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Module: High Frequency Trading

Week 9: Optimal Liquidation: Summary

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The Implementation Shortfall Approach to Trading Cost

- We assume a separation between investment and trading decisions.
- The performance of an actual portfolio (gain, loss, or return).
- The performance of an imaginary paper portfolio in which all trades are made at benchmark prices. A common choice is the average of the bid and ask prices at the time of decision.

Implicit Costs in Trading

- The implicit costs are less visible. They include:
  - Costs of interacting with the market (e.g., bid-ask spread or price impact costs), relative to the benchmark prices.
  - Opportunity costs (the penalty associated with not completing intended trades). Examples of this include:
    - The failure of a limit order to execute because the market has moved away from the limit price.
    - Failure to complete a hedging trade, which may leave the portfolio exposed to additional risk.
  - Delay (failure to fill the order immediately).

Benchmark Prices in Implementation Shortfall

- The implementation shortfall calculation depends crucially on the choice of the benchmark price.
- Pre-trade benchmarks include:
  - The NBBO midpoint at the time the trading or order submission decision was made.
  - The previous day's closing price.
  - When a pre-trade benchmark is used, implementation shortfall is sometimes referred to as slippage.
- Examples of post-trade benchmarks are:
  - The NBBO midpoint five minutes after the trade.
  - The next day's opening price.
- Interval benchmarks are also sometimes used:
  - Time-weighted average price (TWAP) over the day or duration of the order.
  - Volume-weighted average price (VWAP) over the day or duration of the order.

Optimal Liquidation with Temporary Price Impact

- Assuming temporary price impact and aiming for an inventory of 0 at the end:
- The optimal speed of liquidation (or acquiring) is given by:

$$\text{Optimal Speed} = \frac{q(t)}{T - t}$$

where $q(t)$ represents the inventory at time t and T denotes the total time.

Penalty on Final Inventory

- If we don't impose $q(T) = 0$ but instead impose a penalty on the final inventory:

$$EC^{\nu} = \mathbb{E}\Big[\underbrace{\int_t^T \hat{S}_u^{\nu}\, \nu_u \, du}_{\text{Terminal Cash}} + \underbrace{(\mathfrak{N} - Q_T^{\nu})\, S_T}_{\text{Terminal execution at mid}} + \underbrace{\alpha\, (\mathfrak{N} - Q_T^{\nu})^2}_{\text{Terminal Penalty}} \Big] .$$

  where:
  - Q_T represents the shares liquidated or acquired before time T .
  - $S(T)$ is the mid-price at time T.
  - $v(t)$ is the liquidation (acquiring) speed.
  - α is the penalty factor.

Optimal Speed of Liquidation with Penalty

- The revised optimal speed of liquidation (or acquiring) considering the penalty on final inventory is given by:

  $$\text{Optimal Speed} = \frac{q(t)}{T - t - \frac{k}{\alpha}}$$

  where:
  - $q(t)$ represents the inventory at time t.
  - T denotes the total time.
  - k is temporary adverse impact on price.
  - if α is big enough, the optimal speed is equivalent to TWAP

Optimal Liquidation with Permanent Price Impact

- The value function for optimal liquidation considering permanent price impact is given by:

$$H^\nu(t,x,S,q) = \mathbb{E}_{t,x,S,q}\Big[\underbrace{X_T^\nu}_{\text{Terminal Cash}} + \underbrace{Q_T^\nu\,(S_T^\nu - \alpha\, Q_T^\nu)}_{\text{Terminal Execution}} - \underbrace{\phi \int_t^T (Q_u^\nu)^2\, du}_{\text{Inventory Penalty}} \Big],$$

where:

- ϕ is the running penalty.
- Temporary price impact function: $f(\nu) = k\nu$
- Permanent price impact function: $g(\nu) = b\nu$

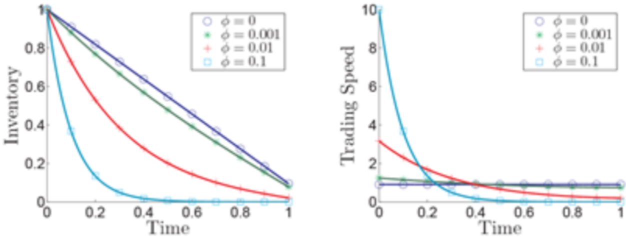
Optimal Liquidation with Permanent Price Impact

The solution is:

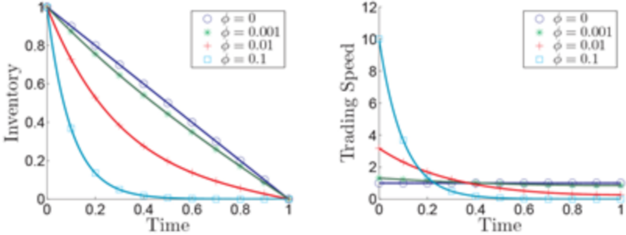
$$\boxed{\nu_t^* = \gamma \, \frac{\zeta \, e^{\gamma\,(T-t)} + e^{-\gamma\,(T-t)}}{\zeta \, e^{\gamma(T-t)} - e^{-\gamma\,(T-t)}} \, Q_t^{\nu^*} \,.}$$

$$\gamma = \sqrt{\frac{\phi}{k}} \quad \text{and} \quad \zeta = \frac{\alpha - \frac{1}{2}b + \sqrt{k\,\phi}}{\alpha - \frac{1}{2}b - \sqrt{k\,\phi}} \,.$$

Optimal Liquidation with Permanent Price Impact



(a) $\alpha = 0.01$



(b) $\alpha = +\infty$