Shear Dispersion Toy Equation

Theory

This code solves a toy equation for sheear dispersion using Fourier-Galerkin spectral methods for periodic boundary conditions

> c_t = nu c_xx + 2 c c_xxxx + c_xxxxxx + 4 c_x c_xxx + 1 - mu c

Let $c = c0 + d$, where d is a small perturbation about $c = c0$. Linearizing about this base state gives

> d_t = nu d_xx + d_xxxxxx + 2 c0 d_xxxx - mu d

Now substitute the following form for the perturbation

> d = a_k(t) exp(i k x)

This gives us the following equation for the Fourier coefficients *ak(t)*

> a_k(t) = exp((2 c0 k^4 - nu k^2 - k^6 - mu)t)

If mu = 0 and nu = 1.0, we see that the system will be linearly unstable if

> 2 c0 k^4 - k^2 - k^6 > 0

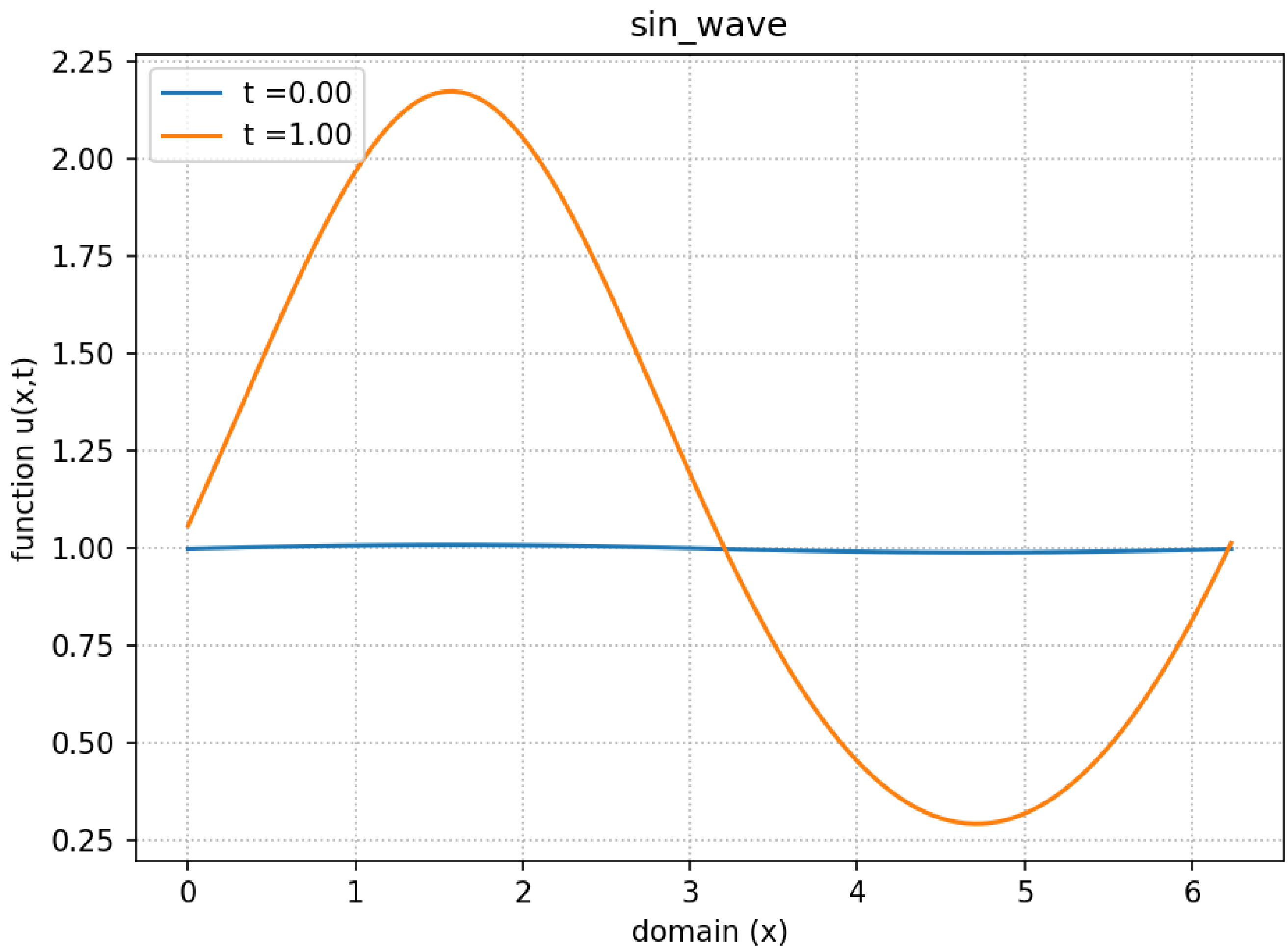
A plot of the function $y = 2\ c0\ k^4 - k^2 - k^6$ tells us that, if $c0 < 1$ system will be stable and will become linearly unstable for a range of wavenumbers if $c0>1$. This instability, propogated by the term c c_xxxx, however, is arrested by terms c_xx and c_xxxxxx and the system settles to a steady state. Simulations for such cases yield a solution that starts to grow and then oscillates with decaying amplitude to a steady state. We can approximate the wavenumber of the final steady state as follows

> d(a_k(t))/dt = (2 c0 k^4 - nu k^2 - k^6 - mu)exp((2 c0 k^4 - nu k^2 - k^6 - mu)t)

and therefore

> d(a_k(t))/dt = 0 if 2 c0 k^4 - nu k^2 - k^6 - mu = 0

For $c0 = 1$, $nu = 1$ and $mu = 0$, we get $k = 1$. It can be seen that this closely approximates the wavenumber for the final steady state obtained from the code. Following figure demonstates the evolution of an initial condition $c0=1+sin(x)$ for a long time.



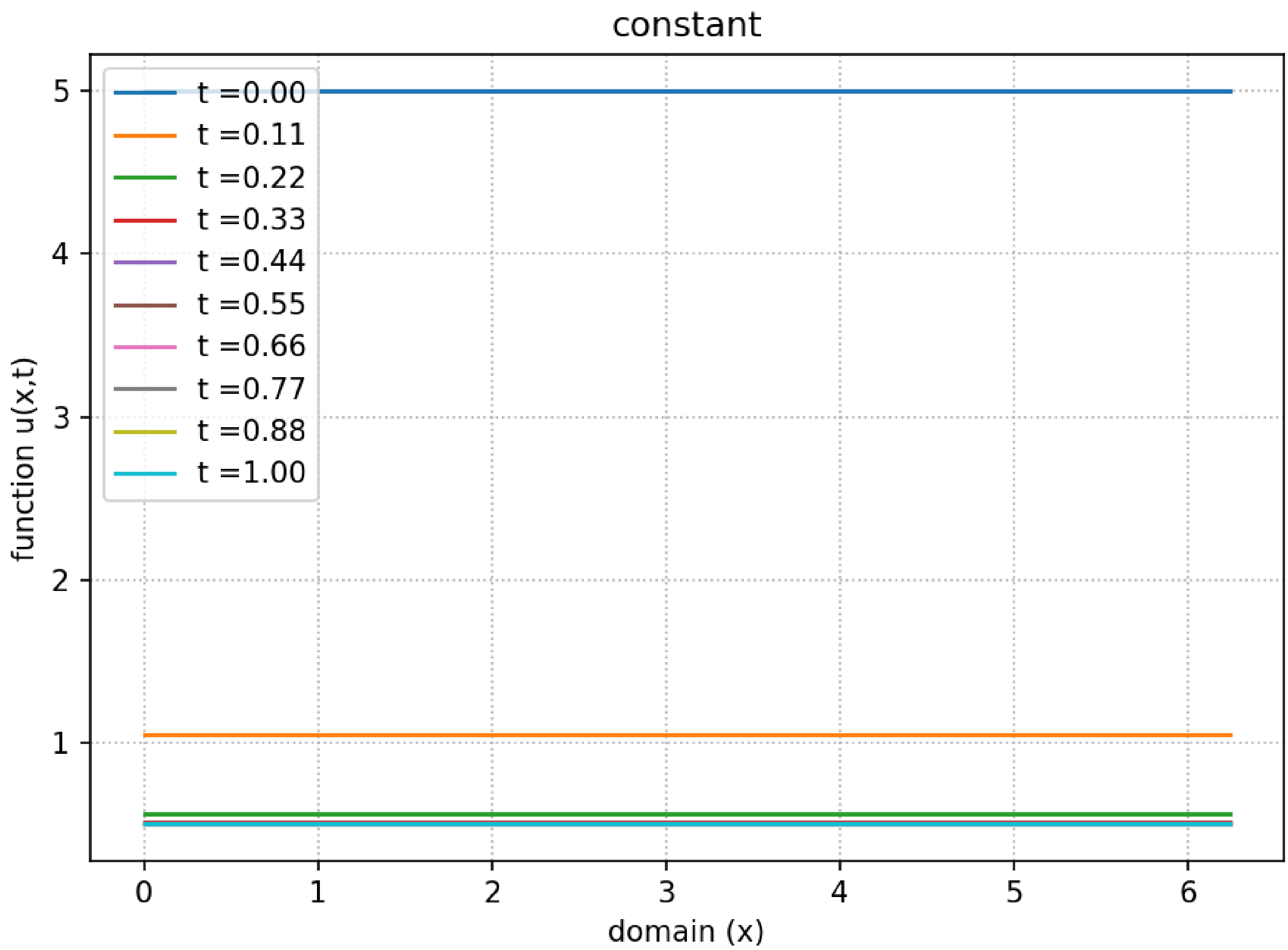
Numerical Formulation

We have evolved the Fourier coefficients of these terms in time using Adams-Bashforth 2 step method for the non-linear terms and the Adams-Moulton 2 step method for the linear terms.

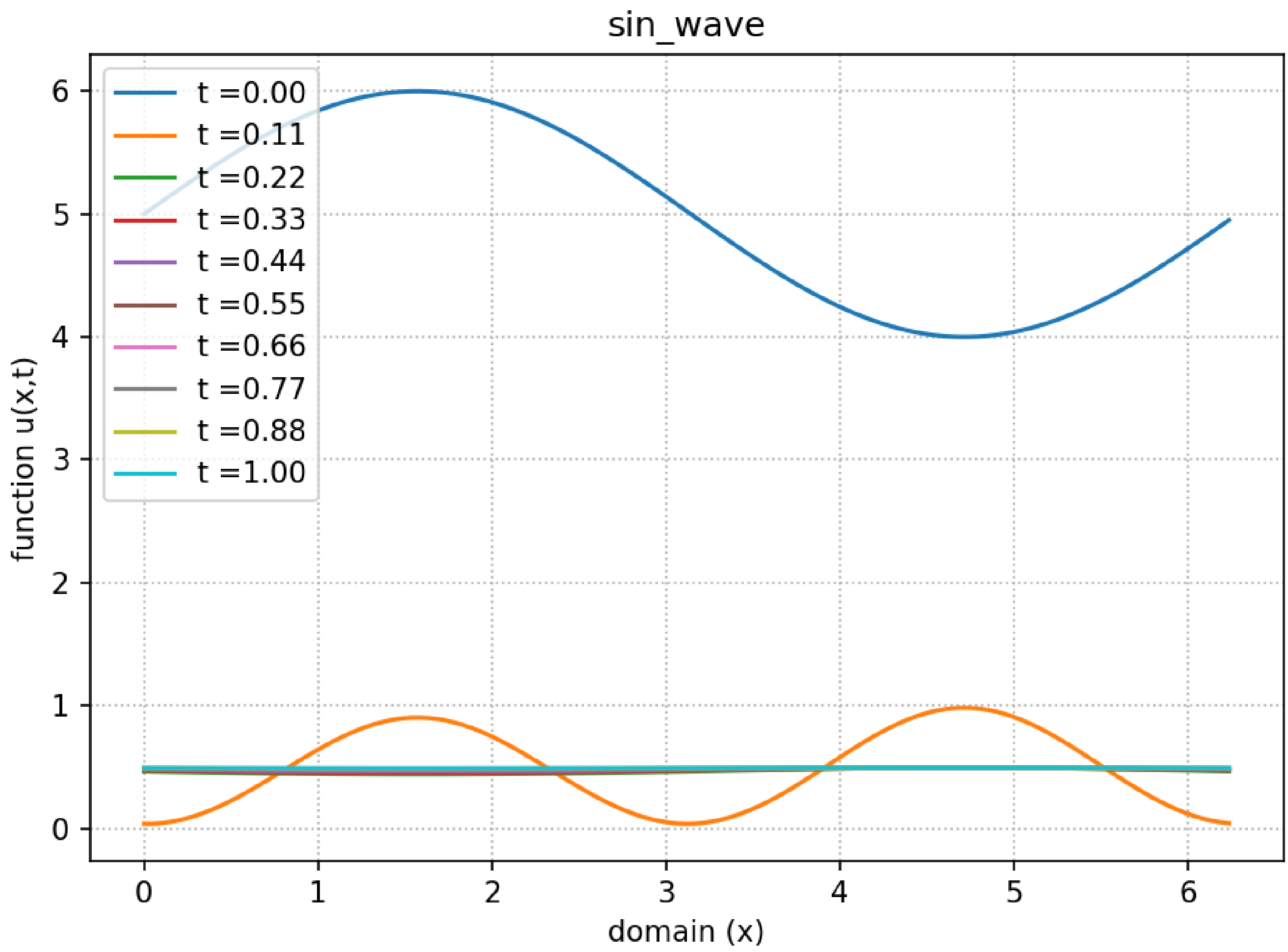
Some Results

A detailed analysis of the code will require a parametric study over the parameter space (mu nu L). Here we will only present a small sample of this space

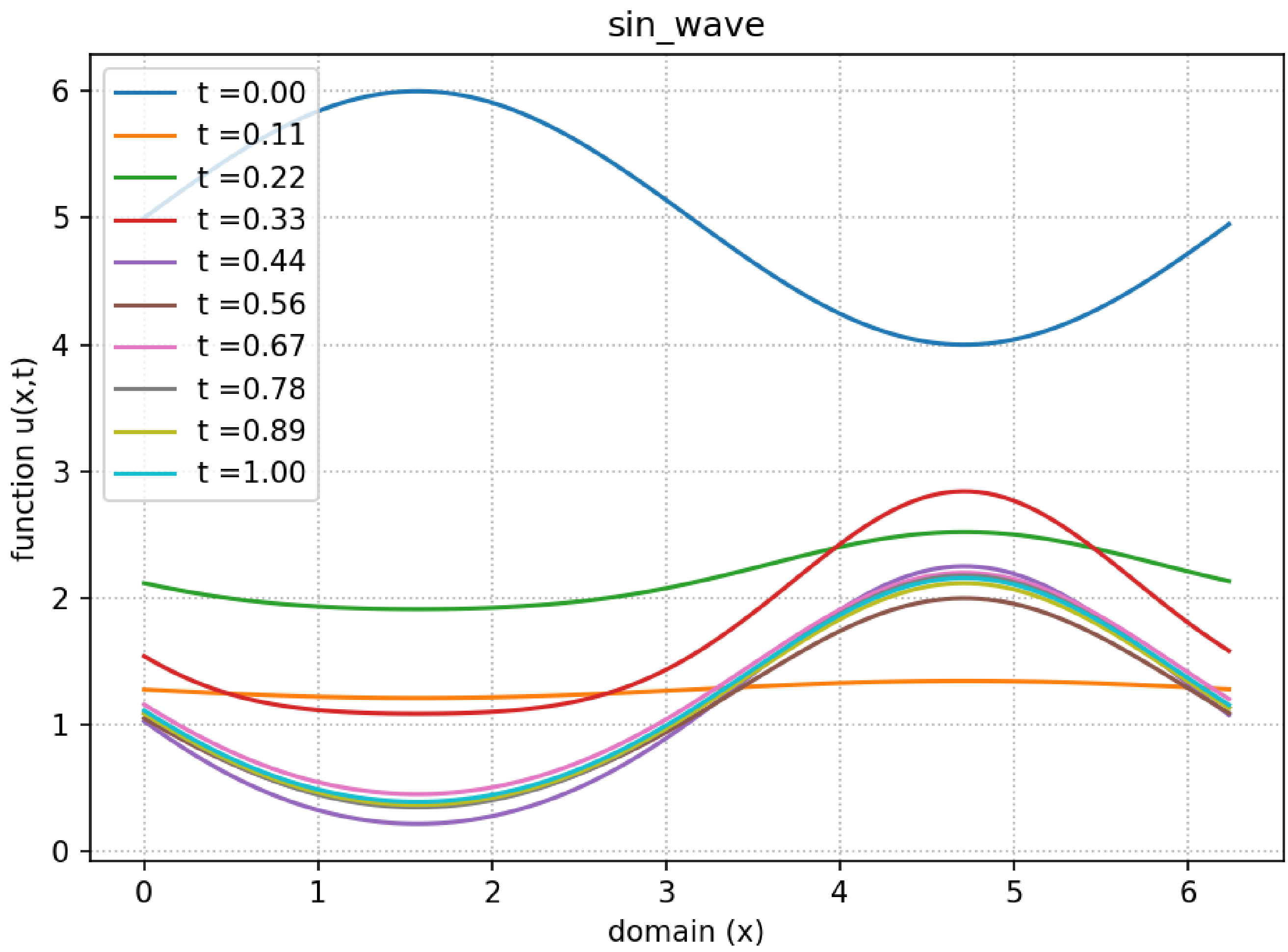
Firstly, we note that a constant initial condition must be eventually decay to 1/mu. This can be seen in the following plot where we start with *c0=5* and system settles to *c0=0.5*



For the above plot, the parameter are *mu=2.0*, *nu=1.0* and *L=2 pi*. Following plot shows the evolution of another initial condition for the same set of parameters. A large value of *mu* causes a rapid decay of the solution



With a smaller value of mu, the decay is slower and the final condition is not constant



To Run The Code

- The root directory must contain -toy_pde.py -post_processing.py -postproc.py -data/
- Run the file `toy_pde.py` with desired parameter values. This will generate a set of datafiles in the folder data/.
- In order to plot the results, run the file `postproc.py`.

User defined functions make_plot() and make_movie() generate the visualizations. Details of the functions are given in the file `post_processing.py`