Westminster Business School
School of Finance and Accounting
7FNCE044 Predictive Analysis for Decision Making
Semesters 1 and 2
Week 7
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Seminar Weeks 7 Sketch Answers

Q1. The answers should illustrate detailed steps:

(a). For any process, y_t, is said to be weak stationary is it has a constant and finite mean and variance and covariances depending on the gap between two observed values of y_t at different time periods. Formally:

$$E(y_t) = \mu$$

$$var(y_t) = \sigma^2 = \gamma_0$$

$$cov(y_t, y_{t-k}) = \gamma_k$$

(b).

i. The process is weakly stationary and has the following properties:

$$E(y_t) = 0$$

$$var(y_t) = \sigma^2(1 + \theta_1 + \theta_2)$$

$$cov(y_t, y_{t-1}) = \sigma^2(\theta_1 + \theta_2\theta_1)$$

$$cov(y_t, y_{t-2}) = \theta_2\sigma^2$$

$$cov(y_t, y_{t-k}) = 0 \qquad \text{for any } k > 2$$

ii. The process is not stationary because the process has time dependent mean:

$$E(z_t) = c + t^2$$

iii. Use forward iteration to express the process as follows:

$$x_t = t\mu + x_0 + \sum_{i=1}^{t} \varepsilon_i$$

which has time dependent mean:

$$E(x_t) = t\mu + x_0 = t\mu$$

iv. The process can also be expressed as follows:

$$w_t = w_0 + \sum_{i=1}^{t} \varepsilon_i = \sum_{i=1}^{t} \varepsilon_i$$

The process has constant mean but time varying variances, thus not weakly stationary:

$$E(w_t) = 0$$

$$var(w_t) = t\sigma^2$$

c). The ACF is defined as: $\rho_j = \frac{\gamma_j}{\gamma_0}$ where $\gamma_j = cov(y_t, y_{t-j})$ and $\gamma_0 = var(y_t)$.

Refer to process (1.b) for methods and steps. The process given is an MA(1). This implies we have the following:

$$E(y_t) = 0$$

$$var(y_t) = \sigma^2(1+\lambda)$$

$$cov(y_t, y_{t-1}) = \lambda\sigma^2$$

$$cov(y_t, y_{t-k}) = 0 \qquad \text{for any } k > 1$$

Thus, the ACF dies off after the first lag:

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\lambda\sigma^2}{\sigma^2(1+\lambda)} = \frac{\lambda}{(1+\lambda)}$$

Q2. This process is the same as 1.c above, except this one has an intercept. Follow the same steps and prove the results given below:

$$E(y_t) = \mu$$

$$Var(y_t) = (1+\theta^2)\sigma^2$$

$$\text{cov}(y_t, y_{t-q}) = \begin{cases} \theta\sigma^2 & q = 1 \\ 0 & q > 1 \end{cases}$$

Use the following properties of the error term (think of them as assumptions);

The error term has the following assumption:

$$E(u_t) = 0$$

$$Var(u_t) = \sigma^2$$

$$Cov(u_t, u_s) = 0$$

This will lead to the following:

$E(y_t) = E(\mu) + \theta E(u_{t-1}) + E(u_t) = \mu$ since the error has zero mean.

$$\text{var}(y_t) = E\left[(y_t - E(y_t))^2\right] = E\left[(\theta u_{t-1} - u_t)^2\right] = (1+\theta^2)\sigma^2$$

$$\text{cov}(y_t, y_{t-s}) = E[(y_t - E(y_t))(y_{t-s} - E(y_{t-s}))] = E[(\theta u_{t-1} - u_t)(\theta u_{t-1} - u_{t-s})]$$

For s=0: $\text{cov}(y_t, y_{t-s}) = \text{var}(y_t)$

For s=1: $\text{cov}(y_t, y_{t-1}) = \theta\sigma^2$

For s>1: $\text{cov}(y_t, y_{t-s}) = 0$

The stationarity should be discussed based on the properties above and the shape of the ACF, which shows that the shock last for just one period and dies off after lag 1.

Q3.

For the given AR(1) process to be covariance stationary, we require the following conditions to be satisfied: $\begin{cases} E(y_t) = \mu \\ \text{var}(y_t) = \sigma^2 < \infty \\ \text{cov}(y_t, y_{t-k}) = \gamma_\kappa \end{cases}$

Impose the above as assumptions.

You need to derive the ACF function:

$$V(y_t) = \frac{\sigma^2}{1-\phi_1^2} = \gamma_0$$

$$Cov(y_t, y_{t-j}) = \frac{\phi_1^j \sigma^2}{1-\phi_1^2} = \gamma_j$$

The autocorrelation function is defined as follows

$$\rho_0 = \frac{\gamma_0}{\gamma_0} = 1$$

$$\rho_j = \frac{\gamma_j}{\gamma_0} = \phi_1^j$$

Q4. Additional Revision Problem

(a) The model is expressed as follows:

$$BUSTRAVL_i = \beta_1 + \beta_2 FARE_i + \beta_3 INCOME_i + \beta_4 POP_i + \beta_5 DENSITY_i + u_i$$

The main assumptions about the explanatory variables are the following:

1.a. The set of explanatory variables are linearly independent. Violating this assumption give a rise to multicollinearity, which causes the OLS estimator to be inefficient.

2.a. The set of explanatory variables are assumed to be fixed and non-random. This implies the explanatory variables are uncorrelated with the errors. Violating this assumption will result in endogeneity. This implies that the OLS estimator is no longer unbiased nor consistent.

(b)

The intercept: reports the average demand for urban transportation per hour.

-295.73: is the estimated effect of an increase in fares. An increase fares by one dollar result in a decrease in demand by around 295 passengers.

-0.203: is the estimated effect of income on demand for urban transportation. It is estimated to be negatively related. The higher the income (by one dollar on average) the less demand for bus services.

1.589: is the estimated average effect of population on demand for bus services per hour. As the population increase by 1000, the demand for bus services increases by 1589 passengers per hour.

0.149: is the estimated average effect of density on demand for bus services. As density increases by one person per square mile, the demand for bus services increases by 0.149 (e.g. an increase of 10 persons by square mile implies an increase of demand by 14.9).

For each coefficient, we have the following hypotheses to test:

$$\mathrm{H_0}: \beta_i = 0$$

$$\mathrm{H_1}: \beta_i \neq 0$$

where i=2 and 4. Failing to reject the null hypothesis, $\mathrm{H_0}$, implies statistical insignificance. Rejecting the null implies the coefficient being tested is statistically significant (i.e. $\mathrm{H_1}$).

The decision rule is as follows:

Reject the null hypothesis, $\mathrm{H_0}$, if $|\hat{t_i}| = \left|\frac{\hat{\beta_i}}{se(\hat{\beta_i})}\right| > t_{(0.25,df)}$

The two-tailed 5% critical value is$t \cong 2.021$. Therefore, we fail to reject the null for FARE and reject the null for POP. The same conclusions are reached using the p-values.

(b) The joint significance hypotheses can be expressed as follows:

$$\mathrm{H_0}: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$

$$\mathrm{H_1}: \text{at least one } \beta_i \neq 0$$

where i=1, 2, 3 and 4. Failing to reject the null hypothesis, $\mathrm{H_0}$, implies the model is jointly insignificant. Rejecting the null implies the model is jointly significant. (i.e. $\mathrm{H_1}$).

The decision rule is as follows:

Reject the null hypothesis, $\mathrm{H_0}$, if $F_{(4,195)} > F^{5\%}_{(4,195)}$

The critical value from the F distribution table is:2.61. Therefore, we reject the null hypothesis because $F_{(4,36)} = 100.44 > F^{5\%}_{(4,36)} \cong 2.61$

The coefficient of determination suggest that about 92% of the variations are explained by the model.

(c)

Here you can simply use a F test to test the following restriction:

$$H_0: \beta_5 = 5\beta_2 \text{ or } \beta_5 - 5\beta_2 = 0$$

$$H_1: H_0 \; is \; false$$

The URSS is already given to you, which is 18480373 and RRSS is 18542143. There is only one restriction. Thus, you can compute the F statistic using the standard formula and obtain the statistic equal to 0.117. The 5% critical value is about 4.12, thus we fail to reject the null. For full mark, the answer should outline all steps including the formula and workings.

(d)

i. $var(u_i) = \sigma^2$ constant for all i.

ii. Figure 1: Different values of the proxy of the variance $\hat{u}_i^2$ observed which represent heteroscedasticity. Figure 2: Mild different dispersion of $\hat{u}_i$ with income signals heteroscedasticity.

(e)

The test used here is a special case Breusch-Pagan test. It is similar to White test but less general. Refer to the text book for detailed discussion on the steps involved.

We have the following:

$$\mathbf{H_0: Homoskedasticity}$$

$$\mathbf{H_1: Heteroskedastictiy}$$

The decision rule is as follows:

Reject the null hypothesis, H_0, if $LM(4) > \chi^2_{(5\%,4)}$

From the χ^2 distribution, the critical value is 5.9915. Therefore, we fail to reject the null of homoskedasticity because $LM(4) = 8.57 < \chi^2_{(5\%,4)} = 9.49$

Consequences of Heteroscedasticity on OLS Estimators: The OLS estimators are still unbiased and consistent. The OLS estimators are inefficient because you could, in principle, find other unbiased estimators with smaller variances. The estimators of the standard errors of the regression coefficients are wrong. They are underestimated, leading to higher values of t and F statistics.

(f) Discuss White's test here.

(g). Discuss Durbin Watson test fully.

The statistic is 1.995. The lower and upper critical values are approximately are 1.29 and 1.72. Thus, there is no autocorrelation. If the other Gauss-Markov assumptions are satisfied, the OLS in Table 1 is unbiased, consistent and efficient the tests above are correct.