Predictive Analysis for Decision Making

Weeks 6 and 7
Modelling Time Series Models: Notation and Concepts

Dr Issam Malki
School of Finance and Accounting
Westminster Business School

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Aims and Objectives

- Most of the models deal with cross sections
- Variables can be time dependent and observed over time for the same entity.
  - Time series models
  - In genera, we follow the same modelling strategies
  - Prominent issues related to time: serial correlation and time varying variance.
- Introduce Time Series Models
  - Modelling long run and short run relations
  - Dealing with stationary data
  - Extension to account for nonstationary components.

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Time Series and Concept of Stationarity

- Stationarity
  - Refers to constant properties of data over time.
  - The presence of stationarity ensures the validity of the methods we developed before (OLS and its properties)
- Key properties (assume a stochastic time series: y_t)
  - The mean: $\mathrm{E}(y_t)$
  - The variance: $\mathrm{var}(y_t) = E\{[y_t - E(y_t)]^2\}$
  - The covariances: $\mathrm{cov}(y_t, y_s) = E\{[y_t - E(y_t)][y_s - E(y_s)]\}$ for $t \neq s$

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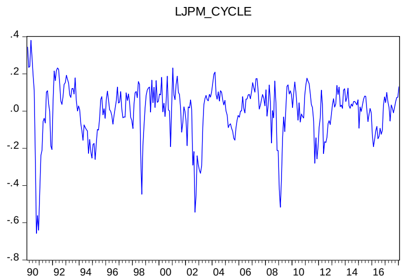
Time Series and Concept of Stationarity



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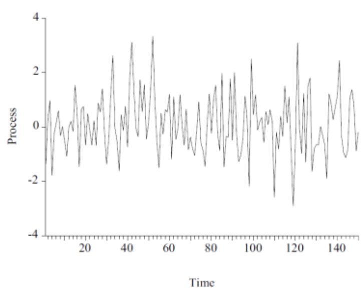
Time Series and Concept of Stationarity



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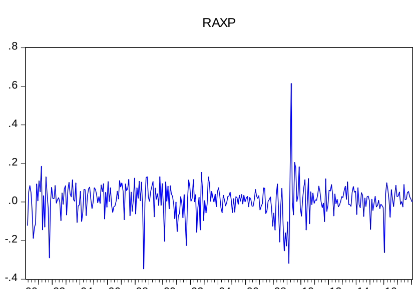

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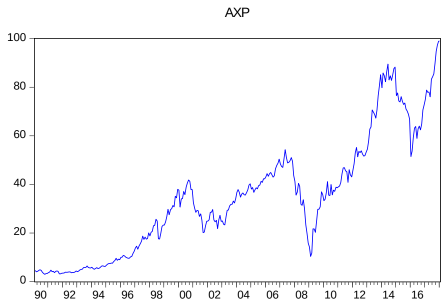

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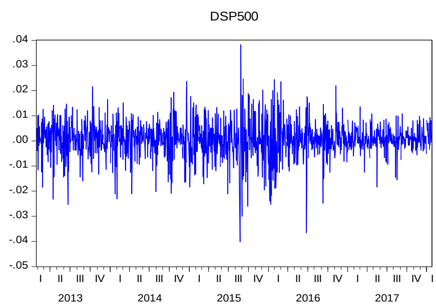

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Time Series and Concept of Stationarity

- Strictly Stationary Process
  - Rarely seen in applications of finance

$$P\{y_{t_1} \le b_1, \ldots, y_{t_n} \le b_n\} = P\{y_{t_1+m} \le b_1, \ldots, y_{t_n+m} \le b_n\}$$

- Covariance (weakly) stationary
  - Commonly used process: the following must be satisfied
    1. $E(y_t) = \mu, \quad t = 1,2,\ldots,\infty$
    2. $E(y_t - \mu)(y_t - \mu) = \sigma^2 < \infty$
    3. $E(y_{t_1} - \mu)(y_{t_2} - \mu) = \gamma_{t_2 - t_1}$ for any t_1 , t_2
- Others include
  - Trend stationary process
  - Cyclical and seasonal processes

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Examples (1)

Suppose we have an *iid* (independently and identically distributed) ε_t with zero mean and variance equal to σ^2. The process is therefore strictly stationary because:

$$E(\varepsilon_t) = 0 \text{ for any } t$$
$$var(\varepsilon_t) = \sigma^2 \text{ for any } t$$
$$cov(\varepsilon_t, \varepsilon_s) = 0 \text{ for any } t \neq s$$

Furthermore, we have: $\varepsilon_t \sim iid(0, \sigma^2)$, $\varepsilon_{t-1} \sim iid(0, \sigma^2)$, …, $\varepsilon_{t-s} \sim iid(0, \sigma^2)$.

In addition, if we take any subset of the series ε_t, the distribution remains the same.

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Examples (1)

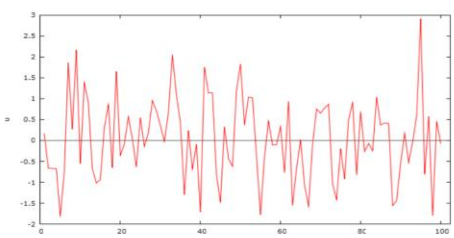

Figure : A realization of an *IID*(0,1).

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Examples (2)

Suppose we have a white noise process, u_t with zero mean and variance equal to 1. The process is therefore strictly stationary because:

$$E(u_t) = 0 \text{ for any } t$$
$$var(u_t) = 1 \text{ for any } t$$
$$cov(u_t, u_s) = 0 \text{ for any } t \neq s$$

Furthermore, we have: $u_t \sim WN(0,1)$, $u_{t-1} \sim WN(0,1)$, …, $u_{t-s} \sim WN(0,1)$.

In addition, if we take any subset of the series u_t, the distribution remains the same.

Note that white noise process is uncorrelated and not independent.

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Examples (3)

Suppose we have the following process:

$$y_t = \mu + t + \varepsilon_t \quad \text{where } \varepsilon_t \sim iid(0, \sigma^2)$$

Then:

$$E(y_t) = \mu + t$$

$$var(y_t) = \sigma^2$$

$$cov(y_t, y_s) = 0 \text{ for any } t \neq s$$

The process is not stationary since the mean is not constant over time. It has, however, constant variance and zero covariances.

The trend is deterministic and therefore the process is called *trend stationary*.

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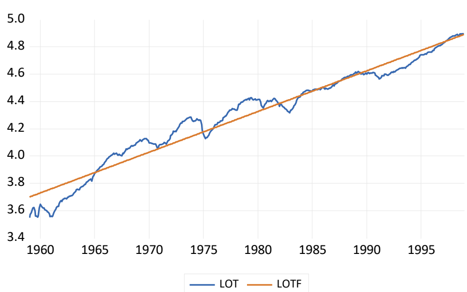
Examples (3)



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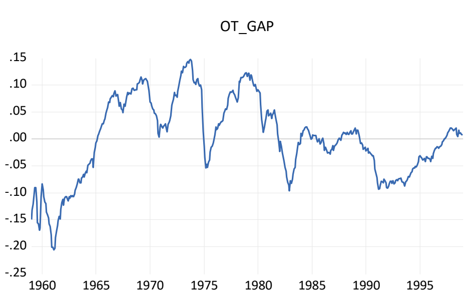
Examples (3)



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EXAMPLES (3)

- We can handle trend stationary process in various ways depending on the purpose.
- Capturing the long run and short run dynamics: decompose the series into:
  - long run: $E(y_t) = \mu + t$, the trend is local and known
  - and short run: $y_t^* = y_t - E(y_t) = \varepsilon_t$, the error if white noise, it is then easy to handle
- Modelling the trend
- Decompose the component using filters such as Hodrick – Prescot or Kalman filters.

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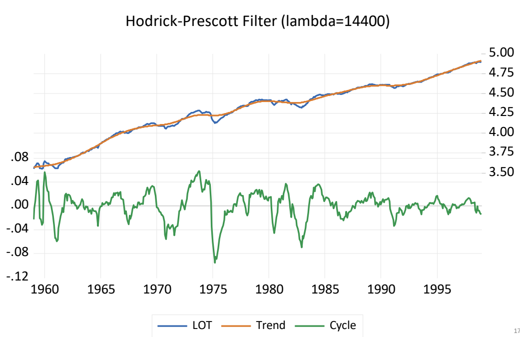
EXAMPLES (3)



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EXAMPLES (4)

Suppose we have the following process:

$$y_t = \phi y_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \sim iid(0, \sigma^2) \text{ and } |\phi| < 1$$

Define the lag operator L where

$$y_{t-1} = L y_t$$
$$y_{t-2} = L y_{t-1} = L^2 y_t$$

Then, we can write the process above as:

$$y_t = \phi L y_t + \varepsilon_t \Leftrightarrow y_t - \phi L y_t = \varepsilon_t,$$

or

$$y_t (1 - \phi L) = \varepsilon_t \Leftrightarrow y_t = \frac{1}{(1-\phi L)} \varepsilon_t$$

$$E(y_t) = \frac{1}{(1 - \phi L)} E(\varepsilon_t) = 0$$

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EXAMPLES (4)

$$var(y_t) = var\left(\frac{1}{(1-\phi)}\varepsilon_t\right) = \frac{1}{1-\phi^2}var(\varepsilon_t) = \frac{1}{1-\phi^2}\sigma^2$$

Both the mean and variance are constant.

We also can show that for this process, we have:

$$cov(y_t, y_{t-k}) = \gamma_k = \phi\gamma_{k-1} = \phi^j\gamma_0$$

where $\gamma_0 = var(y_t)$.

Note also that as $j \to \infty$, we have $\phi^j\gamma_0 \to 0$

This last property is what distinguish white noise process from the weakly stationary process. Note also that these results are dependent on $|\phi| < 1$. 19

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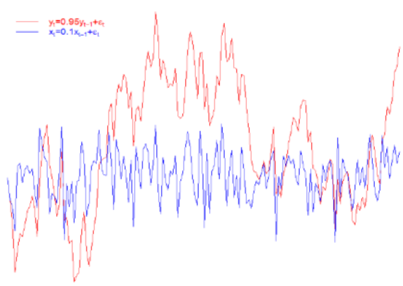
EXAMPLES (4)



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EXAMPLES (5)

Suppose we have the following process:

$y_t = y_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim iid(0, \sigma^2)$ and $y_0 = 0$.

Note that the slope of y_{t-1} is 1. To solve this, we need to rewrite the process recursively using the starting value $y_0 = 0$.

At t=1, we have $y_1 = y_0 + \varepsilon_1 = \varepsilon_1$

At t=2, we have $y_2 = y_1 + \varepsilon_2 = \varepsilon_1 + \varepsilon_2$

At t=3, we have $y_3 = y_2 + \varepsilon_2 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

At t=t, we have $y_t = y_{t-1} + \varepsilon_t = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots + \varepsilon_t$

Then:

$$E(y_t) = E(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots + \varepsilon_t) = 0$$
$$var(y_t) = var(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots + \varepsilon_t) = t\sigma^2$$
$$cov(y_t, y_{t-k}) = 0 \text{ for } k \neq 0$$

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EXAMPLES (5)

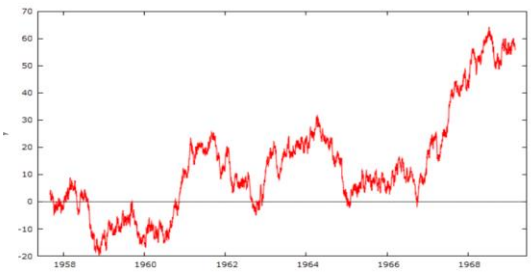

Figure : A realization of a random walk.

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EXAMPLES (5)

The process

$$y_t = y_{t-1} + \varepsilon_t \quad \text{where } \varepsilon_t \sim iid(0, \sigma^2)$$

is known as a (pure) random walk process. The process is a special case of two more general cases:

Random Walk with a Drift:

$$y_t = \alpha + y_{t-1} + \varepsilon_t$$

Random Walk with a Drift and Trend:

$$y_t = \alpha + \beta t + y_{t-1} + \varepsilon_t$$

These processes never converge to the long run equilibrium.

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AUTOCORRELATION AND PARTIAL AUTOCORRELATION

- The partial autocorrelation is simply the conditional correlation captures the serial correlation between two values of the same variable.

$$\rho_j = \frac{\gamma_k}{\gamma_k} = \frac{cov(y_t, y_{t-k})}{var(y_t)}$$

- The autocorrelation time path of the autocorrelation between y_t and y_{t-k} over a pre-specified time horizon.

- The autocorrelation function can be used as an informal test of the stationarity of the data.

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Autocorrelation and Partial Autocorrelation

- The partial autocorrelation is simply the conditional correlation.
- It can be easily obtained by regressing :

y_t on y_{t-1} for the first partial autocorrelation

y_t on y_{t-1}, y_{t-2} for the second partial autocorrelation

$\vdots$

y_t on y_{t-1}, y_{t-2}, …, y_{t-k} for the *k-th* partial autocorrelation

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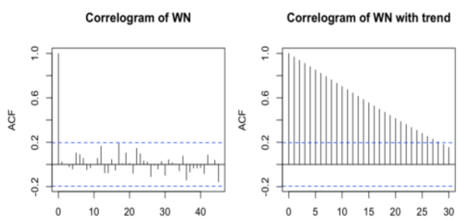
Autocorrelation and Partial Autocorrelation



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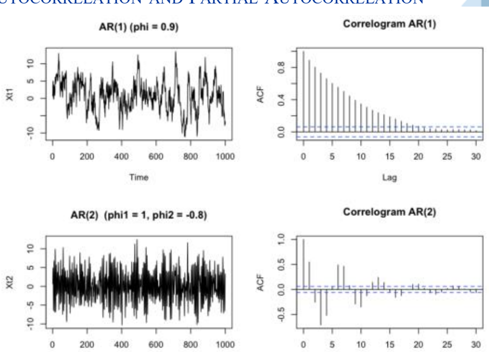
Autocorrelation and Partial Autocorrelation



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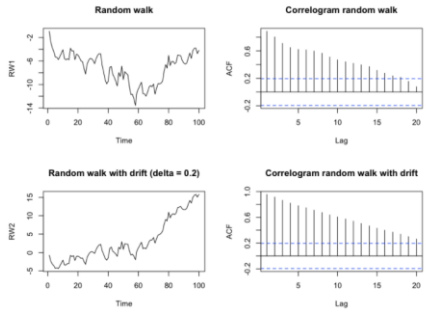
Autocorrelation and Partial Autocorrelation



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THANK YOU

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