Westminster Business School
School of Finance and Accounting
7FNCE044 Predictive Analysis for Decision Making
Semester 2
Week 10
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Seminar Week 10 Sketch Answers

Q1.

a). Define the following models

ARCH(p):

$$r_t = f(\Phi, \Omega_{t-1}) + u_t,\ u_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \ldots + \alpha_p u_{t-p}^2$$

GARCH(1,1):

$$r_t = f(\Phi, \Omega_{t-1}) + u_t,\ u_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2$$

Discussion on the meaning of the terms in all the equations is necessary. In addition, the answer should stress that GARCH(1,1) is equivalent to ARCH(∞).

For both models to be well defined, the variances need to be positive and the coefficients in the conditional variance are constrained to be positive.

b) Testing for the presence of ARCH effects involve the following steps:

The ARCH test (Engle, 1982)

1. Specify the hypotheses (The null: no ARCH effect (i.e. $\alpha_1 = \ldots = \alpha_p = 0$), versus the alternative that: There is an ARCH effect)
2. Estimate an auxiliary regression
   $$\hat{u}_t^2 = \alpha_0 + \alpha_1 \hat{u}_{t-1}^2 + \ldots + \alpha_p \hat{u}_{t-p}^2$$
   Obtain the R^2 of the regression
3. Construct the *LM* statistic: $LM = nR^2 \sim \chi^2_{\alpha,k-1}$
4. Reject the null if $LM = nR^2$ is higher than the critical value.

c) Discuss in detail the use and application of the TGARCH model
   $$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma u_{t-1}^2 d_{t-1} + \beta \sigma_{t-1}^2$$
   Where $d_{t-1} = \begin{cases} 1 \text{ if } u_{t-1} < 0 \text{ (bad news)} \\ 0 \text{ if } u_{t-1} > 0 \text{ (good news)} \end{cases}$

The answer should particularly highlight the difference in magnitude between the bad news and good news (i.e. α and $\alpha+\gamma$) and the conditions under which γ can be used and interpreted.

Q2.

(a) You need to elaborate on the answer and give examples and specify the model. The answer should highlight the following:

ARCH model describes the variance of the current error term as a function of the actual sixes of previous time periods error terms, which is defined – in this model – as the squares of the previous innovations.

Financial time series display time varying volatility and volatility clustering.

(b) The LM test – as proposed by Engle - should be discussed including the specification of the conditional mean, the auxiliary regression, the null and alternative hypotheses, statistics and distribution and the decision rule.

(c-i) The model is Threshold GARCH(1,1). The last term captures the asymmetric effect. Students are expected to provide more details.

(c-ii) it capture the persistence of the shock. The sign is correct and the magnitude implies that the effect of the shock is short lived.

(c-iii) Full computations and interpretation are expected. Positive and negative shocks yields conditional variance 2.17 and 1.96 respectively.

Q3.

a).

The data display clusters of variances suggesting the presence of time varying variance. The kurtosis of the data is above 3 (leptokurtic).
The challenge: classical linear regression is not suitable strategy to model the data. It does not capture the heteroskedastic pattern of the data.
We need to specify the conditional mean and conditional variances. The latter need to be carefully specified from the available structures such as ARCH(p), GARCH(p,q) etc.
For forecasting, we need to ensure that we captured the order of heteroskedasticity. Residuals from the specified model need to be serially uncorrelated.

b).

You need to formally write the specification (write the equation and all the assumptions) The estimated model is a linear model with an intercept and ARCH(1) structure.
The output report two parts. The conditional mean consists of only intercept, which is statistically significant at 5% level.
The conditional variance is defined by an ARCH(1) effect. The ARCH effect suggests that the shock from the past has an estimated effect equal to 0.25.

Define the null and alternative hypotheses for all the tests.

The ARCH(1) effect is statistically significant at 5% level. The intercept is positive and statistically significant.
The ARCH(2) effect hypothesis is rejected. The Q-stat at 10th lag suggests the presence of serially correlated residuals. This latter suggests the model may not be appropriate.

c) Write the down the specification as explained above.

Same as above, define the null and alternative hypotheses of all the tests – or refer back to previous answer is the tests are the same.
The ARCH effect is significant at 10%, while the GARCH effect is significant at all levels of significance. The intercept however is not.
The post estimation tests suggest there is no ARCH(2) effect and residuals are serially uncorrelated. Like the previous model, this one has some weaknesses. The ARCH effect is barely significant, while the intercept is not.

Q4. Plug in all the values into the equation to get: 1.62 and 1.67. Make sure you recognise that I_t is a dummy variable where $I_t = \begin{cases} 1, & if\ u_t < 0 \\ 0, if & u_t \geq 0 \end{cases}$

Q5.

1. The asymmetric effect. It has to be negative and statistically significant to capture the leverage effect.

2. It shows the persistence term [elaborate]

3. Plug in the numbers to find: 0.572 (and 0.4975 for negative effect) and 0.4875 (and 0.5625 for negative effect).