2-D Viewing

THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

Outline

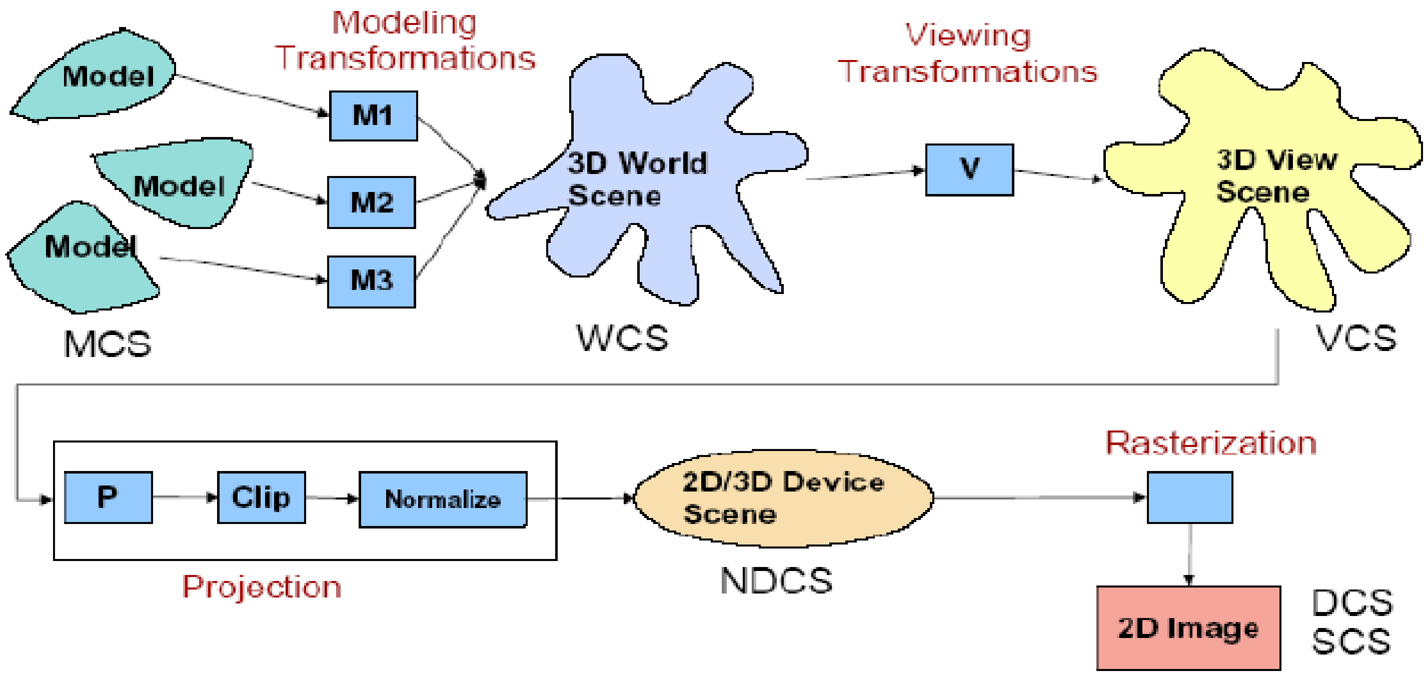
- Viewing
- Viewing Transformation
- Windows to Viewport Transformation

Viewing

- A graphics package allows a user to specify which part of a defined picture is to be displayed and where that part is to be placed on the display device
- 2D viewing deals with the procedures for displaying views of a two-dimensional picture on an output device
  - A view for a 2D picture is selected by specifying a region of the xy plane that contains the total picture or any part of it
  - Picture parts within the selected areas are then mapped onto specified areas of the device coordinates

Viewing Transformation

2-D Viewing Pipeline



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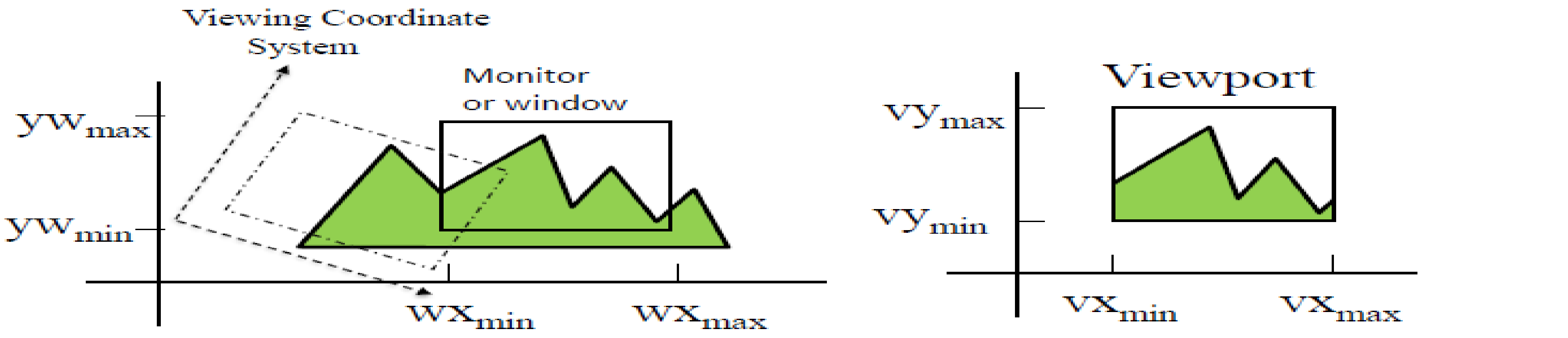
Viewing Transformation(Cont.….)

Viewing transformation

- ➢ Master coordinate system, commonly referred to as the world coordinate system
  - ✓ Clipping window: What do we want to see?
  - ✓ Viewport: Where do we want to see it?



a) World Coordinate System   b) Device Coordinate System

- The mapping of a two-dimensional, world-coordinate scene description to device coordinates is called a **2D viewing transformation**
  - *window-to-viewport transformation* or the *windowing transformation*

Windows to Viewport Transformation

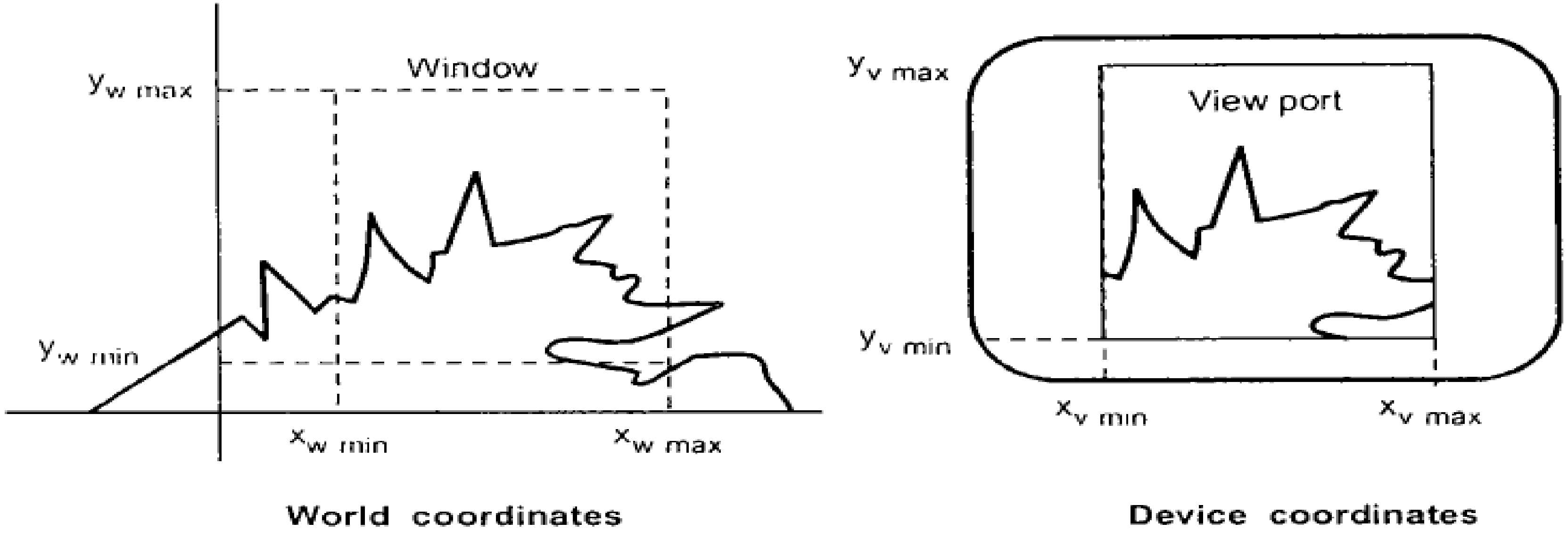
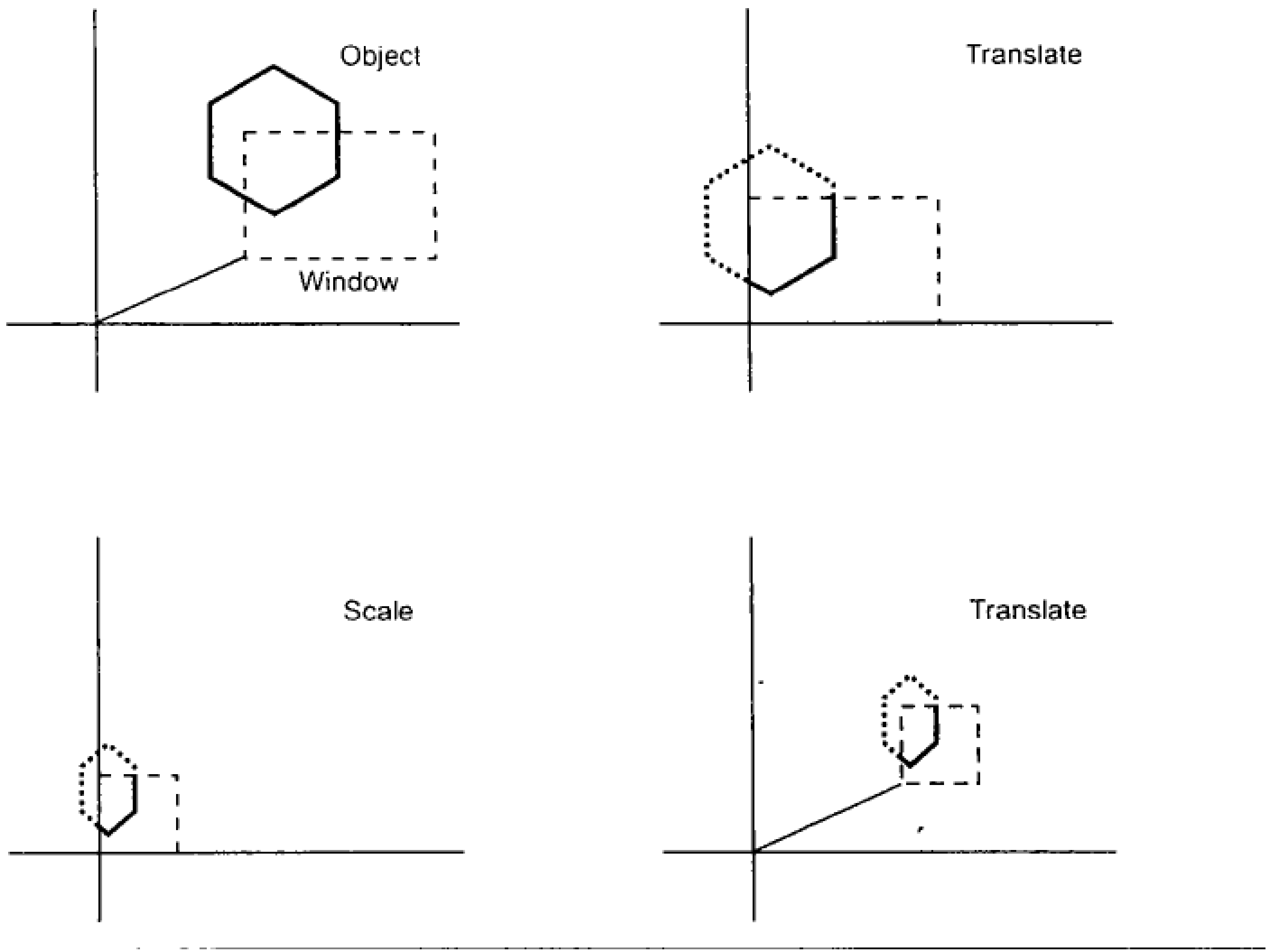


Fig. 5.5 Window and viewport

1. The object together with its window is translated until the lower left corner of the window is at the origin.
2. Object and window are scaled until the window has the dimensions of the viewport.
3. Translate the viewport to its correct position on the screen.

This is illustrated in Fig.5.6.

Windows to Viewport Transformation(Cont.....)



Windows to Viewport Transformation(Cont.....)

The transformation matrices for individual transformation are as given below :

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -X_{w\,min} & -y_{w\,min} & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{where} \quad S_x = \frac{x_{v\,max} - x_{v\,min}}{x_{w\,max} - x_{w\,min}}$$

$$S_y = \frac{y_{v\,max} - y_{v\,min}}{y_{w\,max} - y_{w\,min}}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_{v\,min} & y_{v\,min} & 1 \end{bmatrix}$$

The overall transformation matrix for W is given as

$$W = T \cdot S \cdot T^{-1}$$

Windows to Viewport Transformation(Cont.....)

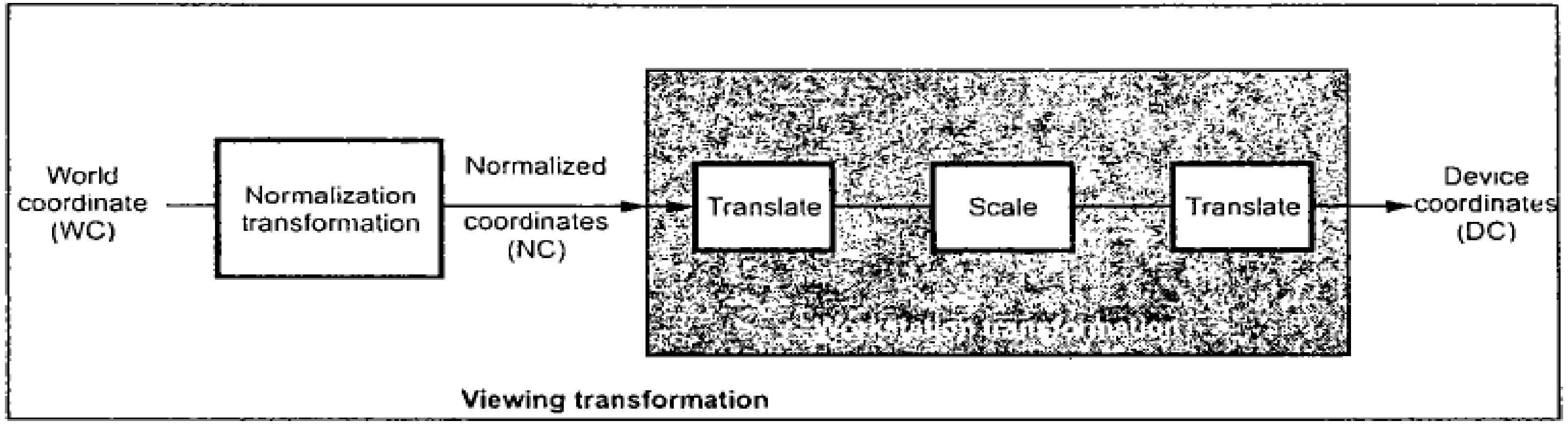
The overall transformation matrix for W is given as

$$W = T \cdot S \cdot T^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_{w\,min} & -y_{w\,min} & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_{v\,min} & y_{v\,min} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ x_{v\,min} - x_{w\,min} \cdot S_x & y_{v\,min} - y_{w\,min} \cdot S_y & 1 \end{bmatrix}$$

The Fig. 5.7 shows the complete viewing transformation.



Windows to Viewport Transformation(Cont.....)

Ex. 5.1 : *Find the normalization transformation window to viewpoint, with window, lower left corner at (1, 1) and upper right corner at (3, 5) onto a viewpoint with lower left corner at (0, 0) and upper right corner at (1/2, 1/2).*

Sol. : Given : Coordinates for window

$$x_{w\,min} = 1 \qquad y_{w\,min} = 1$$
$$x_{w\,max} = 3 \qquad y_{w\,max} = 5$$

Coordinates for view port

$$x_{v\,min} = 0 \qquad y_{v\,min} = 0$$
$$x_{v\,max} = 1/2 = 0.5 \qquad y_{v\,max} = 1/2 = 0.5$$

We know that,

$$S_x = \frac{x_{v\,max} - x_{v\,min}}{x_{w\,max} - x_{w\,min}}$$
$$= \frac{0.5 - 0}{3 - 1}$$
$$= 0.25$$

Windows to Viewport Transformation(Cont.….)

and $$S_y = \frac{y_{v\,max} - y_{v\,min}}{y_{w\,max} - y_{w\,min}}$$

$$= \frac{0.5 - 0}{5 - 1}$$

$$= 0.125$$

We know that transformation matrix is given as

$$T \cdot S \cdot T^{-1} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ x_{v\,min} - x_{w\,min} S_x & y_{v\,min} - y_{w\,min} S_y & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.125 & 0 \\ 0 - (1 \times 0.25) & 0 - (1 \times 0.125) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.125 & 0 \\ -0.25 & -0.125 & 1 \end{bmatrix}$$

Windows to Viewport Transformation(Cont.):Other representation

$$\frac{wx - wx_{\min}}{wx_{\max} - wx_{\min}} = \frac{vx - vx_{\min}}{vx_{\max} - vx_{\min}} \qquad \text{and} \qquad \frac{wy - wy_{\min}}{wy_{\max} - wy_{\min}} = \frac{vy - vy_{\min}}{vy_{\max} - vy_{\min}}$$

Thus

$$\begin{cases} vx = \dfrac{vx_{\max} - vx_{\min}}{wx_{\max} - wx_{\min}}(wx - wx_{\min}) + vx_{\min} \\ vy = \dfrac{vy_{\max} - vy_{\min}}{wy_{\max} - wy_{\min}}(wy - wy_{\min}) + vy_{\min} \end{cases}$$

Since the eight coordinate values that define the window and the viewport are just constants, we can express these two formulas for computing (vx, vy) from (wx, wy) in terms of a translate–scale–translate transformation N

$$\begin{pmatrix} vx \\ vy \\ 1 \end{pmatrix} = N \cdot \begin{pmatrix} wx \\ wy \\ 1 \end{pmatrix}$$

where

$$N = \begin{pmatrix} 1 & 0 & vx_{\min} \\ 0 & 1 & vy_{\min} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \dfrac{vx_{\max} - vx_{\min}}{wx_{\max} - wx_{\min}} & 0 & 0 \\ 0 & \dfrac{vy_{\max} - vy_{\min}}{wy_{\max} - wy_{\min}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -wx_{\min} \\ 0 & 1 & -wy_{\min} \\ 0 & 0 & 1 \end{pmatrix}$$

Windows to Viewport Transformation(Cont.....)

Let

$$s_x = \frac{vx_{max} - vx_{min}}{wx_{max} - wx_{min}} \qquad \text{and} \qquad s_y = \frac{vy_{max} - vy_{min}}{wy_{max} - wy_{min}}$$

Express window-to-viewport mapping in the form of a composite transformation matrix.

SOLUTION

$$N = \begin{pmatrix} 1 & 0 & vx_{min} \\ 0 & 1 & vy_{min} \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -wx_{min} \\ 0 & 1 & -wy_{min} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} s_x & 0 & -s_x wx_{min} + vx_{min} \\ 0 & s_y & -s_y wy_{min} + vy_{min} \\ 0 & 0 & 1 \end{pmatrix}$$

Windows to Viewport Transformation(Cont.....)

Ques. Find the complete viewing transformation that maps a window in world coordinates with x extent 1 to 10 and y extent 1 to 10 onto a viewport with x extent $\frac{1}{4}$ to $\frac{3}{4}$ and y extent 0 to $\frac{1}{2}$ in normalized device space, and then maps a workstation window with x extent $\frac{1}{4}$ to $\frac{1}{2}$ and y extent $\frac{1}{4}$ to $\frac{1}{2}$ in the normalized device space into a workstation viewport with x extent 1 to 10 and y extent 1 to 10 on the physical display device.

Sol. From Prob. 5.1, the parameters for the normalization transformation are $wx_{\min} = 1$, $wx_{\max} = 10$, $wy_{\min} = 1$, $wy_{\max} = 10$, and $vx_{\min} = \frac{1}{4}$, $vx_{\max} = \frac{3}{4}$, $vy_{\min} = 0$, and $vy_{\max} = \frac{1}{2}$. Then

$$s_x = \frac{1/2}{9} = \frac{1}{18} \qquad s_y = \frac{1/2}{9} = \frac{1}{18}$$

Windows to Viewport Transformation(Cont.....)

and

$$N = \begin{pmatrix} \frac{1}{18} & 0 & \frac{7}{36} \\ 0 & \frac{1}{18} & -\frac{1}{18} \\ 0 & 0 & 1 \end{pmatrix}$$

The parameters for the workstation transformation are $wx_{\min} = \frac{1}{4}$, $wx_{\max} = \frac{1}{2}$, $wy_{\min} = \frac{1}{4}$, $wy_{\max} = \frac{1}{2}$, and $vx_{\min} = 1$, $vx_{\max} = 10$, $vy_{\min} = 1$, and $vy_{\max} = 10$. Then

$$s_x = \frac{9}{1/4} = 36 \qquad s_y = \frac{9}{1/4} = 36$$

and

$$W = \begin{pmatrix} 36 & 0 & -8 \\ 0 & 36 & -8 \\ 0 & 0 & 1 \end{pmatrix}$$

The complete viewing transformation V is

$$V = W \cdot N = \begin{pmatrix} 36 & 0 & -8 \\ 0 & 36 & -8 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{18} & 0 & \frac{7}{36} \\ 0 & \frac{1}{18} & -\frac{1}{18} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & -10 \\ 0 & 0 & 1 \end{pmatrix}$$

Windows to Viewport Transformation(Cont.....)

Ques. Find the normalization transformation N which uses the rectangle $A(1, 1)$, $B(5, 3)$, $C(4, 5)$, $D(0, 3)$ as a window [Fig. 5-16(*a*)] and the normalized device screen as a viewport [Fig. 5-16(*b*)].

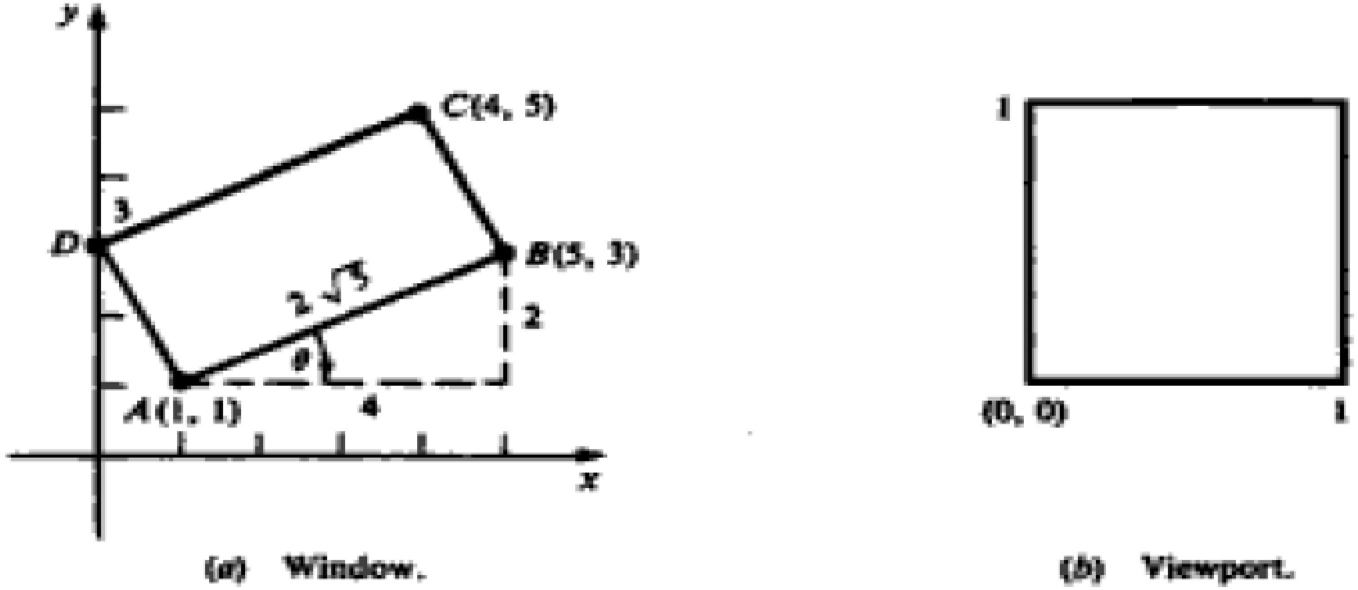


Fig. 5-16

Sol.

Try to solve the problem by your self

Summary

- Objects inside the clipping window are mapped to the viewport; viewport is then positioned within display window
- Changing the position of a viewport, we can view objects at different positions on the display area of an output device
- Changing the size of viewports, we can change the size and proportions of displayed objects; zoom in and zoom out effects
- A section of a two-dimensional scene that is selected for display is called a clipping window because all parts of the scene outside the selected section are "clipped" off

Resources

- https://csis.pace.edu/
- https://www.cg.tuwien.ac.at/courses/CG1
- https://web.iitd.ac.in/~achawla/public_html/429/transformations