Parametric & Geometric Continuity conditions

THAPAR INSTITUTE
OF ENGINEERING & TECHNOLOGY
(Deemed to be University)

Parametric Continuity Condition

For the smooth transition from one section of a piecewise parametric curve to the next, impose continuity condition at the connection points.

Each section of a spline is described with parametric coordinate functions

$$x = x(u)$$

$$y = y(u) \qquad\qquad u_1 \le u \le u_2$$

$$z = z(u)$$

Zero –order Parametric continuity (C^0)

- Simply means that the curves meet. That is x,y and z evaluated at u2 for the first curve section are equal to the values of x,y and z evaluated at u1 for the next curve section.

First –order Parametric continuity (C^1)

- The first parametric derivatives(tangent lines) of the coordinate functions for two successive curve sections are equal at their joining point.

Second –order Parametric continuity (C^2)

- Both first and second parametric derivatives of the two curve sections are the same at the intersection.

Geometric Continuity Condition

- In Geometric Continuity ,only require parametric derivatives of the two sections to be proportional to each other at their common boundary

Zero –order Geometric continuity (G^0)

- Same as Zero –order parametric continuity. That is the two curves sections must have the same coordinate position at the boundary point.

First –order Geometric continuity (G^1)

- The first parametric derivatives(tangent lines) of the coordinate functions for two successive curve sections are proportional at their joining point.

Second –order Geometric continuity (G^2)

- Both first and second parametric derivatives of the two curve sections are proportional at their boundary.

https://www.youtube.com/watch?v=A3lDRn6jafs

Spline Specification

Three methods for specifying a spline representation.

1. We can state the set of boundary conditions that are imposed on the spline.
2. We can state the matrix that characterizes the spline.
3. We can state the set of blending functions.

- Parametric cubic polynomial representation for the x coordinate of a spline section

$$x(u) = a_x u^3 + b_x u^2 + c_x u + d_x \quad , \qquad 0 \le u \le 1$$

- Boundary condition set on the endpoint coordinates x(0) and x(1) and on first parametric derivatives at the endpoints x'(0) and x'(1).

From the boundary condition, obtain the matrix that characterizes the spline curve.

$$x(u) = [u^3 u^2 u \; 1] \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}$$

$$= U \cdot C$$

where U is the row matrix of powers of parameter u, and C is the coefficient column matrix

We can write the boundary conditions in matrix form and solve for the coefficient matrix C as:

$$C = M_{spline} \cdot M_{geom}$$

where M_{geom} is a four-element column matrix containing the geometric constraint values on the spline

M_{spline} is the 4-by-4 matrix that transforms the geometric constraint values to the polynomial coefficients and provides a characterization for the spline curve.

Matrix M_{geom} contains control-point coordinate values and other geometric constraints that have been specified.

Thus, we can substitute the matrix representation for C into equation to obtain

$$X(u)= U.M_{spline}.M_{geom}$$

The matrix M_{spline}, characterizing a spline representation, sometimes called the basis matrix, is particularly useful for transforming from one spline representation to another.

Finally, to obtain the polynomial representation for coordinate x in terms of the geometric constraint parameters

$$x(u) = \sum_{k=0}^{3} g_k . BF_k(\mathrm{u})$$

where g_k are the constraint parameters such as the control- point coordinates and slope of the curve at the control points

$BF_k(\mathrm{u})$ are the polynomial blending functions.