

Ratio and Proportions

Ratio:

- The ratio of two quantities a and b of same units is the fraction x/y , where $b \neq 0$
- The fraction x/y can be represented as $x:y$

Proportion:

1) Proportion is the equality of two ratios.

When $(a : b = x : y)$ is represented as $(a : b :: x : y)$, then a, b, x, y are said to be in proportion.

In $(a : b :: x : y)$, a and y are called as **extremes** and b and x are called as **mean terms**.

Product of means = Product of extremes

2) Mean proportion: Mean proportion between x and y is given as \sqrt{xy}

3) Third proportion: If $p : q = q : s$, then s is called as third proportional to p and q.

4) Fourth proportion: If $u : v = x : y$, then y is the fourth proportional of u, v and x.

Quick Tips and Tricks

1) Comparison of ratios:

$$\text{If } (x : y) > (a : b) \rightarrow \frac{x}{y} > \frac{a}{b}$$

2) Proportion = Equality of two ratios

a, b, c, d are in proportion

$$a : b :: c : d$$

$$a : b = c : d$$

$$\frac{a}{b} = \frac{c}{d}$$

Product of means = Product of extremes

$$b \times c = a \times d$$

Practice Questions Ratio and Proportion

1. Rs. 8400 is divided among A, B, C and D in such a way that the shares of A and B, B and C, and C and D are in the ratios of 2:3, 4:5 and 6:7 respectively. The share of A is

- a. Rs. 1280**
- b. Rs. 8400**
- c. Rs. 8210**
- d. Rs. 1320**

ANSWER: Rs. 1280

Explanation:

Now share of A and B is 2:3

So options B and C are not possible.

Consider Option 1 i.e. **Share of A = Rs. 1280**

$$\frac{\text{Share of A}}{\text{Share of B}} = \frac{2}{3}$$

$$\therefore \text{Share of B} = \frac{3}{2} \times \text{A's Share} = \frac{3}{2} \times 1280 = \text{Rs. 1920}$$

Similarly B:C = 4:5

$$\therefore \text{Share of C} = \frac{5}{4} \times \text{B's Share} = \frac{5}{4} \times 1920 = \text{Rs. 2400}$$

Similarly C:D = 6:7

$$\therefore \text{Share of D} = \frac{7}{6} \times \text{C's Share} = \frac{7}{6} \times 2400 = \underline{\underline{\text{Rs. 2800}}}$$

$$\text{TOTAL} = \text{Rs. 8400}$$

Total is Rs. 8400 so **answer is Option A i.e. Rs. 1280**

12. In a library, the ratio of number of story books to that of non-story books was 4:3 and total number of story books was 1248. When some more story books were bought, the ratio became 5:3. Find the number of story books bought.

- a. 312**
- b. 321**
- c. 936**
- d. 1560**

ANSWER: 312

Explanation:

$$\frac{\text{Story books}}{\text{Non-Story books}} = \frac{4}{3}$$

$$\therefore \text{Non-Story books} = \frac{3}{4} \times \text{Story books} = \frac{3}{4} \times 1248 = 936$$

Let M story books be added. So number of story books = 1248+M

$$\therefore \frac{\text{Story books}}{\text{Non-Story books}} = \frac{5}{3}$$

$$\therefore \frac{1248+M}{936} = \frac{5}{3}$$

$$\therefore 1248 + M = 1560$$

$$\therefore \mathbf{M = 312 = \text{Number of books added}}$$

3. Ajay and Raj together have Rs. 1050. On taking Rs. 150 from Ajay, Ajay will have same amount as what Raj had earlier. Find the ratio of amounts with Ajay and Raj initially.

- a. 3:4**
- b. 7:1**
- c. 1:3**
- d. 4:3**

ANSWER: 4:3

Explanation:

Let initially money with Ajay be A and with Raj be R

So, **$A + R = 1050$**

Also, Money is taken from Ajay, so,

$$A - 150 = R$$

$$\therefore \mathbf{A - R = 150}$$

Adding both equations,

$$2A = 1200$$

$$\therefore \mathbf{A = Rs. 600 = Initial\ money\ with\ Ajay}$$

$$\therefore \mathbf{R = 1050 - 150 = Rs. 450 = Initial\ money\ with\ Raj}$$

$$\therefore \frac{\text{Amount with Ajay}}{\text{Amount with Raj}} = \frac{600}{450} = \frac{4}{3}$$

4. The ratio of market prices of wheat and paddy is 2:3 and the ratio of quantities consumed in a family is 5:4. Find the ratio of expenditure of wheat and paddy.

- a. 6:5**
- b. 5:6**
- c. 1:1**
- d. 8:15**

ANSWER: 5:6

Explanation:

Expenditure = Price x Quantity

$$\frac{\text{Wheat Price}}{\text{Paddy price}} = \frac{2}{3} \quad \text{and} \quad \frac{\text{Wheat Quantity consumed}}{\text{Paddy Quantity consumed}} = \frac{5}{4}$$

Multiplying both ratios

$$\frac{\text{Wheat Price} \times \text{Wheat Quantity consumed}}{\text{Paddy Price} \times \text{Paddy Quantity consumed}} = \frac{2 \times 5}{3 \times 4}$$

$$\frac{\text{Wheat Expenditure}}{\text{Paddy Expenditure}} = \frac{5}{6}$$

5. The ratio of numbers of girls and boys participating in sports of a school is 4:5. If the number of girls is 212, determine the number of boys participating in the sports.

- a. 256**
- b. 265**
- c. 251**
- d. 263**

ANSWER: 265

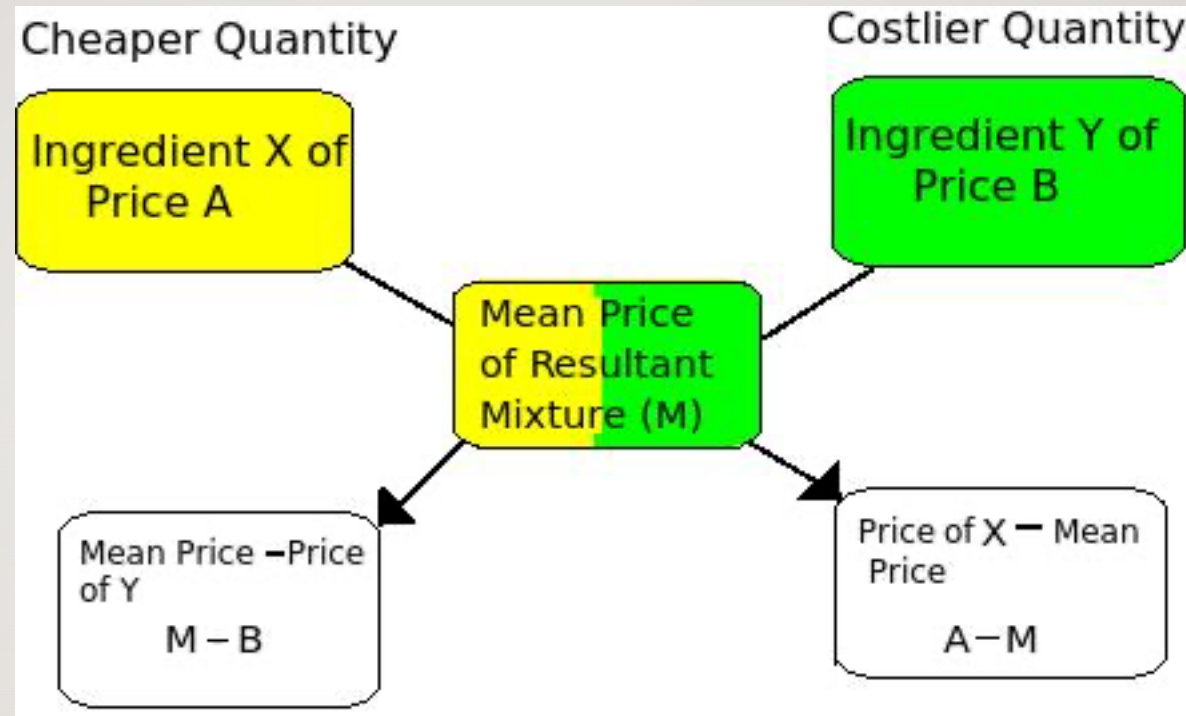
Explanation:

$$\begin{aligned} \text{As per given condition, } \frac{\text{Number of girls}}{\text{Number of boys}} &= \frac{4}{5} \\ \therefore \frac{212}{\text{Number of boys}} &= \frac{4}{5} \\ \therefore \text{Number of boys} &= 265 \end{aligned}$$

ALLIGATION

Alligation: Rule of alligation enables us to find the ratio in which two or more ingredients at a given price must be mixed to produce a resultant mixture of desired price.

Rule of alligation:



$$\text{Ratio} = \frac{(M - B)}{(A - M)} = \frac{(B - M)}{(M - A)}$$

Points to Remember While Using the Rule of Alligation

- The three values alligated should always represent the same variable and should have same units.
- alligation of 3 values of cost gives the ratio in terms of number and vice-versa.
- If two values of cost price and selling price of the mixture are given, then in such cases first calculate the cost price of the mixture and then alligate the 3 values of cost
- A and B represent concentration if the numerical is based on mixing of solutions.

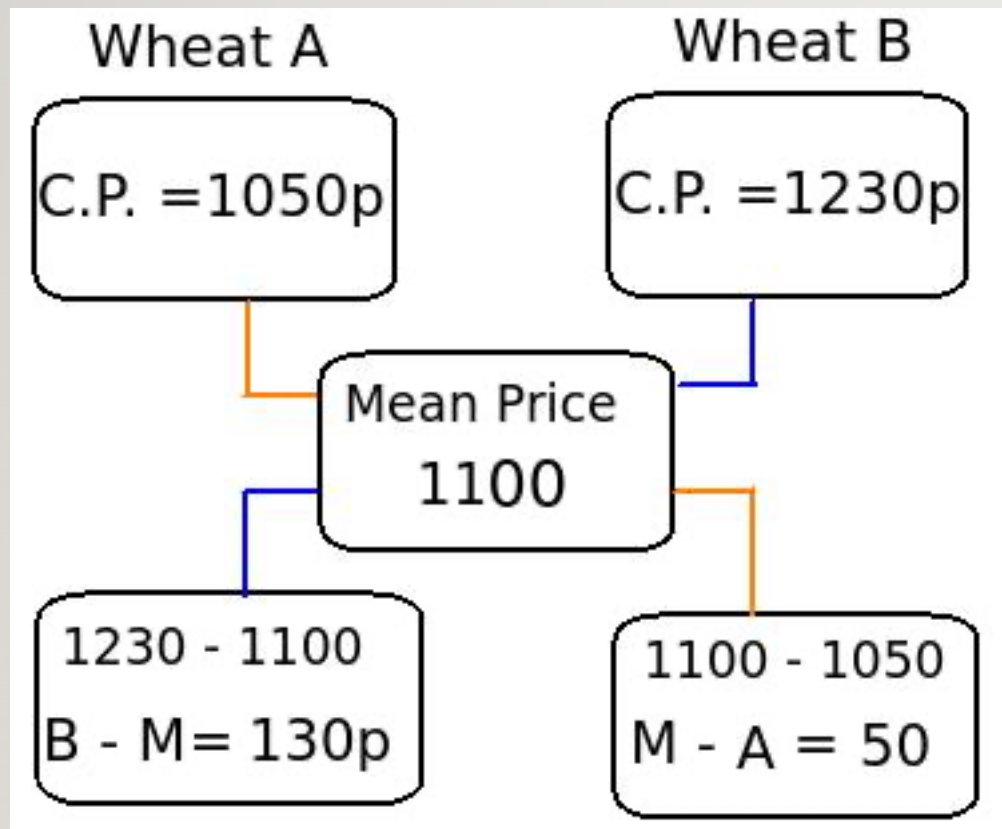
Practice Questions Alligations

1. In what ratio must wheat A at Rs. 10.50 per kg be mixed with wheat B at Rs. 12.30 per kg, so that the mixture be worth of Rs. 11 per kg?

- a. 13 : 5
- b. 18 : 3
- c. 17 : 5
- d. 11 : 5

Correct option :(a)

Convert Rs into paise, to make the calculation easy



$$(B - M)$$

$$(M - A)$$

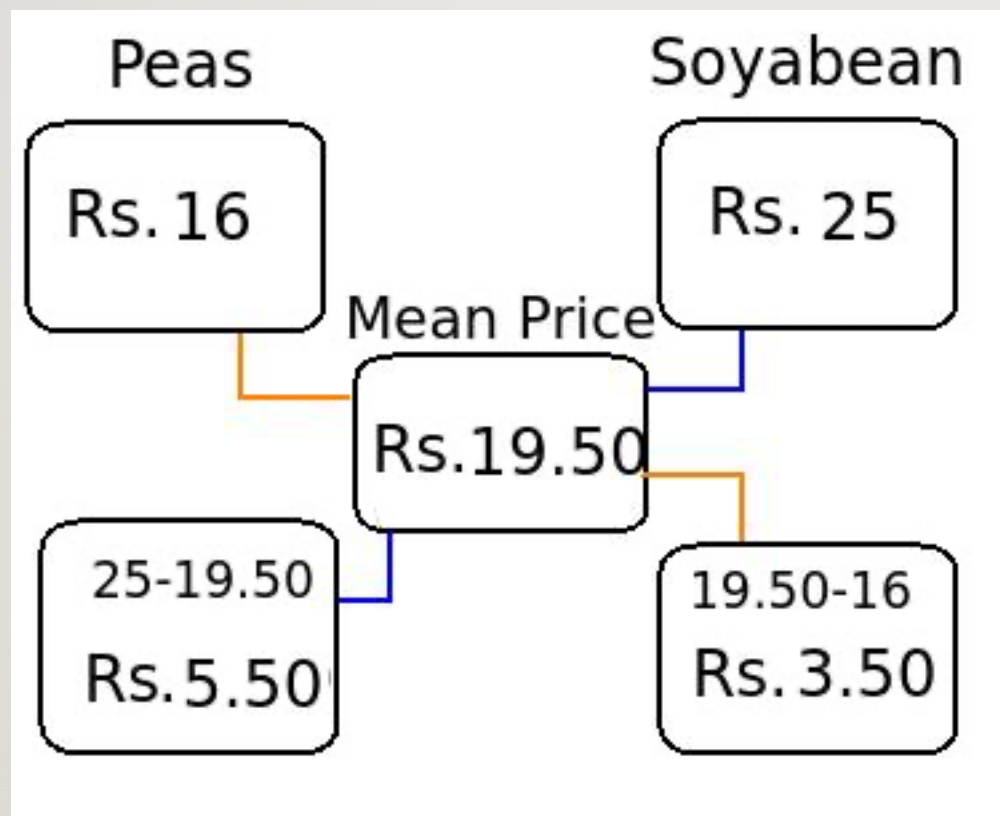
The required ratio = $130 : 50 = 13 : 5$

2. In what ratio must a shopkeeper mix Peas and Soybean of Rs. 16 and Rs. 25 per kg respectively, so as to obtain a mixture of Rs. 19.50 ?

- a. 9 : 5
- b. 7 : 5
- c. 11 : 7
- d. 12 : 8

Correct option: (c)

Use rule of alligation, to determine the ratio



The required ratio of Soybean and Peas = $5.50 : 3.50 = 11 : 7$

3. Chaman has two big cans of wine and water mixture. Chaman mixes the contents of both the cans in a big container. The new mixture has half water and half wine. In what quantity did Chaman mix contents of Can 1 and 2 if Can 2 has wine to water ratio of 2:3 and Can 1 has wine to water ratio 5:3?

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- a. 5:3
- b. 4:5
- c. 5:4
- d. 2:5

ANSWER: 4:5

Explanation:

Can 1 wine : water = 5:3

$$\text{So amount of wine} = \frac{5}{5+3} = \frac{5}{8}$$

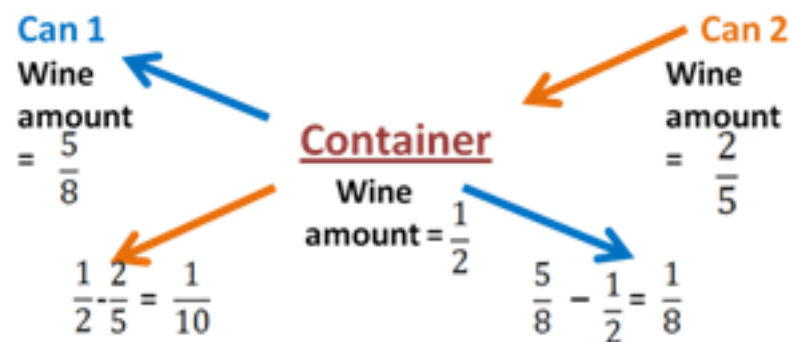
Can 2 wine : water = 2:3

$$\text{So amount of wine} = \frac{2}{2+3} = \frac{2}{5}$$

After mixing Wine Quantity = Water Quantity

\therefore Wine : Water = 1:1

$$\text{So amount of wine} = \frac{1}{1+1} = \frac{1}{2}$$



$$\text{Ratio of contents of Can 1 and Can 2} = \frac{1}{10} : \frac{1}{8} = 8:10 = 4:5$$

4. A milkman had water and milk mixture in a can with water to milk ratio 5:7. He accidentally spills 9 liters of the mixture. He then fills the can with water equal in quantity to spilled mixture. This makes the water to milk ratio 9:7. How much milk did the can initially have?

- a. 21 liters**
- b. 24 liters**
- c. 16 liters**
- d. 20 liters**

ANSWER: 21 liters

Explanation:

Approach 1 – Direct Solving

Here milk : water = 7:5

That means quantity of milk must be 7 or multiple of 7

Only option A i.e. 21 is multiple of 7.

You can check by putting 21 as quantity of milk

Approach 2 – By solving

Water to Milk = 5:7

Let common factor be K, so water = 5K liters and Milk = 7K liters

Now, when we remove 9 liters mixture,

$$\text{Milk removed} = \frac{7}{7+5} \times 9 = 5.25$$

$$\text{Water removed} = \frac{5}{7+5} \times 9 = 3.75$$

Now, 9 liter water is added,

$$\text{So Water quantity} = 5K - 3.75 + 9 = 5K + 5.25$$

$$\text{Milk quantity} = 7K - 5.25$$

$$\text{And, } \frac{\text{Water quantity}}{\text{Milk quantity}} = \frac{9}{7}$$

$$\therefore \frac{5K + 5.25}{7K - 5.25} = \frac{9}{7}$$

$$\therefore K = 3$$

$$\therefore \text{Amount of milk initially} = 7k = 7 \times 3 = 21 \text{ liters}$$

5. Rohit buys some rice at Rs. 10.40 per kg. He mixes it with some rice having price Rs. 8.8 per kg. The final mixture becomes 15kg in weight and with total worth Rs. 146.40. What is the quantity of rice priced at Rs. 8.8 per kg?

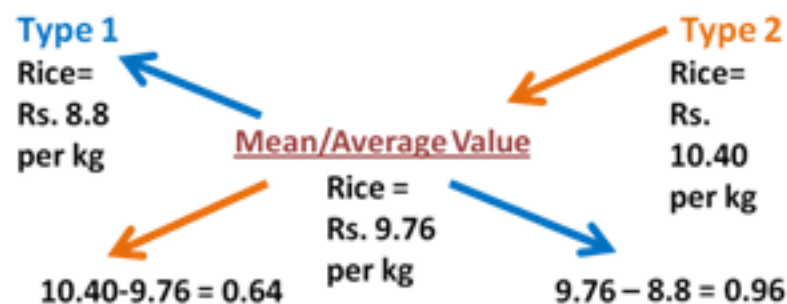
- a. 9 kg
- b. 10 kg
- c. 5 kg
- d. 6 kg

ANSWER: 6 kg

Explanation:

Price of 15kg final mixture = Rs. 146.40

So, **Price of 1 kg final mixture** = $146.40/15 = \text{Rs. } 9.76$



So, **ratio of quantities of type 1 rice to type 2 rice** = $0.64 : 0.96 = 2:3$

Now total mixture is of 15 kg.

Let common factor be K, so rice quantities would be 2K and 3k

$$\therefore 2K + 3K = 15$$

$$\therefore K = 3$$

Quantity of rice priced at Rs. 8.8 per kg = $2K = 2 \times 3 = 6\text{kg}$