

Revision Class

1. Unit Root Tests

You are given a dataset containing time series data for five different stock prices. The data has been transformed using the natural logarithm to stabilize the variance. To determine the stationarity properties of each series, you have run both the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test in R. The output from these tests is provided below.

Questions:

1. For each stock price series, identify the optimal lag length used in the ADF test. Explain the importance of selecting the appropriate lag length in the context of unit root testing.
2. Based on the ADF test results, determine whether the null hypothesis of a unit root is rejected or not rejected for each series at the 5% significance level. State your decision and the corresponding data generating process (stationary or non-stationary) for each series.
3. Examine the KPSS test results for each stock price series. Determine whether the null hypothesis of stationarity is rejected or not rejected at the 5% significance level. State your decision and the corresponding data generating process (stationary or non-stationary) for each series.
4. Considering the results of both the ADF and KPSS tests, determine the order of integration for each stock price series. Explain your reasoning and how you arrived at your conclusions.
5. Identify any contradictory results between the ADF and KPSS tests for each series. Discuss the implications of such contradictions and how you would interpret the overall results. What additional steps or tests could be performed to resolve any discrepancies?
6. Based on the unit root test results, discuss the implications for further analysis of the stock price series. Consider aspects such as the appropriate modelling techniques, the need for data transformations, and the potential presence of long-run relationships among the series.

R Output:

All the series are tested using the following lines:

```
adf_test <- ur.df(series, type = "trend", selectlags = "AIC")  
kpss_test <- kpss.test(series, null = "Trend")
```

Series: Disney

ADF Test:

Optimal Lag Length: 1

ADF Test Statistic: -2.144544

P-value: 0.03271928

KPSS Test:

KPSS Test Statistic: 0.614178

P-value: 0.01

Series: JP Morgan.

ADF Test:

Optimal Lag Length: 1

ADF Test Statistic: -2.503306

P-value: 0.01278659

KPSS Test:

KPSS Test Statistic: 0.758254

P-value: 0.01

2. Expectations Properties

Derive the mean, the variance and the covariances of the following processes:

i.

$$z_t = c + 2(t - 1) + \varepsilon_t,$$

where $\varepsilon_t \sim iid(0,1)$

ii.

$$v_t = v_{t-1} + \varepsilon_t$$

where $v_t \sim iid(0, \delta)$, $\varepsilon_t \sim iid(0, \sigma^2)$ and $cov(\varepsilon_t, v_t) = 0$

3. GARCH Models

Using monthly data for y_t , the FTSE100 index, from January 1992 to December 2005, the following set of results were obtained:

$$\hat{y}_t = 0.08,$$

$$(0.03)$$

$$\hat{\sigma}_t^2 = 0.12 + 0.15u_{t-1}^2 + 0.8\sigma_{t-1}^2 + 0.6u_{t-1}^2I_{t-1}$$

$$(0.02) \quad (0.05) \quad (0.08) \quad (0.20)$$

Values in () are the standard errors of the coefficient estimates and $I_{t-1} = 1$ if $u_{t-1} < 0$, $I_{t-1} = 0$ otherwise. σ_{t-1} is the conditional deviation.

i. What role does the final term in the above conditional variance equation serve?

[5 Marks]

ii. What is the interpretation of the estimated value of lagged conditional variance (σ_{t-1}^2), 0.80?

[5 Marks]

iii. With reference to conditional variance equation, if $\sigma_{t-1}^2 = 1.5$, consider that $\hat{u}_{t-1} = \pm 1.20$. Estimate the value of σ_t^2 , for a positive shock (+1.20) and a negative shock (-1.20).