Computational Methods for Finance
Week 1: Introduction to option II

Yang Yue

University of Westminster

September 2023

Learning Outcomes

At the end of this lecture you will be able to

- Understand the determinants of option prices
- Prove the existence of lower and upper bounds on option prices (premia) and a relationship between call and put option premia.

Determinants of option prices: Key assumptions

- Key Assumptions
  - No transaction costs.
  - All trading profits net of trading losses are subject to the same tax rate.
  - Borrowing and lending are possible at the risk-free interest rate.
  - Arbitrage opportunities are taken immediately (i.e. no arbitrage opportunities exist).

Determinants of option prices: factors

- Six factors affecting option prices
  - Current stock price: S
  - Strike Price: K
  - Time to expiration: T
  - Stock price volatility: σ
  - Risk free interest rate: r

Determinants of option prices: factors

- Directions:

Variable	European Call	European Put	American Call	American Put
S	+	-	+	-
K	-	+	-	+
T	?	?	+	+
σ	+	+	+	+
r	+	-	+	-

Bounds of option values

Table 1: Bounds of option values

Styles	Classes	Lower bounds	Upper bounds
European	Call	$max(S - Ke^{-rT}, 0)$	S
	Put	$max(Ke^{-rT} - S, 0)$	Ke^{-rT}
American	Call	$max(S - Ke^{-rT}, 0)$	S
	Put	$max(K - S, 0)$	K

Bounds of option values: proofs

Table 2: Proof that $p \leq Ke^{-rT}$

		Terminal Value	
Action	Initial Value	$S_T \leq K$	$S_T > K$
Write Put	p	$-(K - S_T)$	0
Lend	$-Ke^{-rT}$	K	K
Total	$p - Ke^{-rT}$	S_T	K

Bounds of option values: proofs

Table 3: Proof that $c \leq S$

		Terminal Value	
Action	Initial Value	$S_T \leq K$	$S_T > K$
Write Call	c	0	-($S_T - K$)
Buy Stock	-S	S_T	S_T
Total	c-S	S_T	K

Bounds of option values: proofs

Table 4: Proof that $c \geq max(S - Ke^{-rT}, 0)$

		Terminal Value	
Action	Initial Value	$S_T \leq K$	$S_T > K$
Buy Call	-c	0	$S_T - K$
Sell Stock	S	$-S_T$	$-S_T$
Lend	$-Ke^{-rT}$	K	K
Total	$S - Ke^{-rT} - c$	$-S_T + K$	0

Bounds of option values: proofs

Table 5: Proof that $p \geq max(Ke^{-rT} - S, 0)$

		Terminal Value	
Action	Initial Value	$S_T \leq K$	$S_T > K$
Buy Put	-p	$K - S_T$	0
Buy Stock	-S	S_T	S_T
Borrow	Ke^{-rT}	$-K$	$-K$
Total	$Ke^{-rT} - S - p$	0	$S_T - K$

Put-Call relationship

For non-dividend paying stock we have: $c + Ke^{-rT} = p + S$

Table 6: Put-call parity

Action	Initial Value	Terminal Value $S_T \leq K$	Terminal Value $S_T > K$
Write Call	c	0	$-(S-K)$
Buy Put	-p	$K - S_T$	0
Buy Stock	-S	S_T	S_T
Borrow	Ke^{-rT}	$-K$	$-K$
Total	$c + Ke^{-rT} - p - S$	0	0

Summary of Week 1

- Concepts of financial market:
  Trading venues, market participants, trading instruments.
- Concepts of option:
  European vs. American options, call vs. put options; their payoffs.
- Option Specifics:
  Underlying assets, contract specifics and moneyness.

Reading

- Chapter 11, Hull (2015)