- Let’s work through some example interview questions that relate to probability theory.
- Often these questions use coins or dice rolls to test your knowledge of probability.

- You are given a fair coin. On average, how many flips would you need to get two of the same flip in a row (either 2 heads in a row or 2 tails in a row)?

- Let's calculate $\mathbf{P_n}$ which we will say is the probability that we get the two consecutive flips (HH/TT) on the **nth** toss.
- Let's examine what $\mathbf{P_n}$ means with a simple example, with n=2, what is $\mathbf{P_n}$?
- What are the odds of getting HH or TT on two tosses?

- Out of 2 tosses
  - HH
  - TT
  - HT
  - TH
  - So the answer is 2/4 or ½
  - Now let's generalize to the nth toss

- Let's calculate $\mathbf{P_n}$ which we will say is the probability that we get the two consecutive flips (HH/TT) on the **nth** toss.
- We know that flips from **1 to n-1** need to be of the form **HTHTHT... or THTHTHT...**
- So what is the probability of that series?

- Probability of THTHTHT... or HTHTHTH... for n-1 tosses is $\frac{2}{2^{n-1}}$ because each is $\frac{1}{2^{n-1}}$
- This is because each flip has a ½ chance of being the flip we need to alternate, and so we need ½ * ½ * ½ ... all the way until n-1 times, which results in $\frac{1}{2^{n-1}}$
- Because there are two possible series: $\frac{2}{2^{n-1}}$

- Probability of THTHTHT... or HTHTHTH... for n-1 tosses is $\frac{2}{2^{n-1}}$ because each is $\frac{1}{2^{n-1}}$
- So we know the nth toss (n≥2) matching up to the previous toss is ½, which means:

$$P(X = n) = \frac{2}{2^{n-1}} \cdot \frac{1}{2} = \frac{1}{2^{n-1}}$$

- We also know that the formula for expected value is:

$$\mathbf{E}[X] = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k.$$

In our case we insert that we know x=n and p=Pn

With our values we end up getting a geometric series that sums up to 3!

$$\sum_{n=2}^{\infty} nP_n = \sum_{n=2}^{\infty} \frac{n}{2^{n-1}} = 3$$

- What is the probability of rolling a total sum of 4 with 2 dice?

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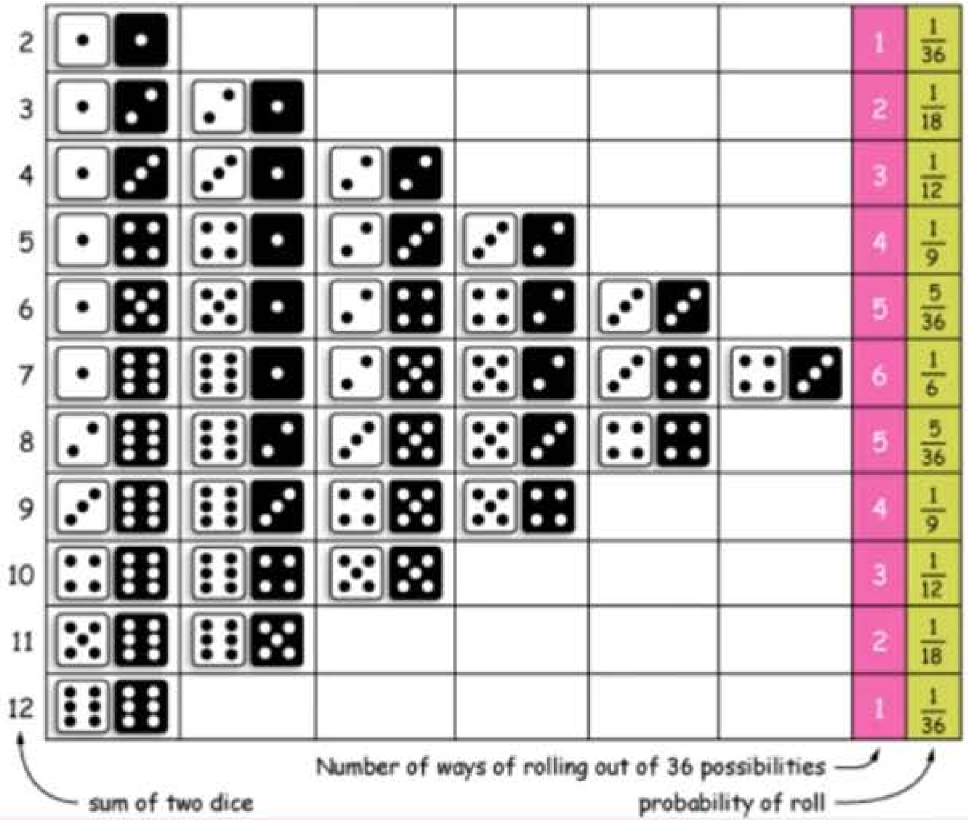

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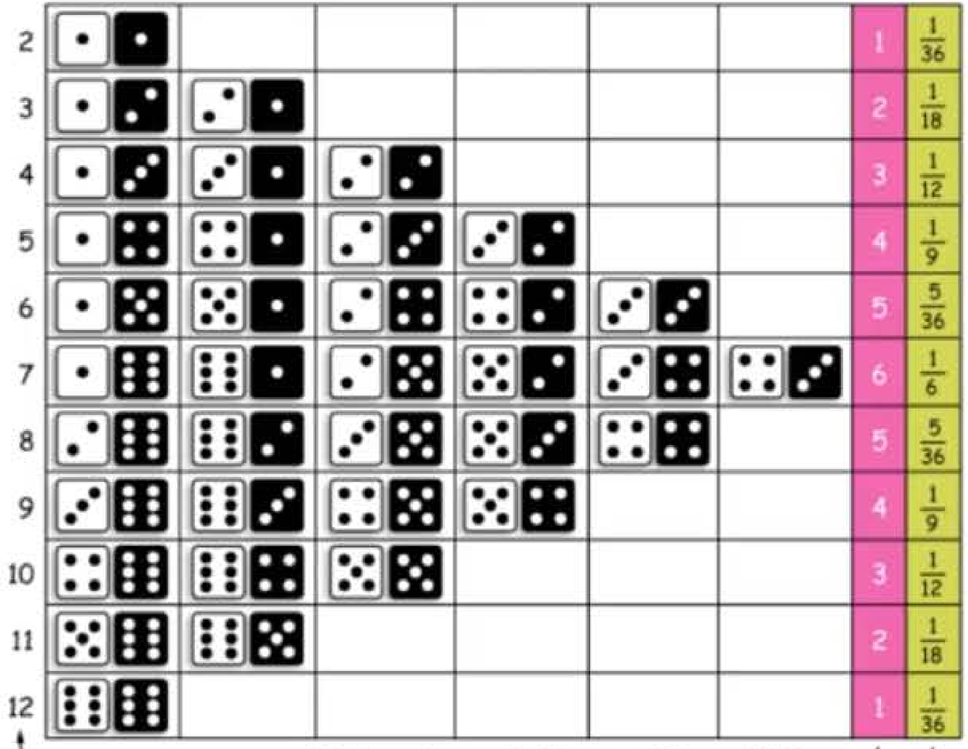
- There are 36 total ways the dice can be thrown.
- The ways the dice can add up to 4 are [(1,3),(2,2),(3,1)] which gives us 3/36.
- So the probability is 1/12.

- What is the probability of rolling at least one 4 with 2 dice?

- For at least one dice we need to think about which combinations are in the form $(4,x),(x,4)$ and $(4,4)$

- This eventually leads to 11 possible ways.
- So the answer is 11/36



- You have two jars, 50 red marbles, 50 blue marbles. You need to place all the marbles into the jars such that when you blindly pick one marble out of one jar, you maximize the chances that it will be red.

- 50 Blue Marbles and 50 Red Marbles
- 2 Jars
- You must place all the marbles in the jar in whatever distribution you prefer
- Then you will randomly pick a jar and then randomly choose a marble.
- Maximize your chance of choosing red!

- In jar one place just one red marble, and place all the rest in jar two. The outcome - red versus blue - takes two steps.
- First, there are 50:50 odds, or a 50% chance the friend will select jar one.

- If he does, then there is a 100% chance of a red outcome. There is also a 50% chance he will choose jar two instead. If he does, there are 49:50 odds, or 49/99 = 49.4949...% of choosing red.

- So on single trial, the odds of a red marble outcome is:
- $1 \times ½ + 49/99 \times ½ = .7475$, or 74.75%.

- If the probability of seeing a car on the highway in 30 minutes is 0.95, what is the probability of seeing a car on the highway in 10 minutes? (Assume a constant default probability)

- Let probability of seeing **NO CAR** in 10 minutes be **P**.
- Therefore the probability of seeing **NO CAR** in 20 min:
  - P(no car in 10min.)× P(no car in 10min.) = **P×P**
  - **No car for 30 mins = P×P×P**

- The probability of seeing at least one car in 30 min = 1 - **P**×**P**×**P** = 0.95
- **P**×**P**×**P** = 0.05
- **P** = 0.37
- Probability of seeing A CAR in 10 min. is
    - 1-**P** = 0.63

- You are given a fair coin. On average, how many flips would you need to get two heads in a row? (Similar to the first question , but now we specifically only want 2 heads)

- There are lots of ways to solve this!
- A general solving for any number of heads in a row is more challenging, but we only care about 2 heads in a row.
- Check out the resource links for more info!

- Keep in mind our following method is not generalized, it only works for 2 heads in a row.

- Let the expected number of coin flips be **x**. The case analysis goes as follows:
- If the first flip is a tails, then we have wasted one flip.
- The probability of this event is **½** and the total number of flips required is **x+1**

- If the first flip is a heads and second flip is a tails, then we have wasted two flips.
- The probability of this event is **1/4** and the total number of flips required is **x+2**

- If the first flip is a heads and second flip is also heads, then we are done.
- The probability of this event is **1/4** and the total number of flips required is 2.

- Adding, the equation that we get is
    - $x = (1/2)(x+1) + (1/4)(x+2) + (1/4)2$
    - $x = 6$

- You are given 10 coins. 9 are fair and 1 is biased. You are told the biased coin has **P>0.5** to be heads. You randomly grab a coin and flip it three times and get **HHT**. What is the probability you flipped the biased coin?

- To solve this equation we can use Baye's Theorem

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

Or the extended alternative:

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|\bar{A}) * P(\bar{A})}$$

Where Ā must be understand as not-A

- Filling what we know (b standing for bias)
- $\mathbf{P(HHT|b)} = P(H|b) \times P(H|b) \times P(T|b)$
- $P(b|HHT) = \mathbf{(P(HHT|b)} \times P(b)\,)/\,P(HHT)$
- Let's now plug in $\mathbf{P(HHT|b)}$

$$\frac{P(H|b) \times P(H|b) \times P(T|b) \times P(b)}{P(HHT|b) \times P(b) + P(HHT|fair) \times P(fair)}$$

Plugging in the probabilities:

$$\frac{P(H|b) \times P(H|b) \times P(T|b) \times P(b)}{P(HHT|b) \times P(b) + P(HHT|fair) \times P(fair)}$$

$$\frac{P(H|b) \times P(H|b) \times P(T|b) \times P(b)}{P(HHT|b) \times P(b) + P(HHT|fair) \times P(fair)}$$

Plugging in the probabilities:

$$\frac{\mathbf{p \times p \times (1\text{-}p) \times 0.1}}{P(HHT|b) \times \mathbf{0.1} + \mathbf{0.5^3 \times 0.9}}$$

$$\frac{P(H|b) \times P(H|b) \times P(T|b) \times P(b)}{P(HHT|b) \times P(b) + P(HHT|fair) \times P(fair)}$$

Plugging in the probabilities:

$$\frac{p \times p \times (1-p) \times 0.1}{p \times p \times (1-p) \times 0.1 + 0.5^3 \times 0.9}$$

$$\frac{P(H|b) \times P(H|b) \times P(T|b) \times P(b)}{P(HHT|b) \times P(b) + P(HHT|fair) \times P(fair)}$$

Plugging in the probabilities:

$$\frac{p^2(1-p) \times 0.1}{p^2(1-p) \times 0.1 + 0.1125}$$

- Given a biased coin with **P>0.5** for heads, how could you simulate a fair coin. In more general words: simulate a fair coin given only access to a biased coin. Note, this is tricky!

- Von Neumann (link in notes) gave a simple solution: flip the coin twice. If it comes up heads followed by tails, then call the outcome HEAD. If it comes up tails followed by heads, then call the outcome TAIL.

- Otherwise (i.e., two heads or two tails occurred) repeat the process.

Throughout we assume that the flips are independent, also this method works regardless of the actual bias (as long as it is not 0 or 1)

- Alice has 2 kids and one of them is a girl. What is the probability that the other child is also a girl? (You can assume that there are an equal number of males and females in the world.)

- With 2 children the possibilities are:
    - BB
    - GG
    - BG
    - GB
    - However you know that there is at least 1 girl!

- So the options are BG,GB,GG meaning we have a ⅓ chance of Alice having another daughter.