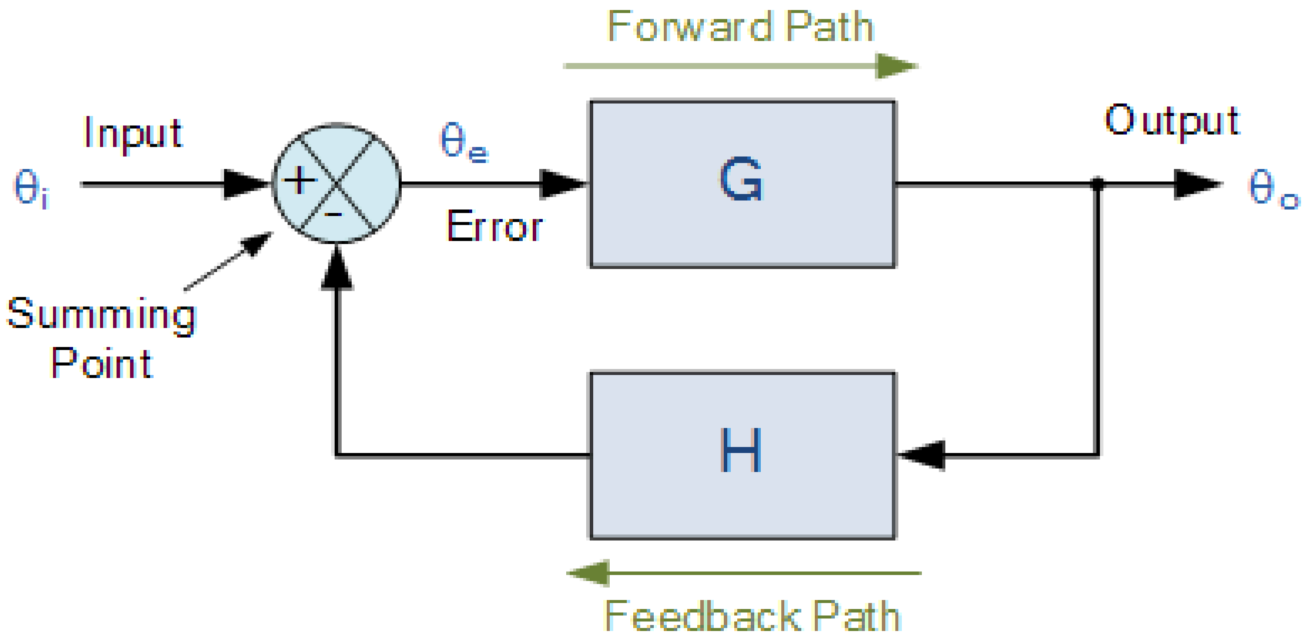
1. Transient Response
2. Steady State Response

Mathematically,

$$c(t) = c_t(t) + c_{ss}(t)$$

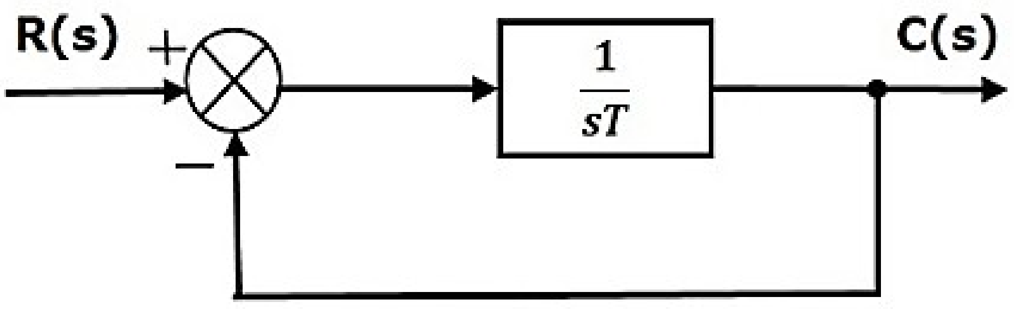


CLOSED LOOP SYSTEM REPRESENTATION



$$\frac{\text{Output}}{\text{Input}} = \frac{\theta_o}{\theta_i} = \frac{G}{1 + GH}$$

RESPONSE OF FIRST ORDER SYSTEM WITH UNIT STEP INPUT



For first order system

$$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

$$C(s) = \frac{1}{sT+1} R(s)$$

Input is unit step $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{s(sT+1)}$$

After partial fraction $C(s) = \frac{1}{s} - \frac{T}{1+sT}$

CONTINUED

Take inverse Laplace

$$c(t) = 1 - e^{-t/T}$$

When t=T $$c(t) = 1 - e^{-T/T} = 1 - e^{-1} = 0.632$$

Where 'T' is known as 'time constant' and defined as the time required for the signal to attain 63.2% of final or steady state value.

Time constant indicates how fast the system reaches the final value.

Smaller the time constant, faster is the system response.

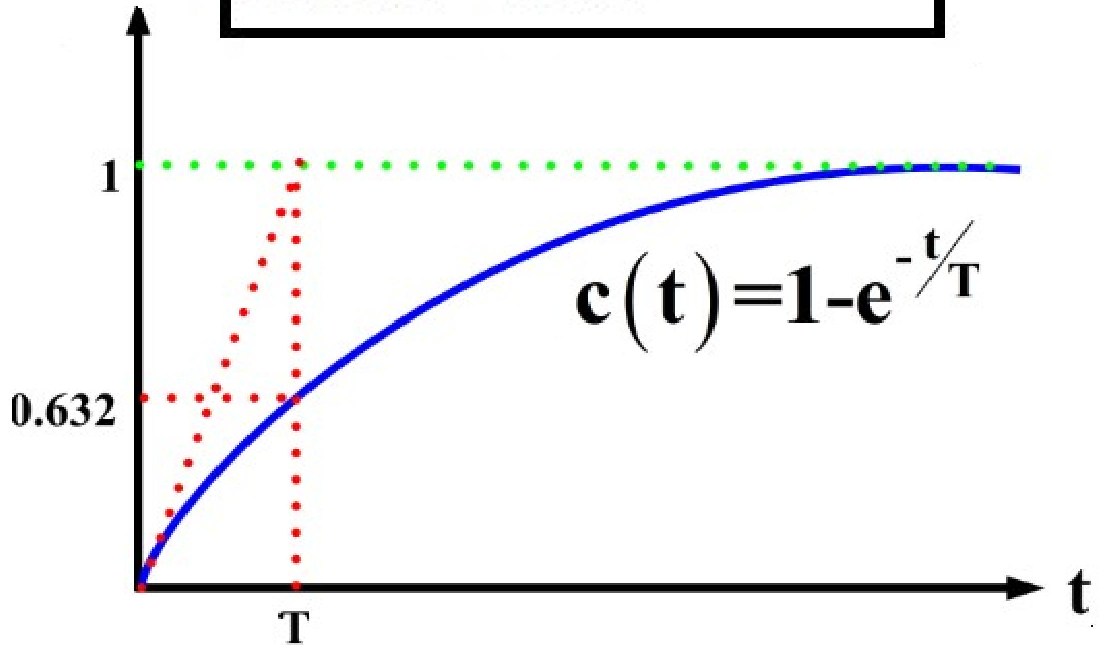
STANDARD SIGNALS

Now the error function is given by :-

$$e(t) = r(t) - c(t)$$
$$= 1 - (I - e^{-t/T}$$
$$e(t) = e^{-t/T}$$

and the steady state Error is -:

$$\boxed{\lim_{t \to \infty} e(t) = \lim_{t \to \infty} e^{-t/T} = 0}$$



RESPONSE OF FIRST ORDER SYSTEM WITH UNIT RAMP FUNCTION:

We know that $$\frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

$$C(s) = \frac{1}{sT+1} R(s)$$

Input is unit Ramp $$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{1}{s^2(1+sT)}$$

After partial fraction $$C(s) = \frac{1}{s^2} - \frac{T}{s} + T\frac{1}{s+\frac{1}{T}}$$

CONTINUED

Take inverse Laplace, we get

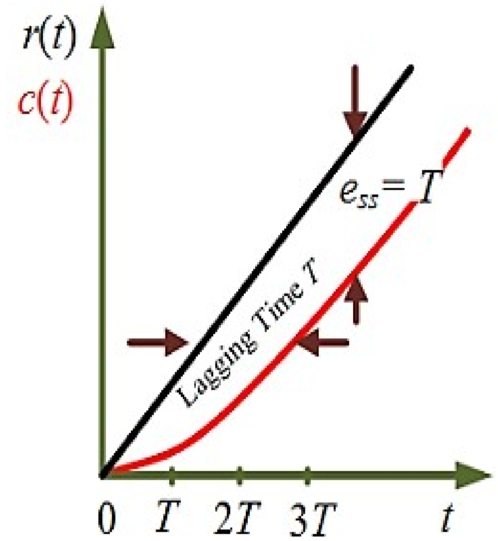
$$c(t) = t - T + Te^{-t/T}$$

Error signal $$e(t) = r(t) - c(t)$$

$$e(t) = t - \left[t - T + Te^{-t/T}\right]$$

$$e(t) = T(1 - e^{-t/T})$$

Steady state error $$= \lim_{t \to \infty}(T - Te^{-t/T}) = T$$



The steady state error is equal to 'T', where 'T' is the time constant of the system.

For smaller time constant steady state error will be small and speed of the response will increase.

RESPONSE OF THE FIRST ORDER SYSTEM WITH UNIT IMPULSE FUNCTION

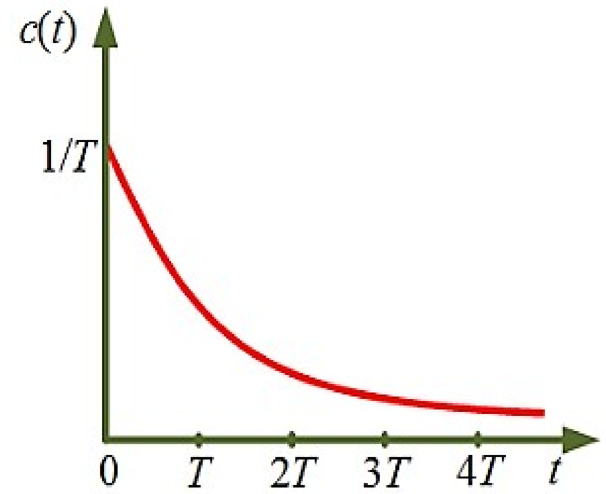
Input is unit impulse function R(s)=1

$$C(s) = \frac{1}{sT+1} R(s)$$

$$C(s) = \frac{1}{sT+1} .1$$

$$C(s) = \frac{1}{T} \frac{1}{s+1/T}$$

Inverse Laplace transform $$c(t) = \frac{1}{T} e^{-t/T}$$



THANK YOU