You're about to get on a plane to Seattle. You want to know if you should bring an umbrella. You call 3 random friends of yours who live there and ask each independently if it's raining.

- Each of your friends has a 2/3 chance of telling you the truth and a 1/3 chance of messing with you by lying.
- All 3 friends tell you that "Yes" it is raining.

- You also know that there is a 25% it’s raining on any given day in Seattle.
- What is the probability that it's actually raining in Seattle?

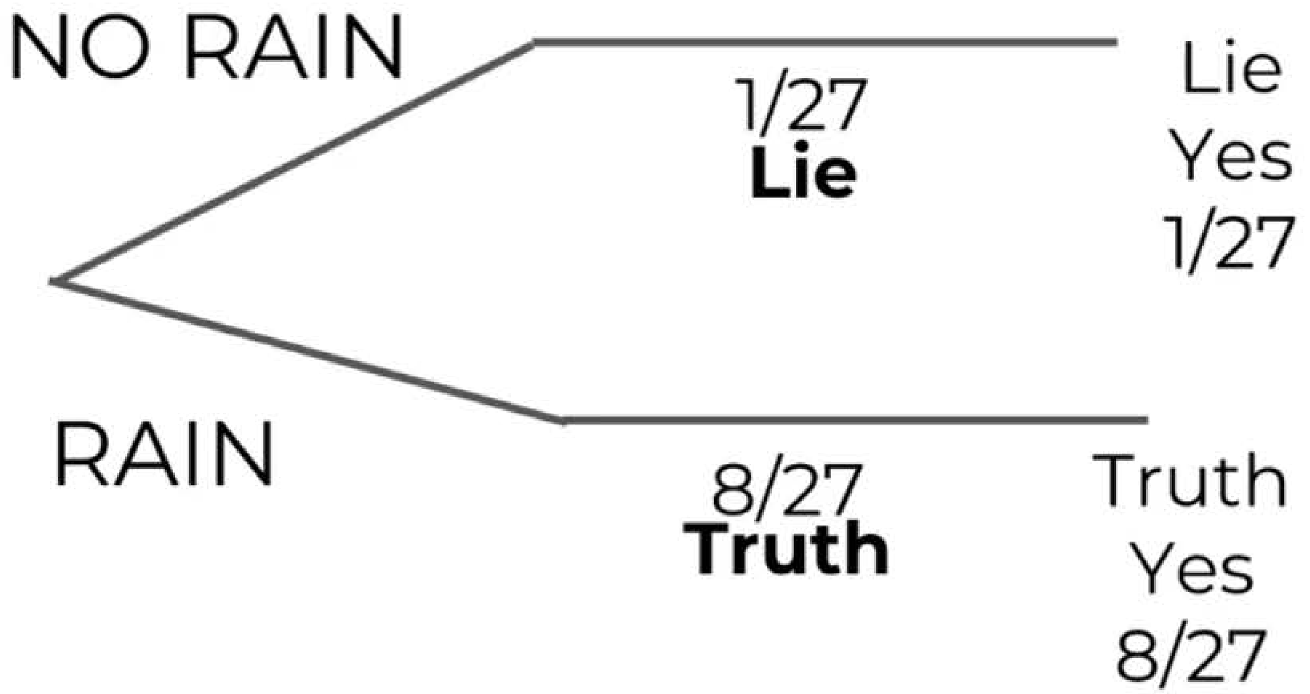
There are two situations where the friends all say “Yes.” One is if it is raining and they are all telling the truth.

Each tells the truth 2/3 of the time, so all three telling the truth is a probability of $(2/3)^3 = 8/27$.

The other situation is if it is not raining and all three are lying.

Each lies 1/3 of the time, so all three lying is a probability of $(1/3)^3 = 1/27$.

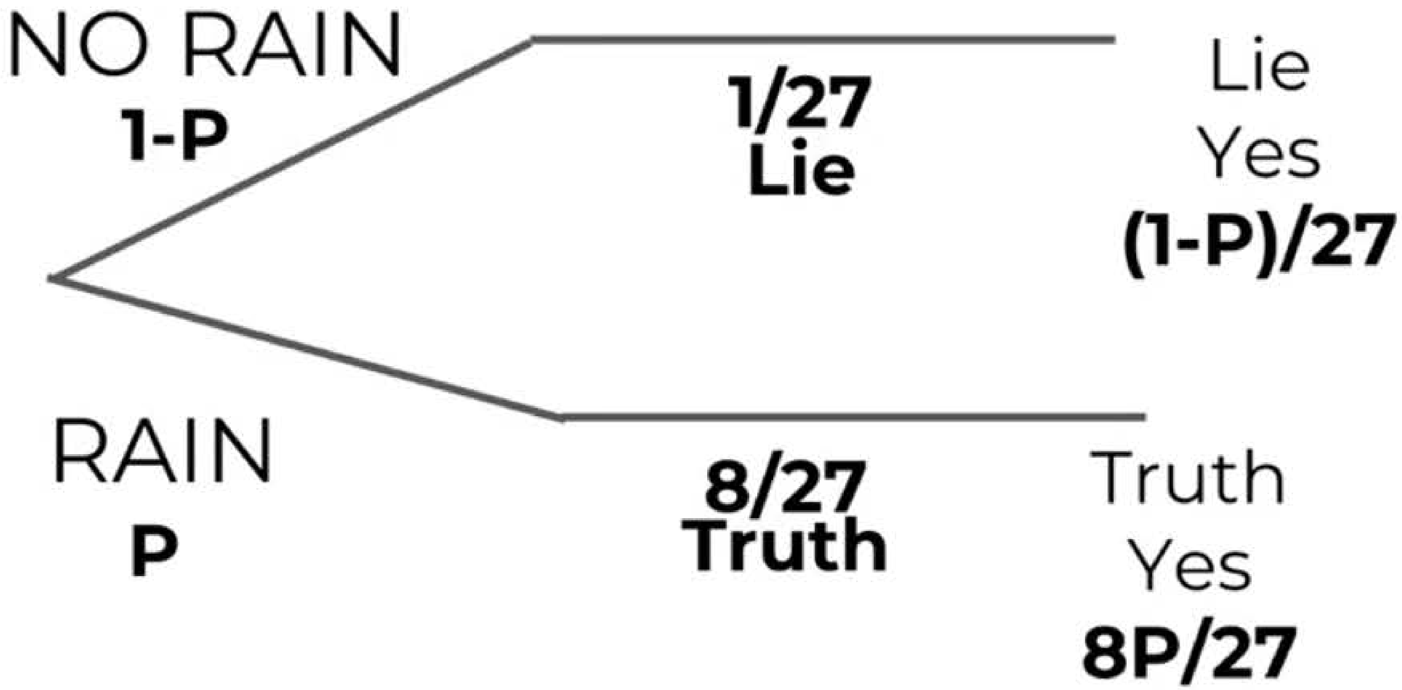
Here is a probability tree of the situation if No Rain and Rain were equally likely.



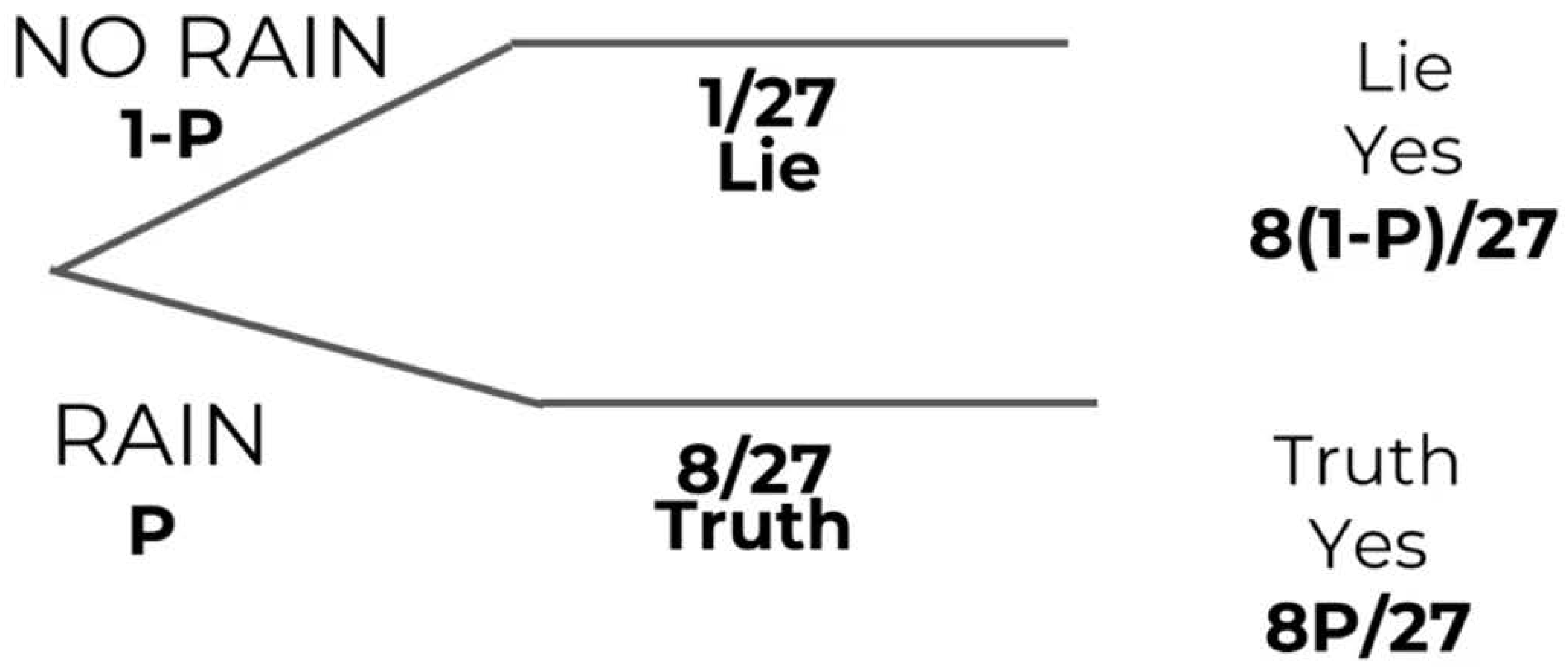
The event “all three say yes” happens $1/3 = 8/27 + 1/27$ of the time.

Out of these times, there is an 88.9% => $8/9 = (8/27)/(1/3)$ chance that it is actually raining, and a $1/9$ chance it is not raining.

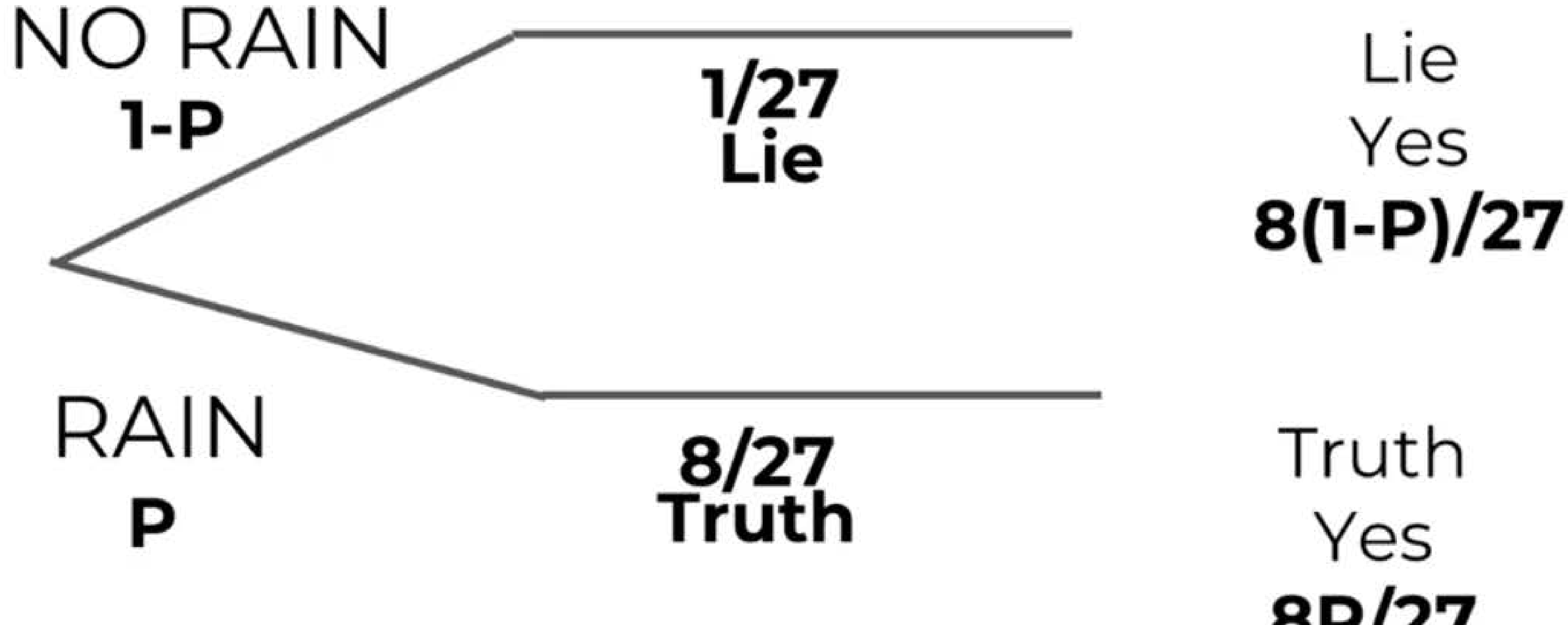
However, we know that the probability of it raining on any given day was not 50/50, here is general case tree:



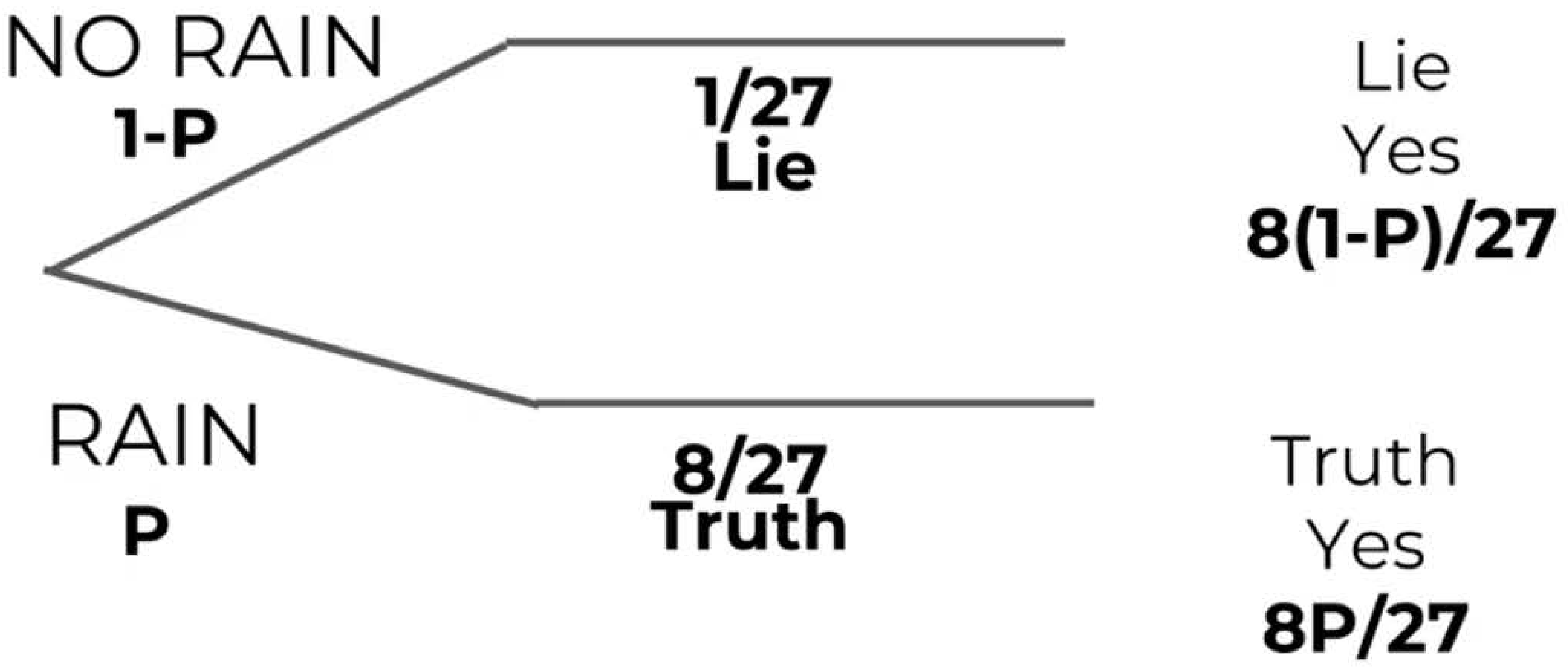
This means the probability that they all say yes (for any situation) is (8P/27) + (8(1-P)/27) = (7p+1)/27.



Which means that out of those times it was actually raining (8P/27) / ((7p+1)/27) = 8P /(7P+1).



Plugging in P=0.25 gives us 8P /(7P+1) → 0.72 chance that it is actually raining .



A new quantum message system has a probability of 0.8 of success in any attempt to send a message through.

Calculate the probability of having 7 successes in 10 attempts.

Hint: You may want to look at the Binomial Distribution to help you with this!

A binomial experiment is one that possesses the following properties:

- The experiment consists of n repeated trials;
- Each trial results in an outcome that may be classified as a success or a failure
- The probability of a success, denoted by p, remains constant from trial to trial and repeated trials are independent.

The number of successes X in n trials of a binomial experiment is called a binomial random variable.

The probability distribution of the random variable X is called a binomial distribution, and is given by the formula:

$$Pr(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

for $k = 0, 1, 2, \ldots, n$, where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- Probability of success **p=0.8** so **q=0.2**

$$\begin{aligned}\text{Probability} &= P(X=7) \\ &= C_7^{10}(0.8)^7(0.2)^{10-7} \\ &= 0.20133\end{aligned}$$

What is the difference between Type I vs Type II error?

Table of error types		Null hypothesis (H_0) is	
		True	**False**
Decision About Null Hypothesis (H_0)	**Reject**	Type I error (False Positive)	Correct inference (True Positive)
	Fail to reject	Correct inference (True Negative)	Type II error (False Negative)

A type I error occurs when the null hypothesis (H0) is true, but is rejected. It is asserting something that is absent, a false hit.

A type I error may be likened to a so-called false positive (a result that indicates that a given condition is present when it actually is not present).

A type II error occurs when the null hypothesis is false, but erroneously fails to be rejected. It is failing to assert what is present, a miss.

A type II error may be compared with a so-called false negative (where an actual 'hit' was disregarded by the test and seen as a 'miss') in a test checking for a single condition with a definitive result of true or false.

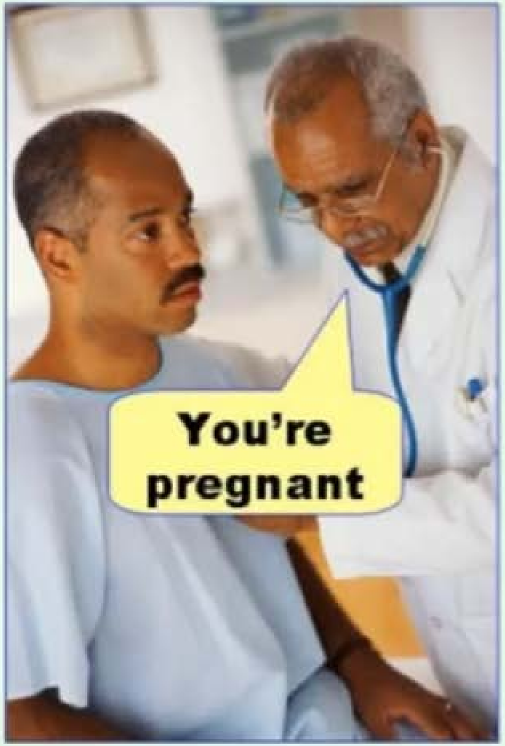
Hypothesis: "Adding water to toothpaste protects against cavities."

Null hypothesis (H0): "Adding water to toothpaste has no effect on cavities."

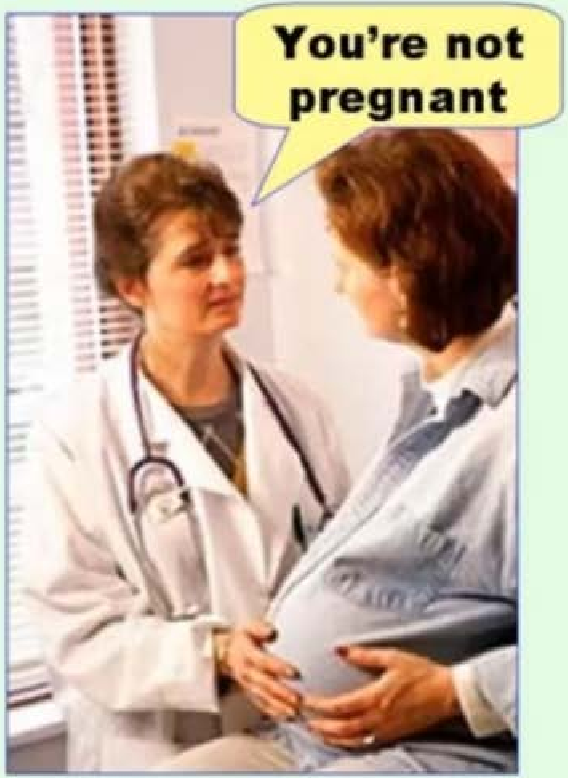
This null hypothesis is tested against experimental data with a view to nullifying it with evidence to the contrary.

- A type I error occurs when detecting an effect (adding water to toothpaste protects against cavities) that is not present.
- The null hypothesis is true (i.e., it is true that adding water to toothpaste has no effect on cavities), but this null hypothesis is rejected based on bad experimental data.

Type I error
(false positive)



Type II error
(false negative)



- A new medical test for a virus has been created.
- 1% of the population has the virus.
- 99% of sick people with the virus test positive (indicating they have the virus).

- 99% of healthy individuals test negative for the virus.
- If a patient tested positive, what is the probability that they have the virus?

This is yet another question where we can use Bayes' Rule (it is very popular in interview questions).

Let's walk through the steps.

We know P(sick)=0.01, we also know that 99% of sick patients test positive, meaning P(positive|sick)=0.99. Also 99% of healthy patients test negative, so P(neg|not sick).

We want to solve for P(sick|positive).

Using Bayes' Rule we have:

$$P(\text{sick}|\text{pos}) = P(\text{sick}) \frac{P(\text{pos}|\text{sick})}{P(\text{pos})}$$

We also can calculate the probability

$$P(pos) = P(pos|sick)P(sick) + P(pos|not\ sick)P(not\ sick)$$

$$P(pos) = 0.99*0.01 + 0.01*(1-0.01) = 0.0198$$

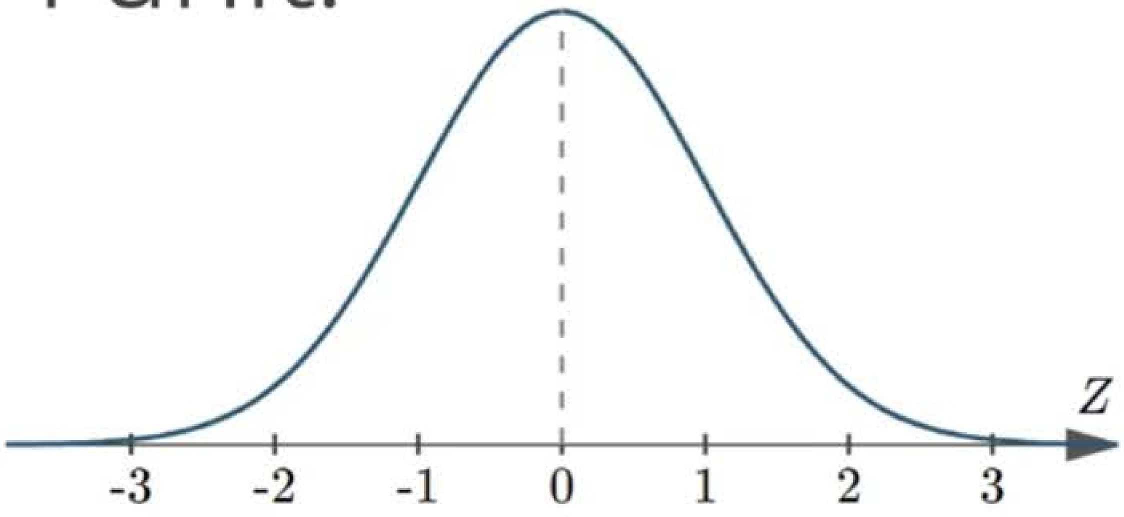
Plugging this back into our previous Bayes' rule we get **$P(sick|pos) = 0.5$**

- The average life of a certain type of motor is 10 years, with a standard deviation of 2 years.
- If the manufacturer is willing to replace only 3% of the motors because of failures, how long a guarantee should she offer?

Quick Note: To fully solve this question you will need to look up some values in a z-table. Treat this as part of a take home task so you can use those resources!

For this question, you will want to make sure you review normal distributions and how to do a look-up in a z-table. We will be using this in order to solve this problem.

We can use a Standard Normal Curve of with a mean of zero and a standard deviation of 1 unit:



Standard Normal Curve $\mu = 0, \sigma = 1$

We can transform all the observations of any normal random variable X with mean μ and variance σ to a new set of observations of another normal random variable Z with mean 0 and variance 1 using the following transformation:

$$Z = \frac{X - \mu}{\sigma}$$

Here we can say **X** is the life of the motor and we are searching for the guarantee period ***x*.**

So we are searching for $P(\mathbf{X} < \boldsymbol{x}) = 0.03$. Using a z-table we find that the corresponding z-score of $\mathbf{z = -1.88}$

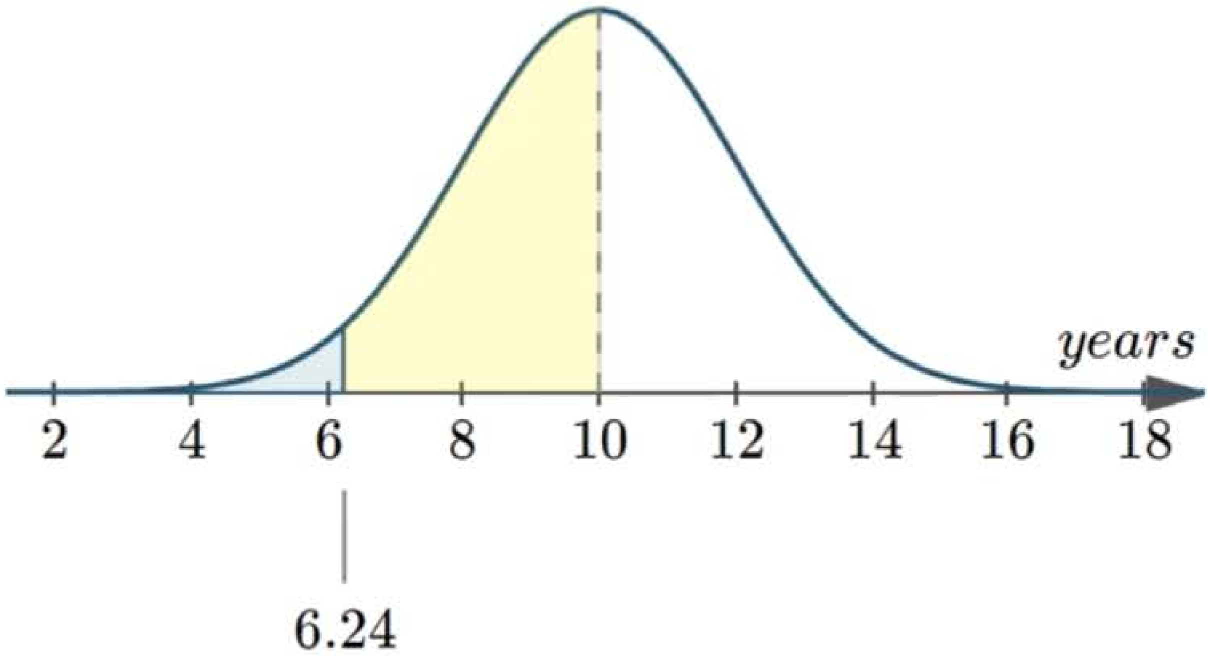
Which means we can then use:

$$Z = \frac{X - \mu}{\sigma}$$

Leaving us with:

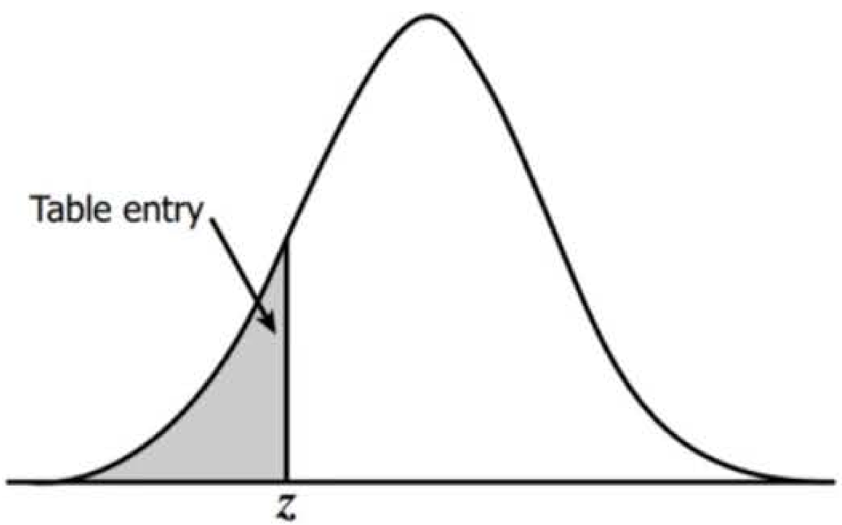
$$\frac{x - 10}{2} = -1.88$$

Solving this leads to x=6.24 , so the guarantee period should be 6.24 years.



Z SCORE TABLE

Z Table



Find values on the left of the mean in this negative Z score table. Table entries for z represent the area under the bell curve to the left of z. Negative scores in the z-table correspond to the values which are less than the mean.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611