Week 11 Bank Market Risk Management

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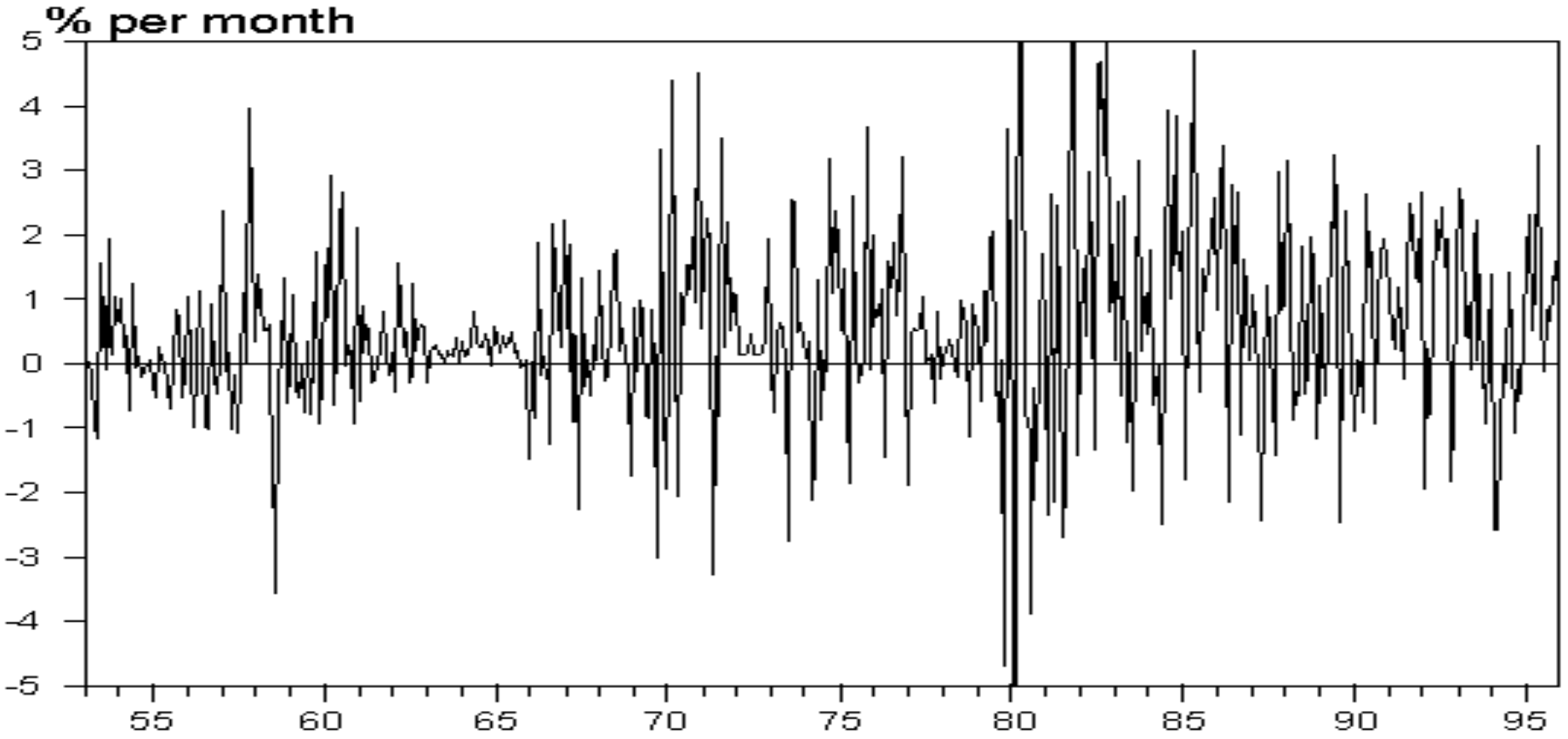
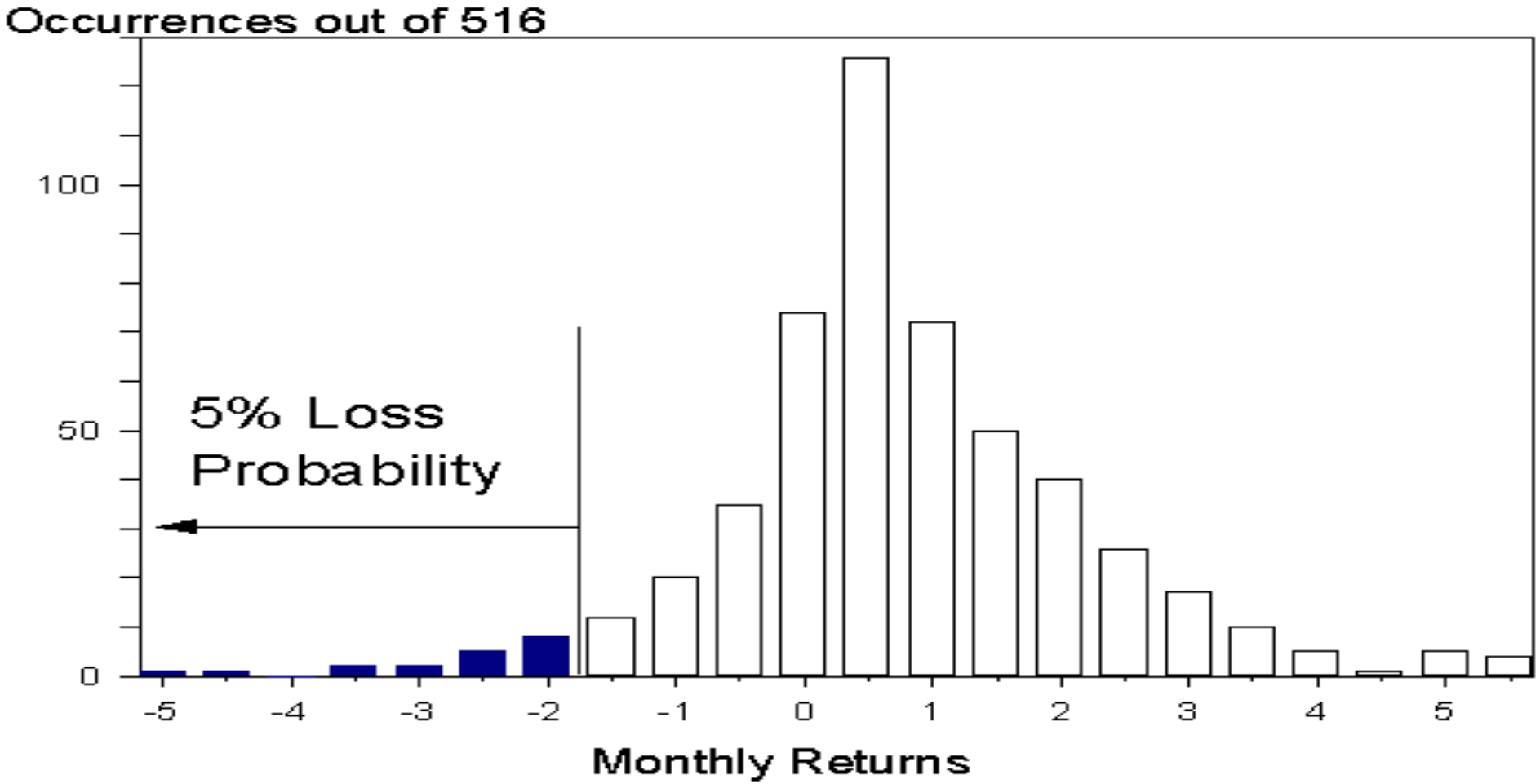
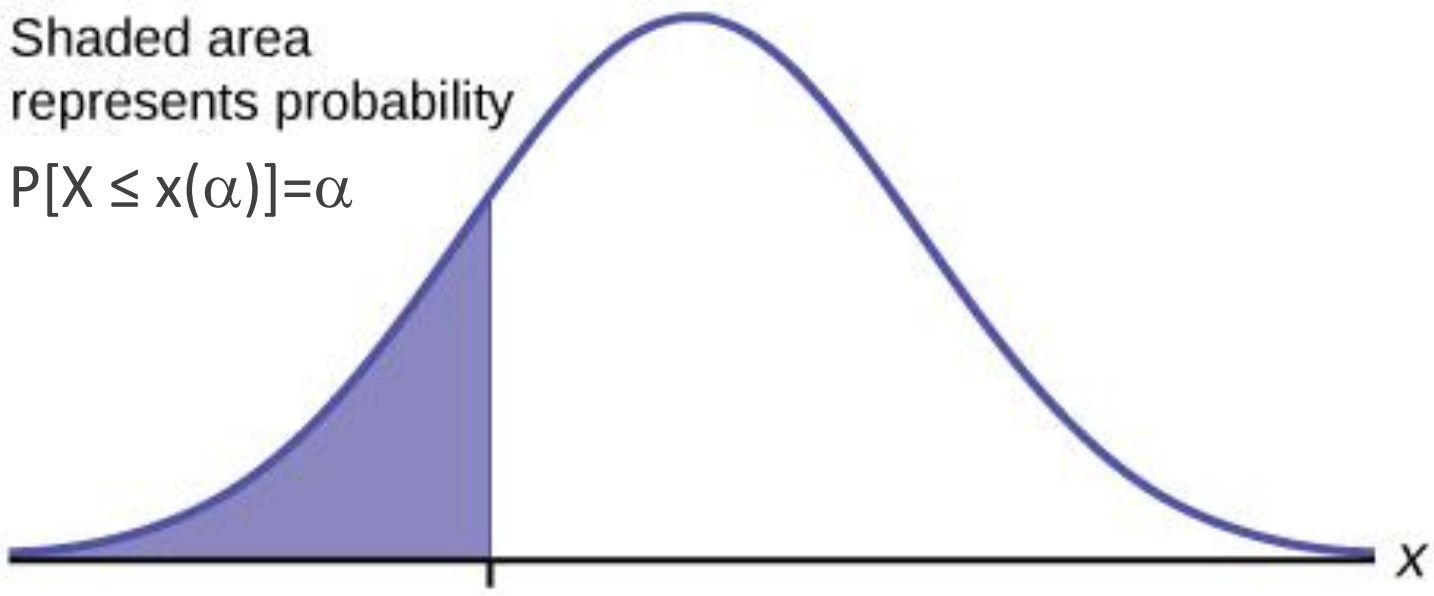
History of medium bond returns



Distribution of medium bond returns



Probability distribution and percentile

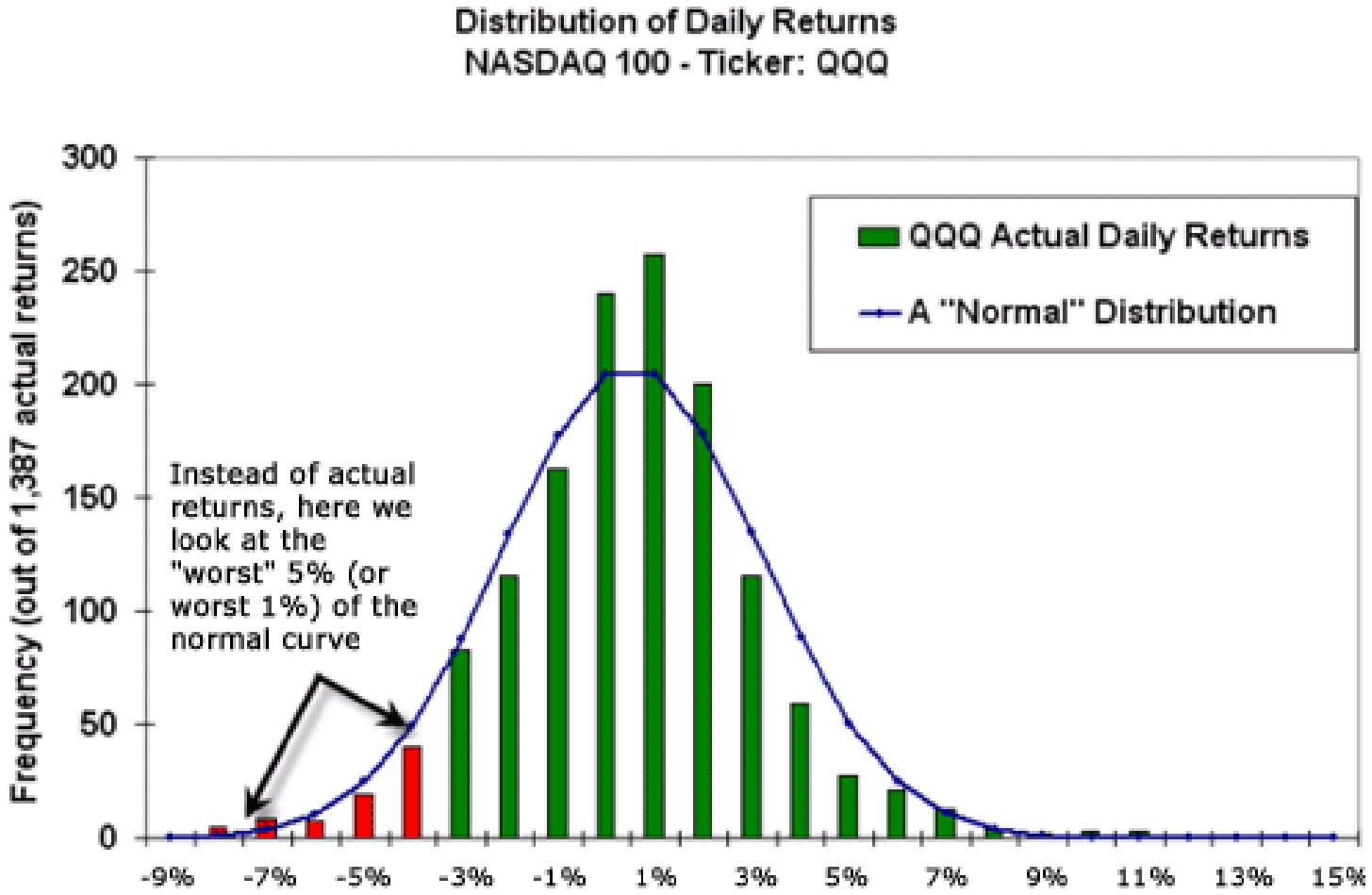


Percentile x(α)

Normal distribution and VaR

For 99% confidence level
→ $\alpha = 2.33$

For 95% confidence level
→ $\alpha = 1.65$



Example

- Long position = $10 million
- Confidence level = 95%
  - Probability that the loss ≥ VaR = 5%
- Standard deviation of *daily* returns = 0.53%

Daily VaR = $V_0 \alpha \sigma$ = $10 million * 1.65 * 0.53% = $87,450

10-day VaR = Daily VaR * $\sqrt{10}$ = $276,541.18

Example 2

- Portfolio VaR in the case of 2 assets:

$$VaR_P = \sqrt{VaR_A^2 + VaR_B^2 + 2VaR_A\ VaR_B\ \rho_{AB}}$$

- VaR_A Value-at-risk of asset A
- VaR_B Value-at-risk of asset B
- ρ_{AB} Correlation between A and B

- This is equivalent to:
  - $VaR_P = V_p\ \alpha\ \sigma_p = V_p\ \alpha \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A\ w_B\ \sigma_{AB}}$

Historical simulation

Based on assumption that history will repeat itself

Going back in time (e.g. over the last 5 years) and applying current weights to a time-series of historical asset returns

- Calculated returns do not represent an actual portfolio
- Reconstruct the history of a hypothetical portfolio using the current position

- Need:

➢A time-series of actual movements for each risk factor

➢Positions on risk factors

Example 1:

- Portfolio of two assets with following current market values:
- V_0 = \$70 and W_0 = \$30 → P_0 = \$100
- Need to "re-value" these assets based on their historical price movements.
- Observe the market prices of these two assets for the past 100 trading days:

Example 2:

Observed market values V_n and W_n, where n = 0,...,100	Observed portfolio value P_n, where n = 0,...., 100	Δ in observed market value $\Delta P = P_n - P_{n-1}$, where n = 0,...., 100	Alternative value $AV = P_0 + \Delta P$
$V_0 = \$70, W_0 = \$30$	$P_0 = \$100$		
$V_1 = \$68, W_1 = \$22$	$P_1 = \$90$	$\Delta P_1 = -\$10$	\$90
$V_2 = \$70, W_2 = \$25$	$P_2 = \$95$	$\Delta P_2 = +\$5$	\$105
$V_3 = \$73, W_3 = \$27$	$P_3 = \$100$	$\Delta P_3 = +\$5$	\$105
$V_4 = \$71, W_4 = \$30$	$P_4 = \$101$	$\Delta P_4 = +\$1$	\$101
etc.	etc.	etc.	etc.
$V_{99} = \$68, W_{99} = \$25$	$P_{99} = \$93$		
$V_{100} = \$67, W_{100} = \$30$	$P_{100} = \$97$	$\Delta P_{100} = +\$4$	\$104

Example 3:

- The 100 sets of daily change in the portfolio value, ΔP_n can be ranked from the day with the worst performance to the day with the best performance:

Daily profit and loss ranked in ascending order:

Rank	1	2	3	4	5	6	....	96	97	98	99	100
Performance	-$15	-$13	-$12	-$9.5	-$9	-$8.8	....	+$7	+$9	+$9.3	+$13	+$14

- To find the Value-at-Risk with 95% confidence level: 95th percentile (one tailed)
- → 5th worst performance = -$9

Weaknesses

- Key disadvantage: Degree of confidence is based on 500 observations; not very many from a statistical standpoint
- Potential solution: increasing number of observations by going back more than 500 days
- Not desirable
- Could weight recent past observations more heavily and go further back

For example

Consider a financial institution with a long position in a one-year, zero-coupon €1,000,000 bond. The current one-year interest rate on the Eurobond is 10%. The current €/$ exchange rate is 0.65. The historical $\sigma_{FX} = 0.0042$ and $\sigma_i = 0.0008$. Evaluate the value at risk for this bond based on changes interest rates and foreign exchange rates over the next 10 days.

No historical data for this Eurobond.

Monte Carlo Simulation

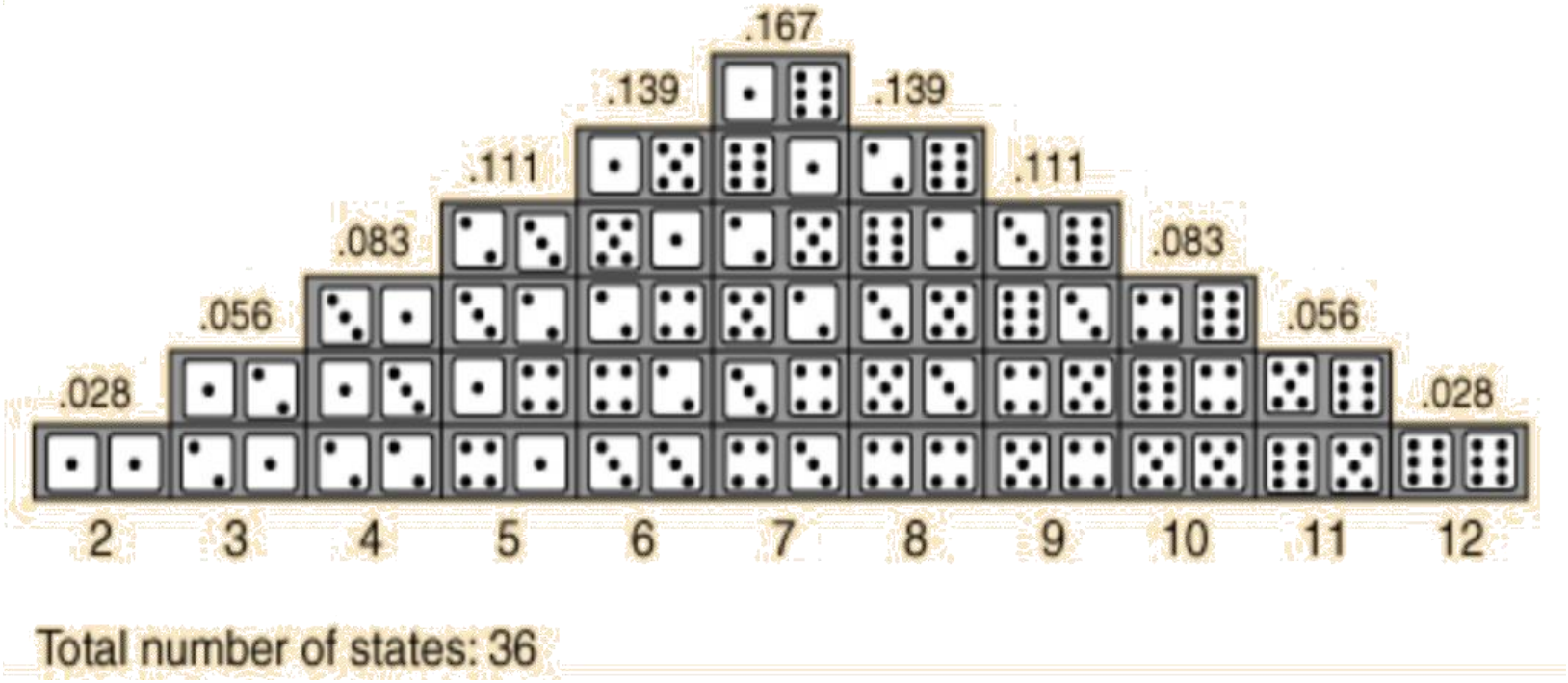
- Two steps:

1. Specify a stochastic process for financial variables as well as process parameters
   - Choice of distributions and parameters such as risk and correlations can be derived from historical data
2. Simulate fictitious price paths for all variables of interest
   - At each horizon considered, portfolio is marked-to-market using full valuation
   - Each of these “pseudo” realizations is then used to compile a distribution of returns, from which a VaR figure can be measured

- Need for each risk factor:
  - specification of a stochastic process (i.e., distribution and parameters)
  - valuation models for all assets in the portfolio
  - positions on various securities

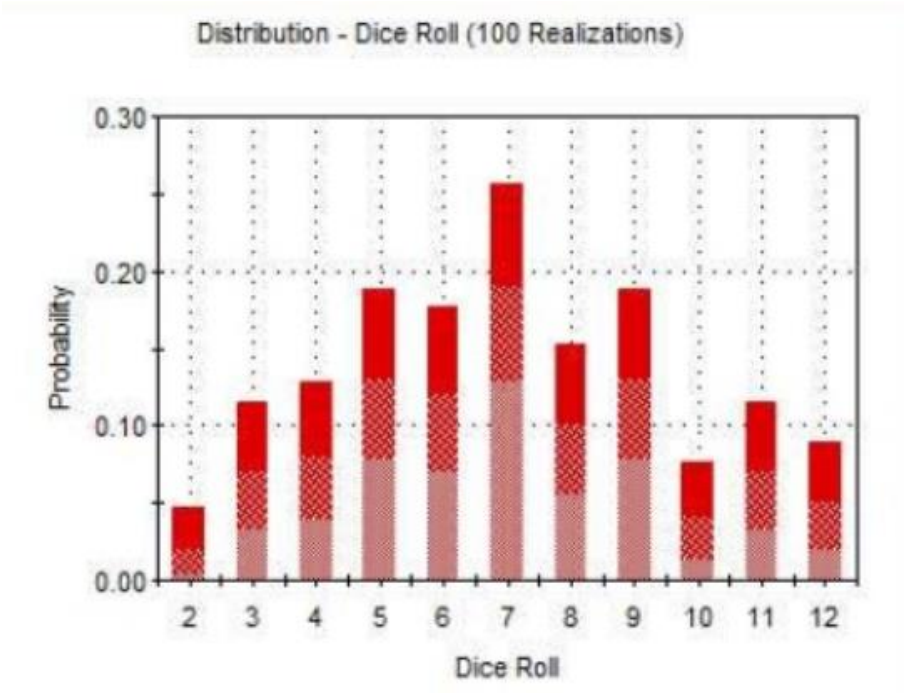
Monte Carlo Simulation 1

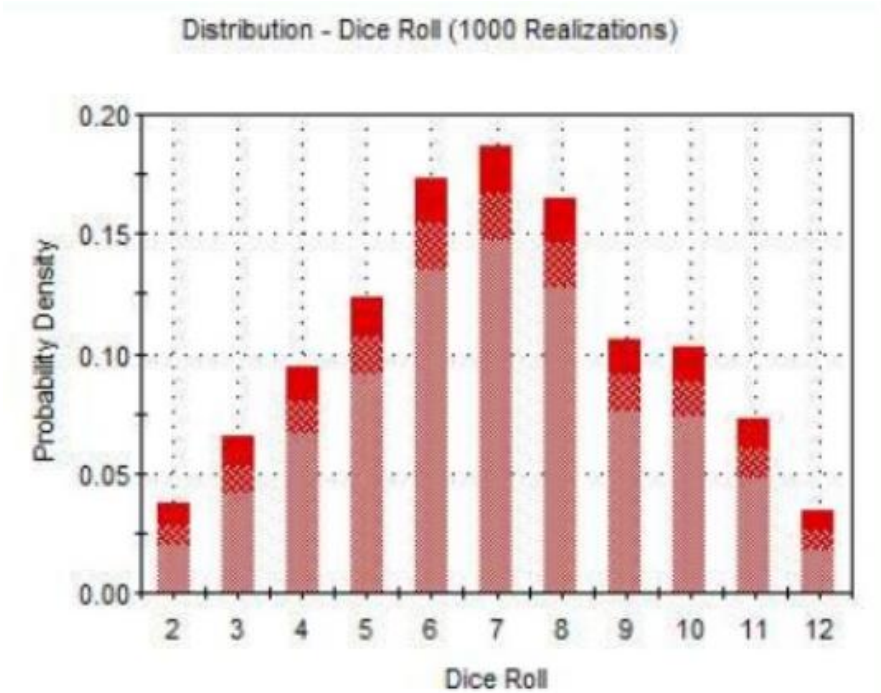
Example: Rolling Dice



Monte Carlo Simulation 2

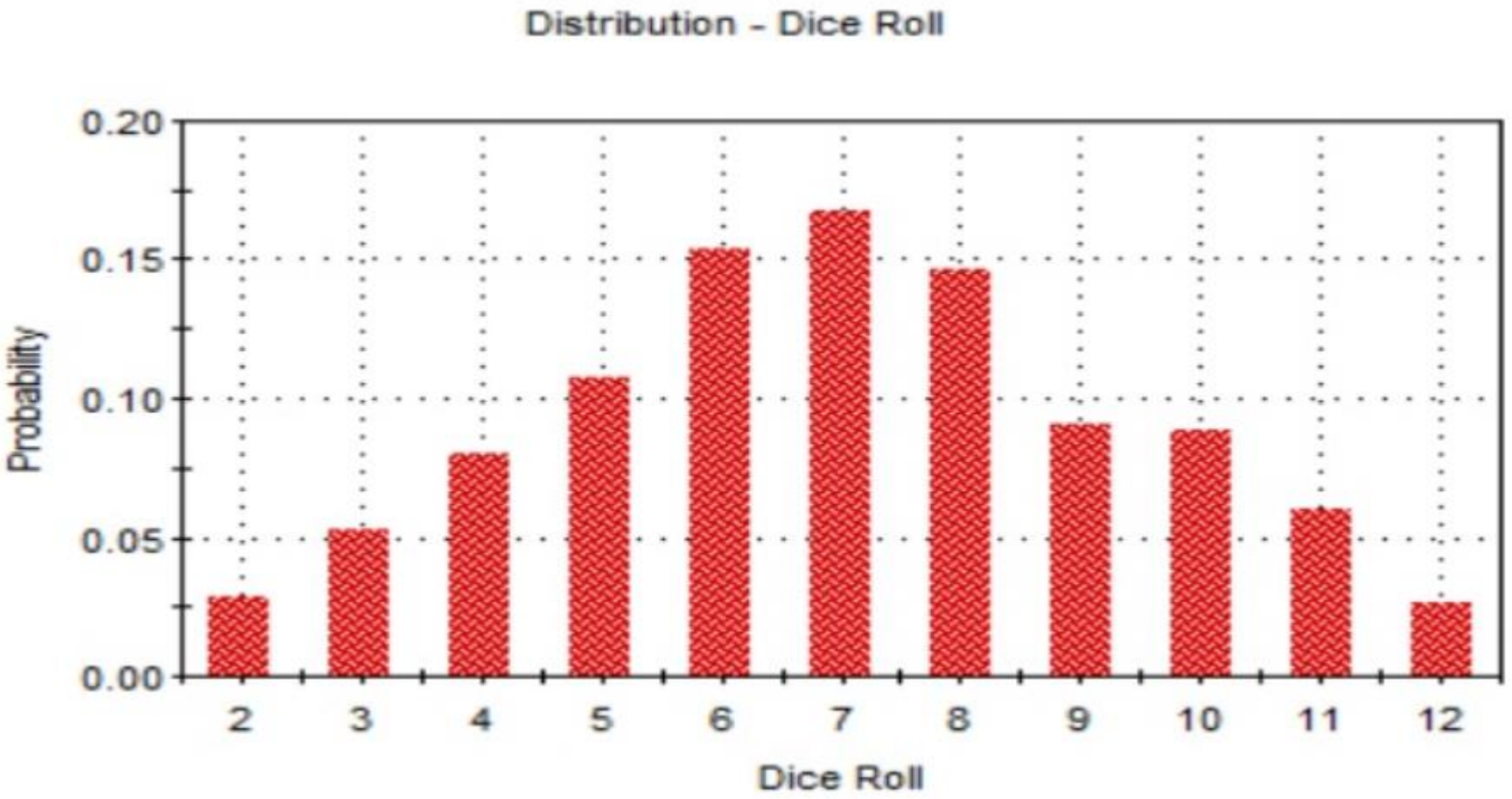
The accuracy of a Monte Carlo simulation is a function of the number of realisations.





Monte Carlo Simulation 3

The output of 10,000 realisation



Comparison of Approaches

- Model building (delta-normal) approach:
  - Advantages: results produced quickly – can be used with volatility updating schemes (e.g: GARCH)
  - Disadvantage: Assumes market variables have a multivariate normal distribution. (e.g: daily changes not normal usually)
- Historical simulation approach:
  - Advantages: historical data determine the joint prob. distribution of the market variables
  - Disadvantage: Computationally slow
- Monte Carlo simulation:
  - Slow – company complete portfolio revalued many times – different instruments

VaR in practice

- VaR has been traditionally used by regulators for setting capital requirements for market risk, but also credit risk and operational risk
- JP Morgan RiskMetrics
  - Allows users to compute a portfolio VaR based on a 95% confidence level over a daily or monthly horizon
- Deutsche Bank Risk Office (formerly RAROC 2020)
  - Provides VaR estimates at the 99% confidence level over an annual horizon, using Monte Carlo method
  - RAROC = Risk-adjusted return on capital

Current modelling practices

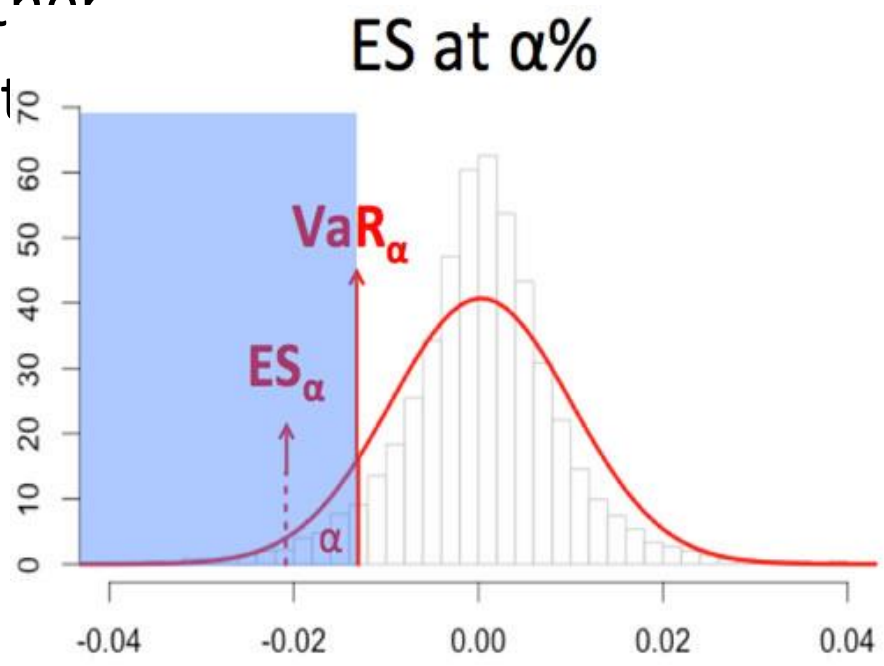
- In a survey of 13 leading banks in Europe and North America, McKinsey (2012) show that:
  - 50% of banks run at least 2 different iterations of VaR, 1 for regulatory reporting and 1 for internal management
  - All calculate VaR on a 1-day holding period
  - 62% of them extrapolate the 10-day VaR (required by regulators) from the 1-day VaR
  - Banks that calculate VaR for internal use do so with shorter holding periods (< 10 days) and sometimes lower confidence interval (< 99%)
  - Most banks use historical analysis to determine whether relative or absolute returns best fit a given asset class

Limitations and challenges of VaR

- VaR estimates and internal modeling are not perfect
  - Inaccurate VaR estimates can expose a bank to excessive risk so that its capital position may turn out not to be large enough to cover actual losses the bank faces
  - The portfolios of the largest banks are so complex with thousands of risk factors it may be impossible to consistently forecast VaR accurately
  - VaR does not measure event risk (e.g., market crash) → portfolio **stress tests** are recommended to supplement VaR
  - VaR does not readily capture liquidity differences among instruments
- Promote "backtesting"
- Even if an individual bank is a good forecaster, there may still be trouble due to systemic risk

Expected Shortfall

- Regulators are switching from VaR to expected shortfall (ES) for calculating market risk
- Also called conditional VaR → one step furth
- Expected loss during time T conditional on t being greater than the X^{th} percentile of the distribution
- Example:
- X = 99; T = 10 days; VaR = £50 million
- ES = average amount lost over a 10-day period assuming that the loss is greater than £50 million



VaR versus ES: case of 2 portfolios

- Is VaR a coherent measure of risk?
  - Risk measure for 2 portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged
- Example:
  - Two independent portfolios (1-year horizon, confidence level = 97.5% → tail of 2.5%)
  - For each: Loss of £10m with p=0.02 and loss of £1m with p=0.98

    → VaR = £1m for each portfolio
  - If combined: Loss of £20m with p=0.0004 (=0.02×0.02), loss of £11m with p=0.0392 (=2×0.02×0.98) and loss of £2m with p=0.9604 (=0.98×0.98)

    → VaR = £11m for combined portfolio
- $VaR\ (P_1+P_2) > VaR\ (P_1) + VaR\ (P_2)$

VaR versus ES: case of 2 portfolios 2

- Expected shortfall:
  - For each portfolio: in the tail of the loss distribution, 2.5% = 2% chance of losing £10m and 0.5% chance of losing £1m

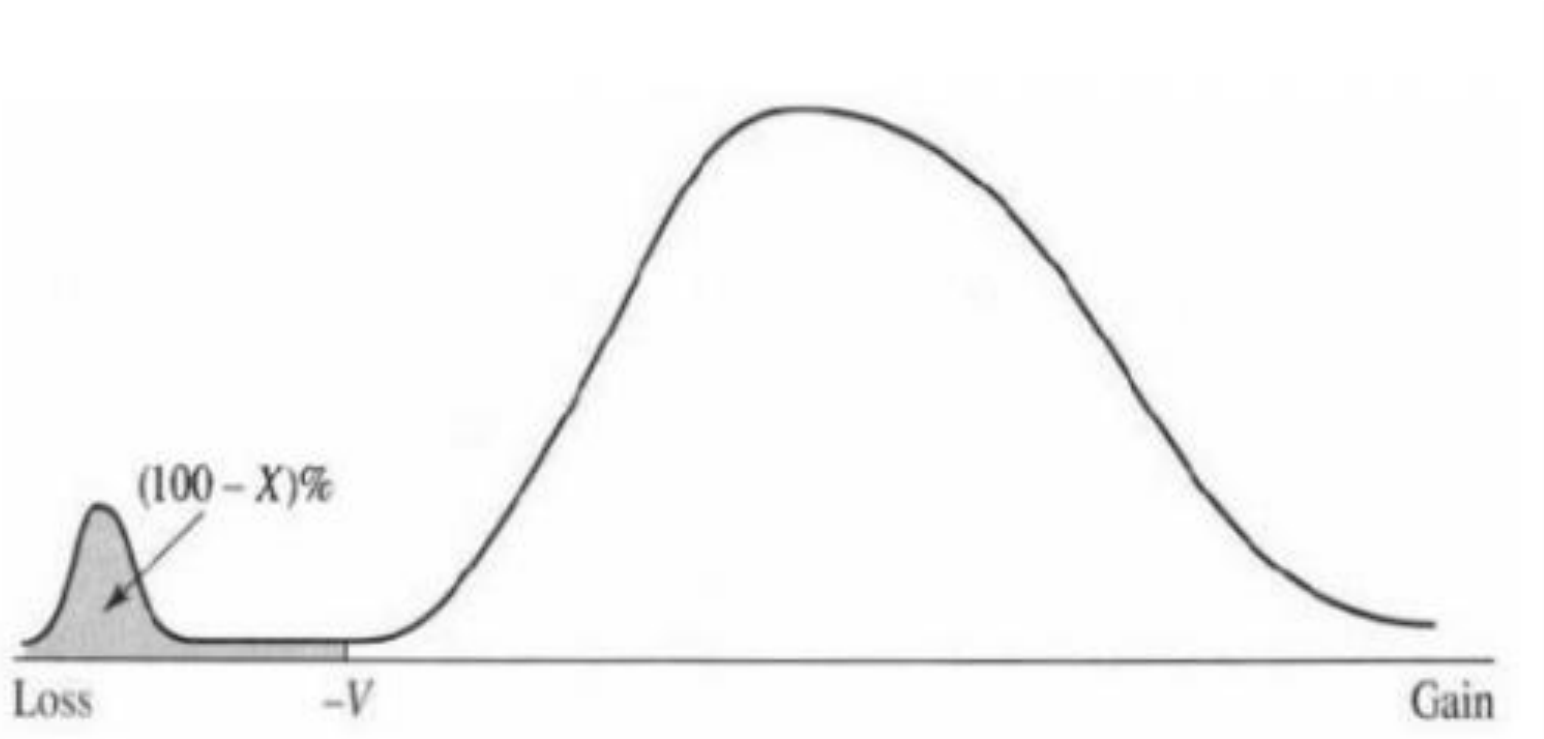
    → Conditional probabilities: Loss of £10m with p=0.8 and loss of £1m with p=0.2

    → ES = £10m × 0.8 + £1m × 0.2 = £8.2m for each portfolio
  - If combined: in the tail of the loss distribution, 2.5% = 0.04% chance of losing £20m and 2.46% chance of losing £11m

    → Conditional probabilities: Loss of £20m with p=0.016 and loss of £11m with p=0.984

    → ES = £20m × 0.016 + £11m × 0.984 = £11.144m for combined portfolio
- ES (P_1+P_2) < ES (P_1) + ES (P_2)

Expected Shortfall (cont.)



BIS standards for ES (2016)

- ES must be computed on a **daily** basis for the bank-wide internal model for regulatory capital purposes
  - In calculating the expected shortfall, a **97.5th percentile confidence level** is to be used
- ES measure must be calibrated to a **period of stress**
  - Must replicate an expected shortfall charge that would be generated on the bank's current portfolio if the relevant risk factors were experiencing a period of stress
- No particular type of **expected shortfall model** is prescribed
  - So long as model used captures all the risks run by the bank, and conforms to each of the capital requirements, banks can use either historical simulation, Monte Carlo simulation, or other appropriate analytical methods

Other alternatives

- VAR-iations:
  - Marginal VaR
  - Incremental VaR
  - Extreme VaR
  - Stressed VaR
- Other tools for managing market risk:
  - Sensitivity/scenario analyses

Stress testing

- **Back Testing:**
- Testing how the VaR estimates would have performed in the past
- **Stress Testing:**
- Estimating how the portfolio would have performed under some of the most *Extreme* market moves seen in last 8 to 25 years
- EX: Extreme movement in US equity prices: set % change in all market variables equal to October 19th 1987 – S&P500 moved 22.3 s.d's
- EX: Yield curve shifts 100, 200 basis points

Regulatory stress testing

- **In Europe**: http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing
- The European Banking Authority's stress test scenario includes key macroeconomic components, such as:
  - Negative GDP growth
  - Abrupt reversal of compressed global risk premia
  - Shadow banking sector that becomes increasingly stressed
  - Shock in residential and commercial real estate prices
  - Foreign exchange risk in central and eastern Europe
  - Misconduct risk

- **In the USA**: Comprehensive Capital Analysis and Review (CCAR) by the Fed

BIS standards for stress testing (2016)

- Banks that use internal model approach for meeting market risk capital requirements must have in place a **rigorous and comprehensive stress testing** program
  - Stress testing to identify events or influences that could greatly impact banks
  - Key component of a bank's assessment of its capital position
- Stress scenarios need to cover a range of factors that can create **extraordinary losses or gains** in trading portfolios, or make the control of risk in those portfolios very difficult
  - Factors include low-probability events in all major types of risk (market, credit, and operational risks)
- Stress tests should be both of a **quantitative and qualitative** nature, incorporating both market risk and liquidity aspects of market disturbances
- The results of stress testing should be routinely **communicated** to senior management and, periodically, to the bank's board of directors

Summary

- For the historical simulation, degree of confidence is based on 500 observations; not very many from a statistical standpoint

- Assets with limited historical data can use Monte Carlo simulation to estimate its VaR.

- Expected shortfall is calculated by averaging all of the returns in the distribution that are worse than the VAR of the portfolio at a given level of confidence.