Predictive Analysis for Decision Making

Week 10
Modelling Time Varying Volatility

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Aims and Objectives

- Linear models and stationary structure of the data do not explain common features in financial time series:
  - Leptokurtic distributions
  - Volatility clustering and volatility pooling.
  - Leverage effects
- Recall the model

$$y_t = \beta_1 + \beta_2 x_{2t} + \ldots + \beta_k x_{kt} + u_t$$
$$u_t \sim N(0,\sigma^2).$$

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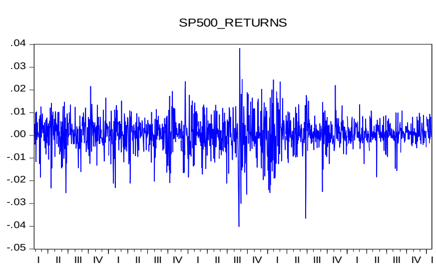
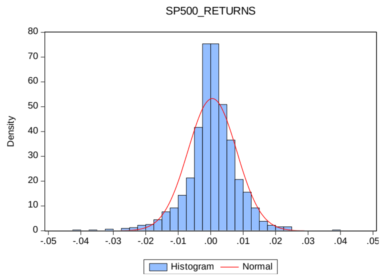
Financial Time Series- Stylized Facts

Clusters of Variances



Financial Time Series- Stylized Facts
Leptokurtic Distribution- Kurtosis>3



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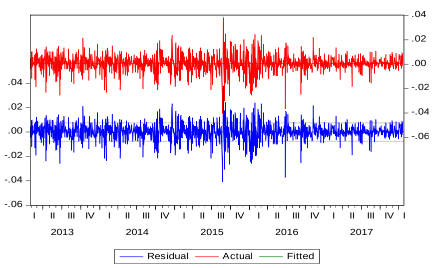
Financial Time Series- Stylized Facts
Residuals are still persistent (only intercept)



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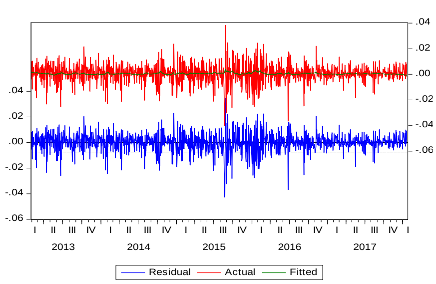
Financial Time Series- Stylized Facts
Residuals are still persistent (ARMA(1,1))



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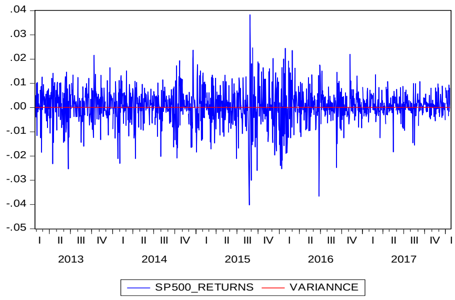
Financial Time Series- Stylized Facts

Constant risk (variance)



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Approaches to Modelling Volatility

ARCH and GARCH Models

- The structure of the variance need to be modified

  Example:

  $y_t = \beta_1 + \beta_2 x_{2t} + \ldots + \beta_k x_{kt} + u_t, \; u_t \sim \mathrm{N}(0, \sigma_t^2)$

- The variance need to be modelled using a suitable structure
- Engle (1982) proposed an Autoregressive Conditional Heteroskedastic structure
  - ARCH models
  - Capture an autoregressive relationship in the model
  - Has similar approach to AR models

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Approaches to Modelling Volatility

ARCH and GARCH Models

- The ARCH(1)

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$$
$$\alpha_0 > 0, \; \alpha_1 \geq 0$$

- The ARCH (q)

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2$$
$$\alpha_0 > 0, \; \alpha_1, \ldots, \alpha_q \geq 0$$

- Note:
  - The condition $\alpha_0 > 0$ implies we have a non-zero variance
  - The condition $\alpha_1, \ldots, \alpha_q \geq 0$ implies that serial the autoregressive terms can be zero but cannot be negative.
  - The coefficients $\alpha_1, \ldots, \alpha_q$ capture the effect of past risk or volatility (called ARCH effects)
  - If the ARCH effects are jointly significant, then we have conditional heteroscedasticity that is time varying.
  - Thus, before we model volatility, we need first to test for the presence of ARCH effect.

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Testing for the ARCH Effect

1. First, run any postulated linear regression of the form given in the equation above, e.g. $y_t = \beta_1 + \beta_2 x_{2t} + ... + \beta_k x_{kt} + u_t$ saving the residuals, $\hat{u}_t$
2. Then square the residuals, and regress them on q own lags to test for ARCH of order q, i.e. run the regression $\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \gamma_2 \hat{u}_{t-2}^2 + ... + \gamma_q \hat{u}_{t-q}^2 + v_t$

   where v_t is iid. Obtain R^2 from this regression
3. The test statistic is defined as TR^2 (the number of observations multiplied by the coefficient of multiple correlation) from the last regression, and is distributed as a $\chi^2(q)$.
4. The null and alternative hypotheses are

   H_0 : $\gamma_1 = 0$ and $\gamma_2 = 0$ and $\gamma_3 = 0$ and ... and $\gamma_q = 0$

   H_1 : $\gamma_1 \neq 0$ or $\gamma_2 \neq 0$ or $\gamma_3 \neq 0$ or ... or $\gamma_q \neq 0$.
5. Reject the null of 'no ARCH effect' if the statistic exceeds the critical value.

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Testing for the ARCH Effect Issues

- How do we decide on q?
- The required value of q might be very large
- Non-negativity constraints might be violated.
  - When we estimate an ARCH model, we require $\alpha_i > 0 \ \forall \ i=1,2,...,q$ (since variance cannot be negative)

A natural extension of an ARCH(q) model which gets around some of these problems is a GARCH model.
  - Generalised Autoregressive Conditional Heteroskedastic model
  - It has been shown that ARCH(∞) is equivalent to GARCH(1,1)
  - GARCH (1,1) has two parts: the ARCH effect and GARCH effect.

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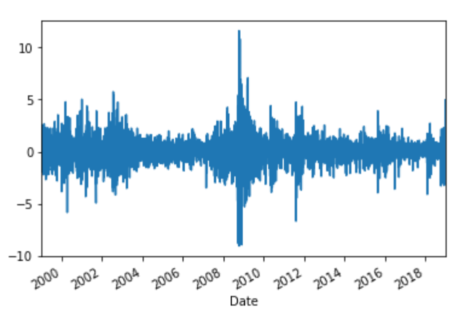
Example 2: ARCH Model



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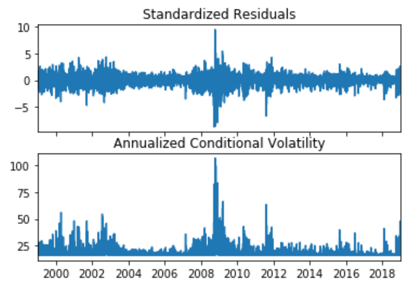
Example 1: ARCH Model

```
                     Constant Mean - ARCH Model Results
==============================================================================
Dep. Variable:                Adj Close   R-squared:                      -0.000
Mean Model:               Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                         ARCH   Log-Likelihood:               -7803.64
Distribution:                    Normal   AIC:                           15613.3
Method:              Maximum Likelihood   BIC:                           15632.9
                                          No. Observations:                 5030
Date:                  Wed, Mar 24 2021   Df Residuals:                     5027
Time:                          00:57:30   Df Model:                            3
                                 Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu             0.0403  1.798e-02      2.240  2.508e-02 [5.038e-03,7.553e-02]
                            Volatility Model
=========================================================================
                 coef    std err          t      P>|t|   95.0% Conf. Int.
-------------------------------------------------------------------------
omega          1.0059  5.585e-02     18.012  1.570e-72 [  0.896,  1.115]
alpha[1]       0.3323  5.381e-02      6.176  6.564e-10 [  0.227,  0.438]
=========================================================================
```

Estimating the ARCH Model Volatility



The GARCH Model

- Combines the ARCH and GARCH effect
  - The general form

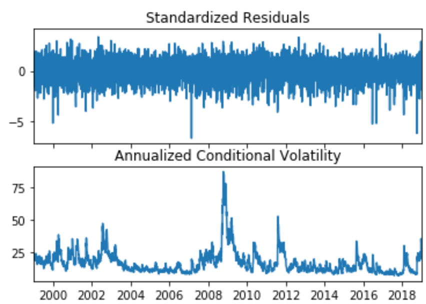
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + v_t$$
$$\alpha_0 > 0 \text{ and } \alpha_1, \beta_1 \geq 0$$

  - The ARCH effect α_1: measures the magnitude of the shock.
  - The GARCH effect β_1: measures the persistence of the shock.

EXAMPLE 2: GARCH MODEL

```
File  Edit  Format  View  Help
                  Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                Adj Close   R-squared:                      -0.001
Mean Model:               Constant Mean   Adj. R-squared:                 -0.001
Vol Model:                        GARCH   Log-Likelihood:               -6936.72
Distribution:                    Normal   AIC:                           13881.4
Method:              Maximum Likelihood   BIC:                           13907.5
                                          No. Observations:                 5030
Date:                  Wed, Mar 24 2021   Df Residuals:                     5026
Time:                          00:57:30   Df Model:                            4
                               Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0564  1.149e-02      4.906  9.302e-07 [3.384e-02,7.887e-02]
                            Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega          0.0175  4.683e-03      3.738  1.854e-04 [8.328e-03,2.669e-02]
alpha[1]       0.1022  1.301e-02      7.852  4.105e-15   [7.665e-02,  0.128]
beta[1]        0.8852  1.380e-02     64.125      0.000   [  0.858,    0.912]
============================================================================
```

ESTIMATING THE GARCH MODEL VOLATILITY



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THE THRESHOLD GARCH MODEL TGARCH/ GJR MODEL

- Due to Glosten, Jaganathan and Runkle

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$$

where $I_{t-1} = 1$ if $u_{t-1} < 0$
$= 0$ otherwise

- For a leverage effect, we would see $\gamma > 0$.
- We require $\alpha_1 + \gamma \geq 0$ and $\alpha_1 \geq 0$ for non-negativity.

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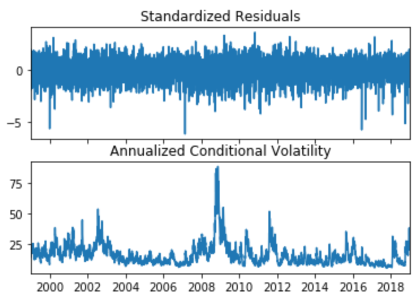
Example 3: TGARCH Model

```
                Constant Mean - TARCH/ZARCH Model Results
==============================================================================
Dep. Variable:                Adj Close   R-squared:                      -0.000
Mean Model:               Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                  TARCH/ZARCH   Log-Likelihood:               -6799.18
Distribution:                    Normal   AIC:                           13608.4
Method:              Maximum Likelihood   BIC:                           13641.0
                                          No. Observations:                 5030
Date:                  Wed, Mar 24 2021   Df Residuals:                     5025
Time:                          01:20:35   Df Model:                            5
                               Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu             0.0143  1.091e-02      1.311      0.190 [-7.080e-03,3.570e-02]
                            Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.0258  4.100e-03      6.299  2.986e-10  [1.779e-02,3.386e-02]
alpha[1]   3.0844e-09  9.156e-03  3.369e-07      1.000 [-1.794e-02,1.794e-02]
gamma[1]       0.1707  1.601e-02     10.664  1.499e-26      [  0.139,  0.202]
beta[1]        0.9098  9.672e-03     94.066      0.000      [  0.891,  0.929]
=============================================================================
```

Estimating the TGARCH Model Volatility



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THANK YOU

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