REPORT

DESIGN AND FABRICATION OF BIO-INSPIRED CELL BLEBBING ROBOTIC SYSTEM

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IIT Bombay
Department of Mechanical Engineering

Introduction

For a long time now researchers have been trying to undertand the motion of micro organisms in the fluid where they are habitated. However the first organised endeavour in this context was made by Dr E.M.Purcell in his much celebrated paper "*life at low reynold's number*" where discussed various constraints to be imposed on micro organisms for its locomotion.

The motion of bacterial amoeboids whose mobility takes place exclusively by material transfer with its sorrounding has long been of interest to both biologists and physicist alike. One of papers in this context explores the shape changes of cell membrane after a volume intake by the bacteria.[1]

In this report we have tried to implement the ideas mentioned above to propose the dynamics of a micro robotic system which consists of three sphere and a two connecting rod between them. the spheres can exchange volume with the sorrounding enviornment to aid in it's locomotion

Literature Review

One of the preliminary works in this area was the proposal of Pushmepullyou[2] robot system. The working schematic of the system is as shown in **fig 1**. It is basically a two sphere system capable of exchanging volume with the other sphere it is connected to and there is a retractable rod connecting both of them.

In other words the control inputs to the system is the volume change ($\dot{v}$) and the retraction rate of the rod($\dot{l}$). One thing to note in this case the sphere is only able to exchange volume with another sphere the total volume of the system is constant($v_1 + v_2 = C$). thus volume change rate is zero for both of the system combined($\dot{v_1} + \dot{v_2} = 0$). thus we can only control the volume flow rate of one of the spheres in this case.

The reason that we are choosing the sphere as the volume changing shape is due to the fact that sphere being one of the most symmetrical and regular shapes and the fact that the velocity flow generated by a sphere flowing in a Newtonian fluid system is very well studied in this case(Stokes Solution,see appendix)

The classical Stokes solution describing the flow around a single sphere of radius a dragged by a force $\vec{f}$ and, in addition, dilated at rate $\dot{v}$

$$\pi\vec{u}(\vec{x};a,f,\dot{v}) = \frac{1}{6\mu|x|}[(3+\frac{a^2}{x^2})\frac{\vec{f}}{4}+(1-\frac{a^2}{x^2})\frac{3}{4}(\vec{f}.\hat{x})\hat{x}]+\frac{\dot{v}}{4x^2}\hat{x}$$

Now we know that as the stokes solution is linear, any linear combination of them is also a solution for the differential equations. thus we can simply say that the velocity profile at any point in the fluid is just simple vector sum of the velocity profile that would have been generated by the sphere if only that sphere had been present. (this only is true if the spheres are far apart $l \gg a$)

Now let us assume that velocities of the sphere is U_i (where i = $1,2$). thus the boundary conditions on the sphere is the fact that velocity on the surface of each of the sphere due to vector sum of each of the sphere should be U_i thus,

$$U_i = \vec{u}(a_i\hat{f};a_i,(-1)^j f,0)+\vec{u}((-1)^i l\hat{f};a_j,(-1)^i f,(-1)^i\dot{v})$$

[1] As the spheres are connected to each other by the center taking the forces as scalars we have the following:

$$2\pi U_i = (-1)^j\frac{1}{3a_i}\frac{f}{\mu}+\frac{\dot{v}}{2l^2}$$

We know that $\dot{l} = U_2 - U_1$ substituting from above in the above equation we have the following :

$$\dot{l} = -\frac{f}{6\pi\mu}(\frac{1}{a_1}+\frac{1}{a_2})$$
$$\Rightarrow f = -6\pi\mu(\frac{1}{a_1}+\frac{1}{a_2})^{-1}\dot{l}$$

We know that $2\dot{X} = U_2 + U_1$ thus we have the following :

$$2\dot{X} = U_1 + U_2 = \frac{f}{6\pi\mu}(-\frac{1}{a_1}+\frac{1}{a_2})+\frac{\dot{v}}{2\pi l^2}$$

Substituting the value of f we get, the final expression gives us the dynamics of the system as follows:

$$2\dot{X} = \frac{a_1-a_2}{a_1+a_2}\dot{l}+\frac{1}{2\pi l^2}\dot{v}$$

[1] Note that the dilation rate term of the i^{th} sphere in the equation($U_i = \vec{u}(a_i\hat{f};a_i,(-1)^j f,\underline{0})+\vec{u}((-1)^i l\hat{f};a_j,(-1)^i f,(-1)^i\dot{v})$) is kept zero, this is because the velocity with which the fluid exchanges volume with the sphere is zero so it is to satisfy this boundary condition we have kept the $\dot{v}$ term in the above equation zero.

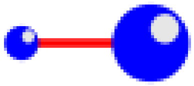

Figure 1: Pushmepullyou

TWO LINK SWIMMER

As shown in the **figure 2** is a schematic of a three sphere two link system. the radius of each of the spheres from left is labeled as a_1, a_2, a_3 respectively. the spheres are capable to exchange volume among themselves but not with the surrounding fluid.also the sphere is connected to the next one with a mass less retractable rod. One of the important assumption here is the fact that the length of the rod is much greater than each of the radius of the sphere.($l_i \gg a_j$ where $i = \{1,2\}$ and $j = \{1,2,3\}$)

As discussed before the stokes solution for a sphere moving in a fluid system is as follows:

$$\pi\vec{u}(\vec{x}; a, f, \dot{v}) = \frac{1}{6\mu|x|}[(3+\frac{a^2}{x^2})\frac{\vec{f}}{4} + (1-\frac{a^2}{x^2})\frac{3}{4}(\vec{f}.\hat{x})\hat{x}] + \frac{\dot{v}}{4x^2}\hat{x}$$

We concluded that if x and y are each solution of the stokes equation, then a linear combination of them is also a solution of the stokes equation. thus,

$$U_1 = u(a_1\hat{i}; a_1, f_1, 0) + u(-l_1\hat{i}; a_2, f_2, \dot{v}_2) + u(-(l_1+l_2)\hat{i}; a_3, f_3, \dot{v}_3)$$
$$U_2 = u(a_2\hat{i}; a_2, f_2, 0) + u(l_1\hat{i}; a_1, f_1, \dot{v}_1) + u((-l_2)\hat{i}; a_3, f_3, \dot{v}_3)$$
$$U_3 = u(a_3\hat{i}; a_3, f_3, 0) + u(l_2\hat{i}; a_2, f_2, \dot{v}_2) + u((l_1+l_2)\hat{i}; a_3, f_1, \dot{v}_1)$$

Simplifying this we get the following

$$U_1 = \frac{f_1}{6\pi\mu a_1} - \frac{\dot{v}_2}{4l_1^2} - \frac{\dot{v}_3}{4(l_1+l_2)^2}$$
$$U_2 = \frac{f_2}{6\pi\mu a_2} + \frac{\dot{v}_1}{4l_1^2} - \frac{\dot{v}_3}{4l_2^2}$$
$$U_3 = \frac{f_3}{6\pi\mu a_3} + \frac{\dot{v}_2}{4l_2^2} + \frac{\dot{v}_1}{4(l_1+l_2)^2}$$

Now we know that $\dot{l}_1 = U_2 - U_1$ and $\dot{l}_2 = U_3 - U_2$ also as the system is not low reynold's number regime $f_1 + f_2 + f_3 = 0$ and $\dot{v}_1 + \dot{v}_2 + \dot{v}_3 = 0$ we get the following:

$$\dot{l}_1 = \frac{1}{6\pi\mu}(\frac{f_2}{a_2} - \frac{f_1}{a_1}) - \frac{\dot{v}_3}{4l_1^2} - \frac{\dot{v}_3}{4l_2^2} + \frac{\dot{v}_3}{4(l_1+l_2)^2} \Rightarrow f_1 = a_1(\frac{f_2}{a_2} - 6\pi\mu(\dot{l}_1 + \frac{\dot{v}_3}{4l_1^2} + \frac{\dot{v}_3}{4l_2^2} - \frac{\dot{v}_3}{4(l_1+l_2)^2}))$$
$$\dot{l}_2 = \frac{1}{6\pi\mu}(\frac{f_3}{a_3} - \frac{f_2}{a_2}) - \frac{\dot{v}_1}{4l_2^2} - \frac{\dot{v}_1}{4l_1^2} + \frac{\dot{v}_1}{4(l_1+l_2)^2} \Rightarrow f_3 = a_3(\frac{f_2}{a_2} + 6\pi\mu(\dot{l}_2 + \frac{\dot{v}_1}{4l_2^2} + \frac{\dot{v}_1}{4l_1^2} - \frac{\dot{v}_1}{4(l_1+l_2)^2}))$$

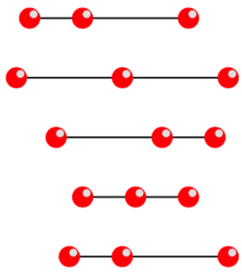

Figure 2: Two link swimmer

Now substituting these values in $f_1 + f_2 + f_3 = 0$ we get,

$$
\begin{aligned}
a_1(\tfrac{f_2}{a_2} - 6\pi\mu(\dot{l}_1 + \tfrac{\dot{v}_3}{4l_1^2} + \tfrac{\dot{v}_3}{4l_2^2} - \tfrac{\dot{v}_3}{4(l_1+l_2)^2})) + f_2 + a_3(\tfrac{f_2}{a_2} + 6\pi\mu(\dot{l}_2 + \tfrac{\dot{v}_1}{4l_2^2} + \tfrac{\dot{v}_1}{4l_1^2} - \tfrac{\dot{v}_1}{4(l_1+l_2)^2})) = 0 \\
\frac{f_2}{a_2} = \frac{1}{a_1+a_2+a_3}(6\pi\mu a_1(\dot{l}_1 + \tfrac{\dot{v}_3}{4l_1^2} + \tfrac{\dot{v}_3}{4l_2^2} - \tfrac{\dot{v}_3}{4(l_1+l_2)^2}) - 6\pi\mu a_3(\dot{l}_2 + \tfrac{\dot{v}_1}{4l_2^2} + \tfrac{\dot{v}_1}{4l_1^2} - \tfrac{\dot{v}_1}{4(l_1+l_2)^2})) \\
\frac{f_1}{a_1} = \frac{1}{a_1+a_2+a_3}(-6\pi\mu(a_2+a_3)(\dot{l}_1 + \tfrac{\dot{v}_3}{4l_1^2} + \tfrac{\dot{v}_3}{4l_2^2} - \tfrac{\dot{v}_3}{4(l_1+l_2)^2}) - 6\pi\mu a_3(\dot{l}_2 + \tfrac{\dot{v}_1}{4l_2^2} + \tfrac{\dot{v}_1}{4l_1^2} - \tfrac{\dot{v}_1}{4(l_1+l_2)^2})) \\
\frac{f_3}{a_3} = \frac{1}{a_1+a_2+a_3}(6\pi\mu a_1(\dot{l}_1 + \tfrac{\dot{v}_3}{4l_1^2} + \tfrac{\dot{v}_3}{4l_2^2} - \tfrac{\dot{v}_3}{4(l_1+l_2)^2}) + 6\pi\mu(a_1+a_2)(\dot{l}_2 + \tfrac{\dot{v}_1}{4l_2^2} + \tfrac{\dot{v}_1}{4l_1^2} - \tfrac{\dot{v}_1}{4(l_1+l_2)^2}))
\end{aligned}
$$

Now we know that $3\dot{X} = U_1 + U_2 + U_3$ which gives us :

$$
\begin{aligned}
3\dot{X} = \frac{1}{a_1+a_2+a_3}((2a_1 - a_2 - a_3)(\dot{l}_1 + \tfrac{\dot{v}_3}{4l_1^2} + \tfrac{\dot{v}_3}{4l_2^2} - \tfrac{\dot{v}_3}{4(l_1+l_2)^2}) + (a_1 + a_2 - 2a_3)(\dot{l}_2 + \tfrac{\dot{v}_1}{4l_2^2} + \tfrac{\dot{v}_1}{4l_1^2} - \\
\tfrac{\dot{v}_1}{4(l_1+l_2)^2})) + \tfrac{2\dot{v}_1}{4l_1^2} + \tfrac{\dot{v}_3}{4l_1^2} - \tfrac{2\dot{v}_3}{4l_2^2} - \tfrac{\dot{v}_1}{4l_2^2} + \tfrac{\dot{v}_1}{4(l_1+l_2)^2} - \tfrac{\dot{v}_3}{4(l_1+l_2)^2}
\end{aligned}
$$

which is nothing but the dynamics of the equation.

APPENDIX

Stokes Solution

Here we are deriving to the velocity profile of a sphere moving in a fluid of velocity U. One of the observations for such a sphere moving in a low reynold's number flow are as follows:

1. As the sphere fluid system is in a low reynold's number flow. We can very easily neglect the inertial elements in the navier stokes equation, we can do this because the reynold's number of a fluid flow is nothing but the ratio of inertial forces to viscous forces. thus, in a low reynold's number flow we have viscous forces dominating the inertial forces. thus the navier stokes equation boils down to something like this.

$$
\begin{aligned}
\nabla \frac{p}{\mu} = \nabla^2 u = -\nabla \text{ x } \omega \\
\nabla . u = 0
\end{aligned} \tag{1}
$$

2. One of the consequence from above is the fact that the divergence of a curl is always zero and the fact that curl of the divergence is always zero. thus taking the curl and divergence of these equations we get,

$$\begin{aligned} \nabla^2 p &= 0 \\ \nabla^2 \omega &= 0 \end{aligned} \tag{2}$$

3. Our choice of coordinate system is one which is at the center of the sphere and the velocity profile of the fluid at infinity is zero.and the velocity of the fluid at the surface of sphere of radius a is U.

$$\begin{aligned} &u = U \\ &u \to 0 \text{ and } p \to p_0 \text{ as } |x| \to \infty \end{aligned} \tag{3}$$

4. One if the important things to note here is also the fact that

It is quite obvious from the discussion above that that eqns(1) and (2) are linear and homogeneous in u, p/μ and U. The expressions of these must thus be linear and homogeneous in U.

It is quite obvious from the symmetry of the system that the pressure profile depends extensively on vector $\vec{x}$ and not on any other linear combination of it's components. Thus, it suffices to say that pressure profile is off the form.

$$p - p_0 = U.\phi(\mathbf{x}) \tag{4}$$

Now as the pressure satisfies the laplace equation we have the following relation. It can be written down as linear combination of spherical harmonics but the only one that satisfies the form (4) is $\nabla(\frac{1}{r})$ thus we have the pressure profile as follows

$$p - p_0 = \frac{CU.x}{r^3} \tag{5}$$

where C is a constant that has to be determined. Now exactly the same kind of argument applies for solving for ω . thus we have the following expression for ω.

$$\omega = \frac{CU \times \mathbf{x}}{r^3} \tag{6}$$

The velocity distribution in the azimuthal direction(ϕ) with respect to the stream function ψ can be easily found out (note: $\theta = 0$ is in the direction of U) as follows:

$$\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta} \tag{7}$$

We know that $u = \nabla \times \psi$ thus, in polar (r,θ) this boils down to,

$$u_r = \frac{1}{r^2 sin(\theta)}\frac{\partial \psi}{\partial \theta}, u_\theta = -\frac{1}{r sin(\theta)}\frac{\partial \psi}{\partial r} \tag{8}$$

Substituting in eqn(1) we have,

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\Big(\frac{1}{\sin\theta}\frac{\partial \psi}{\partial \theta}\Big) = -\frac{CU\sin^2\theta}{r} \tag{9}$$

Now the stream function should satisfy the boundary condition(at $r = a$ $u = \mathbf{U}$) . thus, we can simplify the stream function as,

$$\psi = U\sin^2\theta f(r) \tag{10}$$

thus, from (8) we have,

$$u = U\big(\frac{1}{r}\frac{df}{dr}\big) + \mathbf{x}\frac{x.U}{r^2}\big(\frac{2f}{r^2} - \frac{1}{r}\frac{df}{dr}\big) \tag{11}$$

Substituting eq(10) in eq(9) we come up with the following differential equation in f

$$\frac{d^2 f}{dr^2} - 2\frac{f}{r^2} = -\frac{C}{r} \tag{12}$$

The general solution of this system of differential equation is as follows:

$$f(r) = \frac{1}{2}Cr + Lr^{-1} + Mr^2 \tag{13}$$

Now the $\frac{f}{r^2} \to 0$ as $r \to \infty$ which gives us $M = 0$. Also, we have the following fact that $u_{r|a} = U\cos\theta$ which gives us

$$\begin{aligned} Ucos\theta = \frac{1}{a^2}2U\cos\theta f(a) \implies f(a) = \frac{a^2}{2} \\ \implies L = \frac{1}{2}a^3 - \frac{1}{2}Ca^2 \end{aligned} \tag{14}$$

Similarly, u_θ at $r = a$ gives us

$$-U\sin\theta = -\frac{1}{a\sin\theta}U\sin^2\theta\big(\frac{C}{2} - \frac{L}{a^2}\big) \tag{15}$$

from (14) and (15) we get, $C = \frac{3}{2}a$ and $L = -\frac{1}{4}a^3$ Now putting value of f in eqn (11) we get,

$$u = \frac{Ua}{4r}(3 + \frac{a^2}{r^2}) + \hat{\mathbf{x}}\frac{3aU.\hat{x}}{4r}(1 - \frac{a^2}{r^2}) \tag{16}$$

Now we know that the net viscous force on the sphere $\vec{f} = 6\pi\mu a$.thus substituting in the equation (16) we get,

$$u = \frac{1}{6\pi\mu r}(\frac{\vec{f}}{4}(3 + \frac{a^2}{r^2}) + \hat{\mathbf{x}}\frac{3\vec{f}.\hat{x}}{4}(1 - \frac{a^2}{r^2})) \tag{17}$$

Now in another case consider there is a sphere kept in an infinite fluid which is exchanging volume with the surrounding with a constant rate $\dot{v}$. We are going with the assumption that the sphere is able to exchange volume over the whole surface of the sphere. thus by symmetry arguments it is easy to see that the flow profile due to such a system will be

$$\vec{u} = \frac{\dot{v}}{4\pi x^2}\hat{x} \tag{18}$$

This can be easily seen if we try to satisfy the continuity equation on the spherical control volume of radius x around the sphere that we are talking about

Now we know that any linear combination of these solutions (17) and (18) is also a solution. thus we have the final solution as the following,

$$u = \frac{1}{6\pi\mu r}(\frac{\vec{f}}{4}(3 + \frac{a^2}{r^2}) + \hat{\mathbf{x}}\frac{3\vec{f}.\hat{x}}{4}(1 - \frac{a^2}{r^2})) + \frac{\dot{v}}{4\pi x^2}\hat{x} \tag{19}$$

REFERENCES

1. Coskun, H. & Coskun, H. Bull Math Biol (2011) 73: 658. doi:10.1007/s11538-010-9580-x

2. Avron, J. E., Kenneth, O. & Oaknin, D. H. Pushmepullyou: an efficient micro-swimmer. New J. Phys. 7, 234-234 (2005).

3. Batchelor G K 1967 An Introduction to Fluid Dynamics(Cambridge: Cambridge University Press)

4. Qixuan Wang, Jifebg Hu & Hans Othmer Models of low reynolds number swimmers inspired by cell blebbing (arXiv:1507.08296 [physics.flu-dyn])