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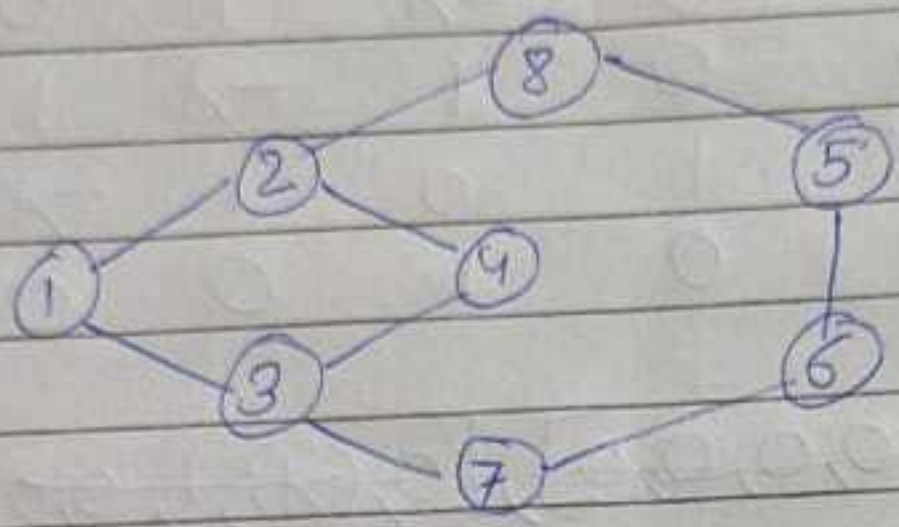
① Calculate Eigenvector for given graph



⇒ A =

0	1	1	0	0	0	0	0
1	0	0	1	0	0	0	0
1	0	0	1	0	0	1	0
0	1	1	0	0	0	1	0
0	0	0	0	0	1	0	1
0	0	0	0	1	0	1	0
0	0	1	1	0	1	0	0
0	1	0	0	1	0	0	0

8×8

⇒ $AX = \lambda X$

Eigenvalue

Eigenvector

⇒ $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\lambda & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\lambda & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -\lambda & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\lambda & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -\lambda & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & -\lambda & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow$ ~~$(-\lambda)\times \| [(-\lambda), 0, 1, 0, 0, 0, 0], [0, (-\lambda), 1, 0, 0, 1, 0],$~~

~~$[1, 1, (-\lambda), 0, 0, 1, 0], [0, 0, 0, (-\lambda), 1, 0, 1], [0, 0, 0, 1, (-\lambda), 1, 0], [0, 1, 1, 0, 1, (-\lambda), 0], [1, 0, 0, 1, 0, 0, (-\lambda)] \|$~~

~~$-1 \times \| [1, 0, 1, 0, 0, 0, 0], [1, (-\lambda), 1, 0, 0, 1, 0],$~~

~~$[0, 1, (-\lambda), 0, 0, 1, 0], [0, 0, 0, (-\lambda), 1, 0, 1], [0, 0, 0, 1$~~

$\Rightarrow (-\lambda)\times$

$$\begin{vmatrix} -\lambda & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\lambda & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -\lambda & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\lambda & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -\lambda & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & -\lambda & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & -\lambda \end{vmatrix}$$

$-1 \times$

$$\begin{vmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & -\lambda & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -\lambda & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\lambda & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -\lambda & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & -\lambda & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\lambda \end{vmatrix}$$

$+1 \times$

$$\begin{vmatrix} 1 & -\lambda & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -\lambda & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\lambda & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -\lambda & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -\lambda & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & -\lambda \end{vmatrix}$$

$$+0 \quad +0 \quad +0 \quad +0 \quad +0 \quad =0$$

$$\Rightarrow (-\lambda)(1+5\lambda-4\lambda^2-12\lambda^3+2\lambda^4+7\lambda^5-\lambda^7) - 1(2\lambda+5\lambda^2-\lambda^3-5\lambda^4+\lambda^6) + 1(\lambda-3\lambda^2-\lambda^3+4\lambda^4-\lambda^6) = 0$$

$$\Rightarrow \lambda^8 - 9\lambda^6 - 2\lambda^5 + 21\lambda^4 + 4\lambda^3 - 13\lambda^2 - 2\lambda = 0$$

$$\Rightarrow \lambda(\lambda-1)(\lambda+1)(\lambda+2)(\lambda^4 - 2\lambda^3 - 4\lambda^2 + 6\lambda + 1) = 0$$

$$\Rightarrow \lambda = \{-2, -1, 0, 1, 0.152, -1.825, 1.385, 2.597\}$$

⇒ For $\lambda = 2.59$

⇒ $A - \lambda I =$

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

-2.59

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

⇒

$$\begin{bmatrix} -2.59 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2.59 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2.59 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -2.59 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -2.59 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2.59 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & -2.59 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & -2.59 \end{bmatrix}$$

⇒ The system associated with the eigenvalue $\lambda = 3.601$

$$\Rightarrow (A - 3.6I) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -0.48 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -0.78 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.96 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -0.99 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1.21 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1.38 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

$\Rightarrow x_1 - 0.48x_8 = 0$

$\Rightarrow x_2 - 0.78x_8 = 0$

$\Rightarrow x_3 - 0.96x_8 = 0$

$\Rightarrow x_4 - 0.99x_8 = 0$

$\Rightarrow x_5 - 1.21x_8 = 0$

$\Rightarrow x_6 - 1.38x_8 = 0$

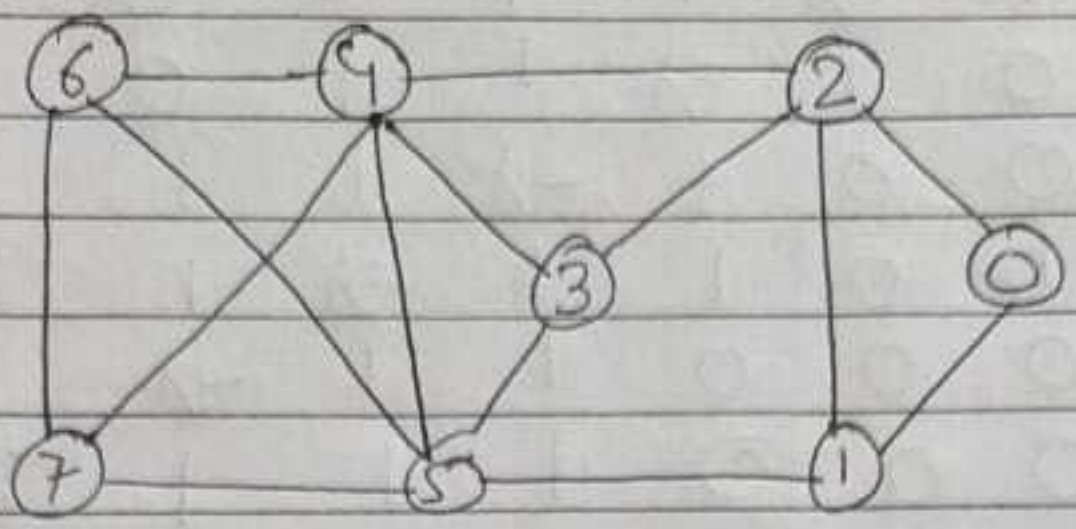
$\Rightarrow x_7 - 1x_8 = 0$

$$\Rightarrow V = \begin{bmatrix} 0.48x_8 \\ 0.78x_8 \\ 0.96x_8 \\ 0.99x_8 \\ 1.21x_8 \\ 1.38x_8 \\ x_8 \\ x_8 \end{bmatrix}$$

Let $x_8 = 1$

$$V = \begin{bmatrix} 1.68 \\ 1.68 \\ 2.68 \\ 2.68 \\ 0.90 \\ 1.34 \\ 2.59 \\ 1 \end{bmatrix}$$

(2)



$$\Rightarrow A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}_{8\times 8}$$

$$\Rightarrow |A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\lambda & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & -\lambda & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\lambda & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\lambda & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & -\lambda & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -\lambda & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (-\lambda)\,x$

$$\begin{vmatrix} -\lambda & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -\lambda & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -\lambda & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\lambda & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & -\lambda & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & -\lambda & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & -\lambda \end{vmatrix}$$

$(-1)\,x$

$$\begin{vmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -\lambda & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -\lambda & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\lambda & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -\lambda & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & -\lambda & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & -\lambda \end{vmatrix}$$

$(+1)\,x$

$$\begin{vmatrix} 1 & -\lambda & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\lambda & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & -\lambda & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & -\lambda & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & -\lambda \end{vmatrix}$$

$+0 \quad +0 \quad +0 \quad +0 \quad +0 \quad +0 \quad =0$

$\Rightarrow (-\lambda)(2-2\lambda-19\lambda^2-14\lambda^3+11\lambda^4+11\lambda^5-\lambda^7) - 1\times(5+9\lambda-5\lambda^2-18\lambda^2 - 9\lambda^4+\lambda^5+\lambda^6) + 1\times(1-3\lambda+3\lambda^2+16\lambda^3+9\lambda^4-\lambda^5-\lambda^6)=0$

$\Rightarrow \lambda^8-13\lambda^6-13\lambda^5+32\lambda^4+53\lambda^3+10\lambda^2-14\lambda-4=0$

$\Rightarrow (\lambda+1)(\lambda^7-\lambda^6-12\lambda^5-\lambda^4+33\lambda^3+20\lambda^2-10\lambda-4)=0$

$\Rightarrow \lambda = \{-1, 0.305, 1.935, 3.601, -2.158, 0.510\}$

$\Rightarrow$ For $\lambda = 3.601$

$\Rightarrow |A-\lambda I| =$

$$\begin{bmatrix} -3.6 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -3.6 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & -3.6 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3.6 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3.6 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & -3.6 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -3.6 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & -3.6 \end{bmatrix}$$

$\Rightarrow$ Now, reduce this matrix

⇒ Now, reduce the matrix

⇒ (A-2.59)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1.68 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1.68 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -2.68 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2.68 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -0.9 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1.34 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.59 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

⇒ $x_1 - 1.68x_8 = 0$
$x_2 - 1.68x_8 = 0$
$x_3 - 2.68x_8 = 0$
$x_4 - 2.68x_8 = 0$
$x_5 - 0.9x_8 = 0$
$x_6 - 1.34x_8 = 0$
$x_7 - 2.59x_8 = 0$

⇒ $V = \begin{bmatrix} 1.68x_8 \\ 1.68x_8 \\ 2.68x_8 \\ 2.68x_8 \\ 0.9x_8 \\ 1.34x_8 \\ 2.59x_8 \\ x_8 \end{bmatrix}$

Let $x_8 = 1$

$V =$	0·48
	0·78
	0·96
	0·99
	1·21
	1·38
	1
	1