

INDIVIDUAL COURSEWORK 2023/ 24

1. General Information and Instructions

- i. This coursework accounts for 40 % of the final mark. The word count must not exceed 2000 words. The coursework is individual and must be your own work.
- ii. Ensure that all theoretical perspectives, mathematical models and working out is detailed and explained.
- iii. Submit your final copy via Turnitin on Blackboard in a WORD document.
- iv. Read the instructions within the paper.
- v. Answer **ALL** questions.

2. Submission and Deadline

To submit your assignment:

- Log on to Blackboard at <http://learning.westminster.ac.uk>;
- Go to the Blackboard site for this module;
- Click on the 'Submit Coursework' link in the navigation menu on the left-hand side
- Click on the link for the assignment;
- Follow the instructions.

Only an electronic copy should be handed in. The copy should be submitted via the blackboard site of the module by **1:00 p.m. on THURSDAY 11 April 2024**.

Question 1

An analyst has the following time series data for the United States (US) over the period January 1959 – December 1998:

r_t	=	interest rate.
lm_t	=	logarithm of money.
lp_t	=	logarithm of prices.
lo_t	=	logarithm of output

The economist aims to analyse the demand for money in the US using a static long run relationship. Figure 1 depicts the dynamics of the four variables while STATA output are given in Table 1:

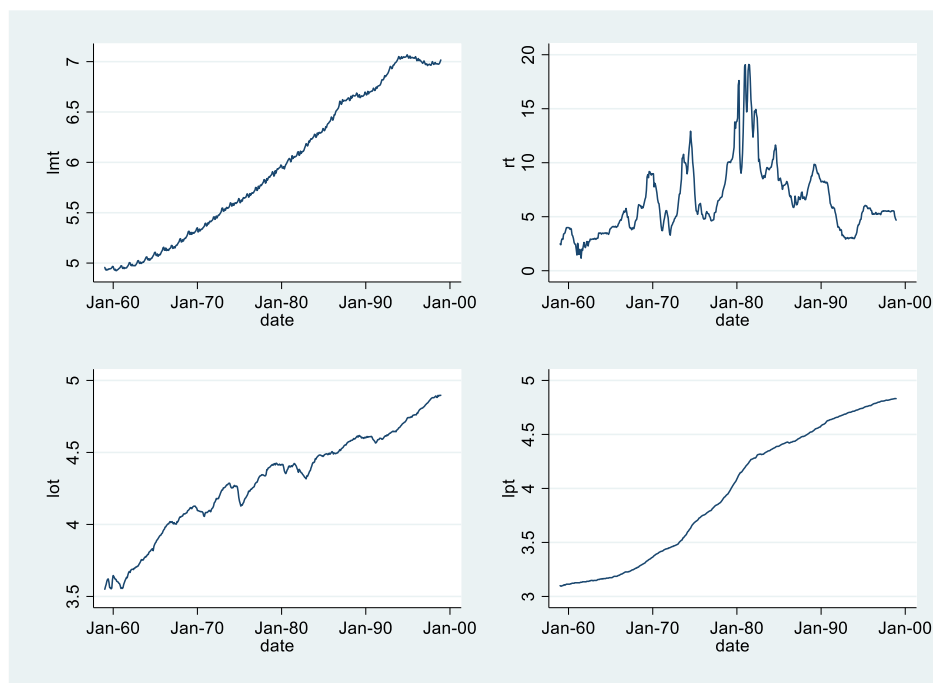


Figure 1

Table 1

lmt	Coef.	St.Err.	t-value	p-value	[95% Conf	Interval]
rt	***	0.001	-26.01	0.000	-0.025	-0.022
lpt	0.961	0.014	***	0.000	0.934	0.989
lot	0.439	0.025	***	0.000	0.390	0.488
Constant	0.416	0.056	7.38	0.000	0.305	0.527
URSS			1.754	RRSS		249.548
R-squared			0.993	Number of obs		480.000
F-test			***	Durbin Watson		0.122
Akaike crit. (AIC)			-1323.557	Bayesian crit. (BIC)		-1306.862

Notes:

1. URSS and RRSS refer to Unrestricted Sum of Squared Residuals and Restricted Sum of Squared Residuals, respectively.
2. Use 5% significance level for all tests.
3. State the null and alternative hypotheses, the test statistic to compute and its distribution, all formulas, and the criteria for rejecting or fail to reject the null hypothesis for all tests.

a). Discuss the main statistical properties of the data. Do they exhibit stationarity? Why?

[5 Marks]

b). Write down a mathematical expression describing the true model of the regression estimated in Table 1. What are the main assumptions imposed on this linear model? Discuss the implications of the invalidity of these assumptions.

[7 Marks]

c). Interpret the coefficient estimates in Table 1. Perform a (two-tailed) test of individual significance of the parameters of all the variables using the critical value of the corresponding distribution and the test p-value. Interpret the test results.

[8 Marks]

d). Explain how you would test for the joint significance of the parameters of all variables. What is your conclusion from this test? Provide an interpretation of the goodness of fit of the model.

[5 Marks]

e). The economist formulates a hypothesis that the effect of logarithm output is half the effect of logarithm prices. Perform an F-test for the economist's hypothesis specifying the null and the equation of the restricted model given that the sum of the squared residuals (RSS) of the restricted model for that test is 1.760. Interpret the test results.

[10 Marks]

f).

- i. Define the concept of unit roots? What are the main implications of presence of unit in the data on the estimated model in Table 1?
- ii. Describe the steps involved to perform Augmented Dickey Fuller (ADF) test.
- iii. Table 2 reports the ADF test for all variables. Discuss the main conclusions of the reported statistics and indicate which variables contain unit roots and which are do not (if any). Explain your answer and show all your working.

[10 Marks]

Table 2

LMT				
Augmented Dickey-Fuller test for unit root		Number of obs = 476		
	Test Statistic	----- Z(t) has t-distribution ----- 1% Critical Value 5% Critical Value 10% Critical Value		
Z(t)	0.048	-2.334	-1.648	-1.283
LOT				
Augmented Dickey-Fuller test for unit root		Number of obs = 476		
	Test Statistic	----- Z(t) has t-distribution ----- 1% Critical Value 5% Critical Value 10% Critical Value		
Z(t)	-0.909	-2.334	-1.648	-1.283
LPT				
Augmented Dickey-Fuller test for unit root		Number of obs = 476		
	Test Statistic	----- Z(t) has t-distribution ----- 1% Critical Value 5% Critical Value 10% Critical Value		
Z(t)	-0.258	-2.334	-1.648	-1.283
RT				
Augmented Dickey-Fuller test for unit root		Number of obs = 476		
	Test Statistic	----- Z(t) has t-distribution ----- 1% Critical Value 5% Critical Value 10% Critical Value		
Z(t)	-2.612	-2.334	-1.648	-1.283

g). Test for the presence of serial correlation in the errors formally using the Durbin Watson statistic. What are your conclusions?

[5 Marks]

Questions continue over page

Question 2

(a) Given that $\varepsilon_t \sim iid(0, \sigma^2)$, derive the mean, the variance and the covariances of the following processes:

i.

$$y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2},$$

where $|\theta_i| < 1, i = 1, 2$

[6 Marks]

ii.

$$w_t = w_{t-1} + \varepsilon_t.$$

where $w_0 = 0$.

[5 Marks]

(b) Which of the above series in (b-i) and (b-ii) are weakly stationary and why?

[4 Marks]

(b) The following regression was run using quarterly data, amounting to 90 observations:

$$\hat{b}_t = 0.49 - 0.27p_t + 0.22y_t$$

(0.84) (0.27) (0.071)

$$\bar{R}^2 = 0.28, DW = 1.20, LM(2)=7.42$$

where (b_t) is the demand for text books, (p_t) is the price of a book and (y_t) is the total level of income, all variables are in logarithms (standard errors in parentheses). DW is the Durbin-Watson test for the first order of autocorrelation and LM(2) is the Lagrange Multiplier test for second order autocorrelation.

(i) Does the above regression suffer from first order autocorrelation?

[4 marks]

(ii) Briefly Describe how you would conduct the LM test for autocorrelation. Does this model suffer from second order autocorrelation?

[6 marks]

Question 3

(a) The following models were fitted to a cross-sectional dataset of 50 firms:

Model 1

$$\hat{y}_i = 2.3 + 0.2x_{1i} - 6.14x_{2i} - 0.01x_{3i} + 1.5x_{4i}$$
$$(0.052) \quad (1.243) \quad (0.015) \quad (1.324)$$
$$R^2 = 0.40, RSS = 0.60$$

Model 2

$$\hat{y}_i = 2.6 + 0.25x_{1i} - 6.14x_{2i}$$
$$(0.42) \quad (2.98)$$
$$R^2 = 0.30, RSS = 0.70$$

where y = rate of return on equity for the firm (ROE), x_1 = market share, x_2 = measure of firm size, x_3 = industrial growth rate, x_4 = level of world trade in industrial products. Numbers in parentheses are coefficient standard errors.

- i. Based on models 1 and 2, examine the joint hypothesis that the coefficients on x_3 and x_4 are both equal to zero. Describe all steps.

[5 Marks]

- ii. Explain how you would test whether financial firms have on average higher ROE.

[5 Marks]

(b) Suppose you wish test for the presence of ARCH effects in a stock market index. Explain how you would conduct a test for the presence of the 4th order ARCH effects.

[3 Marks]

(c) Using monthly data for y_t , the FTSE100 index, from January 1992 to December 2005, the following set of results were obtained:

$$\hat{y}_t = 0.75,$$

$$(0.21)$$

$$\hat{\sigma}_t^2 = 1.76 + 0.07u_{t-1}^2 + 0.24\sigma_{t-1}^2 + 0.87u_{t-1}^2I_{t-1}$$

$$(0.57) \quad (0.02) \quad (0.12) \quad (0.42)$$

$$R^2 = 0.46$$

Values in () are the standard errors of the coefficient estimates and $I_{t-1} = 1$ if $u_{t-1} < 0$, $I_{t-1} = 0$ otherwise. σ_{t-1} is the conditional deviation.

- i. What role does the final term in the above conditional variance equation serve?

[2 Marks]

- ii. What is the interpretation of the estimated value of lagged conditional variance (σ_{t-1}^2), 0.24?

[5 Marks]

- iii. With reference to conditional variance equation, if $\sigma_{t-1}^2 = 0.74$, consider that $\hat{u}_{t-1} = \pm 0.5$. Estimate the value of σ_t^2 , for a positive shock (+0.5) and a negative shock (-0.5).

[5 Marks]

[END OF QUESTIONS]

Distribution Tables

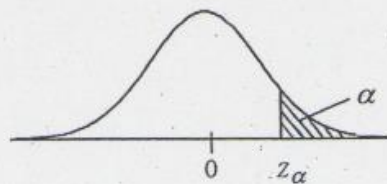
Normal Distribution

Table 4 Percentage Points of the Normal Distribution

The table gives the 100α percentage points, z_α of a standardised normal distribution where

$$\alpha = \frac{1}{\sqrt{2\pi}} \int_{z_\alpha}^{\infty} e^{-z^2/2} dz.$$

Thus z_α is the value of a standardised normal variate which has probability α of being exceeded.



α	z_α	α	z_α	α	z_α	α	z_α	α	z_α
.50	0.0000	.050	1.6449	.030	1.8808	.020	2.0537	.010	2.3263
.45	0.1257	.048	1.6646	.029	1.8957	.019	2.0749	.009	2.3656
.40	0.2533	.046	1.6849	.028	1.9910	.018	2.0969	.008	2.4089
.35	0.3853	.044	1.7060	.027	1.9268	.017	2.1201	.007	2.4573
.30	0.5244	.042	1.7279	.026	1.9431	.016	2.1444	.006	2.5121
.25	0.6745	.040	1.7507	.025	1.9600	.015	2.1701	.005	2.5758
.20	0.8416	.038	1.7744	.024	1.9774	.014	2.1973	.004	2.6521
.15	1.0364	.036	1.7991	.023	1.9954	.013	2.2262	.003	2.7478
.10	1.2816	.034	1.8250	.022	2.0141	.012	2.2571	.002	2.8782
.05	1.6449	.032	1.8522	.021	2.0335	.011	2.2904	.001	3.0902
								.000 1	3.7190
								.000 01	4.2649
								.025	1.9600
								.005	2.5758
								.000 5	3.2905
								.000 05	3.8906
								.000 005	4.4172

Student t Distribution

$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu = 1$	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Chi-Square Distribution

Table A.4 χ^2 Distribution: critical values of χ^2 at 5%, 1%, and 0.1% significance levels

Degrees of freedom	5%	1%	0.1%
1	3.8415	6.6349	10.828
2	5.9915	9.2103	13.816
3	7.8147	11.3449	16.266
4	9.4877	13.2767	18.467
5	11.0705	15.0863	20.515
6	12.5916	16.8119	22.458
7	14.0671	18.4753	24.322
8	15.5073	20.0902	26.125
9	16.9190	21.6660	27.877
10	18.3070	23.2093	29.588
11	19.6751	24.7250	31.264
12	21.0261	26.2170	32.909
13	22.3620	27.6882	34.528
14	23.6848	29.1412	36.123
15	24.9958	30.5779	37.697
16	26.2962	31.9999	39.252
17	27.5871	33.4087	40.790
18	28.8693	34.8053	42.312
19	30.1435	36.1909	43.820
20	31.4104	37.5662	45.315
21	32.6706	38.9322	46.797
22	33.9244	40.2894	48.268
23	35.1725	41.6384	49.728
24	36.4150	42.9798	51.179
25	37.6525	44.3141	52.618
26	38.8851	45.6417	54.052
27	40.1133	46.9629	55.476
28	41.3371	48.2782	56.892
29	42.5570	49.5879	58.301
30	43.7730	50.8922	59.703
40	55.7585	63.6907	73.402
50	67.5048	76.1539	86.661
60	79.0819	88.3794	99.607
70	90.5312	100.425	112.317
80	101.879	112.329	124.839
90	113.145	124.116	137.208
100	124.342	135.807	149.449

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F Distribution

Table A.3 F Distribution: critical values of F with ν_1 and ν_2 degrees of freedom, 5% significance level

$\nu_1 \backslash \nu_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.20	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

DW d Statistic

Table A.5 Durbin-Watson d statistic: d_L and d_U , 5% significance level

n	$k = 1$		$k = 2$		$k = 3$		$k = 4$		$k = 5$	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78