Computational Methods for Finance
Week 7: Monte Carlo Simulation

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Learning Outcomes

At the end of this lecture you will be able to

- Price options using the Monte Carlo Simulation method.

Review

- The payoff for a European option at T
  - European call option: $\max(S_T - K, 0)$
    European put option: $\max(K - S_T, 0)$

Monte Carlo Simulation

- Discounted Expected Cash Flows
  - The price of any asset is given by the discounted expected cash flows (payoffs) to the asset.
  - In the case of derivatives, risk-neutral valuation allows us to assume that the discount rate is given by the risk-free rate; thus,

$$f = e^{-rT} E(f_T),$$

    where f is the price of the derivative, f_T is the payoff to the derivative at maturity (given by its price), and $E(.)$ is an expectations operator.
  - Monte Carlo simulation is used to estimate the expectation. This is achieved by generating a number of possible price paths, and then averaging the implied payoff at maturity.

Monte Carlo Simulation

- Discounted Expected Cash Flows
  For a European put option on a non-dividend paynig asset. the following steps are taken:
  - Step 1: Generate a geometric Brownian motion series of length T.
  - Step 2: Calculate the payoff (f_T) at T using max(K - S_T, 0).
  - Step 3: Repeat Steps 1 and 2 (usually 10,000 times).
  - Step 4: Calculate the arithmetic mean of the f_T's generated to give $\hat{E}(f_T)$.
  - Step 5: Discount this expected value using the risk-free rate to give an estimate of the price of the European put ($\hat{f}$).

Reading

- Chapter 13, Hull (2015)