Westminster Business School
School of Finance and Accounting
7FNCE044 Predictive Analysis for Decision Making
Semesters 1 and 2
Week 7
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Seminar Week 7

Q1.

a) What is the definition of weak (covariance) stationarity?

b) Which of the following series is weak stationary? Justify your answer (for each of the series) and state the assumptions you are using at each step of your derivations.

i). $y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$,

$$\text{where } |\theta_i| < 1, i = 1,2 \text{ and } \varepsilon_t \sim NIID(0, \sigma^2)$$

ii). $z_t = c + t + \varepsilon_t$,

$$\text{where } \varepsilon_t \sim NIID(0, \sigma^2)$$

iii). $x_t = \mu + x_{t-1} + e_t$,

$$\text{where } x_0 = 0 \text{ and } e_t \sim NIID(0, \sigma^2)$$

iv). $z_t = z_{t-1} + u_t$,

$$\text{where } z_0 = 0 \text{ and } u_t \sim NIID(0, \sigma^2)$$

c) Write the autocorrelation function of the following process y_t:

$$y_t = \varepsilon_t + \lambda \varepsilon_{t-1}, |\lambda| < 1, \ \varepsilon_t \sim NIID(0, \sigma^2)$$

Q2. Consider the following model:

$$y_t = \mu + u_t + \theta_1 u_{t-1}$$

Show that:

$$E(y_t) = \mu$$

$$Var(y_t) = (1 + \theta^2)\sigma^2$$

$$\text{cov}(y_t, y_{t-q}) = \begin{cases} \theta\sigma^2 & q = 1 \\ 0 & q > 1 \end{cases}$$

Derive the ACF for this model? Why is this model stationary?

Q3.
Consider the AR(1) process: $y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t$, where ε_t is white noise.

Derive the ACF of the model and comment of its shape.

Q4. Additional Revision Problem

An analyst has the following data for 2010 of 40 cities across de United States:

BUSTRAVL:	the demand for urban transportation by bus in thousands of passengers per hour
FARE:	the bus fare in dollars
INCOME:	the average income per capita in dollars
POP:	the city population in thousands
DENSITY:	the city population density in persons/square mile

She is analysing the effect of bus fares, income, population and density on demand for urban transportation by bus, so she regresses BUSTRAVL on FARE, INCOME, POP and DENSITY. The estimation output are given in Table 1:

Table 1

Dependent Variable: BUSTRAVL

Method: Least Squares

Sample: 1 40

Included observations: 40

	Coefficient	Std. Error	t-Statistic	Prob.
C	3111.181	1071.067	2.904749	0.0063
FARE	-295.7306	424.8354		0.4910
INCOME	-0.202197	0.062564	-3.231821	0.0027
POP	1.588337	0.122654	12.94973	
DENSITY	0.149027	0.035713	4.172925	0.0002

R-squared	0.919868	Mean dependent var	1933.175
Adjusted R-squared	0.910710	S.D. dependent var	2431.757
S.E. of regression	726.6434	Akaike info criterion	16.13122
Sum squared resid	18480373	Schwarz criterion	16.34233
Log likelihood	-317.6243	Hannan-Quinn criter.	16.20755
F-statistic	100.4449	Durbin-Watson stat	1.995180
Prob(F-statistic)	0.000000		

<u>**Notes:**</u>

1) Use 5% significance level for all tests.

2) State the null and alternative hypotheses, the test statistic to compute and its distribution, and the criteria for rejecting or failing to reject the null hypothesis for all tests.

a) Write down the mathematical expression of the estimated model. Outline the key assumptions of the explanatory variables. What are the main implications of violating these assumptions?

b) Interpret the coefficient estimates in Table 1. Perform a (two-tailed) test of individual significance for the parameters of the variables FARE and POP using the critical value of the corresponding distribution and the test p-value. Interpret the test results.

c) Perform an overall significance test for the model in Table 1. What is your conclusion from that test? Provide an interpretation of the value of the coefficient of determination in Table 1.

d) The economist formulates a hypothesis that the effect of the city population density (DENSITY) is five times larger than the effect of the bus fare (FARE), on the variable BUSTRAVL. Perform a Wald test for the analyst's hypothesis specifying the null hypothesis and the equation of the restricted model in the knowledge that the sum of the squared residuals (RSS) of the restricted model for that test is 18542143. Interpret the test results.

e) Figure 1 (on page 4) plots the squared residuals obtained from the regression in Table 1. Figure 2 (page 4) plots INCOME (in the X axis) and the estimated residuals (in the Y axis) from the regression in Table 1.

i) Define the classical linear regression model assumption of heteroscedasticity.

ii) Use these plots to graphically test that assumption. Explain your conclusions.

f) The analyst hypothesises that the residuals of the regression present heteroscedasticity. Using the output in Table 2, explain how would the analyst test for heteroscedasticity and what would his conclusion be? Based on your results of that test, what can you say about the properties of the OLS estimator of the model?

g) Using the estimation results in Table 1: explain how would the analyst test for autocorrelation and what would his conclusion be? Based on your conclusions, comment on the reliability of the OLS estimates and associated statistics in Table 1 and the validity of the test of hypotheses in questions a), b) and c) above.

Figure 1

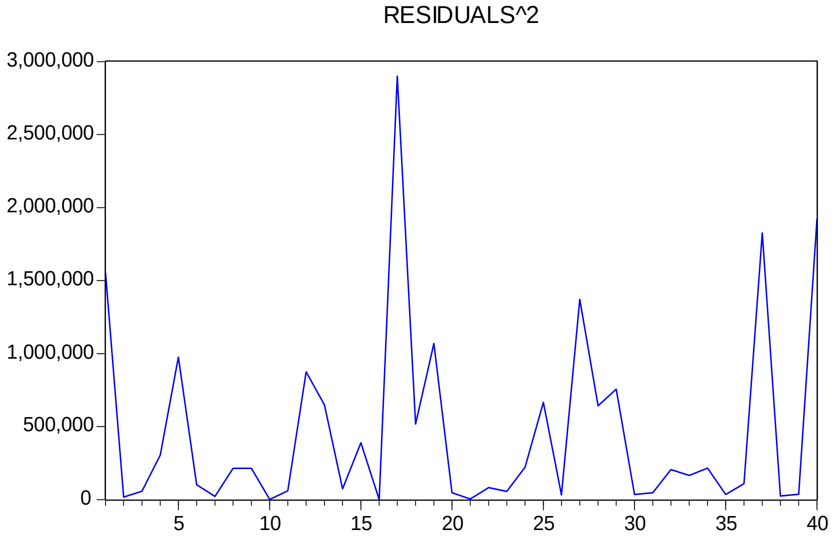

Figure 2

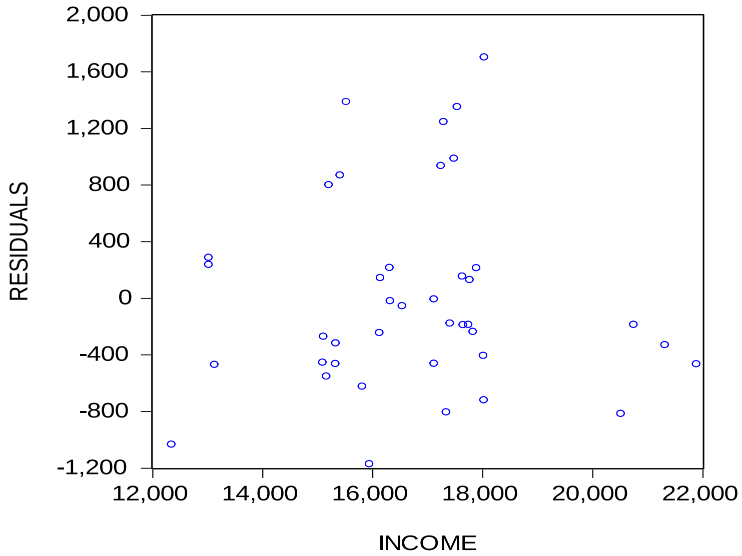

Table 2

F-statistic	2.387061	Prob. F(4,35)	0.0697
Obs*R-squared	8.573396	Prob. Chi-Square(4)	0.0727