PERMUTATIONS AND COMBINATIONS

Permutation formula:

Permutation is defined as arrangement of r things that can be done out of total n things. This is denoted by $^{n}\boldsymbol{P}_{r}$ *which is equal to n!/(n-r)!*

Combination formula

Combination is defined as selection of r things that can be done out of total n things. This is denoted by $^{n}C_{r}$ *which is equal to n!/r!(n-r)!*

As per the Fundamental Principle of Counting, if a particular thing can be done in m ways and another thing can be done in n ways, then either one of the two can be done in m + n ways and both of them can be done in m × n ways.

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Example 1: *How many four-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6 (Repetition of digits not allowed)?*

Solution: *Thousand's place can be filled in 6 ways. Hundred's place can be filled in 5 ways. Ten's place can be filled in 4 ways. Unit's place can be filled in 3 ways. So, using the Fundamental Principle of Counting, we get the answer as $6 \times 5 \times 4 \times 3 = 360$. Or using the formula of Permutations, we need to arrange 4 digits out of total 6 digits. This can be done in* $\mathbf{{}^6P_4 = 360}$ ***ways.***

Example 2: *There are 10 questions in an exam. In how many ways can a person attempt at least one question?*

Solution: *A person can attempt 1 question or 2 questions or .....till all 10 questions. One question out of ten questions can be attempted in* $^{10}C_1 = 10$ *ways. Similarly, two questions out of ten questions can be attempted in* $^{10}C_2 = 45$ *ways. Going ahead by the same logic, all ten questions can be attempted in* $^{10}C_{10} = 1$ *way. Hence the total number of ways = 10 + 45 + 120 +.....10 + 1 = 1023 ways (Using the formula of Combination).*

Alternate Method:

Or some logic can be applied: Every question has 2 options, either it is attempted or not. Going ahead with this logic, since there are 10 questions, and each question has 2 options, so total number of cases = $2^{10} = 1024$*. But this count includes one case in which no question is attempted. This is the violation of the information given. So this case needs to be subtracted. Hence the total number of cases would be 1024 - 1 = 1023.*

Questions in Permutations and Combinations

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16. There are 30 people in a group. If all shake hands with one another , how many handshakes are possible?

a. 870
b. 435
c. 30!
d. 29! + 1

ANSWER: 435

Explanation:

There are 30 people. A handshake needs 2 people.

This simply means in how many ways 2 people can be selected out of 30.

So the answer is ${}^{30}C_2$

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$\therefore {}^{30}C_2 = \frac{30!}{2!(30-2)!} = \frac{30 \times 29}{2} = 435 = \textbf{Number of handshakes}$$

Tip:

If there are n people and they shake hands only once with each other, then,

$$\textbf{Number of handshakes} = {}^{n}C_2 = \frac{n(n-1)}{2}$$

17. In a room there are 2 green chairs, 3 yellow chairs and 4 blue chairs. In how many ways can Raj choose 3 chairs so that at least one yellow chair is included?

a. 3
b. 30
c. 64
d. 84

ANSWER: 64

Explanation:

At least 1 yellow chair means 'Total ways – No yellow chairs.'

Tip:

SELECT = Combination $= {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

SELECT and ARRANGE = Permutation $= {}^{n}P_{r} = \frac{n!}{(n-r)!}$

Total ways to select 3 chairs from the total (2+3+4=) 9 chairs $= {}^{9}C_{3}$

$${}^{9}C_{3} = \frac{9!}{3!6!} = \textbf{84 ways}$$

Now, we do not want even one yellow chair. So,

Now, we should select 3 chairs from 6 chairs (2 green and 4 blue) $= {}^{6}C_{3}$

$${}^{6}C_{3} = \frac{6!}{3!3!} = \textbf{20 ways}$$

Ways to select at least 1 yellow chair = 84-20 = 64 ways

18. 17 students are present in a class. In how many ways, can they be made to stand in 2 circles of 8 and 9 students?

a. $^{17}C_9$ x 9! X 8!
b. $^{17}C_9$ x 8! X 7!
c. 8! X 7!
d. $^{17}C_8$ x 8! X 9!

ANSWER: $^{17}C_9$ x 8! X 7!

Explanation:

Here, we first have to select 9 students from 17 students.

Tip:

SELECT = Combination $= {}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$

SELECT and ARRANGE = Permutation $= {}^{n}P_{r} = \dfrac{n!}{(n-r)!}$

Select = Combination

∴ Select 9 students $= {}^{17}C_9$

Tip:

We can arrange 'n' things in 'n!' ways.
But if they are to be arranged in a circle,
then we can arrange them in (n-1)! ways.

Arrange 9 students in circle = 9-1 = **8! ways**

17-9 = 8 students remain.

Arrange remaining 8 students in another circle = 8-1 = **7! ways**

∴ Total ways to arrange 17 students in 2 circles = $^{17}C_9$ x 8! X 7!

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19. In Daya's bag there are 3 books of History, 4 books of Science and 2 books of Maths. In how many ways can Daya arrange the books so that all the books of same subject are together?

a. 9
b. 6
c. 8640
d. 1728

ANSWER: 1728

Explanation:

There are **3 types of books** – History, Science and Maths.

These can be arranged in 3! Ways = 3 x 2 x 1 = **6 ways.**

Also in each type, books can be arranged as follows -

3 history books can be arranged in 3! Ways = **6 ways**

4 Science books can be arranged in 4! Ways = 4 x 3 x 2 x 1 = **24 ways**

2 Maths books can be arranged in 2! Ways = **2 ways**

Total ways = 6 x 6 x 24 x 2 = **1728**

20. 4 members form a group out of total 8 members.
(i) In how many ways it is possible to make the group if two particular members must be included.
(ii) In how many ways it is possible to make the group if two particular members must not be included?

a. 15 and 360
b. 15 and 15
c. 30 and 360
d. 360 and 360

ANSWER: 15 and 15

Explanation:

Tip:

SELECT = Combination $= {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

SELECT and ARRANGE = Permutation $= {}^{n}P_{r} = \frac{n!}{(n-r)!}$

(1) Compulsorily include 2 particular members.

When we include these 2 members, we are left with 4-2 = 2 spots in the group

Also, number of members which remain are 8-2 = 6

So now we need to select 2 people out of 6.

$${}^{6}C_{2} = \frac{6!}{2!4!} = \mathbf{15 = number\ of\ ways}$$

(2) Compulsorily exclude 2 particular members.

When we exclude these 2 members, we are left with 8-2 = 6 members

So now we need to select 4 people out of 6.

$${}^{6}C_{4} = \frac{6!}{4!2!} = \mathbf{15 = number\ of\ ways}$$

21. A box contains 4 black, 3 red and 6 green marbles. 2 marbles are drawn from the box at random. What is the probability that both the marbles are of the same color?

a. 12/74
b. 24/78
c. 13/78
d. None of these

ANSWER: 24/78

Explanation:

Total marbles in a box = 4 black + 3 red + 6 green marbles = 13 marbles

2 marbles are drawn from 13 marbles at random. Therefore,

$n(S) = {}^{13}C_2 = 78$ ways

Let A be the event that 2 marbles drawn at random are of the same color. Number of cases favorable to the event A is

$n(A) = {}^{4}C_2 + {}^{3}C_2 + {}^{6}C_2 = 6 + 3 + 15 = 24$

Therefore, by definition of probability of event A,

$P(A) = n(A)/n(S) = 24/78$

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22. A box contains 2 white, 3 black and 5 red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw?

a. 29
b. 36
c. 48
d. 85
e. None of these

ANSWER: 85

Explanation:

Total balls in a box = 2W + 3B + 5R = 10 balls.

Therefore total number of ways of drawing 3 balls from 10 balls is $^{10}C_3$ = 120 ways ....(1)

This includes all types of color combinations of white, black and red.

Now total balls in a box of white and red color = 2W + 5R = 7 balls.

Therefore total number of ways of drawing 3 balls from 7 balls is $^{7}C_3$ = 35 ways ....(2)

This includes all types of color combinations of white and red only.

Therefore from (1) and (2), we get the total number of ways of drawing 3 balls, which includes at least one black ball = 120 – 35 = 85 ways.

23.A person can go from place "P" to "Q" by 7 different modes of transport, but is allowed to return back to "P" by any mode other than the one used earlier. In how many different ways can he complete the entire journey?

a. 42
b. 30
c. 11
d. 5^6

ANSWER: 42

Explanation:

The person can travel from “P” to “Q” in any of the 7 different modes of transport. However, while returning, he cannot use the mode which he used earlier. Thus, while returning, he has the option of only 6 different modes of transport. Hence, the total number of ways would be 7 * 6 = 42.

24. How many words can be formed by using all letters of word ALIVE.

a. 86
b. 95
c. 105
d. 120

ANSWER: 120

Explanation:

The word ALIVE contains 5 different letters

Therefore,

Required number of words = $^5p_5$ = 5!

= (5*4*3*2*1) = 120

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25. In how many different ways can the letters of the word 'GEOMETRY' be arranged so that the vowels always come together?

a. 720
b. 4320
c. 2160
d. 40320

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ANSWER: 4320
Explanation:

There are in all 8 letters in the given word of which 3 are vowels. As the vowels should always be together, considering the 3 vowels as one letter, there are in all 6

letters which can be arranged in 6! ways = 720
Also the 3 vowels can be arranged in 3! ways = 6
Total number of arrangements = 720*6 = 4320

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