UNIVERSITY OF WESTMINSTER

MSc Fintech with Business Analytics

2022-2023

COMPUTATIONAL METHODS FOR FINANCE

Lecturer: [redacted]

ANALYSIS ON POLKADOT (DOT-USD)

PRICING VANILLA EUROPEAN PUT OPTION & GREEKS ANALYSIS

Individual Coursework – Semester 1

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Word Count: 2.522

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ABSTRACT

Polkadot is chosen and introduced in this report because of its reputational technology and achievements in the crypto space in recent years. The first session will give brief background of the project and analysis on price movement of its currency - Polkadot (DOT-USD) with time period of 2 years (Sep 2020- Aug 2022).

In recent years, option valuation methods are very important in the theory of finance and increased wildly in the practice field. Options for stock is already proving its position in financial market, but not yet in cryptocurrency. Hence, this report will discover this area by discover various approaches on option prices valuation included binomial tree models, Monte Carlo simulation and Black-Scholes. Vanilla European Put option will be written and priced by Binomial tree and Monte-Carlo simulation, in which final price will be concluded based on literature and mathematical analysis.

Furthermore, with the complexity of numerical computation, the last session will analyze 5 Greeks measure to provide deeper understanding of derivative option pricing, how the option value moves according to various independent variables and the risk associated with the option.

Github link: https://github.com/joy-bb/CMF-W1613280.git

POLKA DOT OVERVIEW

1.Project background

Polkadot is a software launched in 2020, is one of the newest that seeks to incentivize a global network of computers to operate a blockchain on top of which users can launch and operate their own blockchains. It aims at growing an ecosystem of cryptocurrencies, which similar like other well-known project like Ethereum (ETH), Cosmos (ATOM) and EOSIO (EOS). Because Polkadot allows any type of data to be shared trustlessly or send any asset or token trustlessly between any type of blockchain, it unlocks a wide range of real-world use cases.

To date, Polkadot has successfully raised around $200 million from investors from the sales of its cryptocurrency DOT-USD, making it one of the most well-funded blockchain projects in history, and ranking in top 20 coins over the last 2 years (CoinMarketCap.com).

The Polkadot network enables the development of three different blockchain variants.

- The primary Polkadot blockchain, where transactions are closed off more quickly, is called the Relay Chain. According to 2020 testing, this architecture enables Polkadot to perform more than 1,000 transactions/second
- Parachains - Parachains are specialised blockchains that employ the computer power of the relay chain to validate the legitimacy of operations.
- Bridges - Bridges enable communication between the Polkadot network and other blockchains. The team is establishing connections with other blockchains, including as Ethereum and Bitcoin, so that tokens may be traded decentralised exchange-free.

What Distinguishes Polkadot from Ethereum?

Polkadot is regarded as Ethereum 2.0, the updated version of Ethereum, because they both have prominent founders and share many characteristics, however Polkadot is superior in both design and functioning.

Both networks have a primary blockchain that settles transactions and permits the development of several auxiliary blockchains that make use of it. Both systems also employ staking in place of mining to maintain network synchronisation.

The main area of research is how to make network-to-network transactions compatible. Parity, for instance, has created technology for users who would want to launch apps that make use of the Ethereum code and community but that would operate on Polkadot. Additionally, programmers may mimic a replica of the Ethereum blockchain using Polkadot's development model so that it can be utilised in their own unique blockchain architecture.

2.Polkadot price movement of the chosen period

This report is analyzed price DOT-USD movement in the period of Sep 2020-Aug 2022.

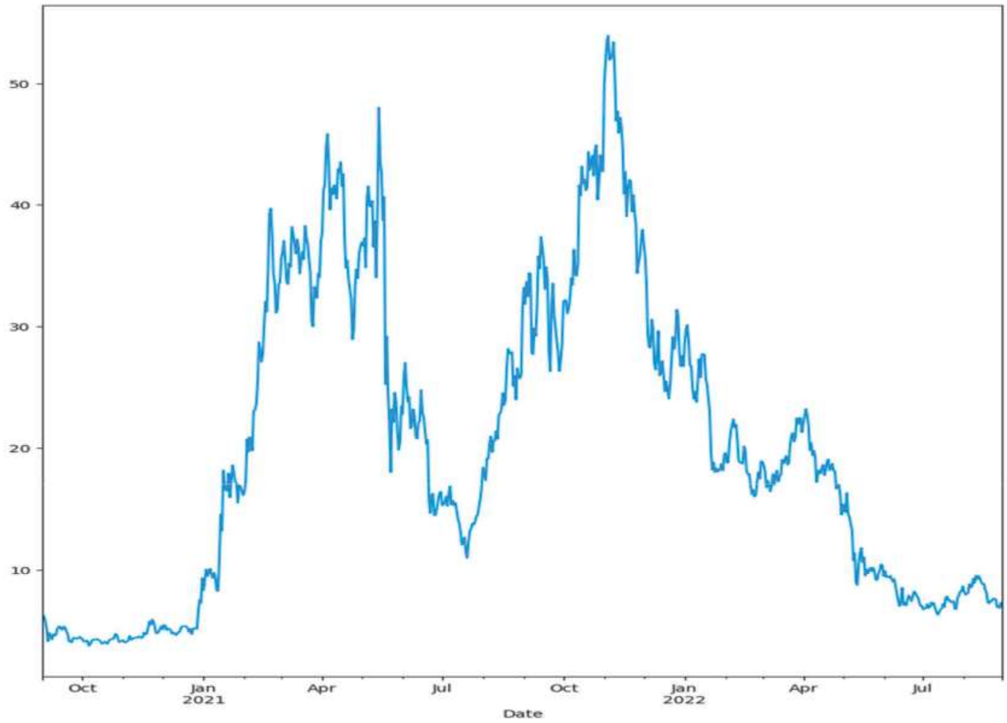

Figure 1. Polka Dot Price movement over the 2 year period of Sep 2020-Aug 2022

Descriptive analysis on Polka Dot price during chosen period

Period	*Mean*	*Std*	*Min*	*Max*	*25%*	*50%*	*75%*
Sep 2020-Dec 2022	**20.13**	**12.60**	**3.76**	**8.13**	8.13	18.34	29.61

3.Technical analysis

- *Log Return* is one of three methods used to calculate asset’s return, assuming that returns are continuously compounded. It is calculated by taking the natural log of the ending value divided by the beginning value.

In crypto world, since the volatility is extremely higher compared to other assets (stocks), this report is using 3 months data *(Jun 2022-Aug 2022)* to calculate Polka Dot log return, instead of 1 year to archie more accurate and updated impact.

As calculated, ***the log-return of Polka Dot from the 3month-annalysied period is -0.0030*** and is plotted in figure 2.

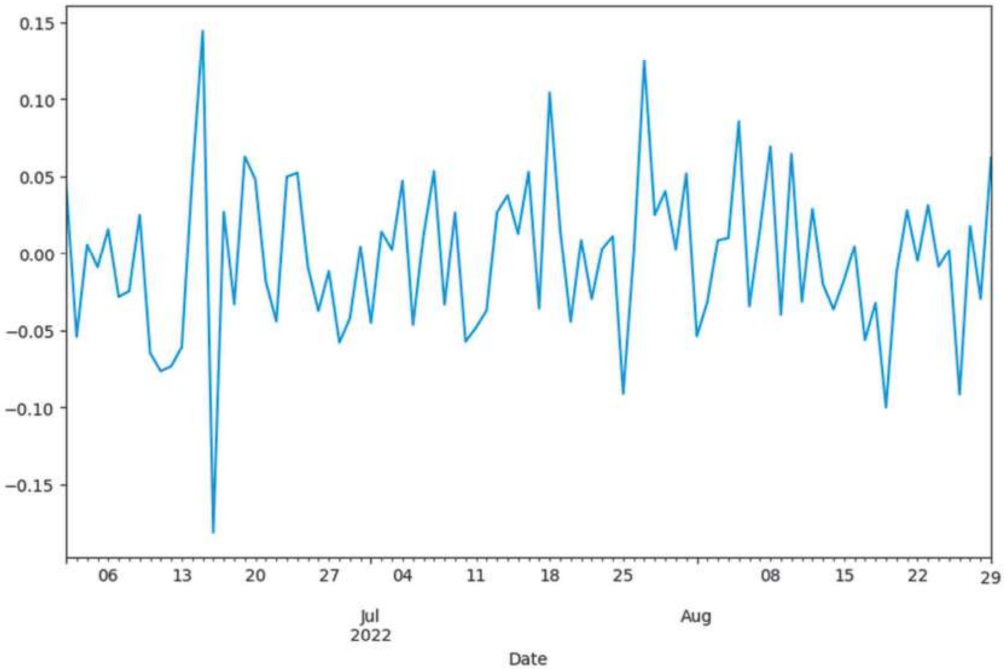

Figure 2. Polka Dot's time series of Log return (Jun-Aug 2022)

- *Annualized volatility* is statistical and historical volatility, based on historical prices and represents the fluctuations of underlying asset by measuring price changes over predetermined periods of time.

  In this case, from the figure 2, there is a bit higher fluctuation in 15 Jun (8.5103) compared to other time in that period, means the volatility is higher and Polka Dot price ranging further away from the standard deviation.

  Formula: $vol = \sigma\sqrt{T}$

  HG 3

  where:

  - v = volatility over some interval of time

- σ =standard deviation of returns
- T = number of periods in the time horizon

As calculated, ***the annualized volatility of Polka Dot Jun-Aug/2022 period is 97.39%***

Volatility is a key factor in pricing options, measuring the fluctuation of the asset price from now until maturity date. Volatility is coefficient within option-pricing formulas.

OPTION PRICING MODELS

This report is for the sell-side, writing Vanilla European Put Option. The option pricing will be calculated, analyzed and decided with application of Binomial Tree model and Monte-Carlo Simulation, based on following figures:

	Note	*Details*
Spot price	Adjust close price on 30/08/2022	***#S: 7.00***
Strike price		***#K: 8.00***
Time to maturity	*1 week*	***#T: 1/52***
Interest rate	Source: US. 10 Year Treasury Rate, updated Apr 2022	***#r: 0.03***
Volatility	Calculated using 3months-data of Jun-Aug 2022	***#σ: 97.39% or 0.97***

1.Binomial Tree

The binomial option pricing model uses an iterative procedure, producing the specification of nodes in time during the time period between the valuation date and the date to maturity. At every step, the asset price either goes up or goes down, hence the model predicts and creates a binomial distribution of underlying asset prices. Therefore, it represents theoretically possible paths that the asset price will take in the option period.

Consequently, the option prices at each step of Binomial tree are calculated backward from expiration to the present. Any changes to asset prices or option value are considered to figure out the result at specific node in time.

- Step 1: Creating Binomial tree by working forward from valuation date to expiration. The up and down factors are calculated with following formula:

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}.$$

$$S_n = S_0 \times u^{N_u - N_d},$$

Where N_u is the number of up ticks and N_d is the number of down ticks.

- Step 2: Finding option value at each final node

Max [$(K - S_n)$, 0], for a put option,

- Step 3: Finding option value at earlier node using risk-neutral probability

$$p = \frac{e^{r\delta t} - v}{u - v}.$$

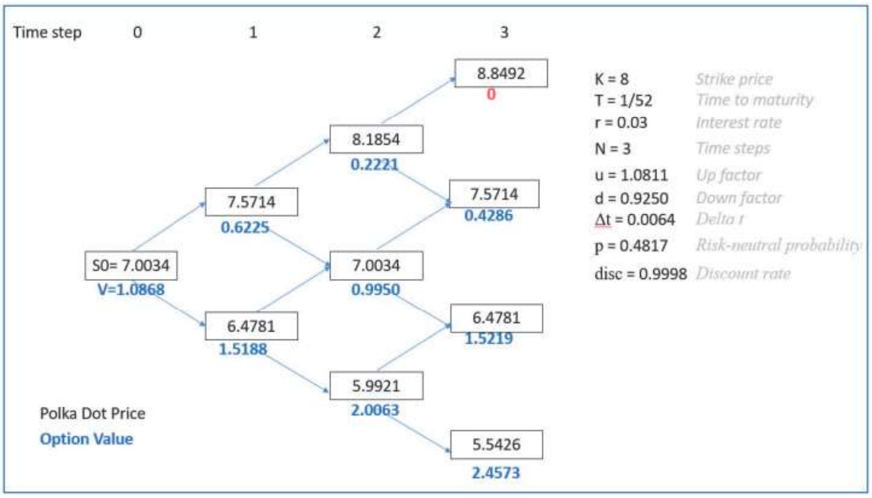

Figure 3. Multiplicative Binomial Tree Valuation of a Europe put call for Polka Dot

Polka Dot has spot price of \$7, put option strike price of \$8, one-week expiration date, and interest rate (r) of 3%. The option price is calculated on each node according to the Polka Dot prices moving on each node. When price move above strike price - node 3,1 – the option will be worthless hence the option price is 0. In the other hands, when the price movement is still below the strike price, the put option is in-out-money, therefore having the according values stated on figure 3.

Based on the **Bionimal Tree** model, the option price for chosen European put is computationally calculated as **1.0868.**

2.Monte-carlo simulation

Computational modelling is used in Monte Carlo simulation to forecast results. A random number is initially produced by the model using a probability distribution. The stock price is then generated by the random number using the extra inputs of volatility and time to expiration. The value of the option is then determined using the produced stock price at the time of expiration. The program then repeatedly estimates outcomes using a new set of random values drawn from the probability functions. Before it is finished, a Monte Carlo simulation may require thousands of computations, depending on the model, the amount of uncertainty, and the probability distributions utilised. The average of all the estimated results is often used by Monte Carlo simulation for option models as the option price.

In setup the underlying of the option to be valued follows the stochastic differential equation.

$$dS_t = rS_t dt + \sigma S_t dW_t$$

This stochastic differential equation can be discretized over equidistant time intervals and simulated according to the following equation, which represents an Euler scheme. In this case, z is a standard normally distributed random number. For M time intervals (M = 1000), the length of the time interval is given as $\Delta t \equiv \frac{T}{M}$ where T is the time horizon for the simulation (for example, the maturity date of an option to be valued).

$$S_t = S_{t-\Delta t} \exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}z\right)$$

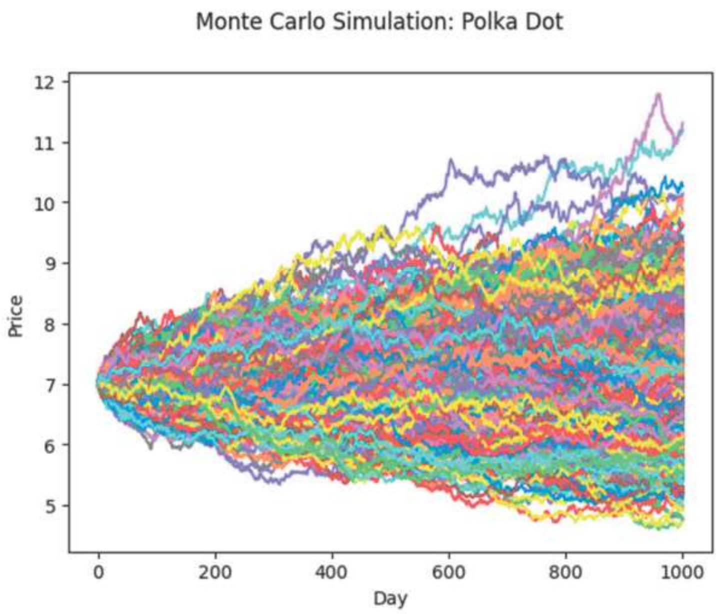

Figure 4. Monte Carlos Simulation – Polka Dot

The frequencies of different outcomes generated by this simulation will form a normal distribution, that is, a bell curve. The most likely return is in the middle of the curve, meaning there is an equal chance that the actual return will be higher or lower.

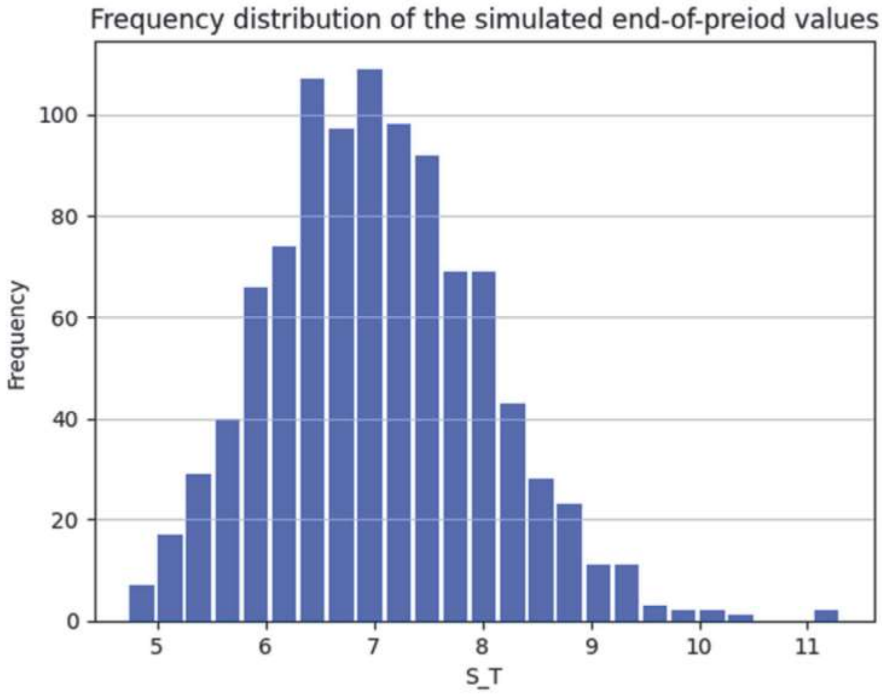

Figure 5. Frequency distribution of simulated

Based on the **Monte Carlo Simulation** model, the option price for chosen European put is computational calculated as **1.0603.**

3.Comparision and Final Price

Binomial Tree is good for a American option, because it can provides insight as to when exercising the option, for which can be exercised at any time before the expiration date. The pros of this multi-period view is making clear visualization asset price's movement from period to period and evaluate the option value accordingly regarding decisions made at different time node. With the efficiency and accuracy Binomial tree provided, the model is better used when there are a small number of options values without dividends. In addition, there are only two possibilities for underlying asset price while in reality, it can be worth any number of within any given range.

Monte Carlo, however, taking to account of randomness and should be seen as supplement method of Binomial Tree model, because the increasing of a variety of complexity in financial instruments, especially in crypto space. It's flexible and more powerful method because it calculates a multi-dimension integral and used to solve the problem of high dimension. However, Monte Carlo simulation also has drawbacks like ignoring other factors which is not price-movement-related such as economy change, company's management, market, market stimulations, etc. Also, the computation with many times and pricing option gives different results for every calculation since it's heavily based on random movement.

Binomial Tree	Monte-Carlo Simulation	Final Price
Option price: 1.0868	Option price: 1.0603	**1.0736**

Since this report aims at writing European Put option for a cryptocurrency (Polka Dot), at spot price S0=$7, strike price K=$8, maturity 1 week, with risk free rate 3% - the final option's premium is chosen by taking average price from both models above – results in **$1.0736**. The reason is to take advantages of Monte-Carlo model as below, and using Binomial Tree's price to stabilize the random move of Monte-Carlo (since Monte-Carlo simulation provide different prices after every calculation). Hence, average price from both model is recommended in this report.

- The option can only be exercised at maturity date, so the advantage of Binomial tree is not applicable in the case for Europe option in consideration of its shortcomings.
- Advantage of Monte-Carlo model suitable more for the complex, high volatility and uncertainty like crypto space, since the model is simulated 1000 times and the frequency coming at the highest compared to Binomial tree.

As reference, the final price 1.0736 is quite close to the price calculated from Black-Scholes model 1.0717 (will be discussed in next session), making the result is more reliable.

5 GEEKS ANALYSIS

The Greeks are calculated to measure the sensitivity of option's price (Black-scholes model) to independent variables such as underlying asset price, volatility, time to maturity and interest rate. Therefore, these Greeks are utilized in sensitivity analysis, designed to better understand how option prices change. They are necessary for determining how to properly hedge a portfolio and are therefore important for risk management.

The key Greeks – Delta, Gamma, Theta, Rho and Vega are the most popular in all derivatives and will be discussed in this report.

Name	Dependent Variable	Independent Variable
Delta	Option price	Value of underlying asset
Gamma	Delta	Value of underlying asset
Vega	Option price	Volatility
Theta	Option price	Time to maturity
Rho	Option price	Interest rate

This report will cover Greeks in the Black-Scholes environment, where option is only exercised at maturity date and have zero dividend. Also, underlying asset's price movement will be assumed to follows a geometric Brownian motion process. This means the underlying variables are the following: the stock price, volatility, the risk-free rate, and time.

Formula **Put = N (−d2) K exp[−r(T−t)] − N(−d1)S0**

Where

$$d_1 = \frac{\log(S/K) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\log(S/K) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

Based on the **Black-scholes** model, the option price for chosen European put is computationally calculated as **1.0717.**

This session will analyze Greeks measures for the above Vanilla European Put Option.

1.Delta

Delta (Δ) is designed to measure the sensitivity of option value changing accordingly to the changes in the underlying asset's price. In other words, if the underlying asset's price increases by $1, the price of the option will change by Δ amount.

The delta of a call option has a range between 0 and 1, while the delta of a put option has a range between 0 and -1.

Formula: $\boldsymbol{\Delta = \frac{\partial V}{\partial S} = -e^{-q(T-t)} N(-d_1)}$

Where:

- ∂ – the first derivative
- V – the option's price (theoretical value)
- S – the underlying asset's price

Result calculated: -0.81944862

Explain: if polka dot price increase by $1, the European put option's premium will then decrease by $0.8194.

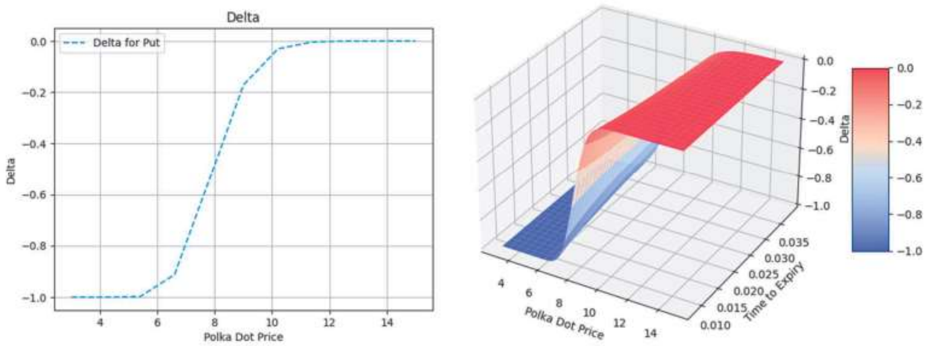

Figure 6. Delta 2D movement & 3D Surface

2.Gamma

Gamma (Γ) is designed to measure the Delta changing accordingly to the changes in the underlying asset's price. In other words, if the underlying asset's price changes by $1, the option delta will change by gamma amount. The main application of gamma is the assessment of the option's delta.

Higher gamma indicates that delta could change significantly in response to even small movements in asset price. Gamma is higher for options at-the-money and lower for options in- and out-of-the-money. The further away from expiration date, the smaller the Gamma, in other words, the closer the expiration date, the higher the gamma.

Formula: $\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial^2 S} = \frac{e^{-q(T-t)} N'(d_1)}{\sigma S \sqrt{T-t}}$

Where:

- ∂ – the first derivative
- V – the option's price (theoretical value)
- S – the underlying asset's price

Result calculated: 0.277949

Explain: if polka dot price increase by $1, the put option's delta will then increase by 0.2779. The gamma fluctuation will be wider when the strike price is very close to asset's price. And in this case, spot price = $7 and strike price = $8.

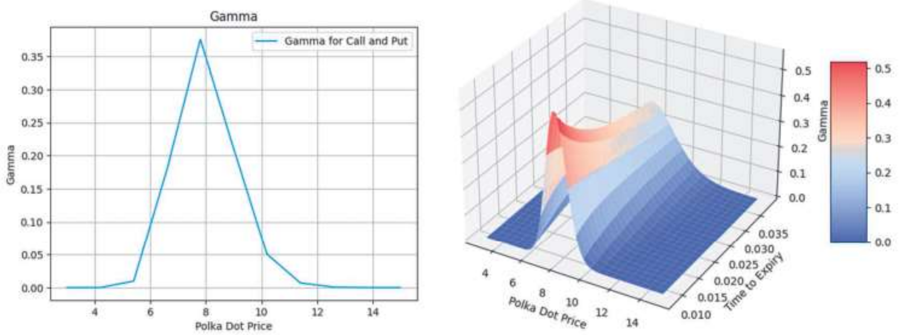

Figure 7. Gamma 2D movement & 3D Surface

3.Theta

Theta (θ) is used to measure option price's sensitivity relative to change of option's time to maturity. If the option's time to maturity decreases by one day, the option's premium will change by the theta amount.

Theta is generally expressed as a negative number because the closer to date of expiration, the option's value will decrease. In this case, the theta is positive because this report is calculated from the sell-side perspective instead of buyer/trader.

Formula: $\theta = \frac{\partial V}{\partial T} = -\frac{\sigma S e^{-q(T-t)} N'(-d_1)}{2\sqrt{T-t}} + qSN(-d_1)e^{-q(T-t)} - rKe^{-r(T-t)}N(-d_2)$

Where:

- ∂ – the first derivative
- V – the option's price (theoretical value)
- τ – the option's time to maturity

Result calculated: 6.670054

Explain: if the time to maturity decrease by 1 year, the option value will decrease by \$6.67. In other words, if the time to maturity decreases by 1 day, the option value will decrease by \$6.67 / 365 = \$0.0183.

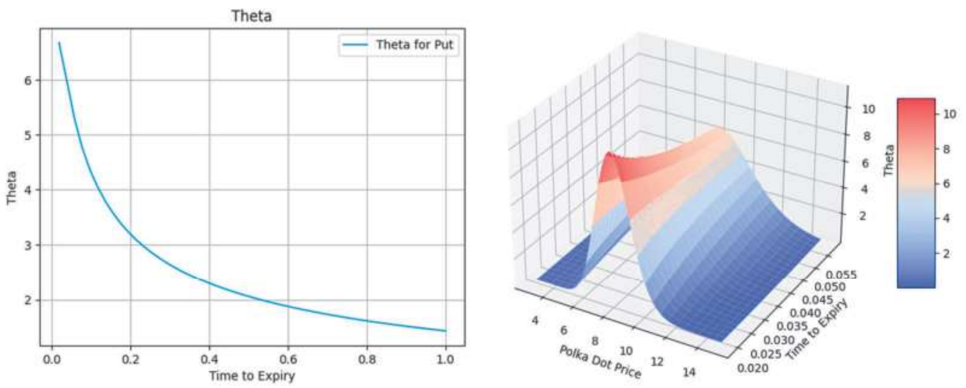

Figure 7. Theta 2D movement & 3D Surface

4.Vega

Vega (ν) is an option Greek that measures the sensitivity of an option price relative to the volatility of the underlying asset. Volatility plays a major role pricing the option since it measures how the price of the option changes with a 1% change in volatility.

Formula: $\nu = \frac{\partial V}{\partial \sigma} = S\sqrt{T-t}e^{-q(T-t)}N'(d_1)$

Where:

- ∂ – the first derivative
- V – the option's price (theoretical value)
- σ – the volatility of the underlying asset

Result calculated: v = 0.255333

Explain: if the volatility increases by 1%, the option's value will increase by $0.0026

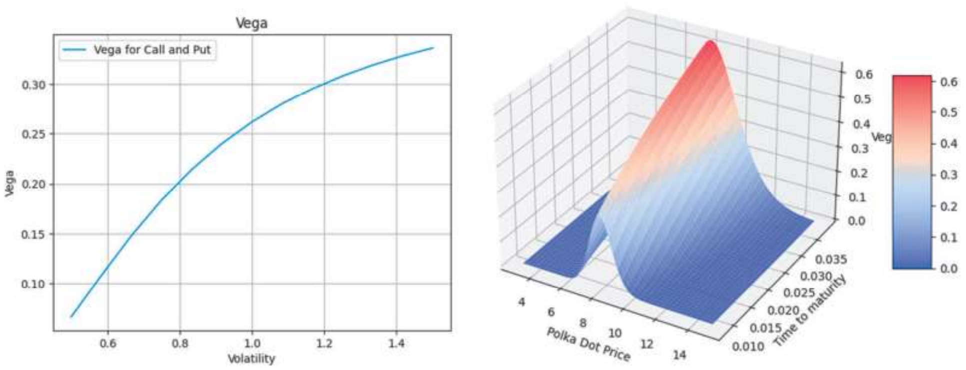

Figure 8. Vega 2D movement & 3D Surface

5.Rho

Rho (ρ) measures the sensitivity of the option price according to the change in interest rates. If the risk-free interest rate increases by 1%, the option price will change by the rho amount.

The rho is considered the least significant among other option Greeks because option values are generally less sensitive to interest rate changes than to changes in other parameters.

Formula: $\rho = \frac{\partial V}{\partial r} = -K(T-t)e^{-r(T-t)}N(-d_2)$

Where:

- ∂ – the first derivative
- V – the option's price (theoretical value)
- r – interest rate

Result calculated: -0.13111746

Explain: if the risk-free rate increases by 1%, the put option will decrease by \$0.0013

Figure 6. Delta 2D movement & 3D Surface

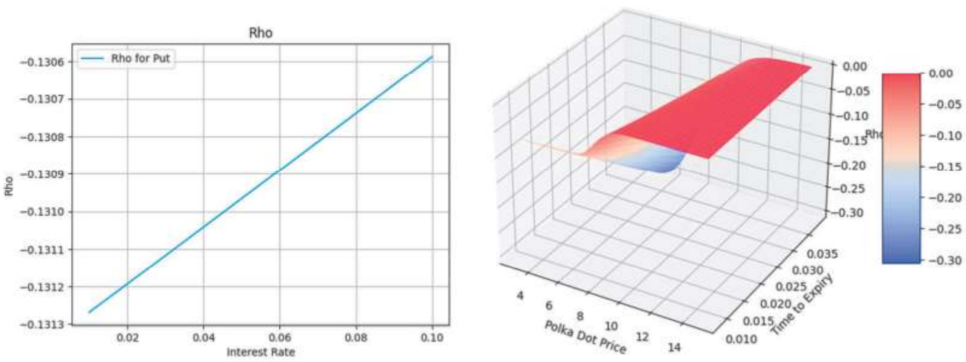

Figure 9. Rho 2D movement & 3D Surface