DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

ME415: Computational Fluid Dynamics & Heat Transfer **Autumn 2016**

Assignment # 1: Computational Heat Conduction for Cartesian Geometry

Weightage: 10% **Instructor**: Prof. Atul Sharma

Date Posted: 22nd Aug. (Monday) **Due Date: 31st Aug. (Wednesday, Early Morning 2 AM)**

ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed): Create a single zipped file consisting on (a) filled-in answer sheet of this doc file converted into a pdf file and (b) all the computer programs. The name of the zipped file should be **rollnumber_A1**

Note: Both problem and answer sheet are provided below. **SCILAB or MATLAB** should *preferably* be used for programming as well as generating graphical results.

Refer http://spoken-tutorial.org/tutorial-search/?search_foss=Scilab&search_language=English, for getting started to SCILAB for programming as well as generating graphical results. To save figure: Go to "Graphic window number", click on "File", then click on "Export to", select "Windows BMP image" in the "Files of type". Make sure to save the file in the same location where you have this file. More details are given in the next page.

1. **Flux based methodology for CFD development and code-verification for 1D unsteady state heat conduction problem, on a uniform grid.**

   Consider 1D conduction in a long stainless-steel (density ρ: **7750 kg/m3**, specific-heat c_p: **500 J/Kg K**, thermal-conductivity k: **16.2 W/m-K**) sheet of thickness $L=1$ *cm*. The sheet is initially at a uniform temperature of 30^oC and is suddenly subjected to a constant temperature of $T_{wb} = 0^oC$ on the west and $T_{eb} = 100^oC$ on east boundary.

   Using the flux based solution methodology of CFD development, a computer program (**A1_1D_Prob1.sci**) for *explicit method on a uniform* 1-D Cartesian grid is given along with this assignment sheet. Present a testing of the code for a volumetric heat generation of 0 and 100, MW/m^3. Consider maximum number of grid points as $imax=12$ and the steady state convergence tolerance as $\varepsilon_{st}=10^{-4}$. Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution.

2. **Flux based methodology for CFD development and code-verification for 2D unsteady state heat conduction problem, on a uniform grid.**

   Consider 2D conduction in a square shaped ($L_1=1m$ and $L_2=1m$) long stainless-steel plate. The plate is initially at a uniform temperature of 30^oC and is suddenly subjected to a constant temperature of $T_{wb} = 100^oC$ on the west boundary, $T_{sb} = 200^oC$ on the south boundary, $T_{eb} = 300^oC$ on the east boundary, and $T_{nb} = 400^oC$ on north boundary.

i. Using the flux based solution methodology of CFD development, develop a computer program for *explicit method on a uniform* 2-D Cartesian grid. Use the steady state stopping criterion for non-dimensional temperature, given (slide # 5.58 & 5.59) as

$$\left(\frac{\partial\theta}{\partial\tau}\right)_{i,j} = \frac{l_c^2}{\alpha\Delta T_c}\left(\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t}\right)_{\max \text{ for } i,j} \le \varepsilon_{st} \left\{l_c = L_1 \;\&\; \Delta T_c = T_{nb} - T_{wb}\right\}$$

ii. Present a CFD application of the code for a volumetric heat generation of **0** and **50 kW/m³**. Consider maximum number of grid points as $imax \times jmax = 12 \times 12$ and the steady state convergence tolerance as $\varepsilon_{st} = 10^{-4}$. Plot the steady state temperature contours with and without volumetric heat generation.

3. **Coefficient of LAEs based methodology for CFD development and code-verification for 1D unsteady state heat conduction problem, on a non-uniform grid.**

Consider 1D conduction in a long stainless-steel sheet of thickness $L = 1\ cm$. The sheet is initially at a uniform temperature of **30°C** and is suddenly subjected to a constant temperature of $T_{wb} = 0^oC$ on the west and $h = 1000\ W/m^2.K$ and $T_\infty = 100^oC$ on east boundary.

i. Generate a non-uniform 1D Cartesian grid, using an algebraic method (can be found in slide no. 5.63 to 5.66). The method involves a transformation of a uniform grid, in a ξ- coordinate based 1D computational domain of unit length, to a *x*- coordinate based physical domain of length L, using an algebraic equation given (*Hoffmann and Chiang, 2000*) as

$$x = L\frac{(1+\beta)\left[(\beta+1)/(\beta-1)\right]^{(2\xi-1)} - (\beta-1)}{2\left\{1 + \left[(\beta+1)/(\beta-1)\right]^{(2\xi-1)}\right\}}$$

This equation results in a grid which is finest near the two ends of the domain and gradually become coarser at the middle of the domain. It is called as equal *clustering* of grids at both the ends of the domain. Consider maximum number of grid points as $imax = 12$ and $\beta = 1.2$ (which controls the non-uniformity in the grid size).

ii. Using the coefficient of LAEs based solution methodology of CFD development, a Gauss-Seidel method based computer program for *implicit method on the non-uniform* 1-D Cartesian grid (**A1_1D_Prob3.sci**) is given along with this assignment sheet. Present a CFD application of the code for a volumetric heat generation of 0 and 100, MW/m³. Consider the convergence tolerance as $\varepsilon_{st} = 10^{-4}$ for steady state, and $\varepsilon = 10^{-4}$ for iterative solution. Plot the steady state temperature profiles with and without volumetric heat generation, and compare with the exact solution.

4. **Coefficient of LAEs based methodology for CFD development and code-verification for 2D unsteady state heat conduction problem, on a non-uniform grid.**

Consider 2D conduction in a square shaped (***L_1=1m*** and ***L_2=1m***) long stainless-steel plate. The plate is initially at a uniform temperature of ***30^oC*** and is suddenly subjected to a constant temperature of ***T_{wb} = 100^oC*** on the west boundary, insulated on the south boundary, constant incident heat flux of ***q_W = 10 kW/m^2*** on the east boundary, and ***h=100 $W/m^2.K$*** and ***T_∞ = 30^oC*** on north boundary.

i. Generate a non-uniform 2D Cartesian grid, using an algebraic method, using the equation given above for the non-uniform grid generation in the x-direction. However, this equation is also used in the y-direction to generate the 2D grid. This equation results in a grid which is finest near the two ends (east and west as well as north and south boundary) of the domain and gradually become coarser at the middle of the domain. It is called as equal clustering of grids at both the ends of the domain. Consider maximum number of grid points (for the temperature) as ***imax×jmax=12×12*** and ***β=1.2***.

ii. Using the coefficient of LAEs based solution methodology of CFD development, develop a Gauss-Seidel method based computer program for the *implicit method on a non-uniform* 2-D Cartesian grid. Use the stopping criterion presented in the previous problem, with ***$\Delta T_c = T_{wb} - T_\infty$***.

iii. Using the non-uniform grid and the program, present a CFD application of the code for a volumetric heat generation of **0** and ***50 kW/m^3***. Consider the convergence tolerance as ***$\varepsilon_{st}=10^{-4}$*** for the steady state, and ***$\varepsilon=10^{-4}$*** for iterative solution by the Gauss-Seidel method. Plot the steady state temperature contours with and without volumetric heat generation.

Best Wishes for your success in the insightful field of Computational Fluid Dynamics

Keep Playing with the codes in future also.

NOTE: CODE EXECUTION IN SCILAB

To open a *Scilab console*:

Linux: Applications→Programming→Scilab or Applications→Science→Scilab

Windows: Start→All programs→Scilab→Scilab.exe

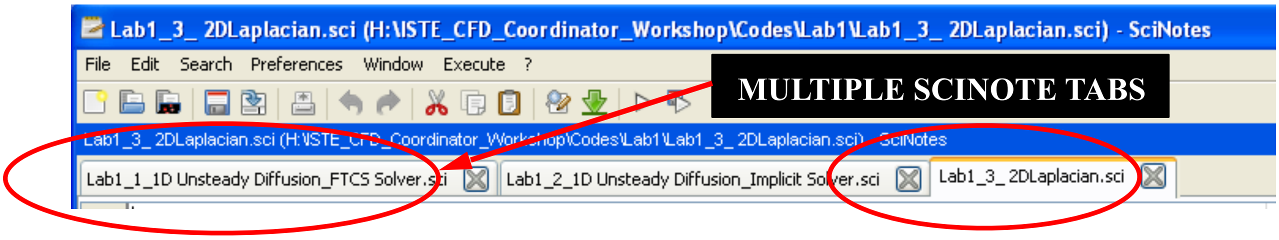
To load a source code (or a *Scinote* file):

In the Scilab console, go to top menu bar

File→Open a file…→(Browse for the *.sci file path)

The source code opens in a new *Scinote* window.

NOTE: Once this window is open, subsequent loading of a new *.sci file will open a new tab in the same *Scinote* window.

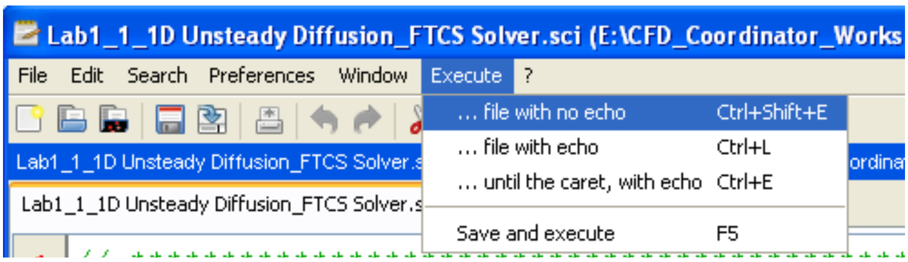


To execute a program written in *Scinote*:

In the Scinote window (with the desired program tab open), go to top menu bar

Execute→…file with no echo (please do not select …file with echo)

The execution begins in the *Scilab console* window.



NOTE: Only one *Scilab* code can be executed at a time. Once an execute command is given and the code is to be stopped at an intermediate stage, use the method given below. If another execute command is given without completing/aborting the previous run, erroneous results may be produced.

To abort a running program:

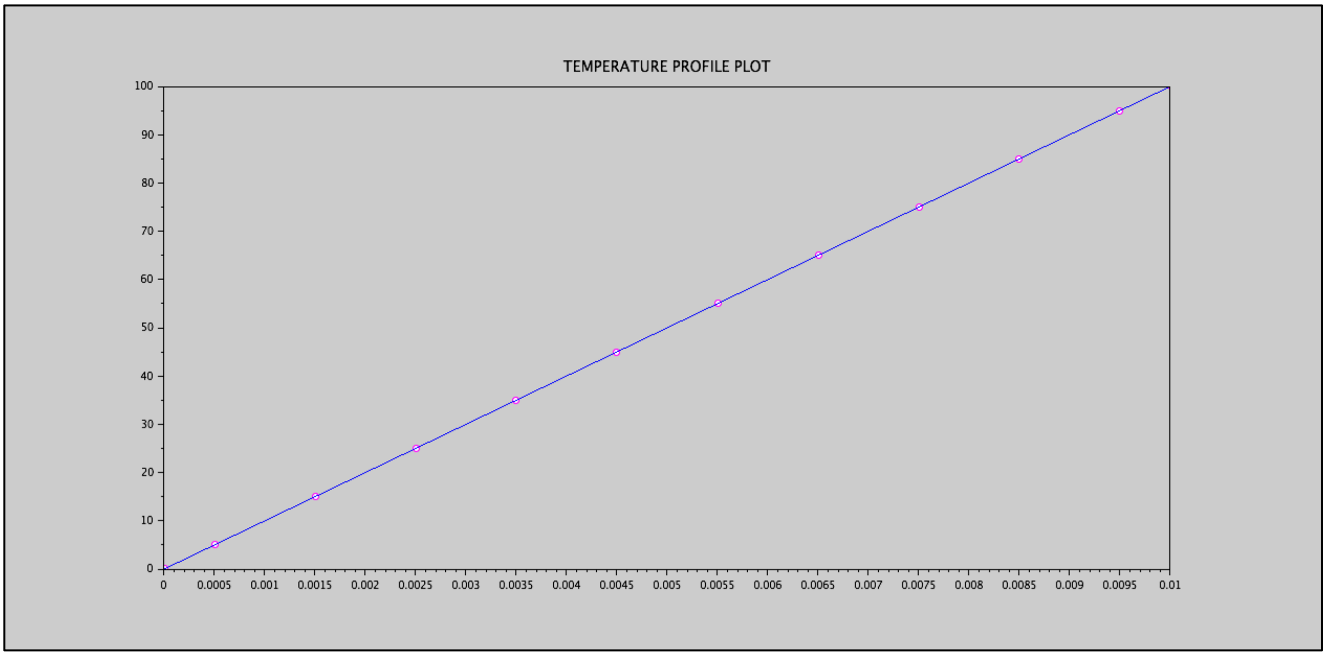
Go to the *Scilab console* window and press **CTRL+C**. This interrupts the code execution. A prompt appears asking for user input. Enter "abort" here to stop the code execution.

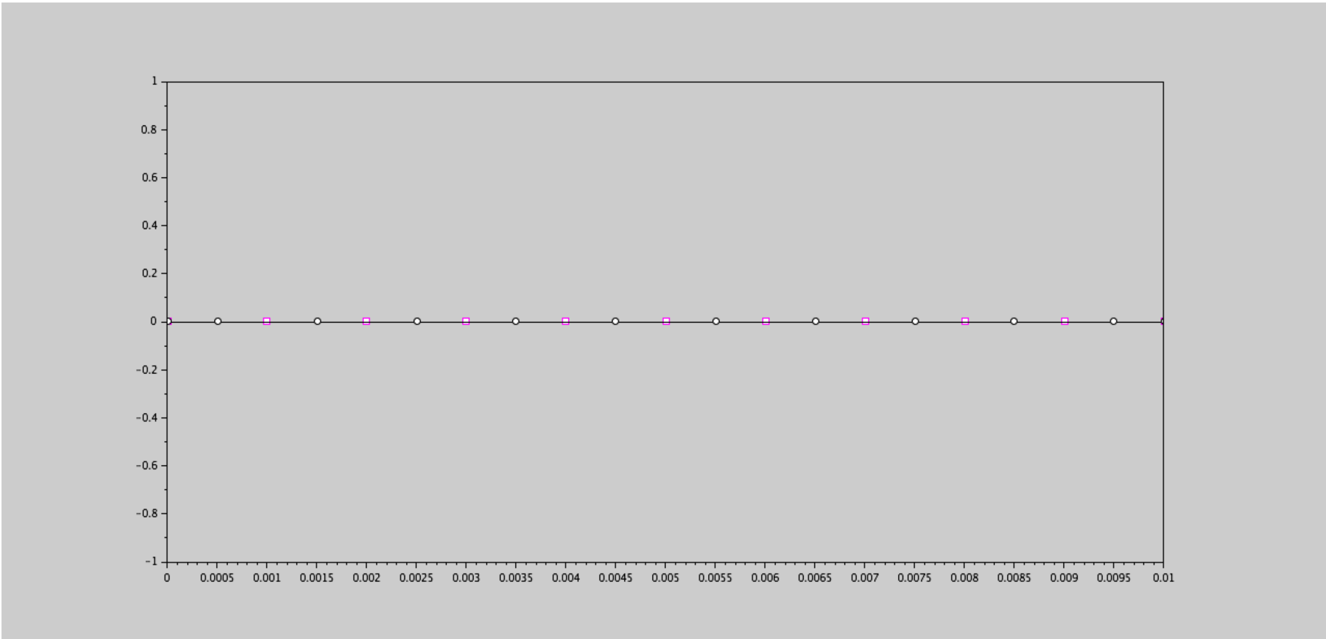
Answer Sheet

Problem # 1: Flux based methodology, with explicit method and uniform grid: 1D Conduction

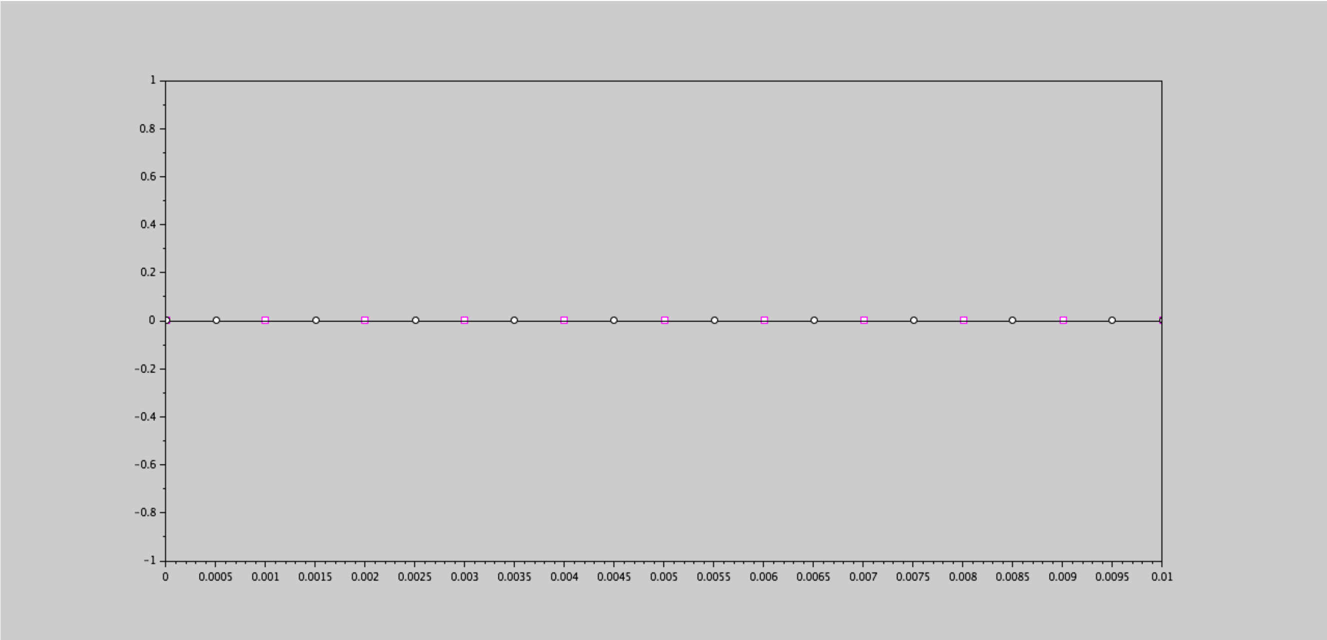
Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution. (2 figures).

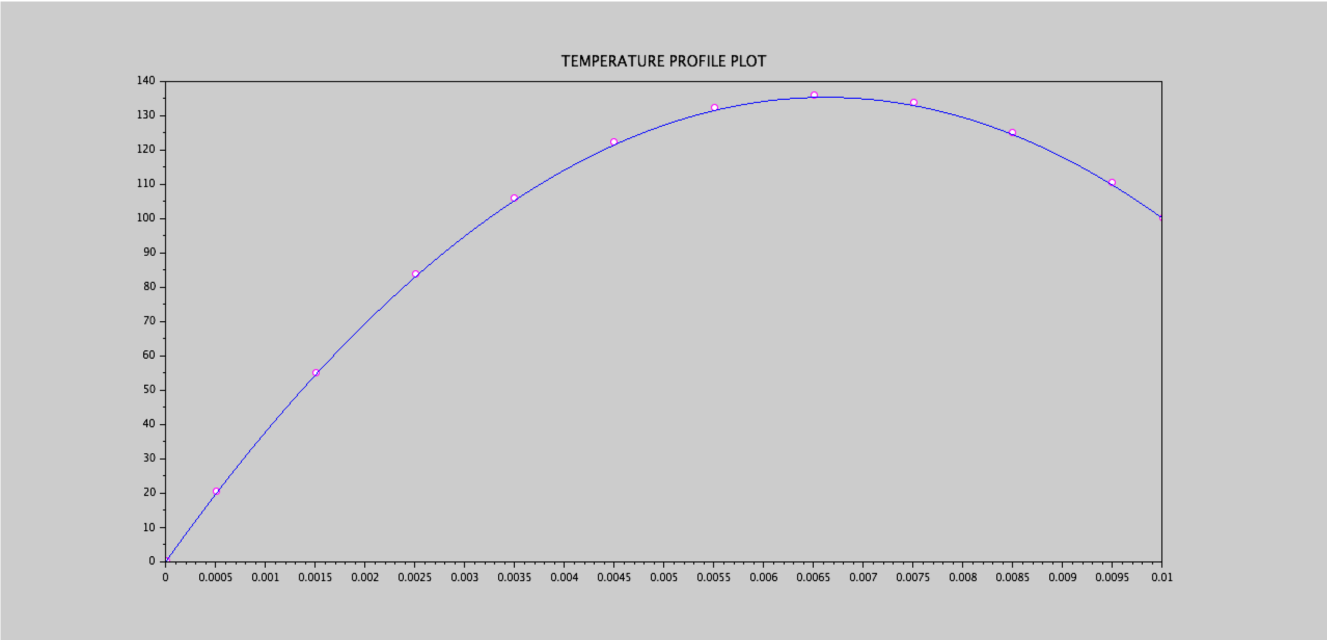
(a) without volumetric heat generation





(b) with volumetric heat generation (100 MW)





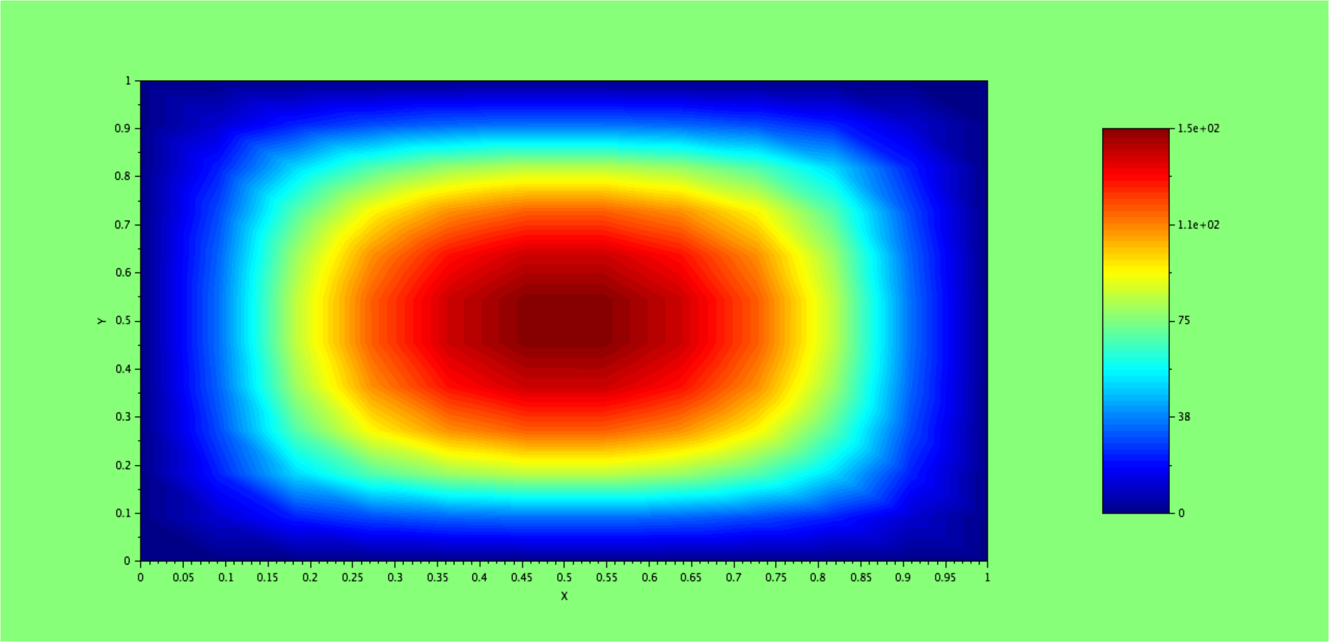
Discussion on the Fig. 1.1: Write your answer, limited inside this text box only

We see that as compared to the case with no heat generation there is a considerable difference in temperature profile with distance over the whole solid bar (almost parabolic) this may be due to the fact that there is already a fixed temperature at the boundaries so that any heat generated has to be limited at the weighted mean of the rod so that whenever we star going towards the boundary of the rod they start decreasing and tend towards the Boundary conditions hence the parabolic type of profile

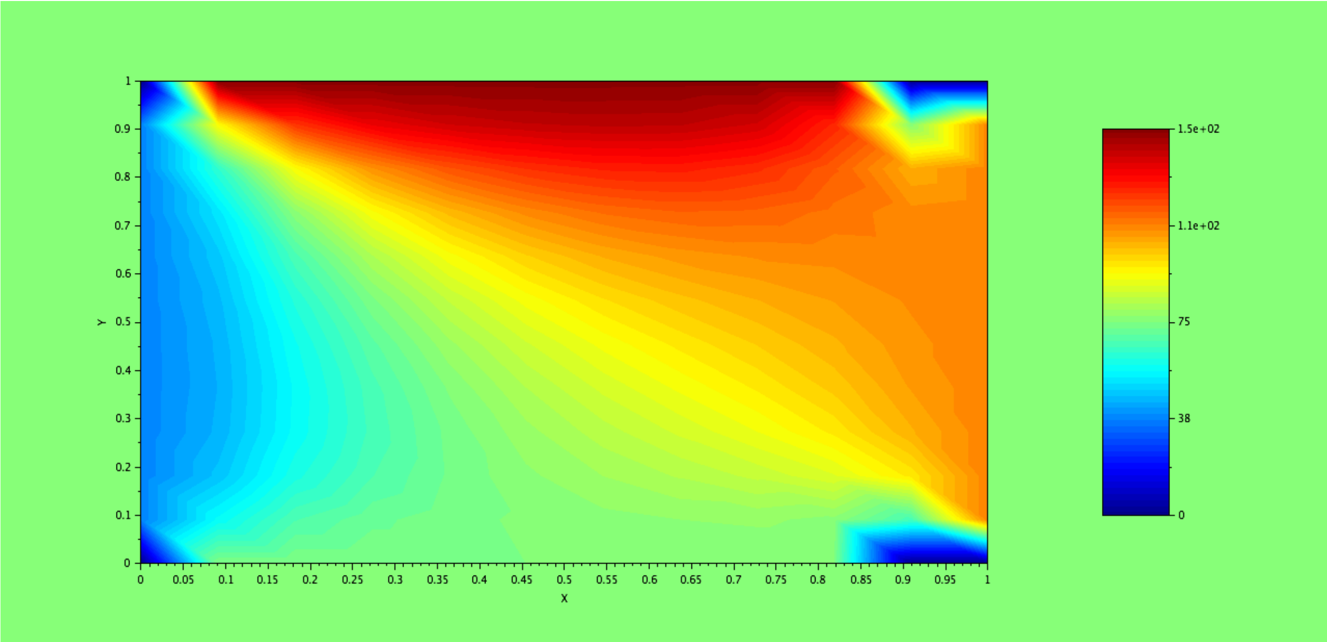
Problem # 2: Flux based methodology, with explicit method and uniform grid: 2D Conduction

Plot the steady state temperature contours with and without volumetric heat generation. (2 figures).

(a) with volumetric heat generation



(b) without volumetric heat generation



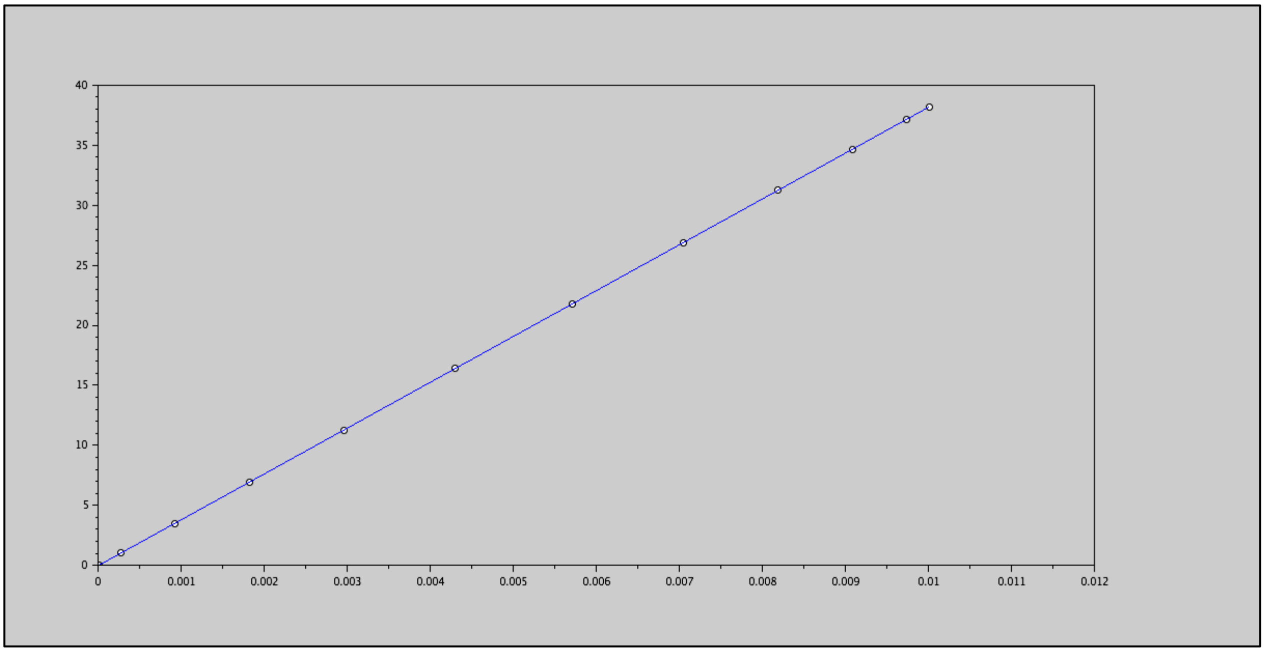
Discussion on the Fig. 1.2: Write your answer, limited inside this text box only

We see that as compared to the case with no heat generation there is a considerable difference in temperature profile with distance over the whole solid bar (almost parabolic) this may be due to the fact that there is already a fixed temperature at the boundaries so that any heat generated has to be concentrated at the weighted mean of the plate so that whenever we star going towards the boundary of the rod they start decreasing to satisfy the boundary condition

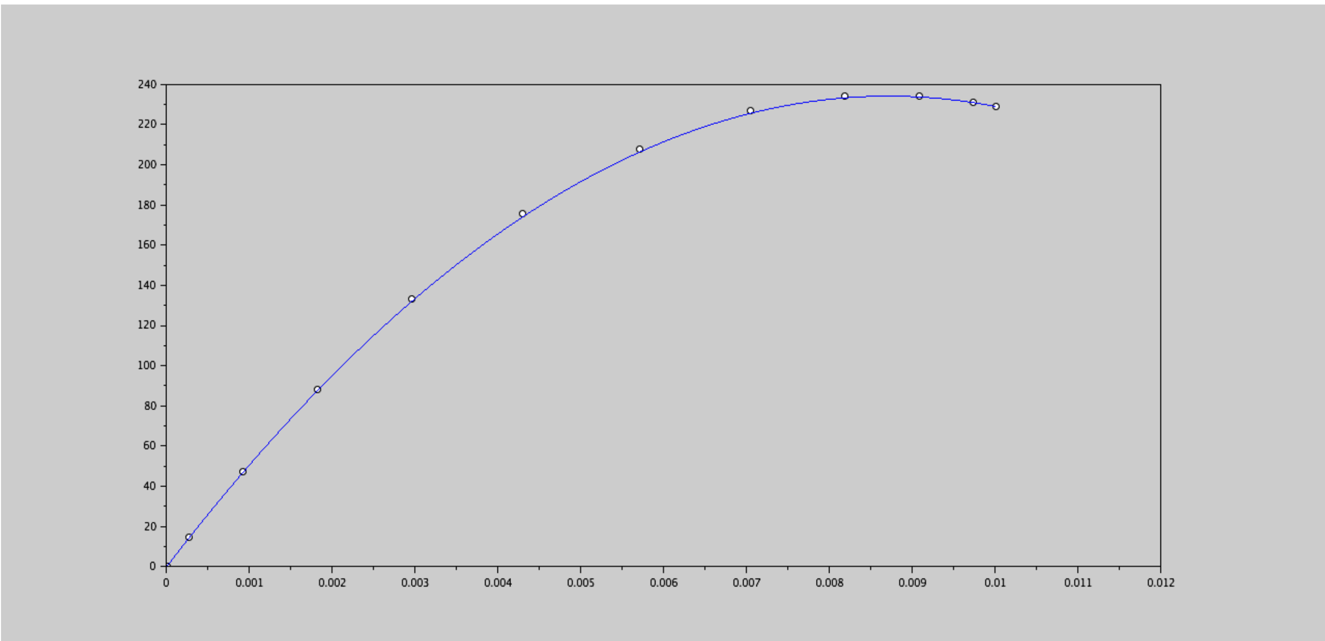
Problem # 3: Coefficient of LAEs based methodology, with implicit method and non-uniform grid: 1D conduction

Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution. (2 figures).

(a) without volumetric heat generation



(b) with volumetric heat generation 100 MW

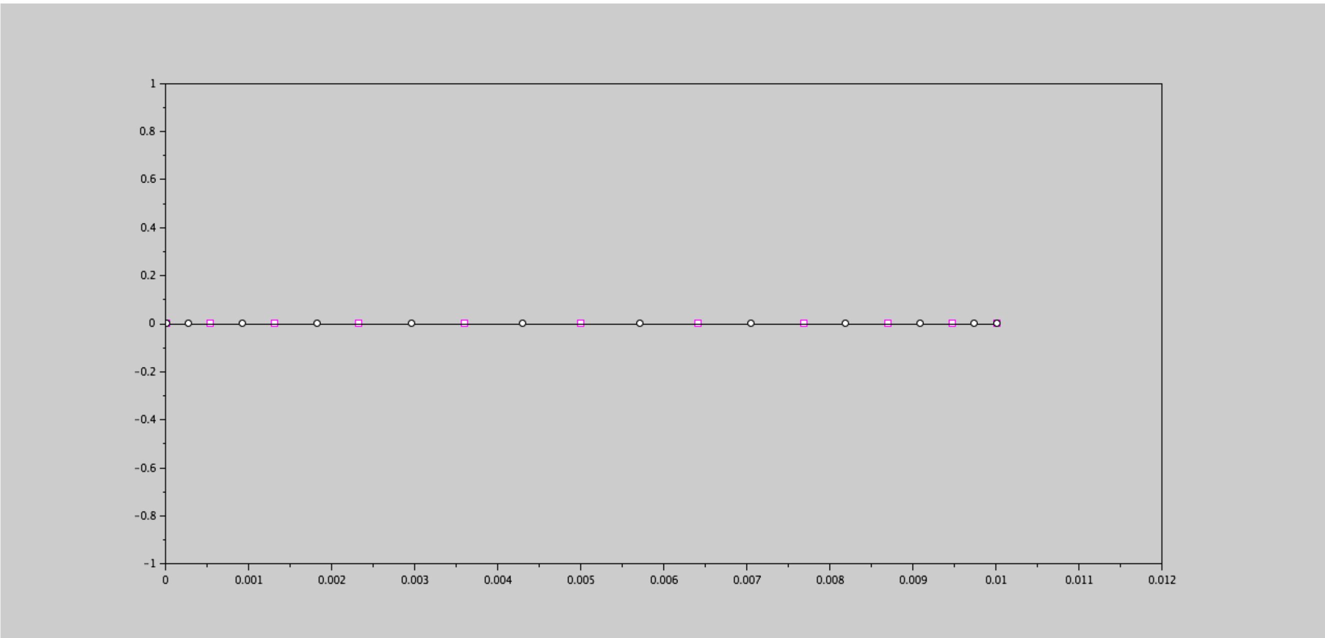


Discussion on the Fig. 1.3: Write your answer, limited inside this text box only

We see that as compared to the case with no heat generation there is a considerable difference in temperature profile with distance over the whole solid bar (almost parabolic) this may be due to the fact that there is already a fixed boundary conditions so that any heat generated has to be limited at the weighted mean of the rod so that whenever we star going towards the boundary of the rod they start decreasing and tend towards the Boundary conditions hence the parabolic type of profile

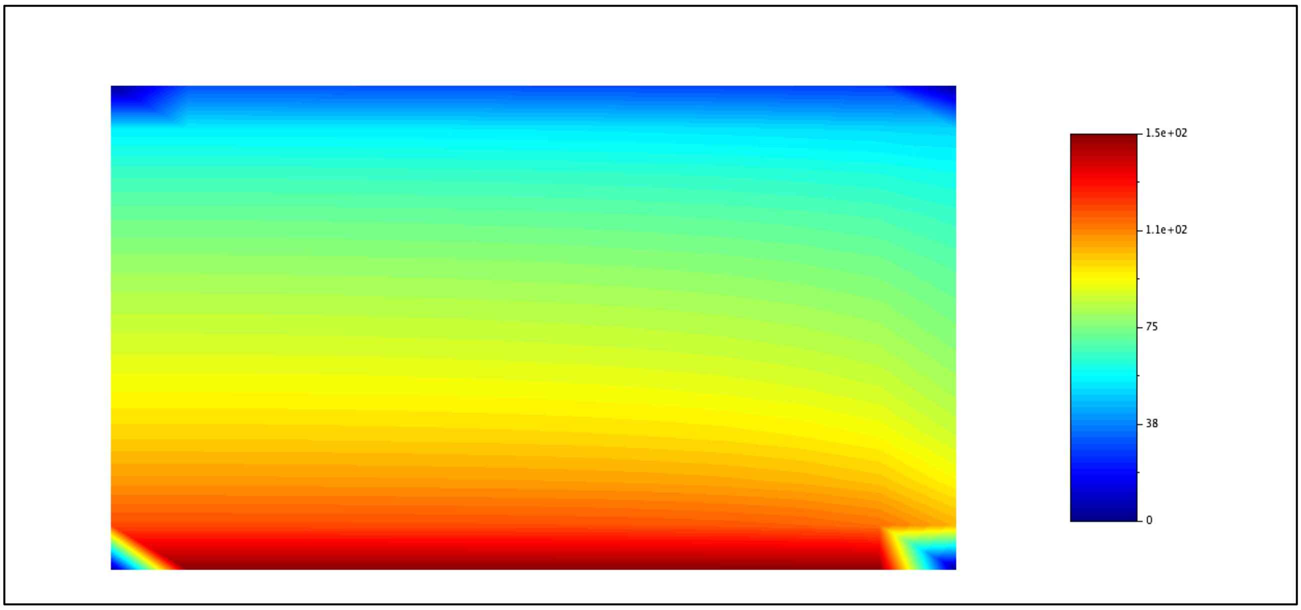
and this closely satisfies the actual analytic solution that we come up with

Problem # 4: Coefficient of LAEs based methodology, with implicit method and non-uniform grid: 2D conduction

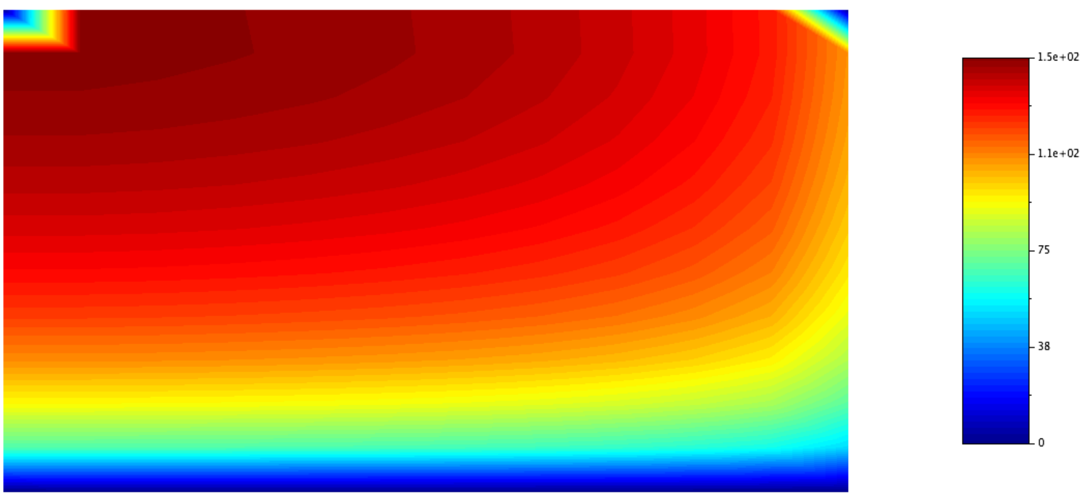


Plot the steady state temperature profiles with and without volumetric heat generation. (2 figures).

(a) without volumetric heat generation



(b) with volumetric heat generation



Discussion on the Fig. 1.4: Write your answer, limited inside this text box only

We see that as compared to the case with no heat generation there is a considerable difference in temperature profile with distance over the whole solid bar (almost parabolic) this may be due to the fact that there is already a fixed temperature at the boundaries so that any heat generated has to be limited at the weighted mean of the plate so that whenever we star going towards the boundary of the rod they start decreasing and tend towards the Boundary conditions hence the parabolic type of profile