7FNCE025 High Frequency Trading
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Week 3 Seminar Questions

1. If $dS = \mu S dt + \sigma S dZ$, and A and n are constants, find the stochastic equations satisfied by
(a) $f(S) = AS$;
(b) $f(S) = S^n$.

2. By expanding df in Taylor series to $\mathcal{O}dt$ and using that $(dZ)^2 = dt$, prove that

$$\int_{t_0}^{t} Z(\tau) dZ(\tau) = \frac{1}{2}\left(Z(t)^2 - Z(t_0)^2\right) - \frac{1}{2}(t - t_0).$$

3. Consider the general stochastic differential equation

$$dG = A(G,t)dt + B(G,t)dZ.$$

Use Itô's Lemma to show that it is theoretically possible to find a function $f(G)$ which itself follows a random walk but with zero drift.

4. There are n assets satisfying the following stochastic differential equations

$$dS_i = \mu_i S_i dt + \sigma_i S_i dZ_i \qquad \text{for } i = 1, \ldots, n.$$

Recall that the Wiener process dZ_i satisfies

$$\mathbb{E}[dZ_i] = 0, \qquad dZ_i^2 = dt$$

as usual, but the asset price changes are correlated with

$$dZ_i dZ_j = \rho_{ij} dt$$

where $-1 \leq \rho_{ij} = \rho_{ji} \leq 1$.[1]
Derive Itô's Lemma for a function $f(S_1, \ldots, S_n)$ of the n assets $S_1, \ldots, S_n$.

[1]Note that we do not need to take the expected value. In general if two Wiener processes $Z_1(t)$ and $Z_2(t)$ are correlated we have that $dZ_1(t)dZ_2(t) = \rho dt$ where ρ is the correlation coefficient