

Liang Barsky Algorithm for 2-D Line Clipping



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Outline

- Liang Barsky Algorithm
- Summary

Liang Barsky Algorithm

5.3.2.4 Liang-Barsky Line Clipping Algorithm

In the last section we have seen Cyrus-Beck line clipping algorithm using parametric equations. It is more efficient than Cohen-Sutherland algorithm. Liang and Barsky have developed even more efficient algorithm than Cyrus-Beck algorithm using parametric equations. These parametric equations are given as

$$x = x_1 + t\Delta x$$

$$y = y_1 + t\Delta y, \quad 0 \leq t \leq 1$$

where

$$\Delta x = x_2 - x_1 \text{ and } \Delta y = y_2 - y_1$$

The point clipping conditions (Refer section 5.3.1) for Liang-Barsky approach in the parametric form can be given as

$$x_{wmin} \leq x_1 + t\Delta x \leq x_{wmax} \text{ and}$$

$$y_{wmin} \leq y_1 + t\Delta y \leq y_{wmax}$$

Liang-Barsky express these four inequalities with two parameters p and q as follows :

$$tp_i \leq q_i \quad i = 1, 2, 3, 4$$

where parameters p and q are defined as

$$p_1 = -\Delta x, \quad q_1 = x_1 - x_{wmin}$$

$$p_2 = \Delta x, \quad q_2 = x_{wmax} - x_1$$

$$p_3 = -\Delta y, \quad q_3 = y_1 - y_{wmin}$$

$$p_4 = \Delta y, \quad q_4 = y_{wmax} - y_1$$

Liang Barsky Algorithm(Cont.....)

Following observations can be easily made from above definitions of parameters p and q .

- If $p_1 = 0$: Line is parallel to left clipping boundary.
- If $p_2 = 0$: Line is parallel to right clipping boundary.
- If $p_3 = 0$: Line is parallel to bottom clipping boundary.
- If $p_4 = 0$: Line is parallel to top clipping boundary.
- If $p_i = 0$, and for that value of i ,
 - If $q_i < 0$: Line is completely outside the boundary and can be eliminated.
 - If $q_i \geq 0$: Line is inside the clipping boundary.
- If $p_i < 0$: Line proceeds from outside to inside of the clipping boundary.
- If $p_i > 0$: Line proceeds from inside to outside of the clipping boundary.

Therefore, for nonzero value of p_i , the line crosses the clipping boundary and we have to find parameter t . The parameter t for any clipping boundary i can be given as

$$t = \frac{q_i}{p_i} \quad i = 1, 2, 3, 4$$

Liang-Barsky algorithm calculates two values of parameter t : t_1 and t_2 that define that part of the line that lies within the clip rectangle. The value of t_1 is determined by checking

Liang Barsky Algorithm(Cont.....)

the rectangle edges for which the line proceeds from the outside to the inside ($p < 0$). The value of t_1 is taken as a largest value amongst various values of intersections with all edges. On the other hand, the value of t_2 is determined by checking the rectangle edges for which the line proceeds from the inside to the outside ($p > 0$). The minimum of the calculated value is taken as a value for t_2 .

Now, if $t_1 > t_2$, the line is completely outside the clipping window and it can be rejected. Otherwise the values of t_1 and t_2 are substituted in the parametric equations to get the end points of the clipped line.

Algorithm

1. Read two endpoints of the line say $p_1 (x_1, y_1)$ and $p_2 (x_2, y_2)$.
2. Read two corners (left-top and right-bottom) of the window, say $(x_{wmin}, y_{wmax}, x_{wmax}, y_{wmin})$
3. Calculate the values of parameters p_i and q_i for $i = 1, 2, 3, 4$ such that

$$p_1 = -\Delta x \quad q_1 = x_1 - x_{wmin}$$

$$p_2 = \Delta x \quad q_2 = x_{wmax} - x_1$$

$$p_3 = -\Delta y \quad q_3 = y_1 - y_{wmin}$$

$$p_4 = \Delta y \quad q_4 = y_{wmax} - y_1$$

Liang Barsky Algorithm(Cont.....)

4. if $p_i = 0$, then
 { The line is parallel to i^{th} boundary.
 Now, if $q_i < 0$ then
 { line is completely outside the boundary, hence
 discard the line segment and goto stop.
 }
 else
 { Check whether the line is horizontal or vertical and accordingly
 check the line endpoint with corresponding boundaries. If line
 endpoint/s lie within the bounded area then use them to draw
 line otherwise use boundary coordinates to draw line. Go to stop.
 }
 }
5. Initialise values for t_1 and t_2 as
 $t_1 = 0$ and $t_2 = 1$
6. Calculate values for q_i/p_i for $i = 1, 2, 3, 4$.
7. Select values of q_i/p_i where $p_i < 0$ and assign maximum out of them as t_1 .
8. Select values of q_i/p_i where $p_i > 0$ and assign minimum out of them as t_2 .
9. If $t_1 < t_2$, then
 { t_1 is the value of t at which the line enters the boundary.
 }

Liang Barsky Algorithm(Cont.....)

9. If ($t_1 < t_2$)
{ Calculate the endpoints of the clipped line as follows :
 $xx_1 = x_1 + t_1 \Delta x$
 $xx_2 = x_1 + t_2 \Delta x$
 $yy_1 = y_1 + t_1 \Delta y$
 $yy_2 = y_1 + t_2 \Delta y$
Draw line (xx_1, yy_1, xx_2, yy_2)
}
10. Stop.

Advantages

1. It is more efficient than Cohen-Sutherland algorithm, since intersection calculations are reduced.
2. It requires only one division to update parameters t_1 and t_2 .
3. Window intersections of the line are computed only once.

Ex. 5.5 Find the clipping coordinates for a line p_1p_2 where $p_1 = (10, 10)$ and $p_2 = (60, 30)$, against window with $(x_{wmin}, y_{wmin}) = (15, 15)$ and $(x_{wmax}, y_{wmax}) = (25, 25)$.

Sol. : Here,

$$x_1 = 10 \quad x_{wmin} = 15$$

$$y_1 = 10 \quad y_{wmin} = 15$$

Liang Barsky Algorithm(Cont.....)

$$x_2 = 60$$

$$x_{wmax} = 25$$

$$y_2 = 30$$

$$y_{wmax} = 25$$

$$p_1 = -50$$

$$q_1 = -5$$

$$p_1/q_1 = 0.1$$

$$p_2 = 50$$

$$q_2 = 15$$

$$p_2/q_2 = 0.3$$

$$p_3 = -20$$

$$q_3 = -5$$

$$p_3/q_3 = 0.25$$

$$p_4 = 20$$

$$q_4 = 15$$

$$p_4/q_4 = 0.75$$

$$t_1 = \max(0.25, 0.1) = 0.25$$

since for these values $p < 0$

$$t_2 = \min(0.3, 0.75) = 0.3$$

since for these values $p > 0$

Here, $t_1 < t_2$ and the endpoints of clipped line are :

$$xx_1 = x_1 + t_1 \Delta x$$

$$= 10 + 0.25 \times 50$$

$$= 22.5$$

$$yy_2 = y_1 + t_1 \Delta y$$

$$= 10 + 0.25 \times 20$$

$$= 15$$

$$xx_2 = x_1 + t_2 \Delta x$$

$$= 10 + 0.3 \times 50$$

$$= 25$$

Liang Barsky Algorithm(Cont.....)

$$x_1 + t\Delta x \geq x_{\min}$$

$$x_1 + t\Delta x \leq x_{\max}$$

$$y_1 + t\Delta y \geq y_{\min}$$

$$y_1 + t\Delta y \leq y_{\max}$$

$$t\Delta x \geq x_{\min} - x_1 \quad -①$$

$$t\Delta x \leq x_{\max} - x_1$$

$$t\Delta y \geq y_{\min} - y_1 \quad -②$$

$$t\Delta y \leq y_{\max} - y_1$$

Multiply eq. ① & ② by -ve sign

$$-t\Delta x \leq x_1 - x_{\min}$$

$$t\Delta x \leq x_{\max} - x_1$$

$$-t\Delta y \leq y_1 - y_{\min}$$

$$t\Delta y \leq y_{\max} - y_1$$

LIANG-BARSKY LINE CLIPPING ALGORITHM

$$x = x_1 + t(x_2 - x_1)$$

$$x = x_1 + t\Delta x$$

$$y = y_1 + t(y_2 - y_1)$$

$$y = y_1 + t\Delta y$$

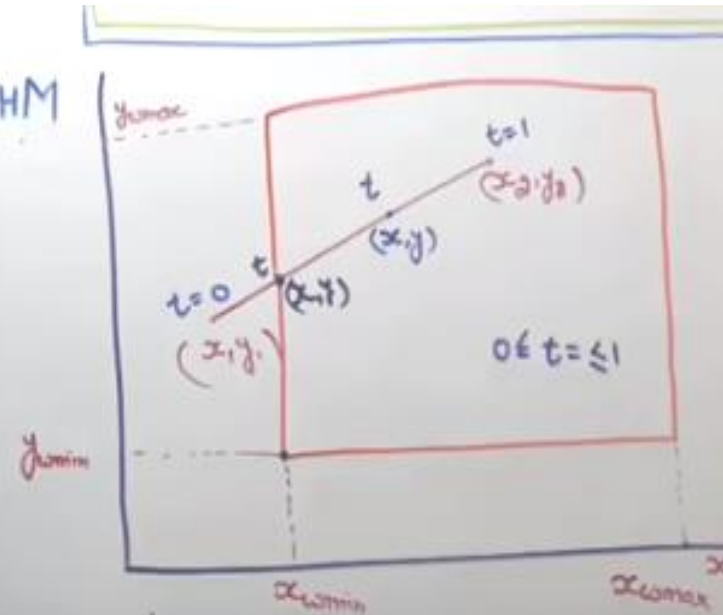
As per point Clipping Algorithm

$$x_{\min} \leq x \leq x_{\max}$$

$$y_{\min} \leq y \leq y_{\max}$$

$$x_{\min} \leq x_1 + t\Delta x \leq x_{\max}$$

$$y_{\min} \leq y_1 + t\Delta y \leq y_{\max}$$



Line Parametric Equation

$$x = (1-t)x_1 + tx_2$$

$$y = (1-t)y_1 + ty_2$$

Liang Barsky Algorithm(Cont.....)

LIANG-BARSKY
LINE CLIPPING ALGORITHM

General Equation form
 $t P_k \leq q_k \quad \{k=1,2,3,4\}$
 $P_1 = -\Delta x \quad q_1 = x_1 - x_{wmin}$
 $P_2 = \Delta x \quad q_2 = x_{wmax} - x_1$
 $P_3 = -\Delta y \quad q_3 = y_1 - y_{wmin}$
 $P_4 = \Delta y \quad q_4 = y_{wmax} - y_1$

$t = \min(1, \frac{q_k}{P_k})$
 → Now check $t_1 > t_2$ line
 Completely outside & Reject it
 IF $t_1 < t_2$ then we will use it
 $x = x_1 + t \Delta x$
 $y = y_1 + t \Delta y$

IF $P_k = 0$ Line parallel
 IF $q_k < 0$ Line outside
 IF P_k other than Nonzero
 IF $P_k < 0$ then t_1 Value find
 the new Value of t
 $t = \max(0, \frac{q_k}{P_k})$
 else mean $P_k > 0$
 then t_2 find

Line Parametric Equation
 $x = (1-t)x_1 + tx_2$
 $y = (1-t)y_1 + ty_2$
 $x = x_1 + t \Delta x$
 $y = y_1 + t \Delta y$

Liang Barsky Algorithm(Cont.....)

General Equation form **NUMERICAL LIANG-BARSKY LINE CLIPPING ALGORITHM**

$t P_k \leq q_k \quad \{k=1,2,3,4\}$

$P_1 = -70 \quad q_1 = x_1 - x_{\min} = -10$
 $P_2 = 70 \quad q_2 = x_{\max} - x_1 = 80$
 $P_3 = -60 \quad q_3 = y_1 - y_{\min} = 10$
 $P_4 = 60 \quad q_4 = y_{\max} - y_1 = 40$

$t = \min(1, \frac{q_k}{P_k})$

Now check $t_1 > t_2$ line
 Completely outside & Reject it: y_{\min}

If $t_1 < t_2$ then we will use it

If $P_k = 0$ Line parallel
 If $q_k < 0$ Line outside
 If P_k other than Nonzero
 If $P_k < 0$ then t_1 Value find
 the new Value of t
 else mean $P_k > 0$
 then t_2 find

for $k=1,2,3,4$

$x = x_1 + t \Delta x$
 $y = y_1 + t \Delta y$

$P_k < 0$
 P_1, P_3
 $t_1 = \max(0, \frac{10}{-70}, \frac{10}{-60})$
 $t_1 = 1/7$
 $x = 10 + \frac{1}{7} \times 70 = 20$
 $y = 30 + \frac{1}{7} \times 60 = 38.57$

$P_k > 0$
 P_2, P_4
 $t_2 = \min(1, \frac{80}{70}, \frac{40}{60})$
 $t_2 = \frac{2}{3}$
 $x = 10 + \frac{2}{3} \times 70 = 56.67$
 $y = 30 + \frac{2}{3} \times 60 = 70$

Window A(20,20), B(90,20), C(90,70)
 D(20,70), Line $P_1(10,30), P_2(80,90)$
 $x_{\min}=20, y_{\min}=20, x_{\max}=90, y_{\max}=70$
 $x_1=10, y_1=30, \Delta x=80-10=70, \Delta y=90-30=60$
 $x_2=80, y_2=90$

Summary

- It is more efficient than cohen-sutherland algorithm, since intersection calculations are reduced
- Require one division to update parameters t_1 and t_2

Resources

- [https:// iq.opengenus.org/liang-basrky-line-clipping-algorithm/](https://iq.opengenus.org/liang-basrky-line-clipping-algorithm/)
- [https:// https://www.tutorialandexample.com/line-clipping/](https://www.tutorialandexample.com/line-clipping/)
- <https://www.cs.helsinki.fi/group/goa/viewing/leikkaus/lineClip.html>