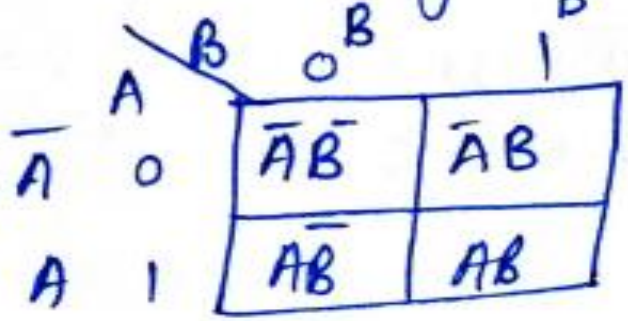
Karnaugh - Map (K-Map) simplification

Kmap is a general with graphical method of simplifying a boolean equation.

Kmap structure :-

a) The structure of a 2 inputs (variables) Kmap is



Here A & B are I/P
0 & 1 are values of A or B
Inside box, we can enter output value of Y.

(b) 3 Variables of K-map :-

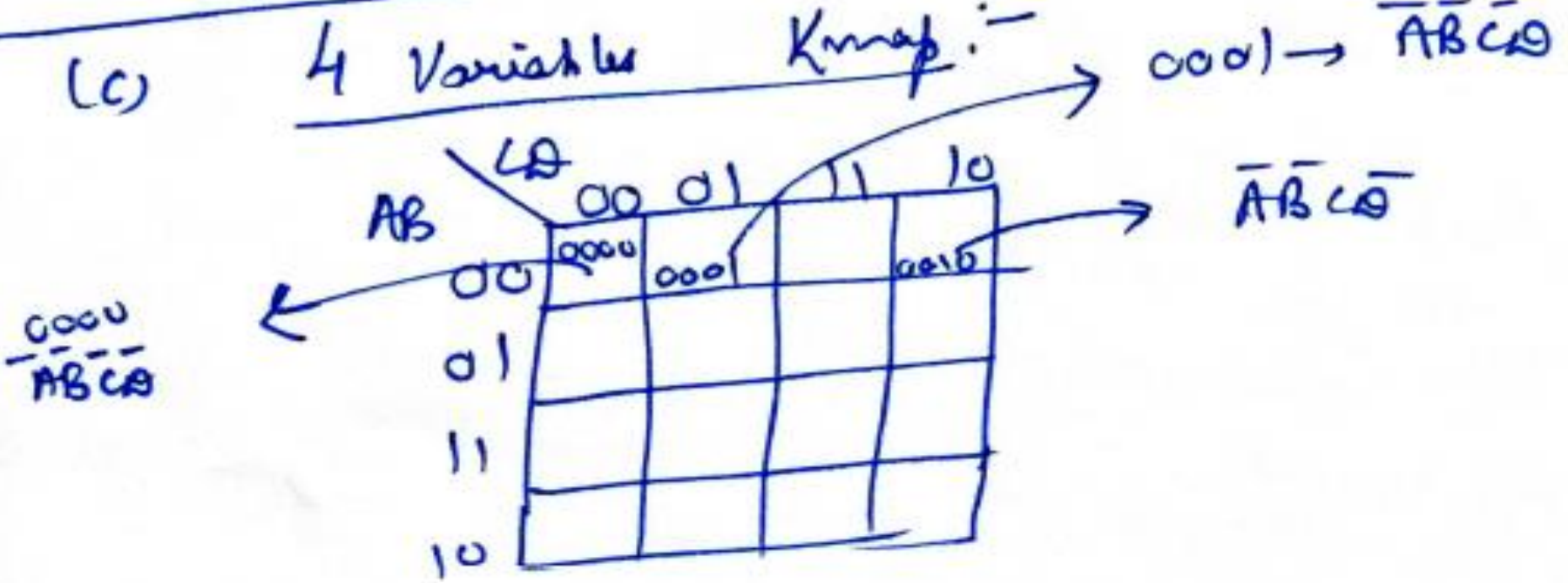
Gray code

0 → $\bar{A}$

1 → A

A \ BC	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
1	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

(c) 4 Variables Kmap :-



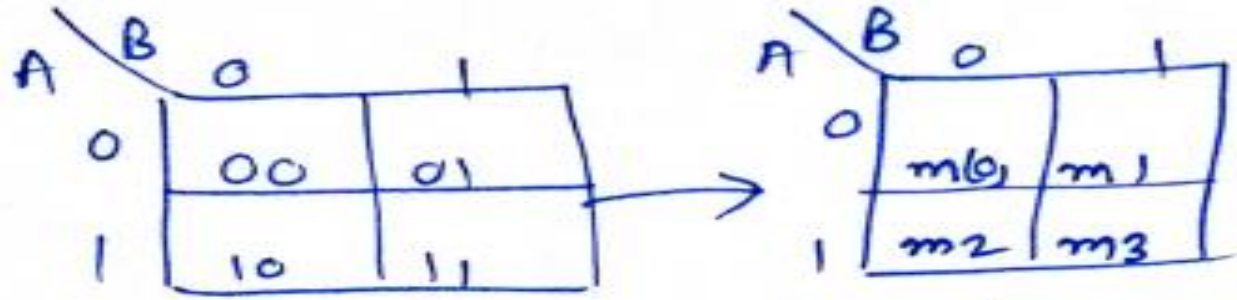
00

01

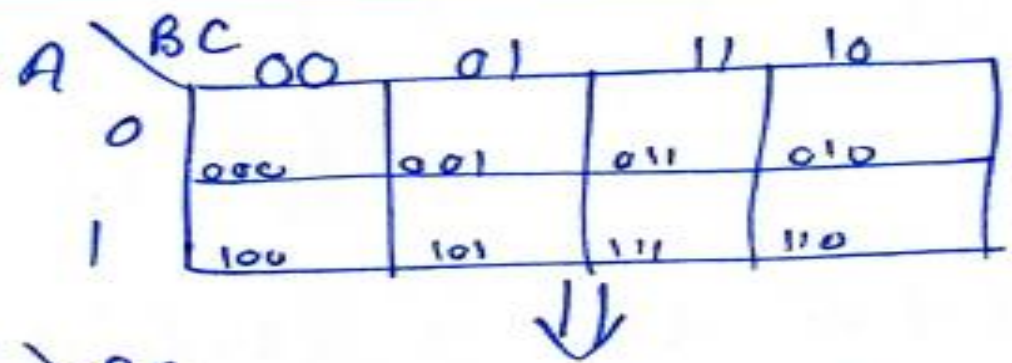
11

10

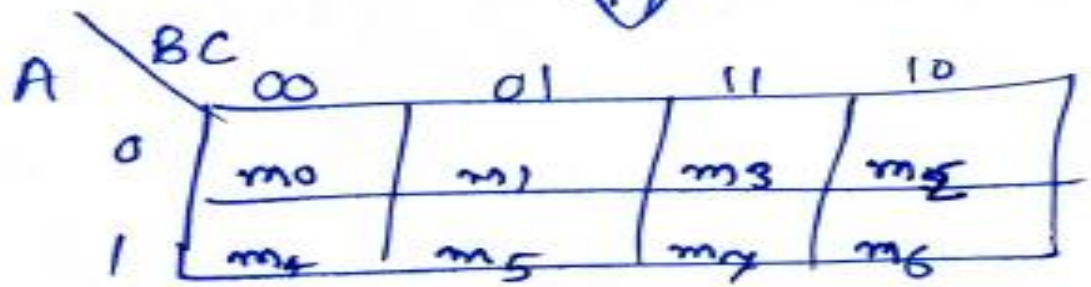
Gray code

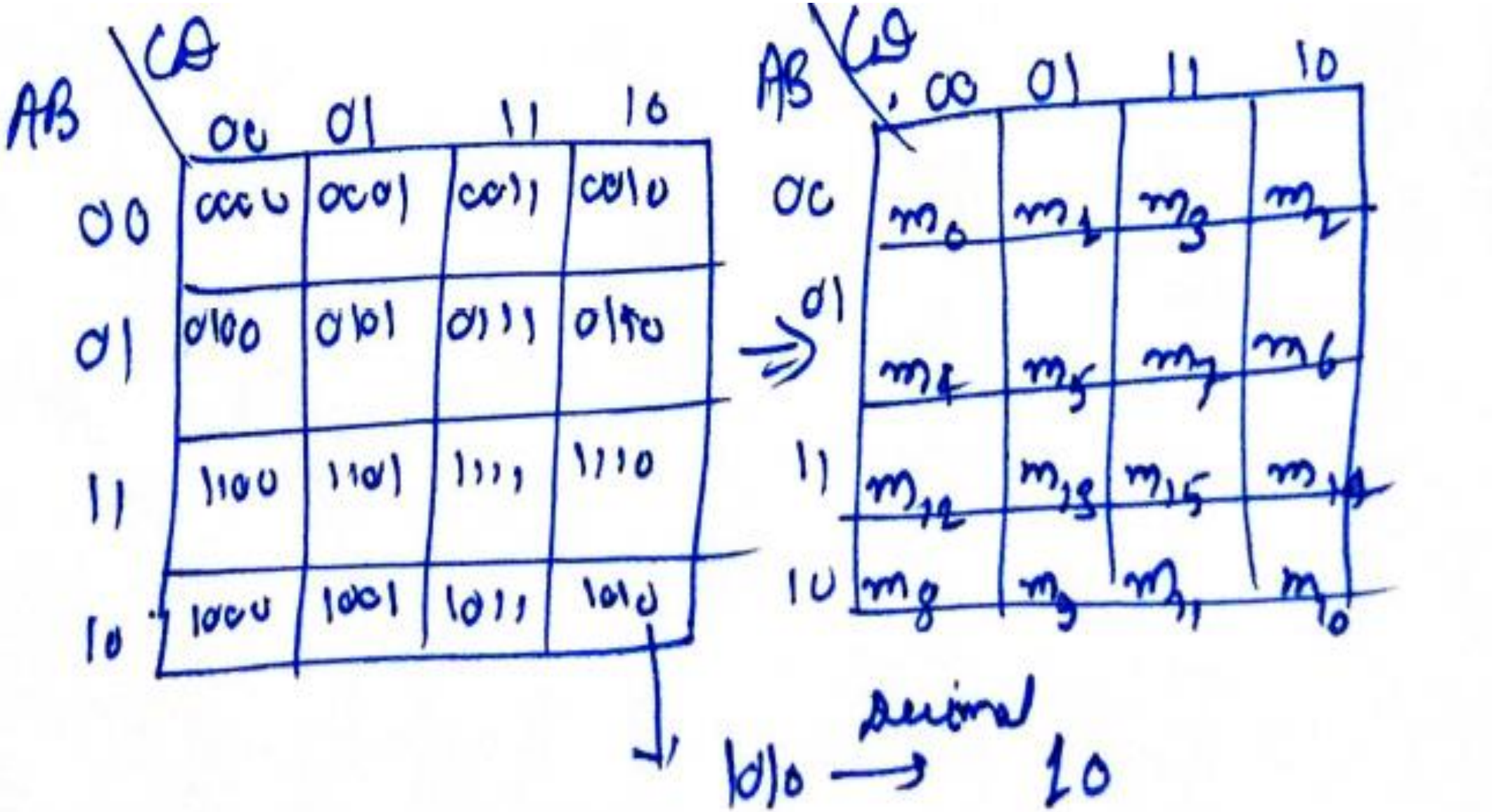


	Decimal
00 →	0
01 →	1
10 →	2
11 →	3



	Decimal
000 →	0
001 →	1
011 →	3
010 →	2




AB
CD
00 01 11 10
00 0000 0001 0011 0010
01 0100 0101 0111 0110
11 1100 1101 1111 1110
10 1000 1001 1011 1010
AB
CD
00 01 11 10
00 m0 m1 m3 m2
01 m4 m5 m7 m6
11 m12 m13 m15 m14
10 m8 m9 m11 m10
1010 → decimal 10

Poll

2. There are _____ cells in a 4-variable K-map.

a) 12

b) 16

c) 18

d) 8

Solu

2. There are _____ cells in a 4-variable K-map.

a) 12

b) 16

c) 18

d) 8

View Answer

Answer: b

Explanation: There are 16 = (2^4) cells in a 4-variable K-map.

Relation between truth table & Karnaugh

	A	B	Y
m_0	0	0	0
m_1	0	1	1
m_2	1	0	0
m_3	1	1	1

A \ B	0	1
0	0	1 (01)
1	0	1 (11)

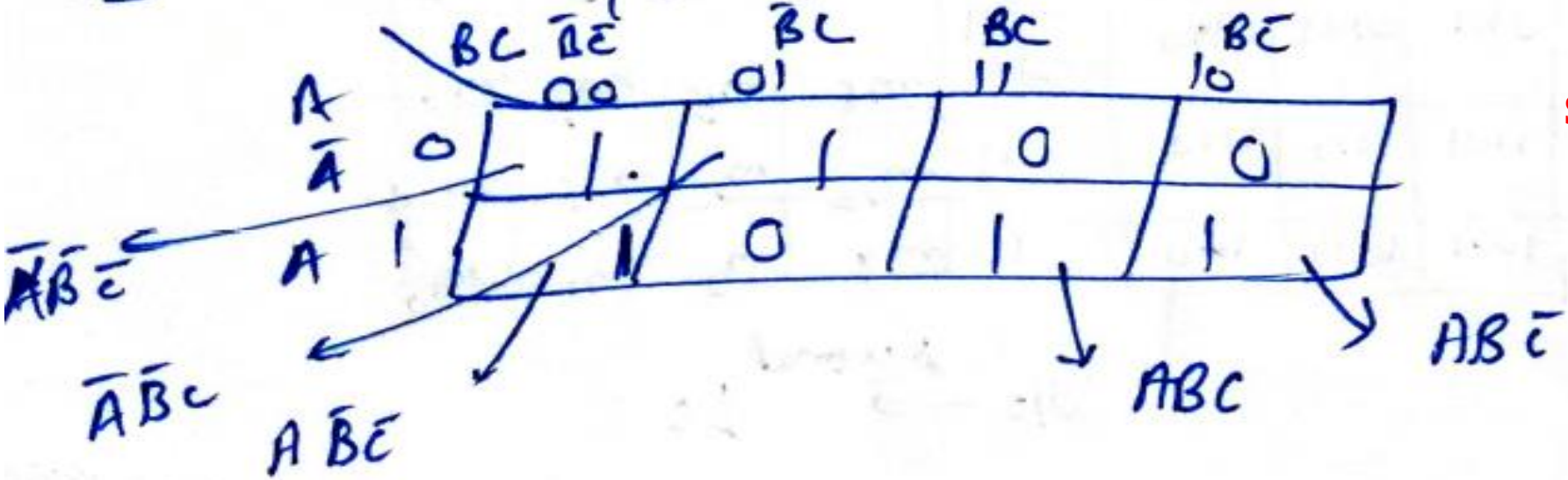
	A	B	C	Y
m_0	0	0	0	1
m_1	0	0	1	0
m_2	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

A \ BC	00	01	11	10
0	0: 1	1: 0	3: 1	2: 0
1	4: 0	5: 1	7: 1	6: 0

Q. Represent the equation given below on K-map

Sol:-

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

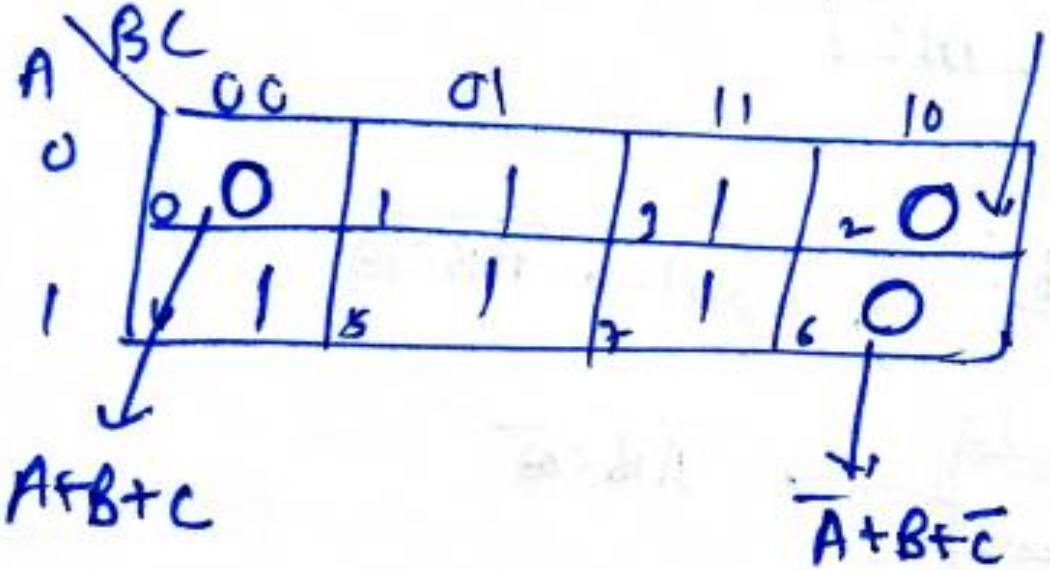


SOP=FILL LOGIC 1

Q: Represent the following standard POS expression on K-Map

$$Y = (A+B+C)\cdot(A+\bar{B}+C)\cdot(\bar{A}+\bar{B}+C)$$

POS=FILL LOGIC 0



$A+B+C \Rightarrow 000 \rightarrow M_0$

$A+\bar{B}+C \Rightarrow 010 \rightarrow M_2$

$\bar{A}+\bar{B}+C \Rightarrow 110 \rightarrow M_6$

Logic

$0 \rightarrow A$

$1 \rightarrow \bar{A}$

Simplification of Boolean Expression Using K-Map

A) A group of two adjacent 1's or 0's is called pair.

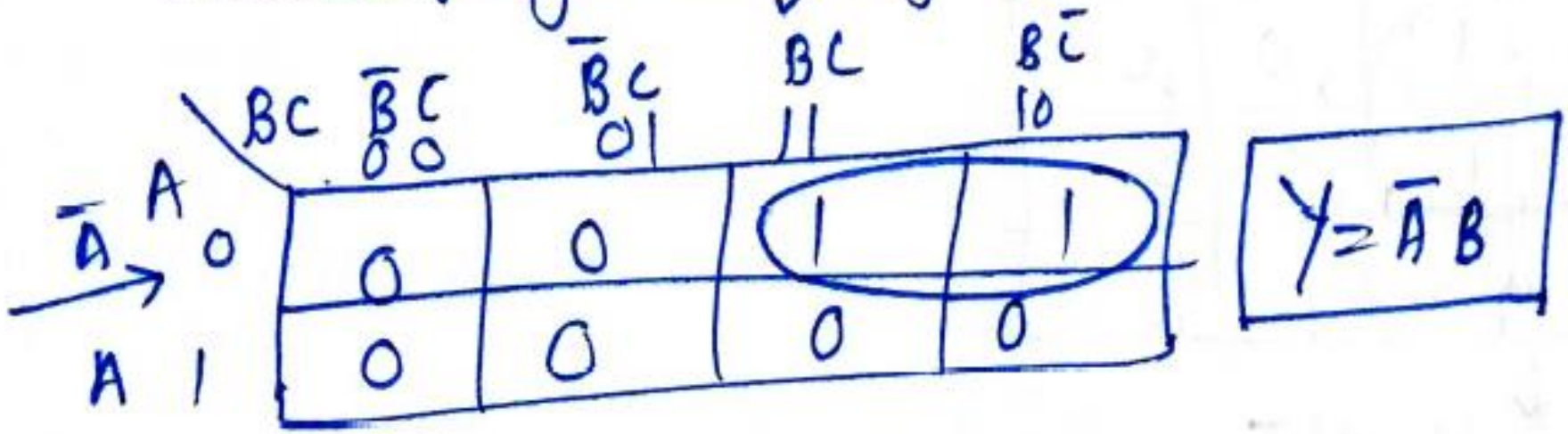
B) Quad :- A group of four adjacent 1's or 0's called quad.

C) Octal :- A group of eight adjacent 1's or 0's called as octet.

OCTET

Grouping of two adjacent ones (1's) Pair

A)



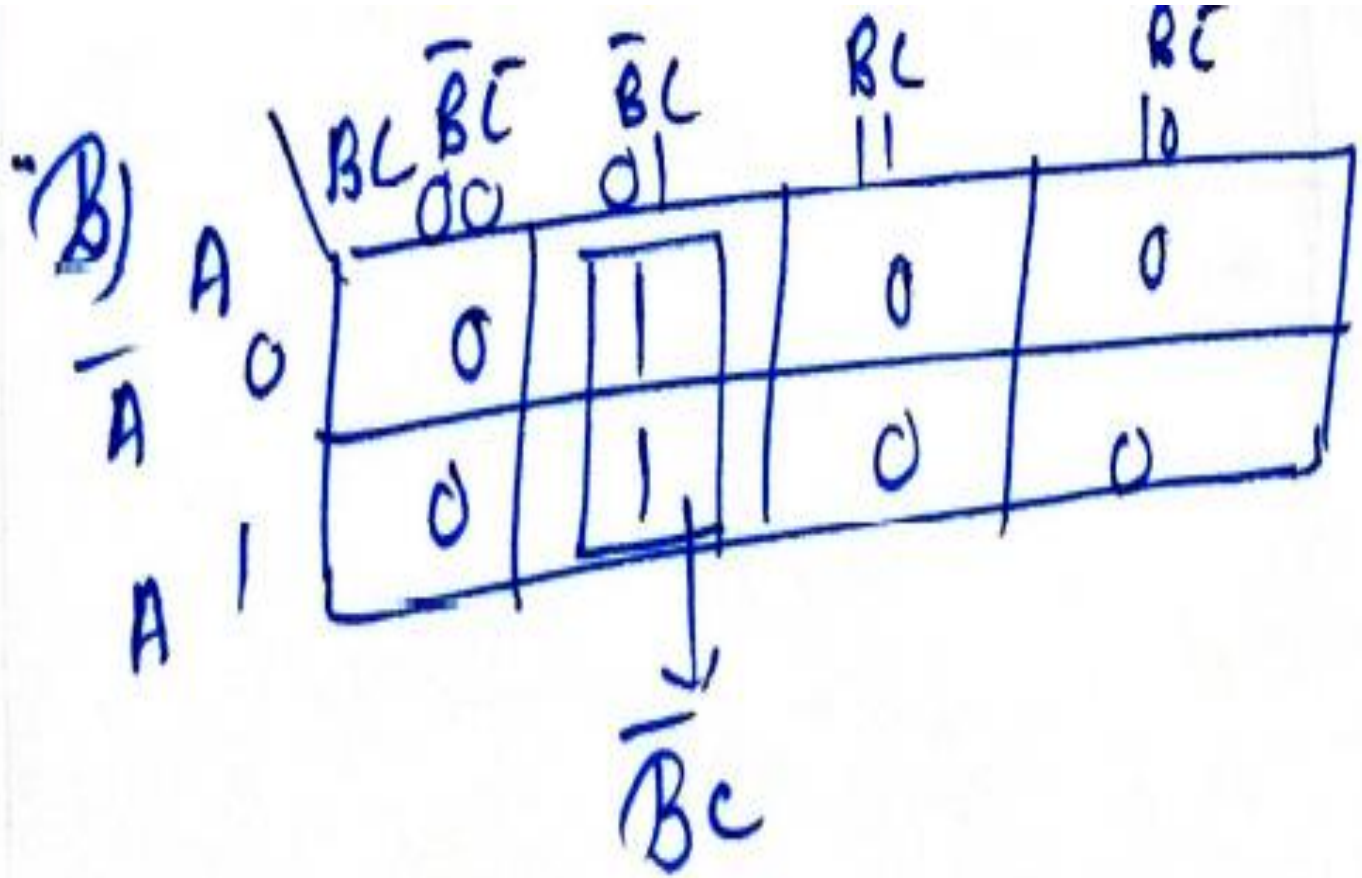
$Y = \bar{A}B$

Otherwise :-

$$Y = \bar{A}BC + \bar{A}B\bar{C}$$

$$\Rightarrow \bar{A}B(C+\bar{C}) \Rightarrow \bar{A}B$$

B)

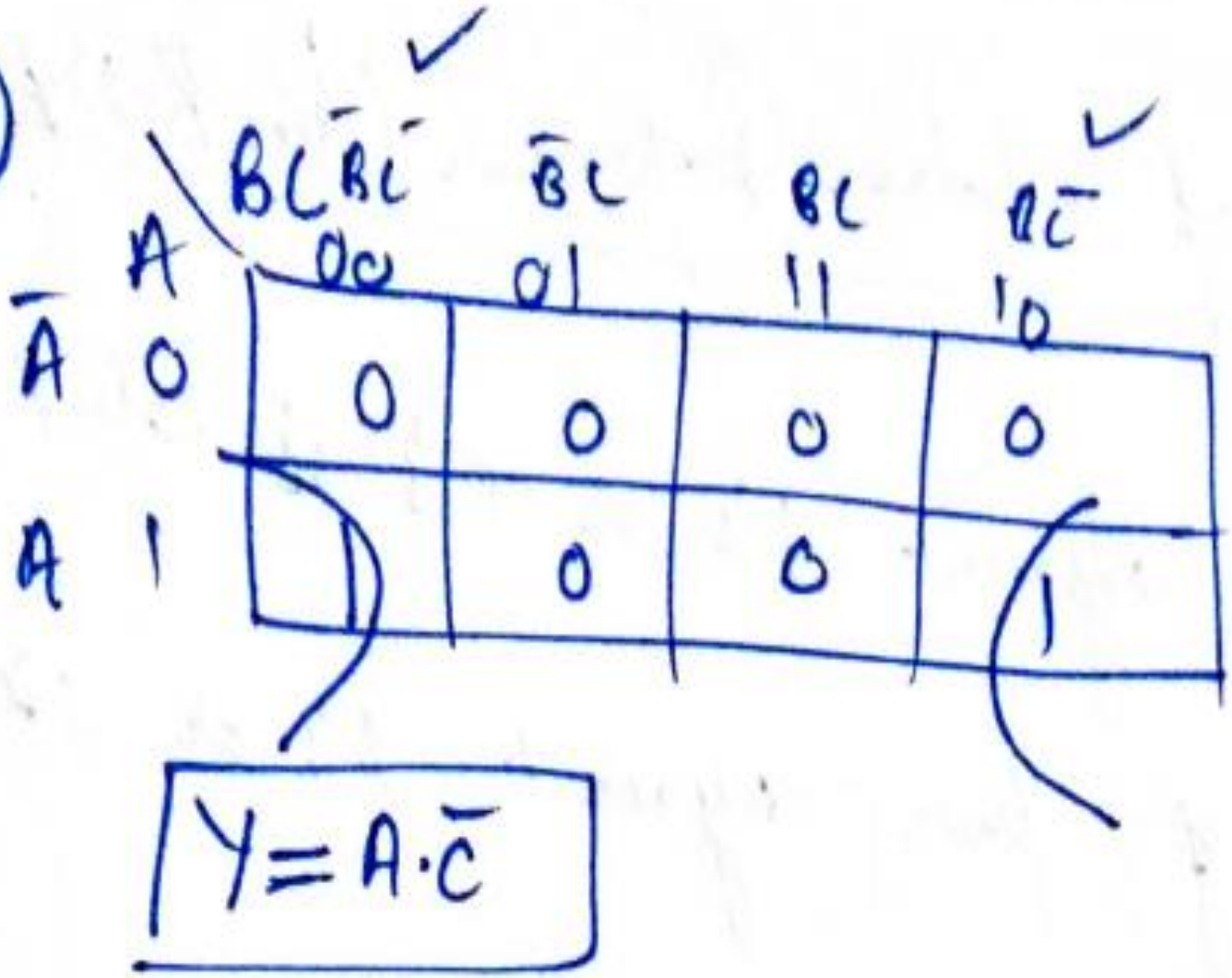


$$Y = \bar{A}\bar{B}C + A\bar{B}C$$

$$\Rightarrow \bar{B}C(\bar{A}+A)$$

$$\Rightarrow \bar{B}C$$

(3)

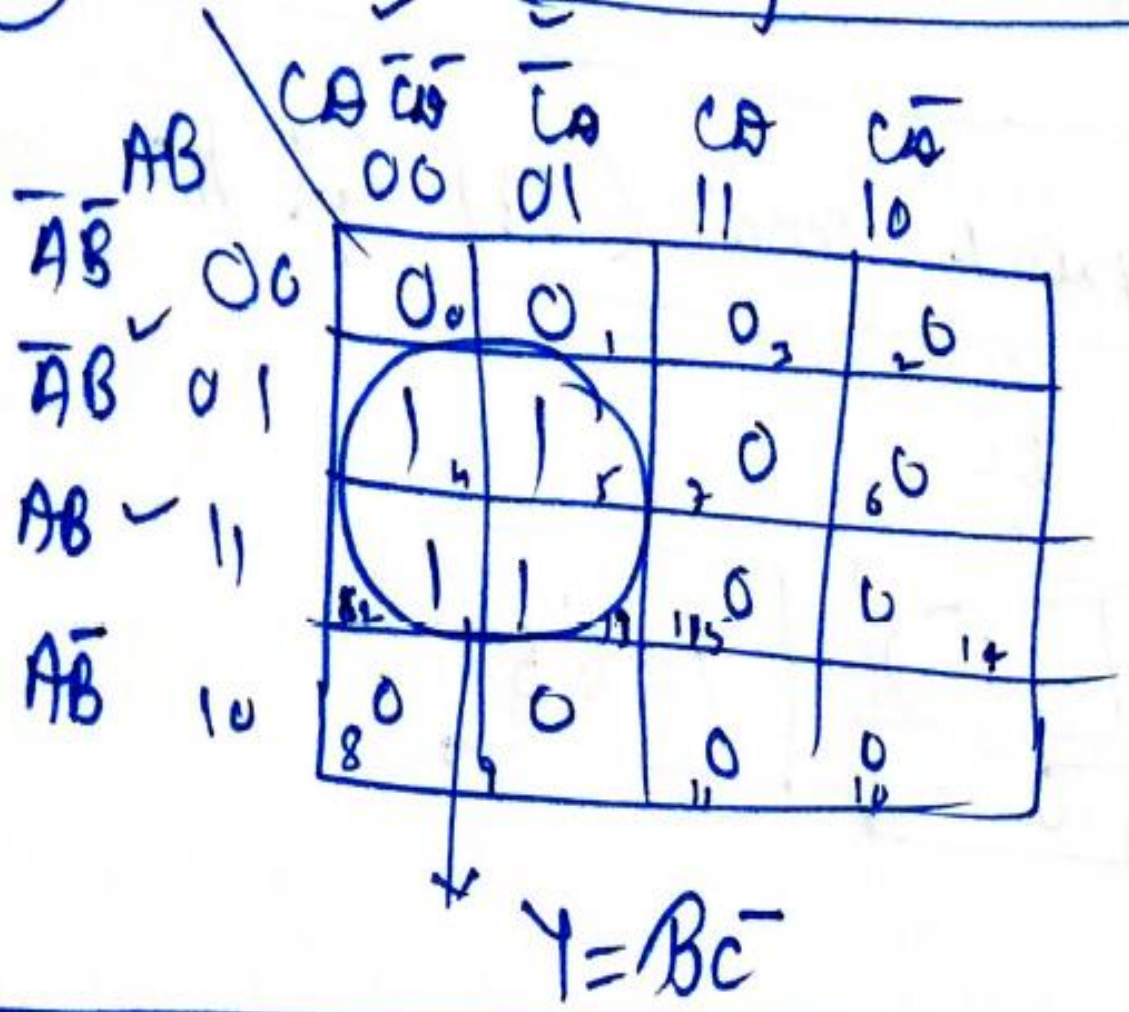


$Y = A\bar{B}\bar{C} + AB\bar{C}$

$\Rightarrow A\bar{C}(\bar{B}+B$

$\Rightarrow A\bar{C}$

K Map for 4-Variables

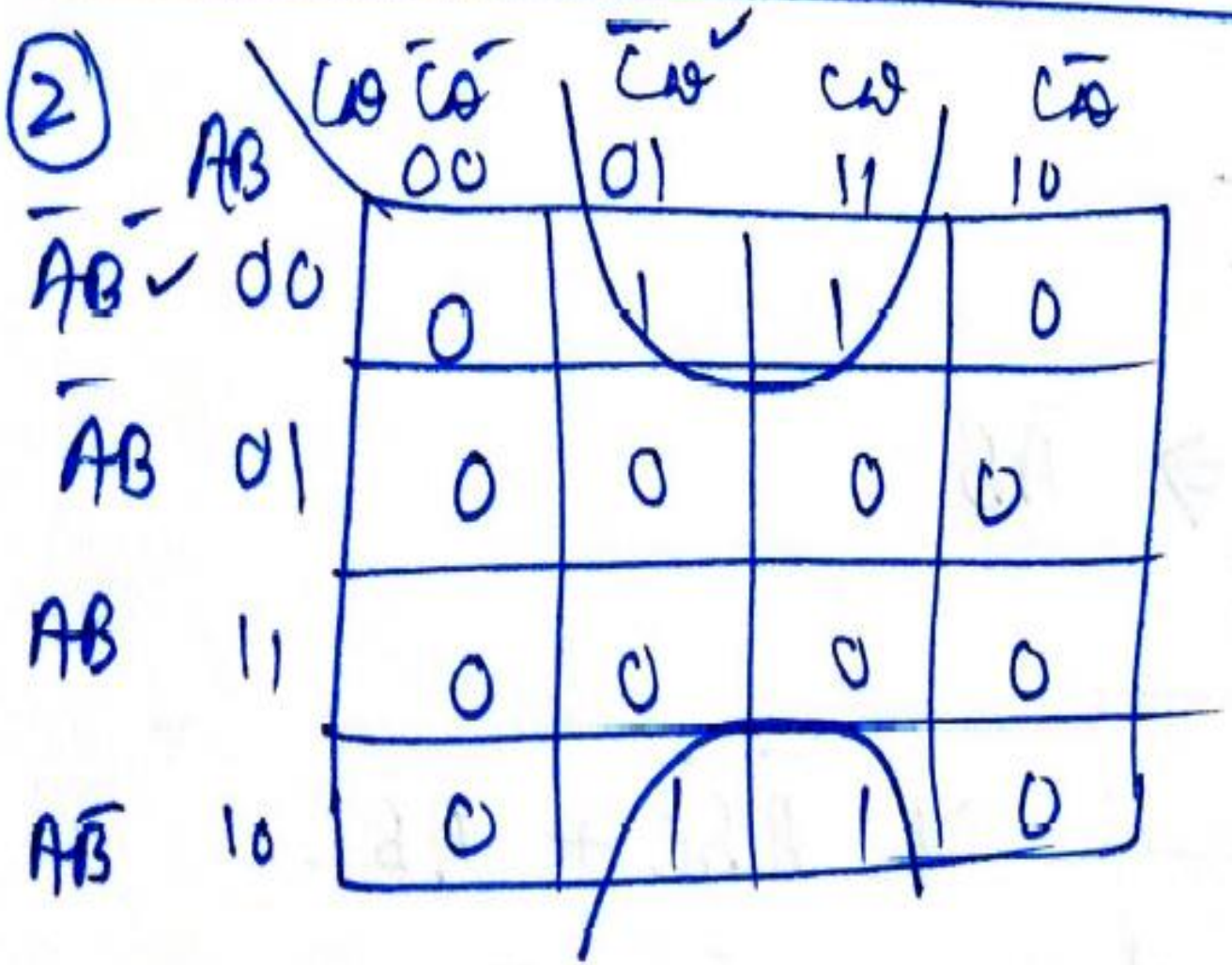


$Y(A,B,C) = \Sigma m(4,5,12,13)$

Y(A,B,C,D)

SUM minterm means fill '1' logic

(2)



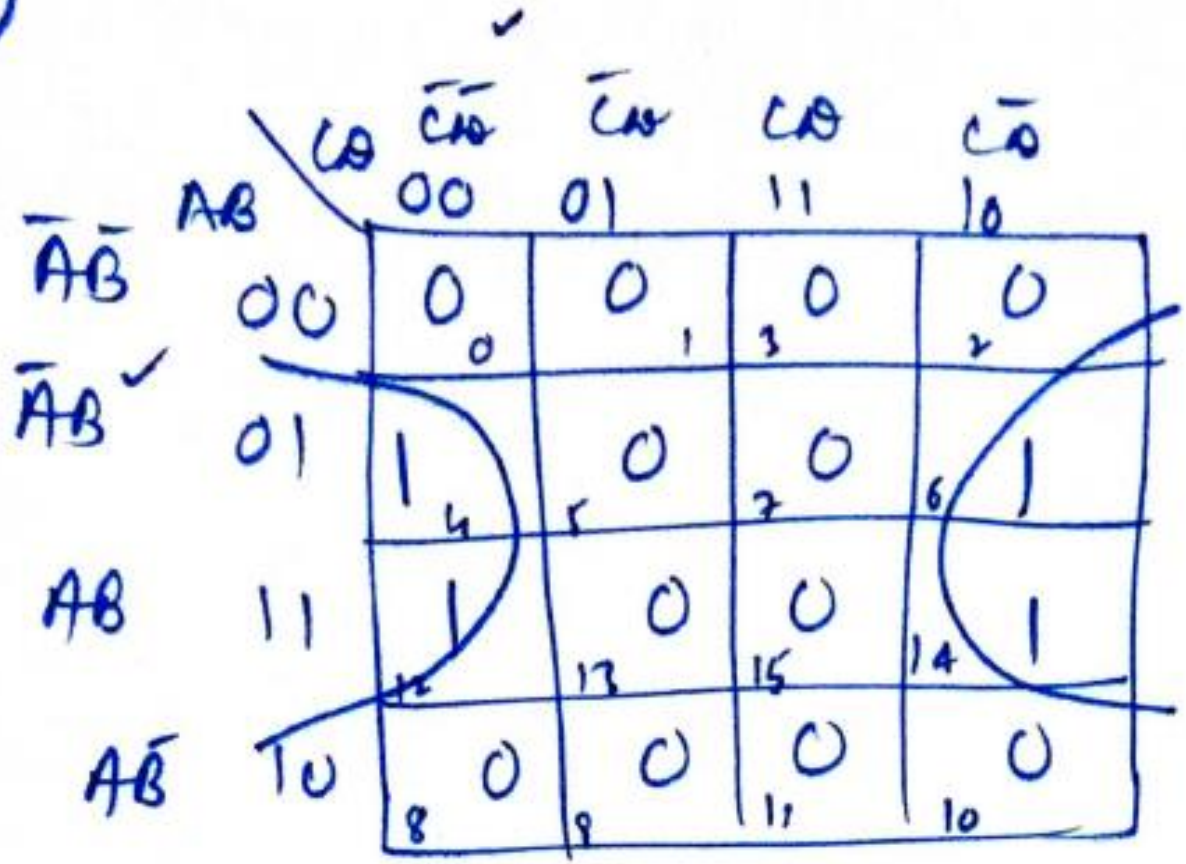
$Y = \bar{B}\bar{D}$

$Y(A,B,C,D) = \Sigma m(1,3,9,11)$

Y(A,B,C,D)

SUM minterm means fill '1' logic

(3)



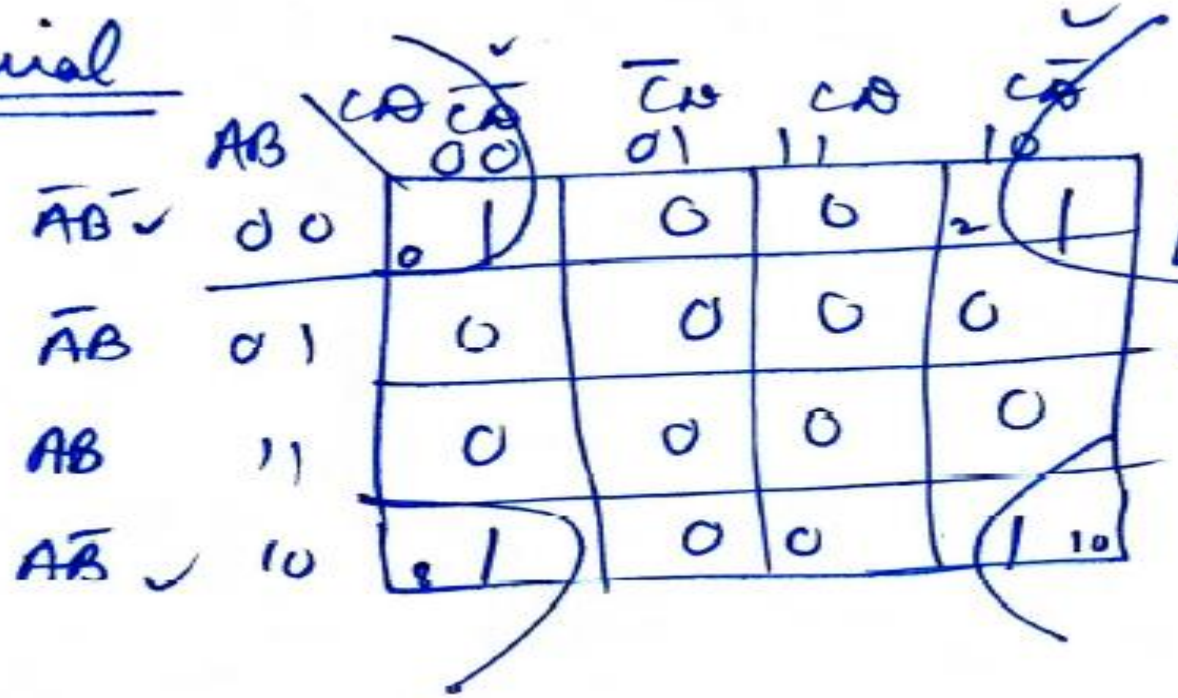
$Y(A,B,C) = \Sigma m(4,6,12,14)$

Y(A,B,C,D)

SUM minterm means fill '1' logic

$Y = B\bar{D}$

(4) Special



$Y = \bar{B}\bar{D}$

$Y(A,B,C) = \Sigma m(0, 2, 8, 10)$

Y(A,B,C,D)

SUM minterm means fill '1' logic

Simplification of Boolean expression

Otherwise :-

$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}$

$Y = \bar{B}\bar{C}\bar{D}(\bar{A}+A) + \bar{B}\bar{C}\bar{D}(\bar{A}+A)$

$Y = \bar{B}\bar{C}\bar{D} + \bar{B}\bar{C}\bar{D} \Rightarrow \bar{B}\bar{D}(C+\bar{C}) \Rightarrow \bar{B}\bar{D}$

Advantage

Poll

3. The Boolean expression Y = (AB)' is logically equivalent to what single gate?

a) NAND

b) NOR

c) AND

d) OR

Solutions

3. The Boolean expression Y = (AB)' is logically equivalent to what single gate?

a) NAND

b) NOR

c) AND

d) OR

View Answer

Answer: a

Explanation: If A and B are the input for AND gate the output is obtained as AB and after inversion we get (AB)', which is the expression of NAND gate. NAND gate produces high output when any of the input is 0 and produces low output when all inputs are 1.

Map the following standard SOP expression on a Karnaugh map:

$$\bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + AB\bar{C}D + ABCD + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D}$$

Solu

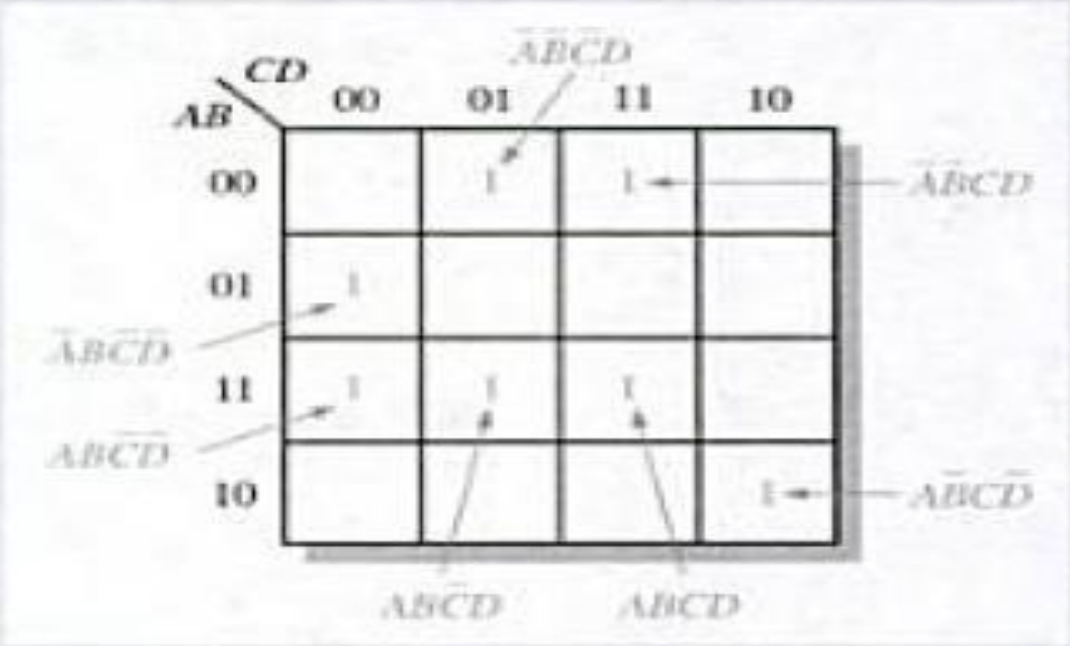
Map the following standard SOP expression on a Karnaugh map:

$$\bar{A}\bar{B}CD + \bar{A}B\bar{C}D + AB\bar{C}D + ABCD + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D}$$

Solution The expression is evaluated as shown below. A 1 is placed on the 4-variable Karnaugh map in Figure 4–25 for each standard product term in the expression.

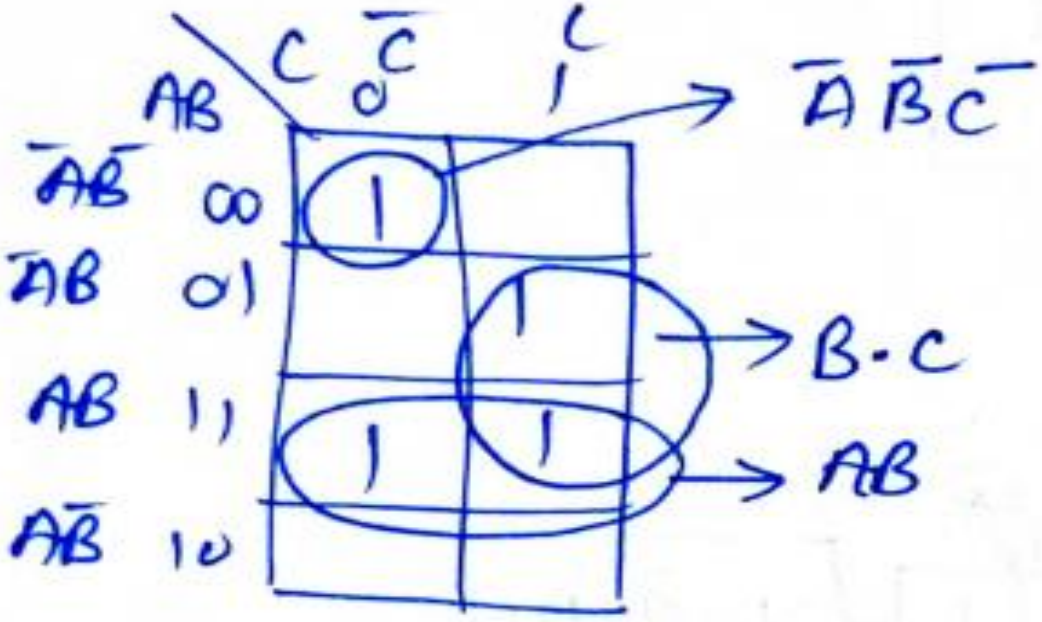
$$\underset{0011}{\bar{A}\bar{B}CD} + \underset{0100}{\bar{A}B\bar{C}D} + \underset{1101}{AB\bar{C}D} + \underset{1111}{ABCD} + \underset{1100}{AB\bar{C}\bar{D}} + \underset{0001}{\bar{A}\bar{B}\bar{C}D} + \underset{1010}{A\bar{B}C\bar{D}}$$

► **FIGURE 4–25**



Q:- Determine the product-terms of Karnaugh's table and the resulting minimum SOP expressions.

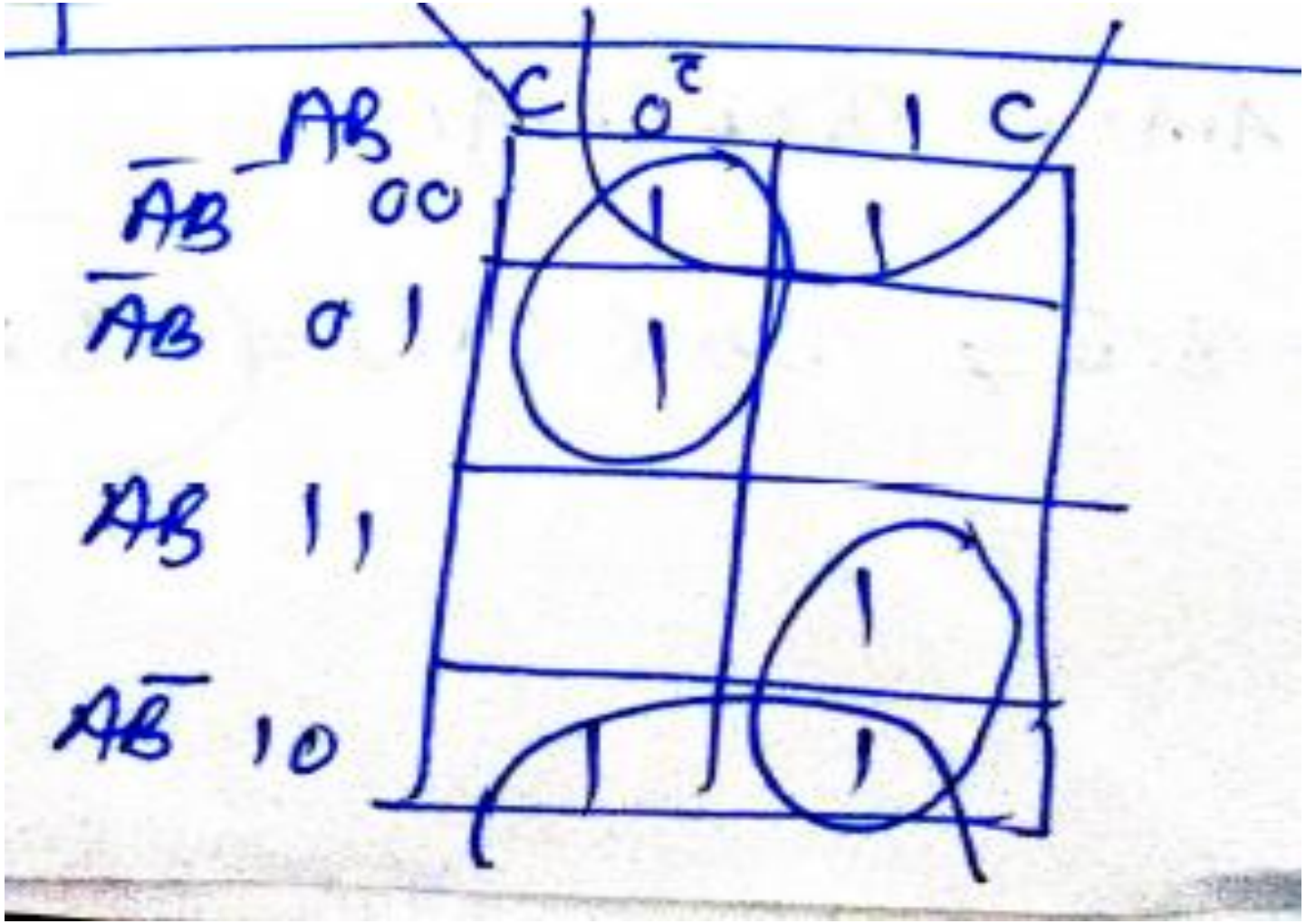
Solution :-



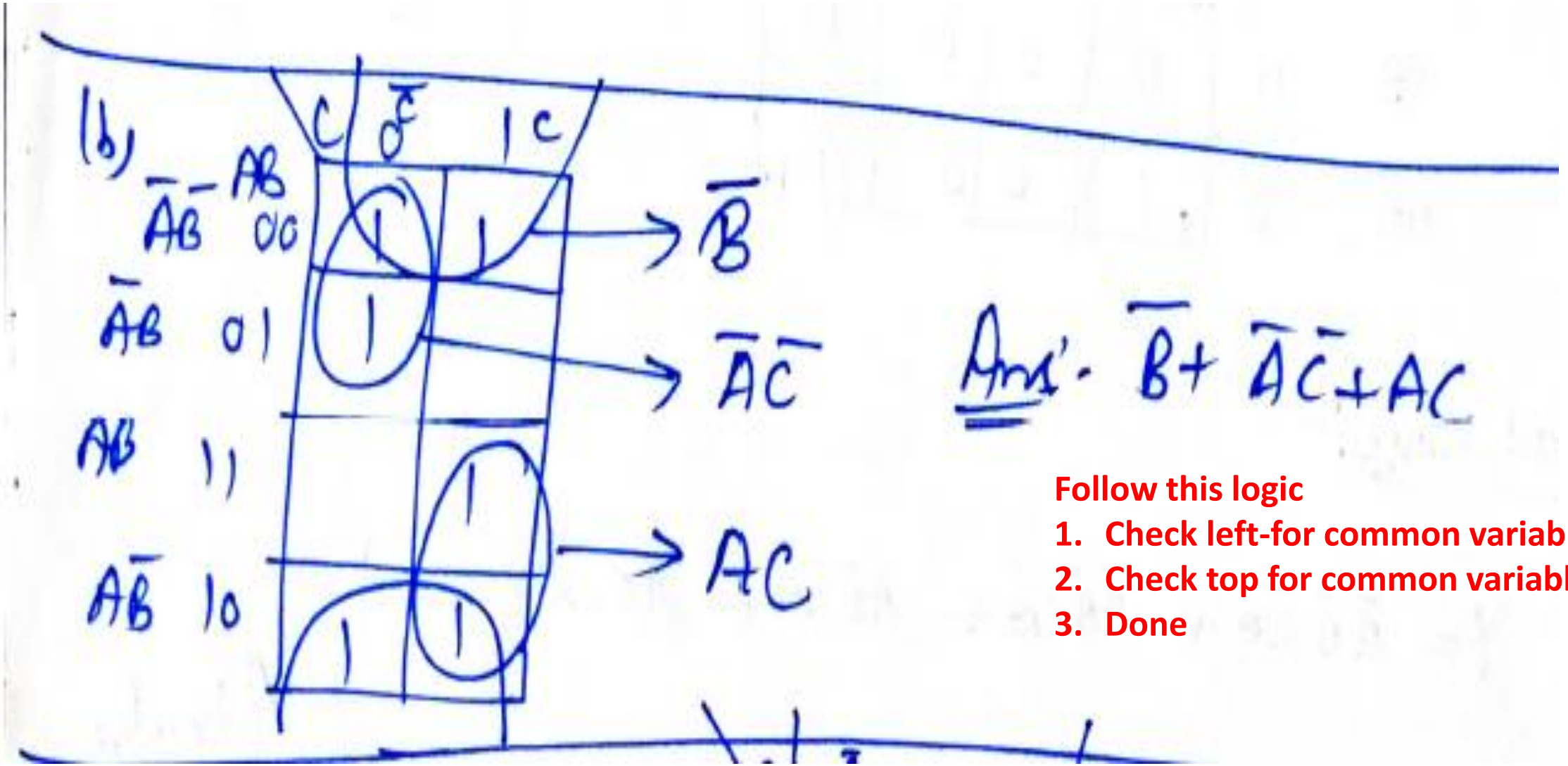
1. **Overlapping pairs are allowed like this**
2. **No logic 1 should be left**
3. **Go from higher (octet) to lower (single)**

Ans:- $AB + BC + \bar{A}\bar{B}\bar{C}$

Question



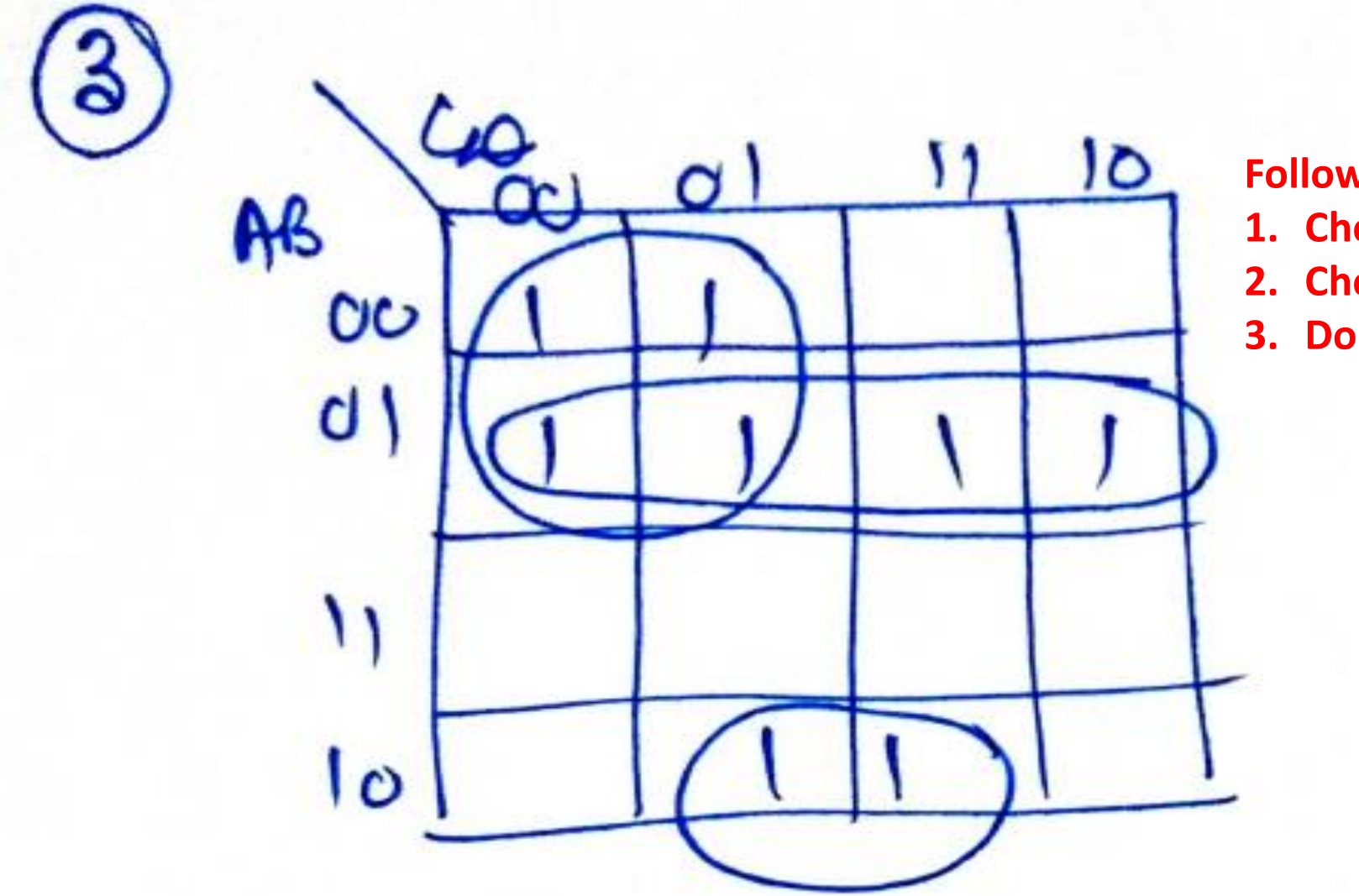
Solution



Follow this logic

1. **Check left-for common variable**
2. **Check top for common variable**
3. **Done**

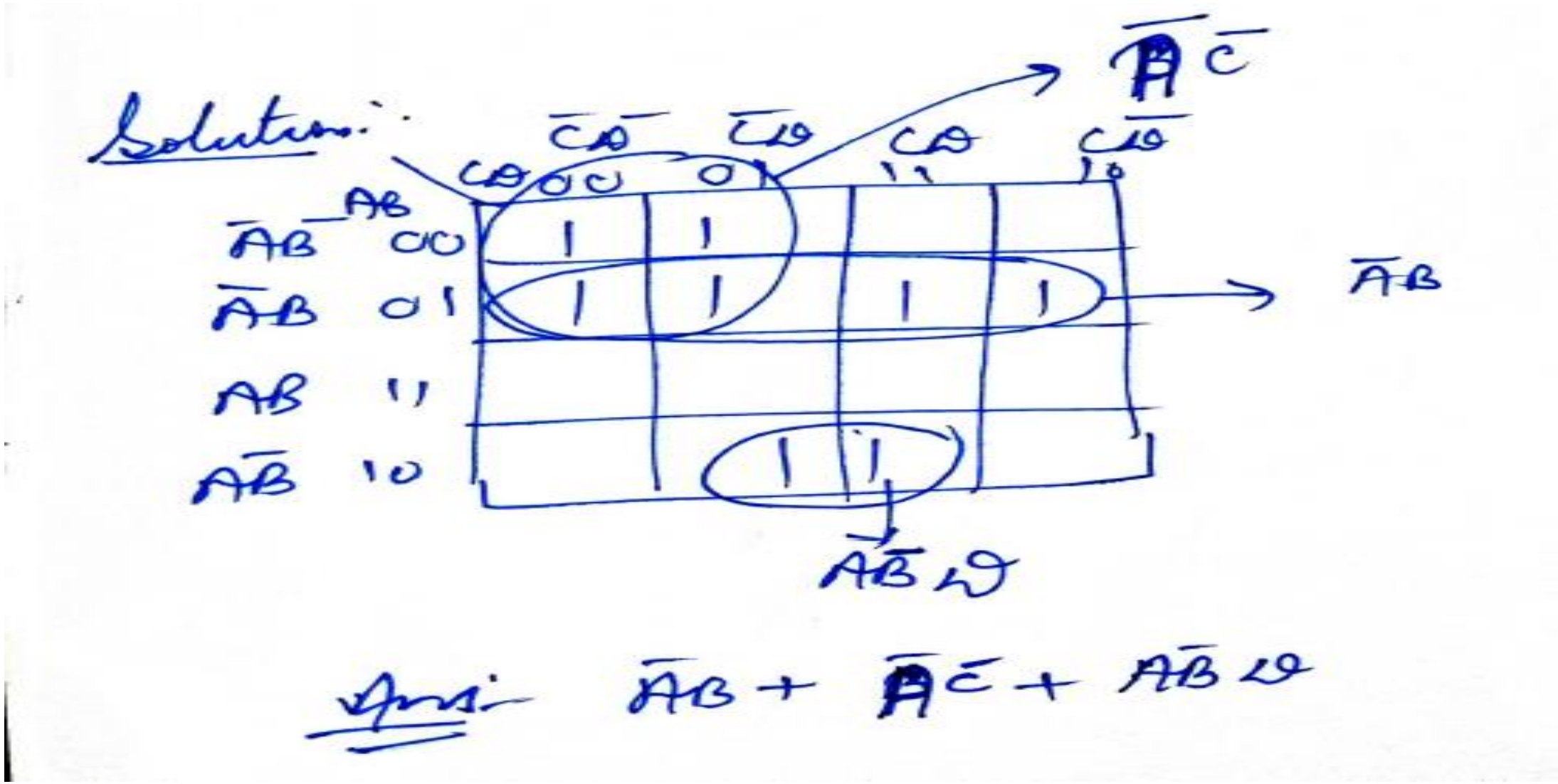
Questions



Follow this logic

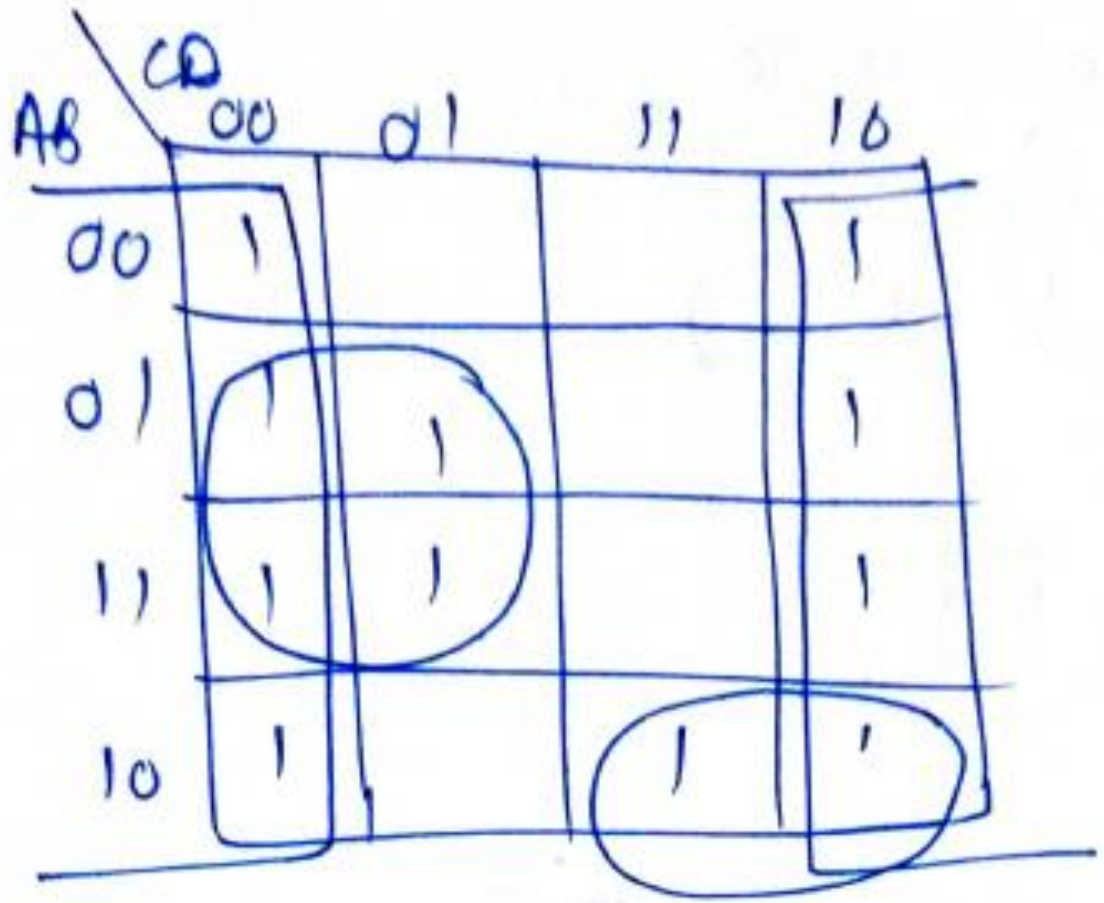
1. **Check left-for common variable**
2. **Check top for common variable**
3. **Done**

Solution

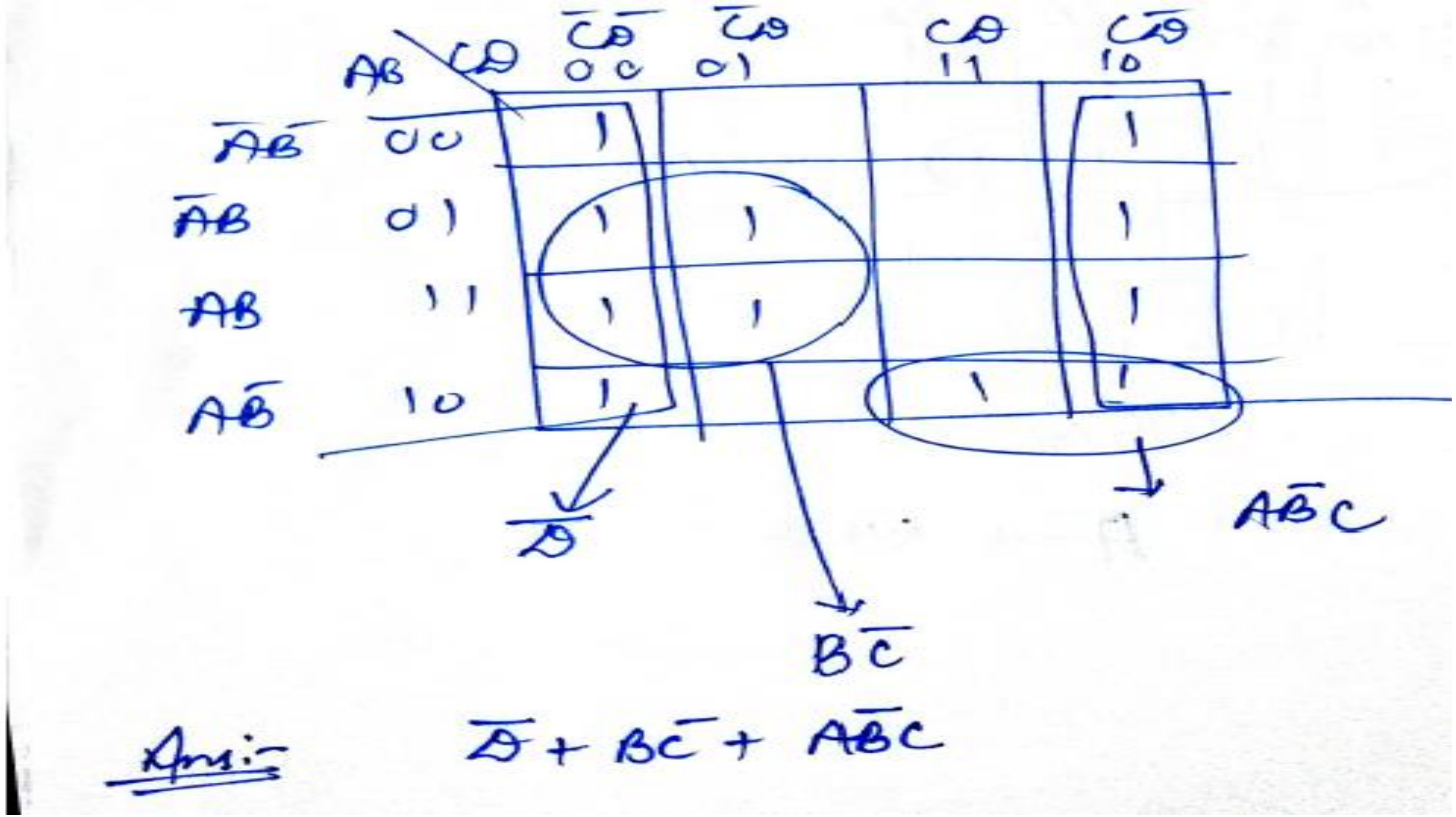

Solution:
$\bar{A}\bar{C}$
$\bar{C}\bar{D}$
$\bar{C}D$
$C\bar{D}$
CD
AB
$\bar{A}\bar{B}$ 00
$\bar{A}B$ 01
AB 11
$A\bar{B}$ 10
$\bar{A}B$
$A\bar{B}D$
Ans $\bar{A}B + \bar{A}\bar{C} + A\bar{B}D$

Questions

Q.



Solution


AB
CD
$\bar{C}\bar{D}$
$\bar{C}D$
CD
$C\bar{D}$
00
01
11
10
$\bar{A}\bar{B}$
$\bar{A}B$
AB
$A\bar{B}$
$\bar{D}$
$B\bar{C}$
$A\bar{B}C$
Ans:-
$\bar{D} + B\bar{C} + A\bar{B}C$

Use a Karnaugh map to minimize the following standard SOP expression:

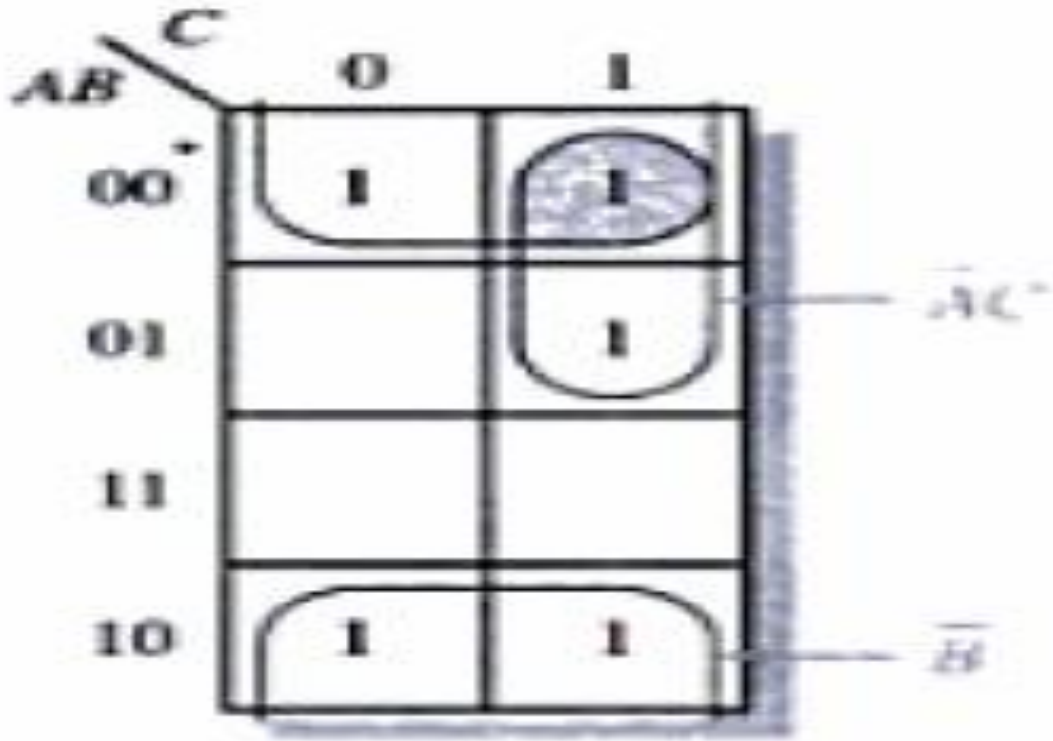
$$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

Solu

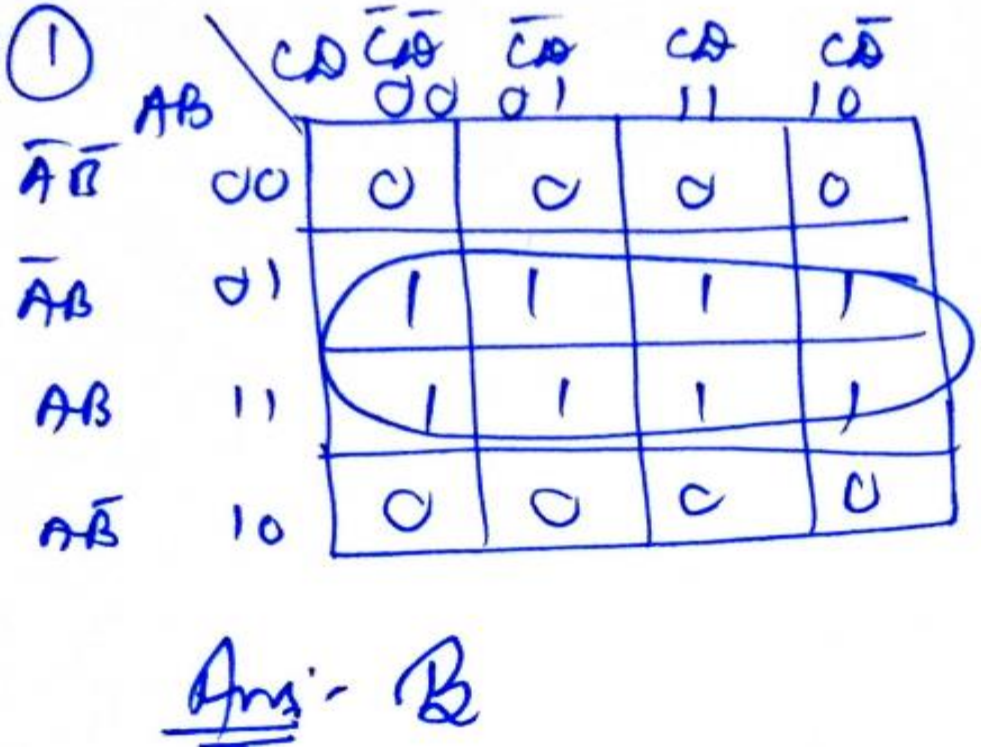
The binary values of the expression are

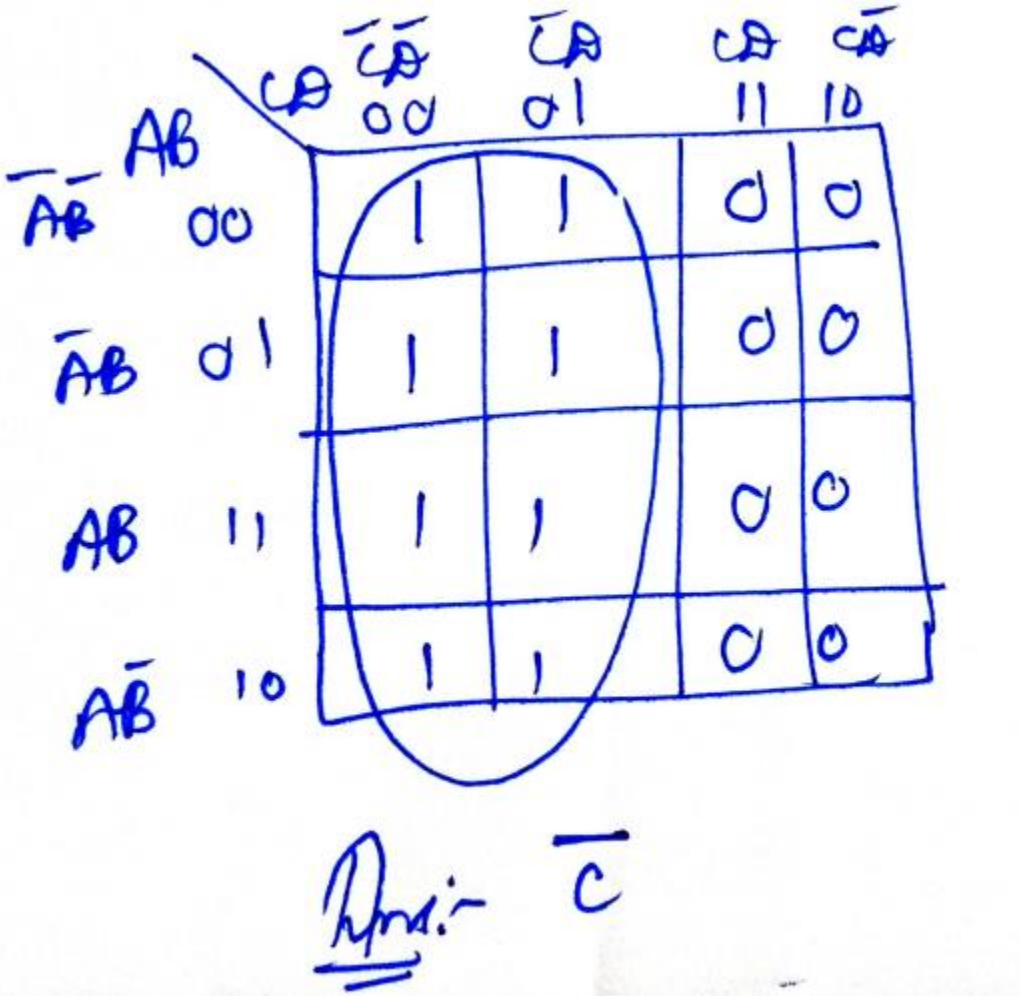
001

101 + 011 + 011 + 000 + 100



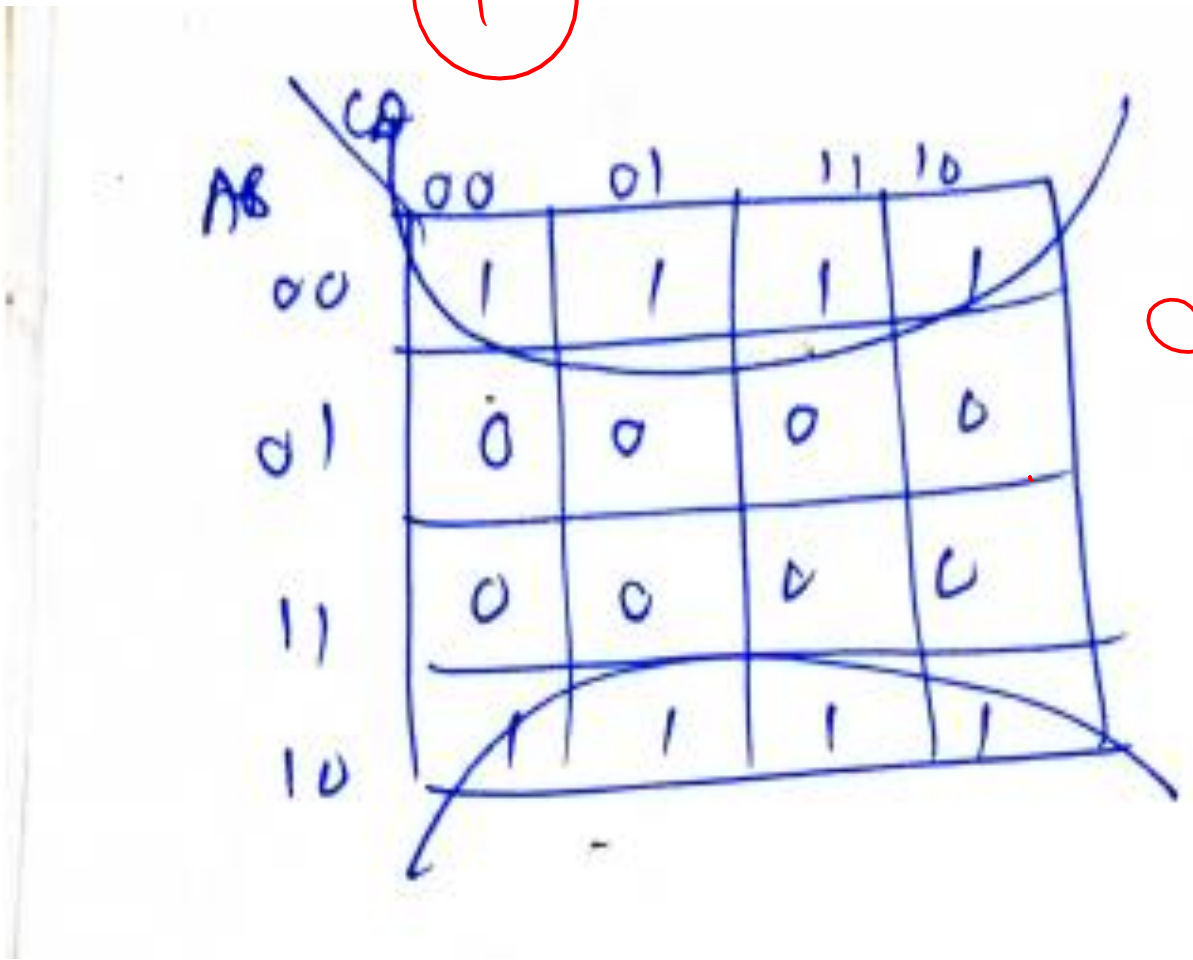
Octet Conditions





Questions

1



choose the correct option

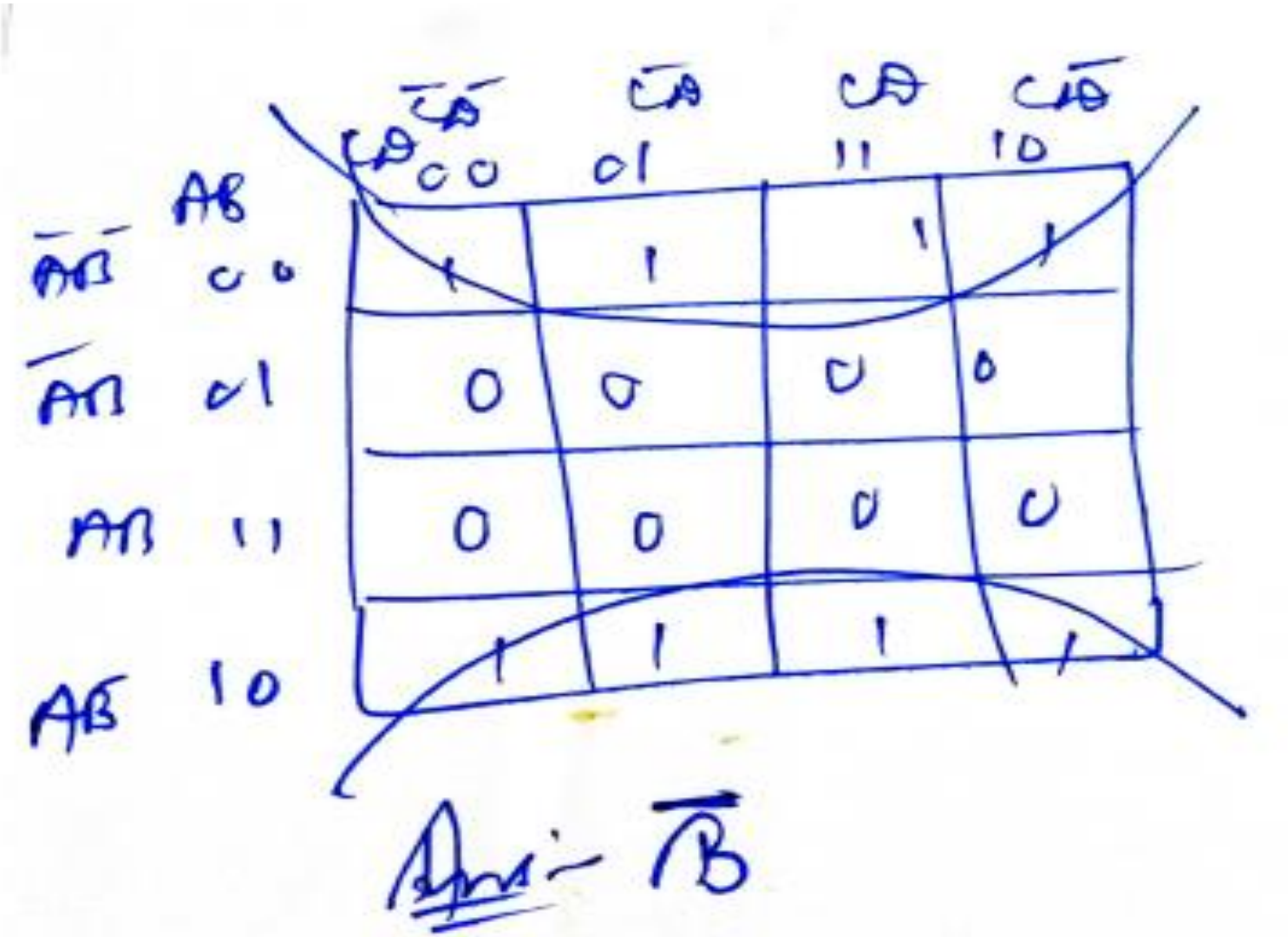
(1) $\bar{A}$

(2) $\bar{B}$

(3) $\bar{C}$

(4) None

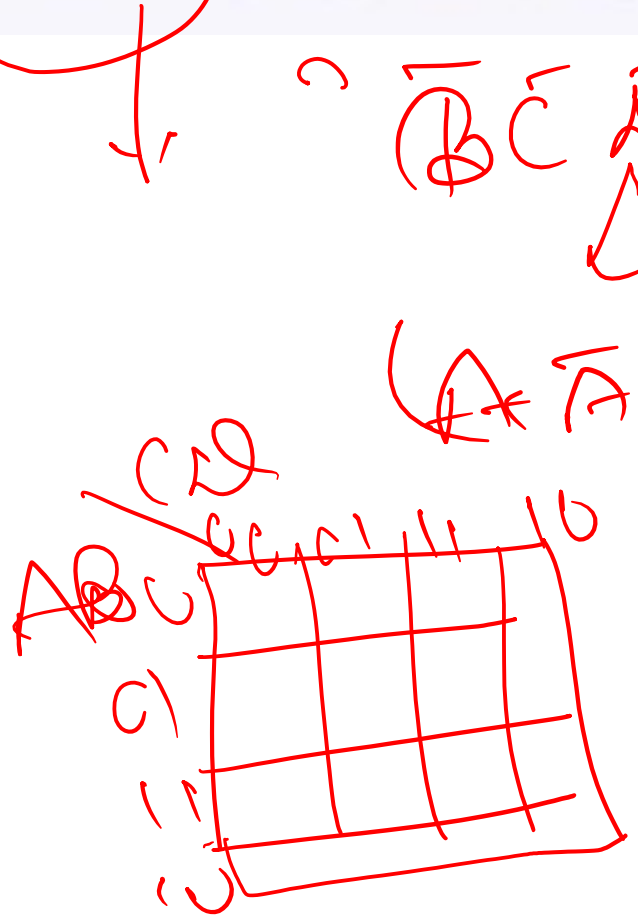
Solu


$\bar{C}\bar{D}$ $\bar{C}D$ CD $C\bar{D}$
AB 00 01 11 10
$\bar{A}\bar{B}$ 00
$\bar{A}B$ 01
AB 11
$A\bar{B}$ 10
1 1 1 1
0 0 0 0
0 0 0 0
1 1 1 1
Ans- $\bar{B}$

Ques

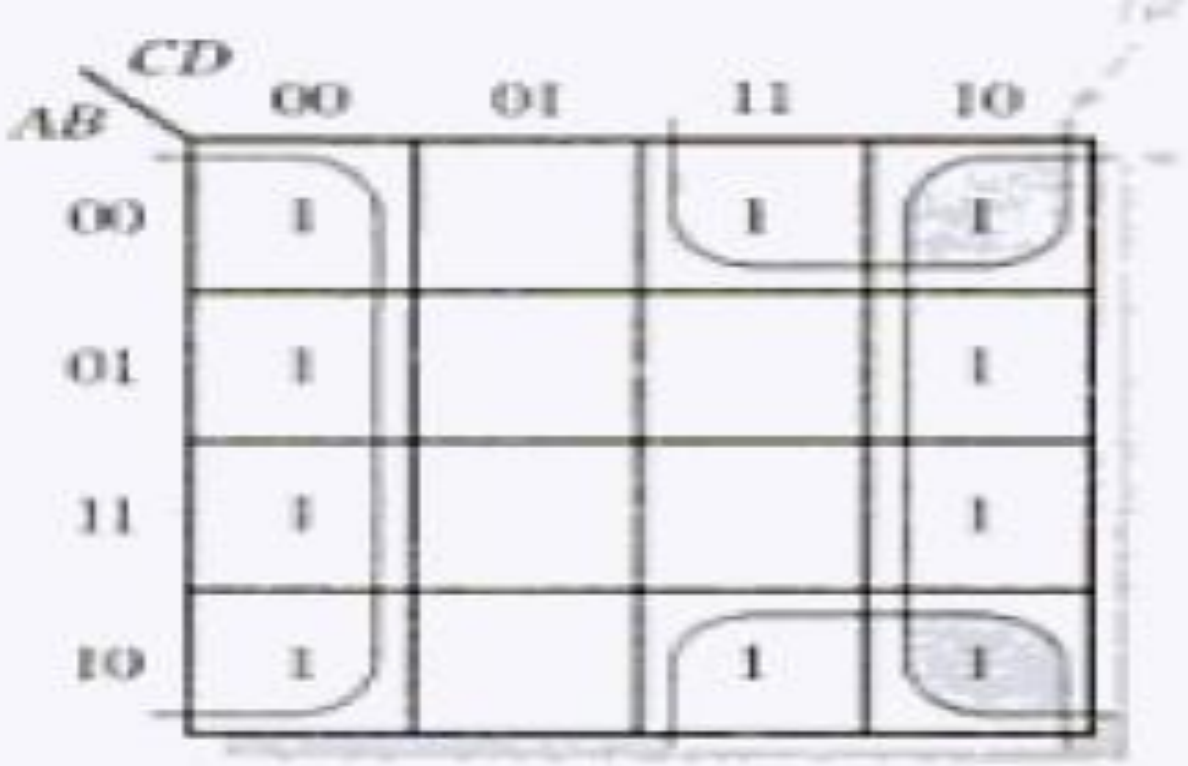
Use a Karnaugh map to minimize the following SOP expression:

$$\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + A\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + A\bar{B}C\bar{D}$$



Solutions

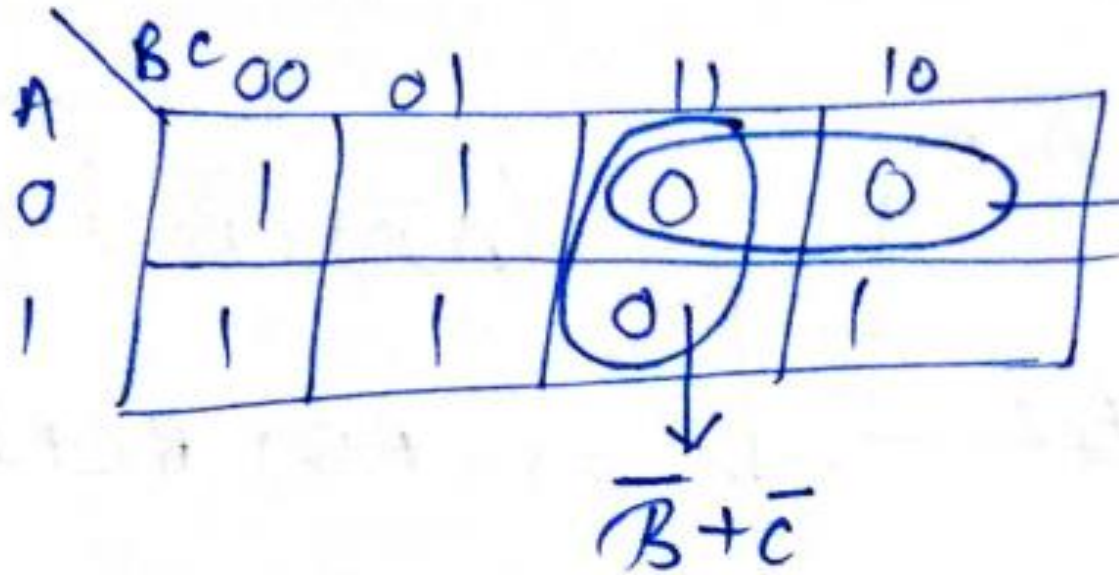
The first term $\overline{B}\overline{C}\overline{D}$ must be expanded into $A\overline{B}\overline{C}\overline{D}$ and $\overline{A}\overline{B}\overline{C}\overline{D}$ to get the standard SOP expression, which is then mapped; and the cells are grouped as shown in Figure 4–33.



$$\overline{D} + \overline{B}C$$

Simplification of Standard POS form using Kmap

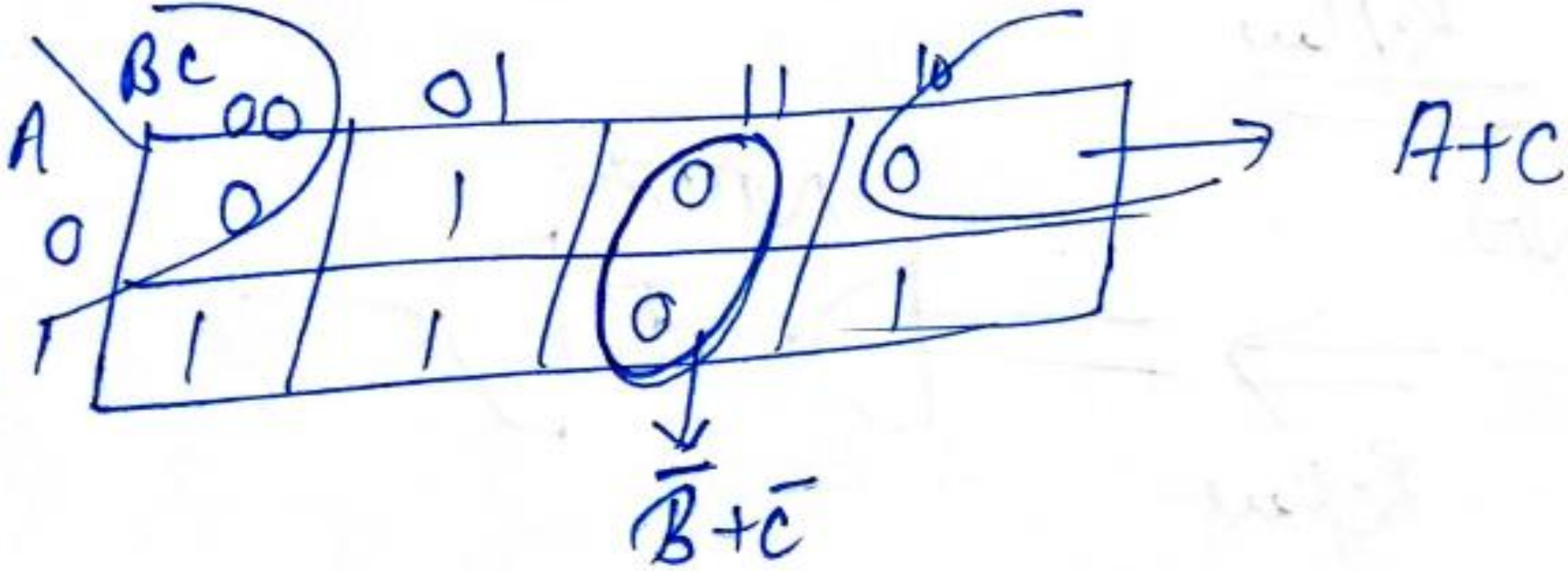
(1) Find the expression in POS form for Kmap given



POS
$0 \rightarrow A$
$1 \rightarrow \bar{A}$

Ans- $Y = (A+\bar{B})(\bar{B}+\bar{C})$

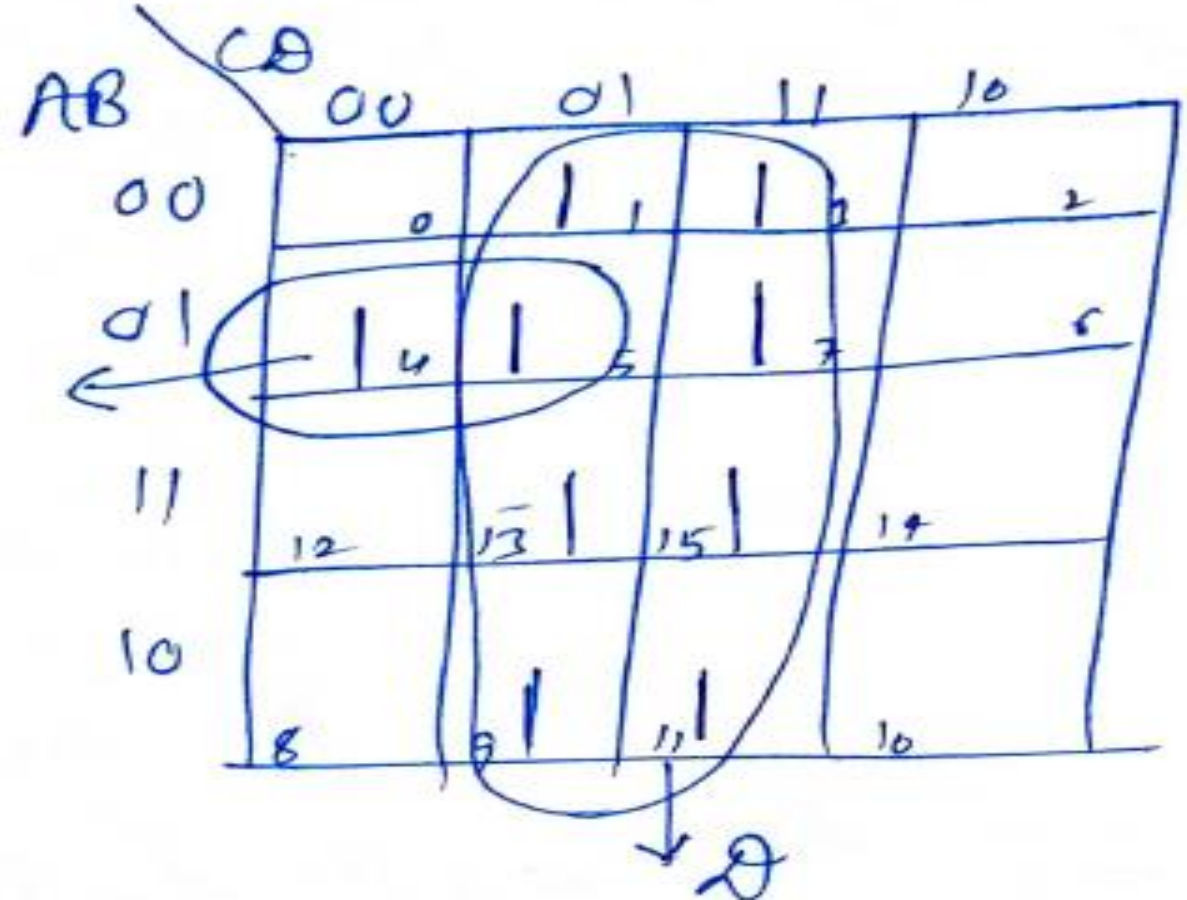
(2) $Y = \Pi M (0,2,3,7)$



$$Y = (A+C)(\bar{B}+\bar{C})$$

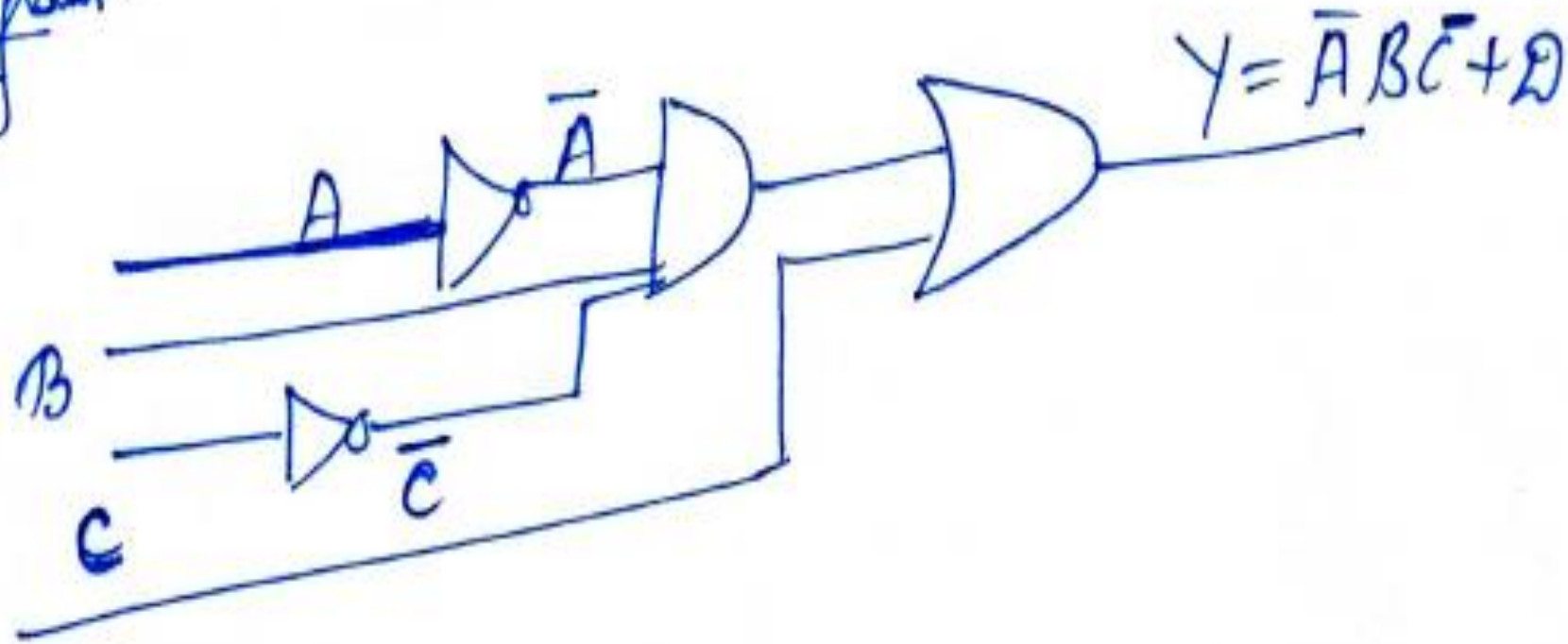
Using Karnaugh map solve the following expression using gates.

$Y(A,B,C,D) = \Sigma m(1,3,4,5,7,9,11,13,15)$



$Y = \bar{A}B\bar{C} + D$

Diagram :-



Poll

10. Which of the following expressions is in the sum-of-products form?

a) (A + B)(C + D)

b) (A * B)(C * D)

c) A* B *(CD)

d) A * B + C * D

Solutions

10. Which of the following expressions is in the sum-of-products form?

a) (A + B)(C + D)

b) (A * B)(C * D)

c) A* B *(CD)

d) A * B + C * D

View Answer

Answer: d

Explanation: Sum of product means that it is the sum of all product terms. Thus, the number is multiplied first and then it is added: A * B + C * D.

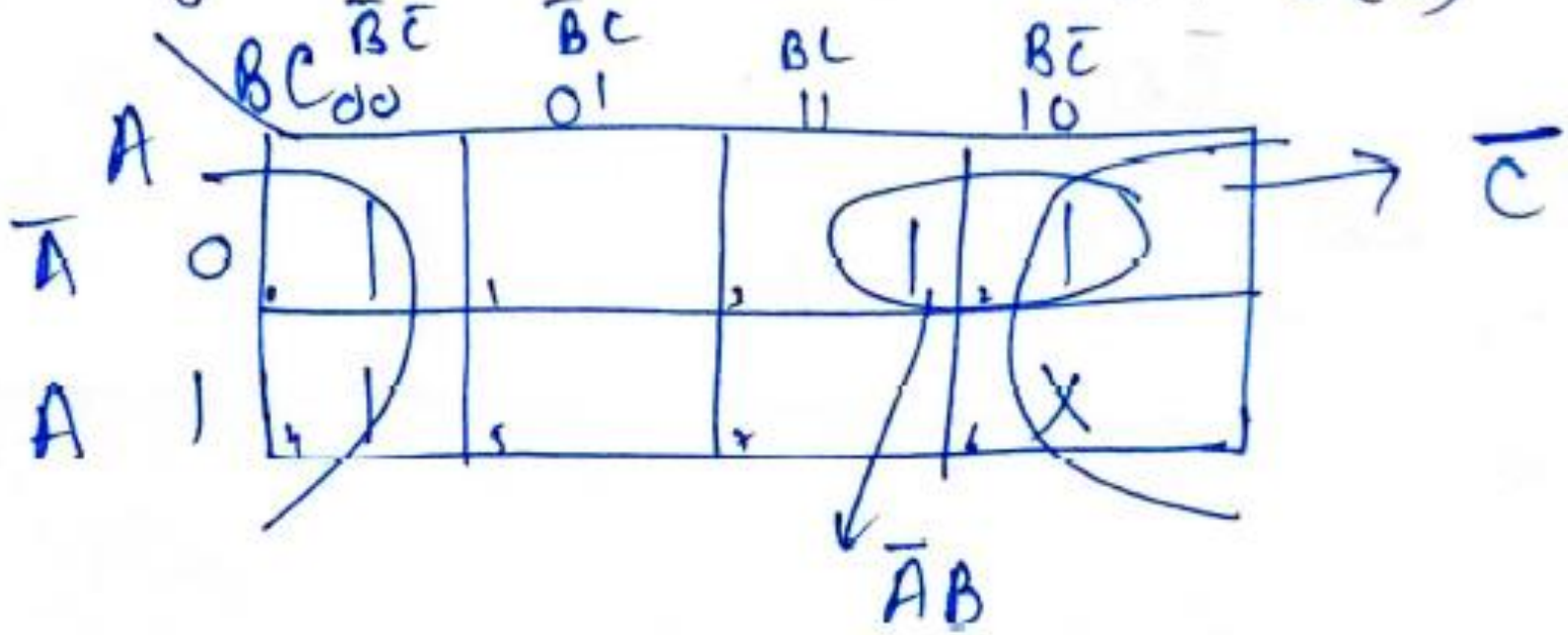
Do not care conditions

(1) These are referred as don't care condition because in some cases the output for certain I/P combinations has no effect on overall output.

(2) 'X' are to be adjusted in such a way that the function output will be minimized expressions.

Q:- Minimize the following function on Kmap

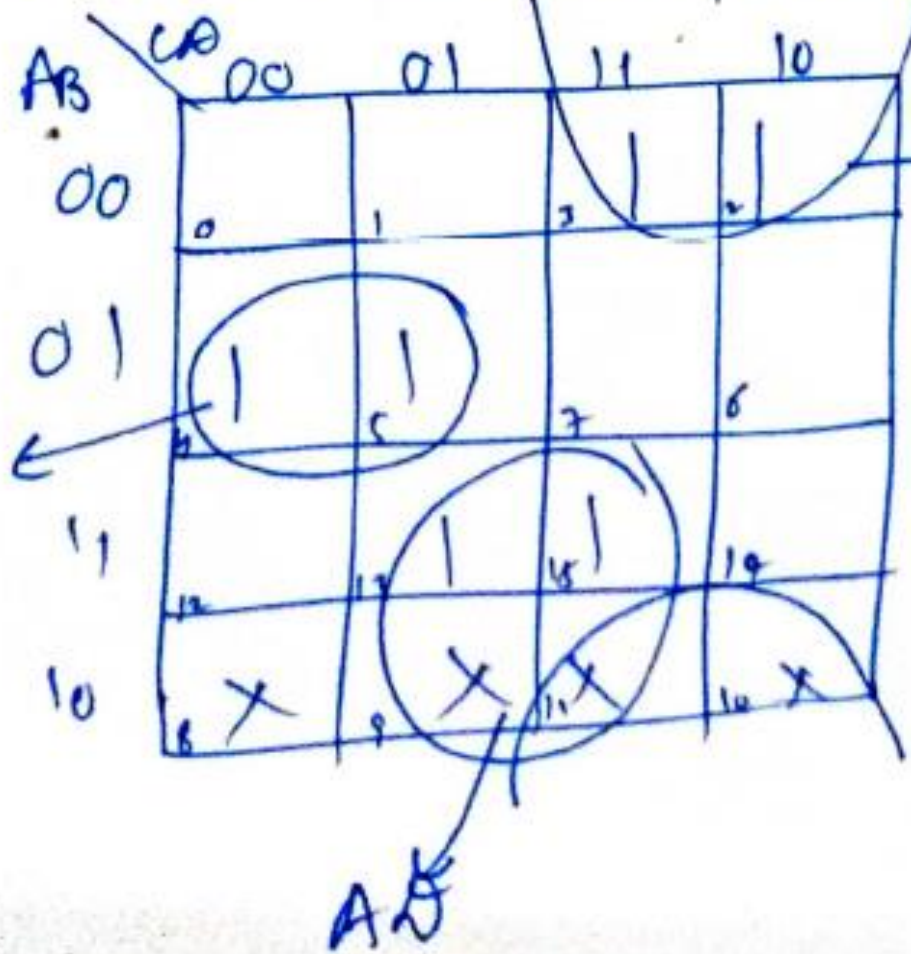
$$Y(A,B,C) = \sum m(0,2,3,4) + d(6)$$



$$Y = \bar{A}B + \bar{C}$$

Q. Minimize the following expression on Kmap ?

$Y(A,B,C,D) = \sum m(2,3,4,5,13,15) + \sum d(8,9,10,11)$

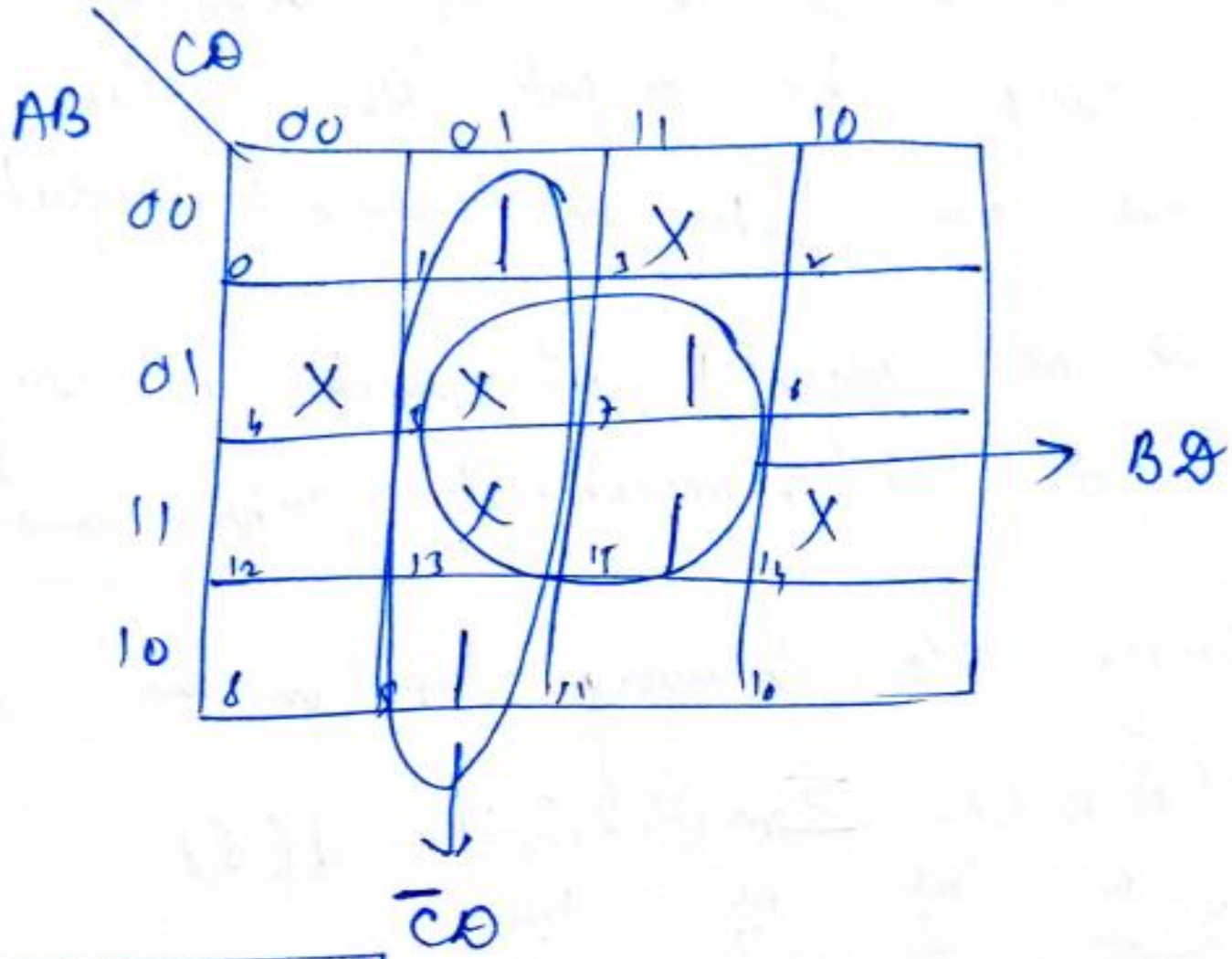


$Y = AD + \bar{B}C + \bar{A}B\bar{C}$

(2)

(3)

$Y(A,B,C,D) = \sum m(1,7,9,15) + \sum d(3,4,5,13,14)$



$$Y = \bar{C}D + BD$$

Poll

12. Which of the following expressions is in the product-of-sums form?

a) (A + B)(C + D)

b) (AB)(CD)

c) AB(CD)

d) AB + CD

Solutions

12. Which of the following expressions is in the product-of-sums form?

a) (A + B)(C + D)

b) (AB)(CD)

c) AB(CD)

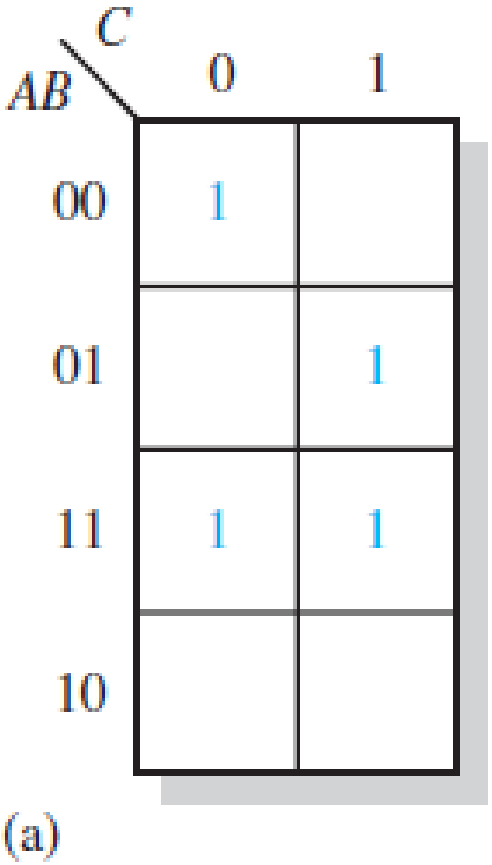
d) AB + CD

View Answer

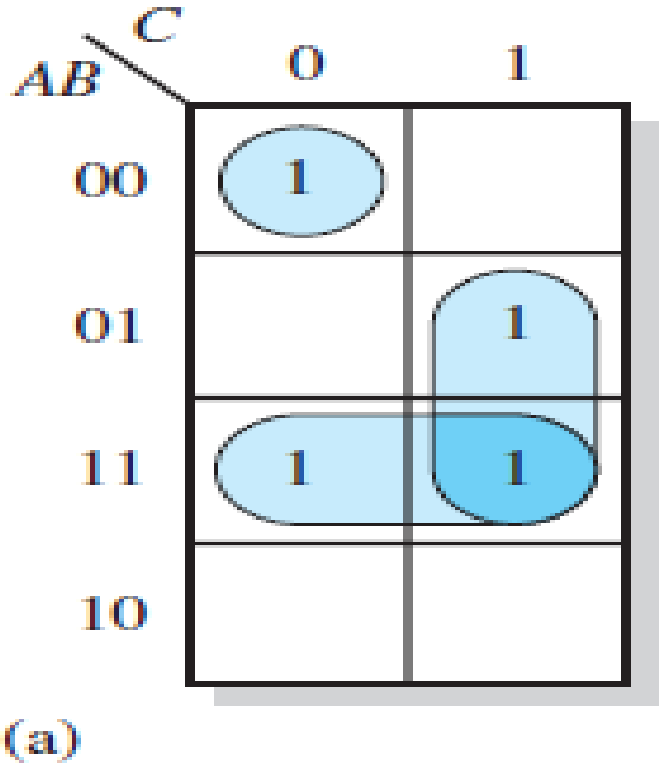
Answer: a

Explanation: (A + B)(C + D) represents the product-of-sums form.

Group the 1s in each of the Karnaugh maps in Figure 4–33.



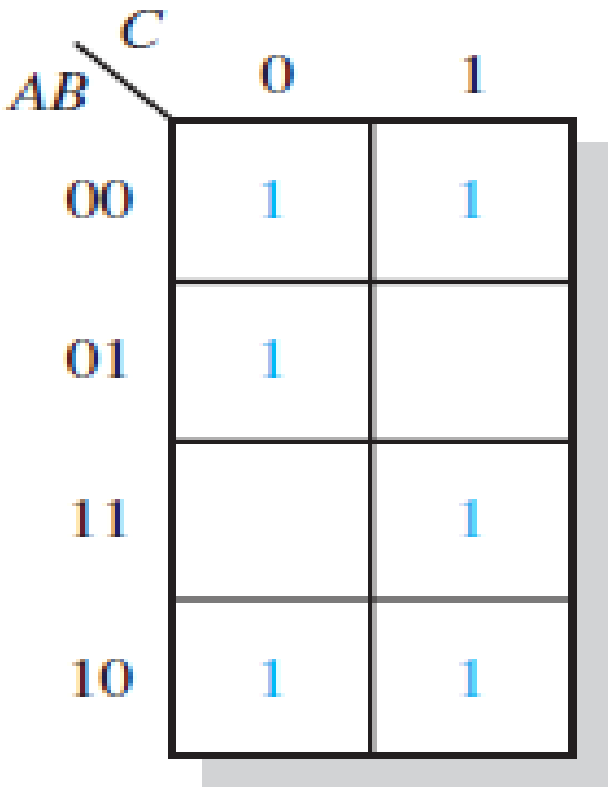
(a)



(a)

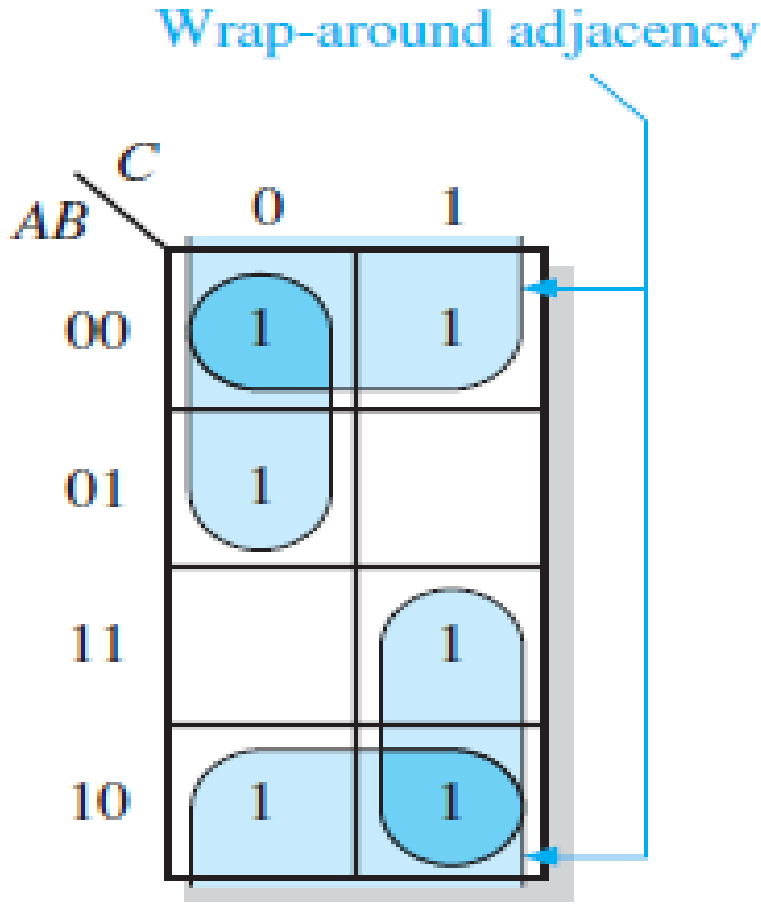
FIGURE 4–34

Group the 1s in each of the Karnaugh maps in Figure 4–33.



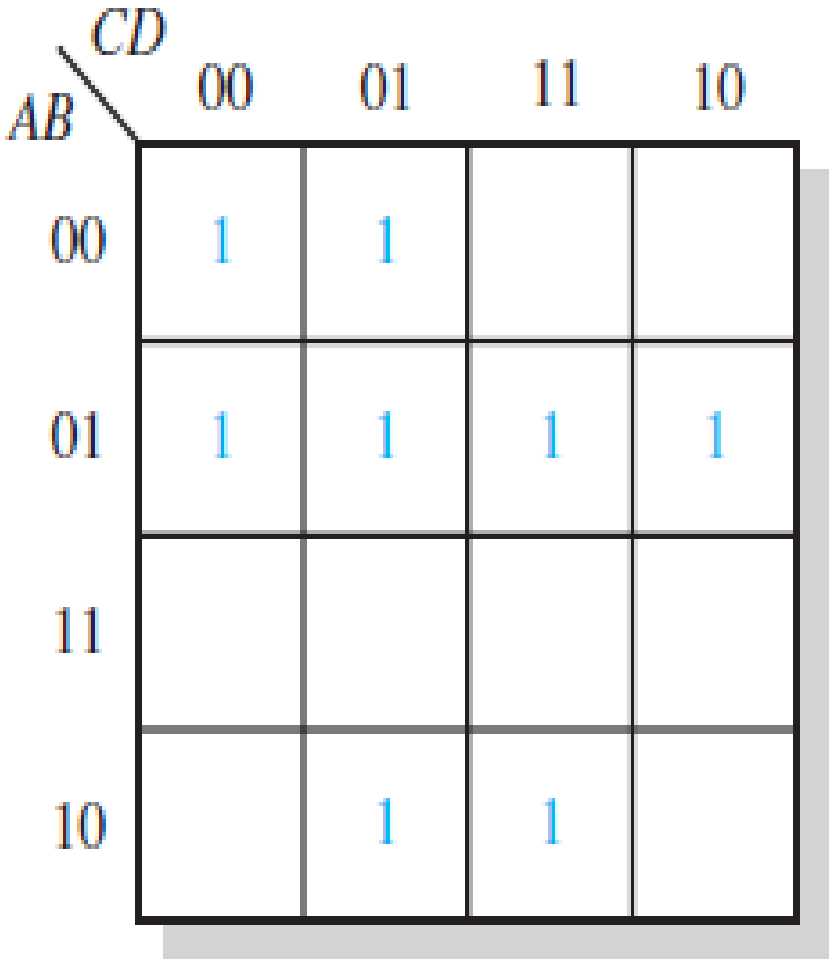
(b)

Group the 1s in each of the Karnaugh maps in Figure 4–33.

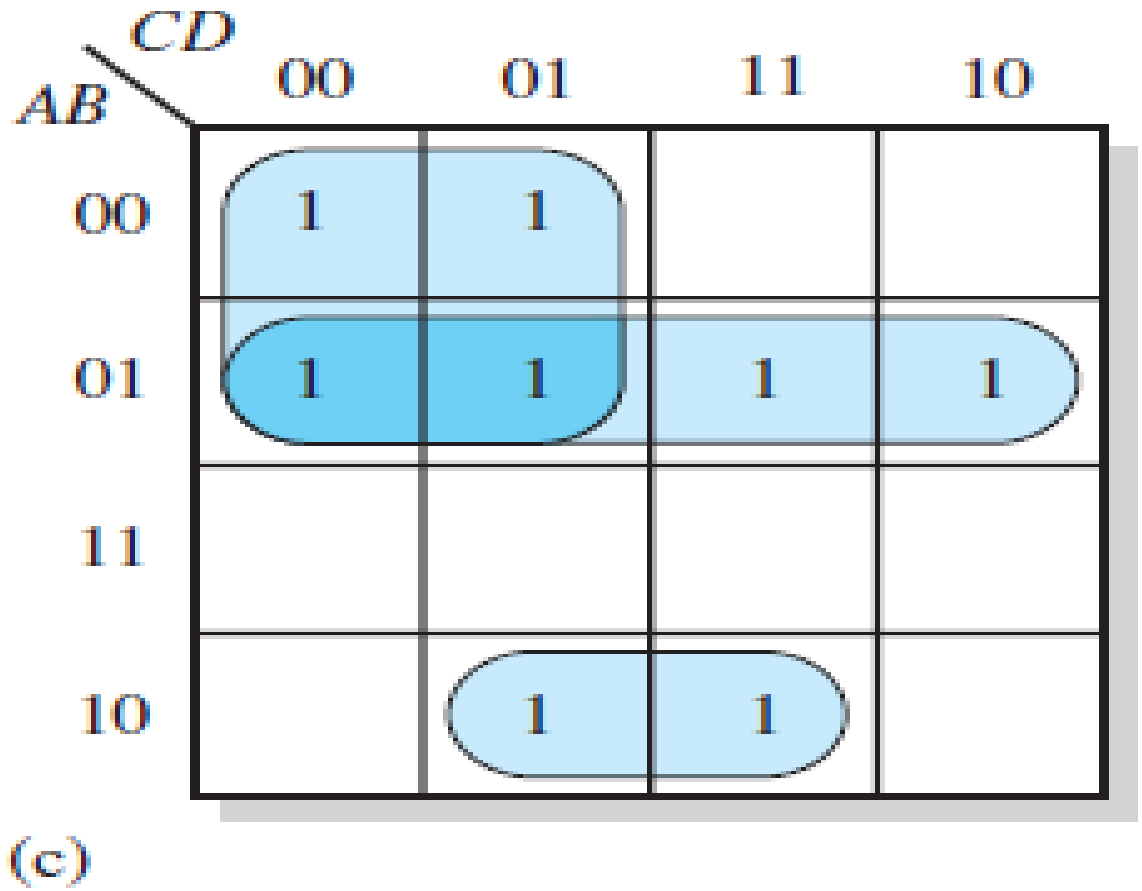


(b)

Group the 1s in each of the Karnaugh maps in Figure 4–33.

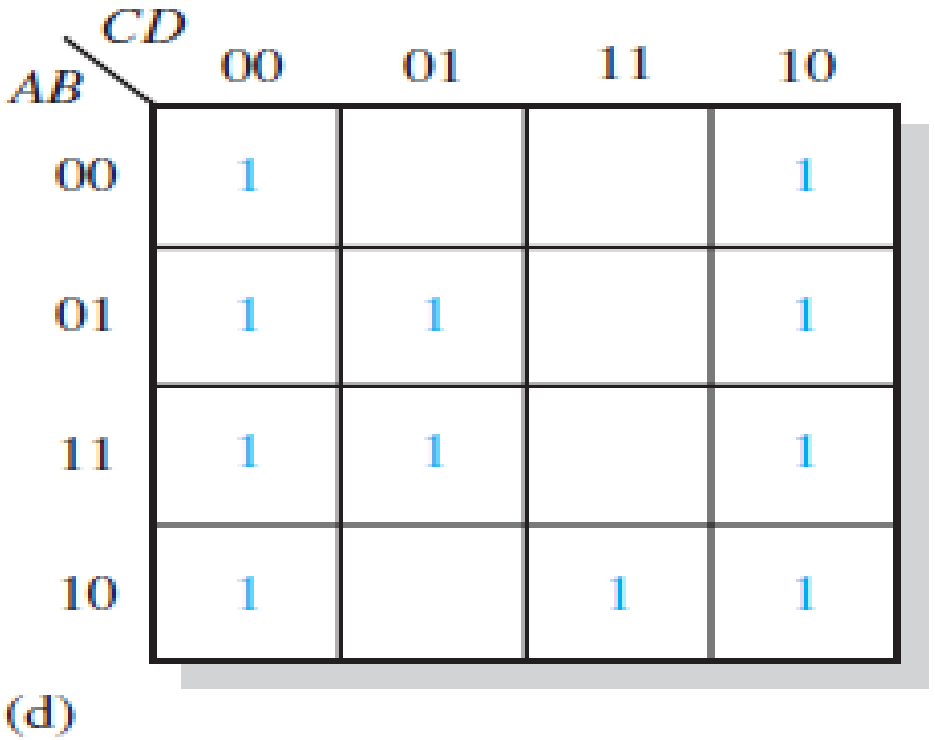


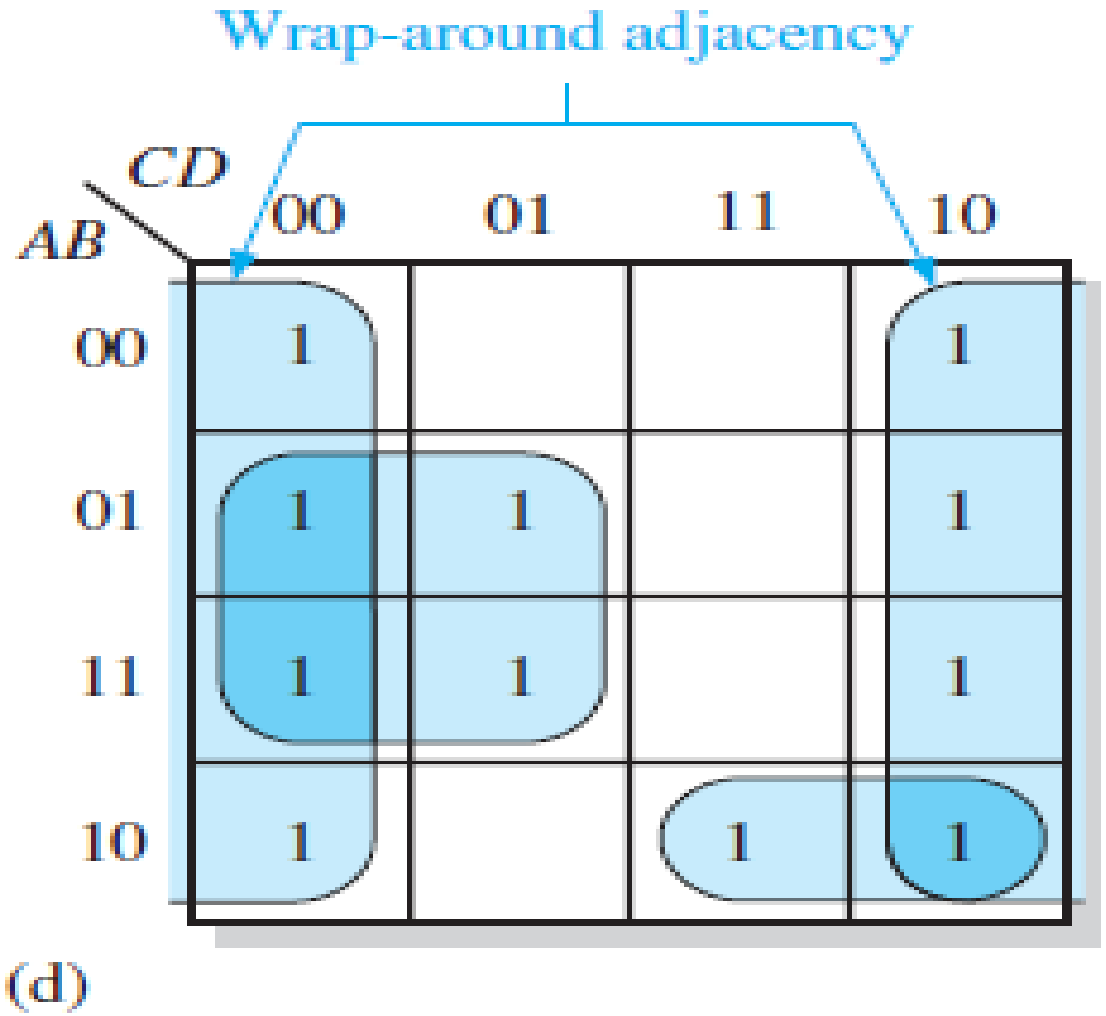
(c)



(c)

Group the 1s in each of the Karnaugh maps in Figure 4–33.




Wrap-around adjacency
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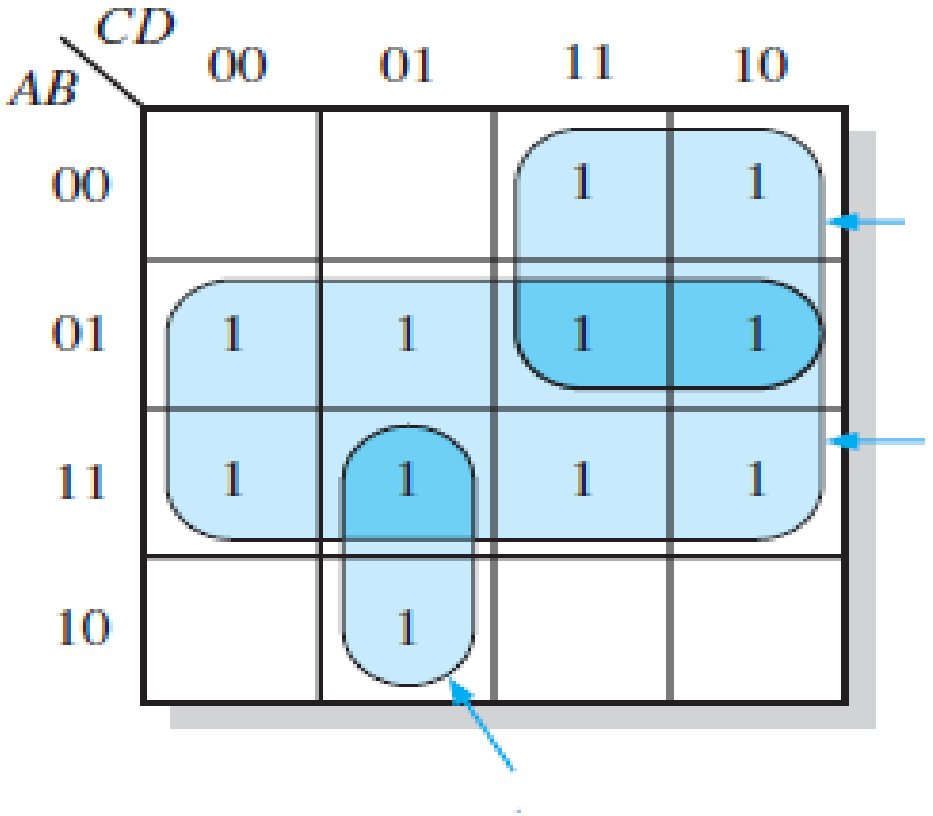
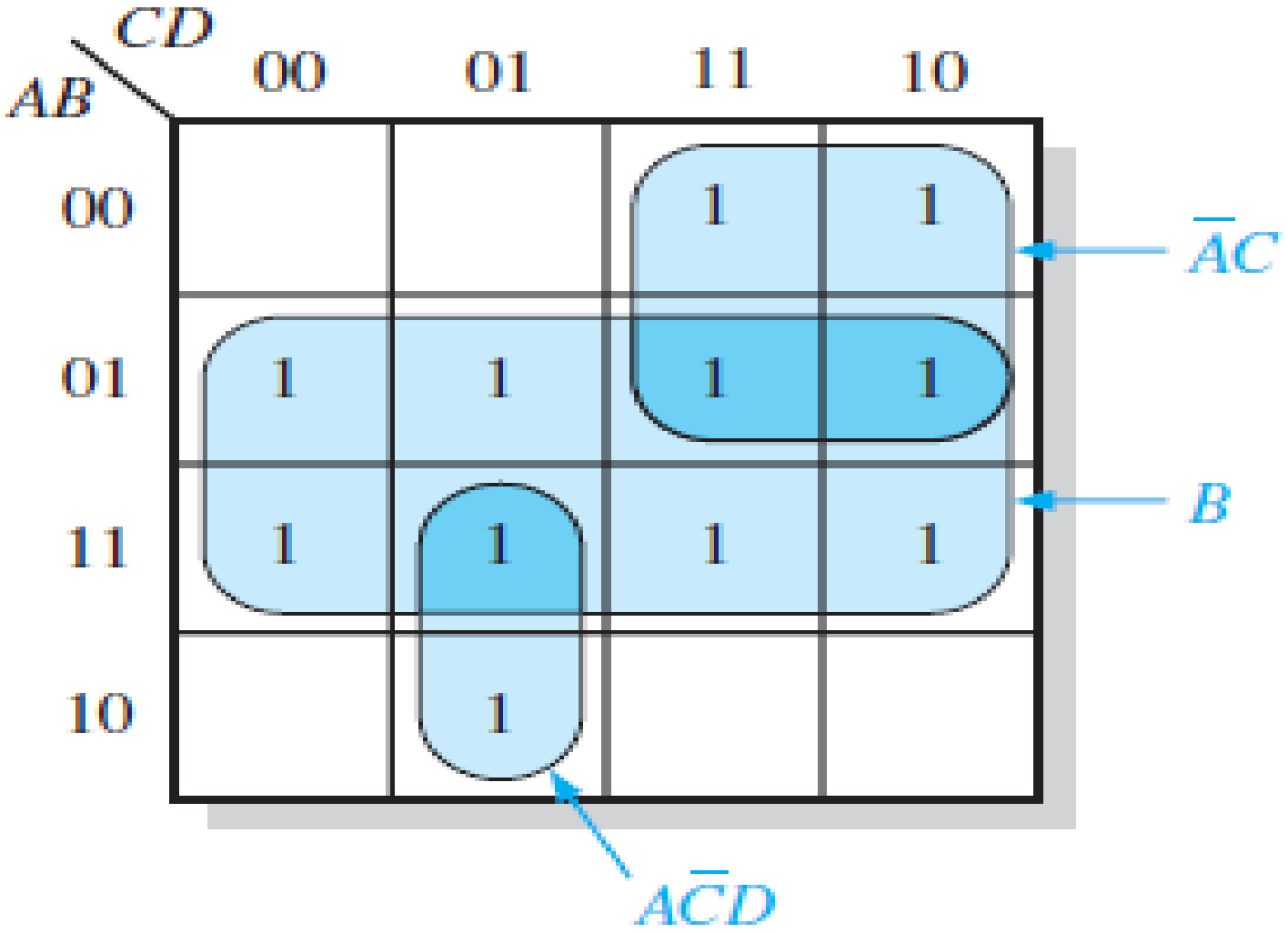
(d)

Determine the product terms for the Karnaugh map in Figure 4–35 and write the resulting minimum SOP expression.



Solutions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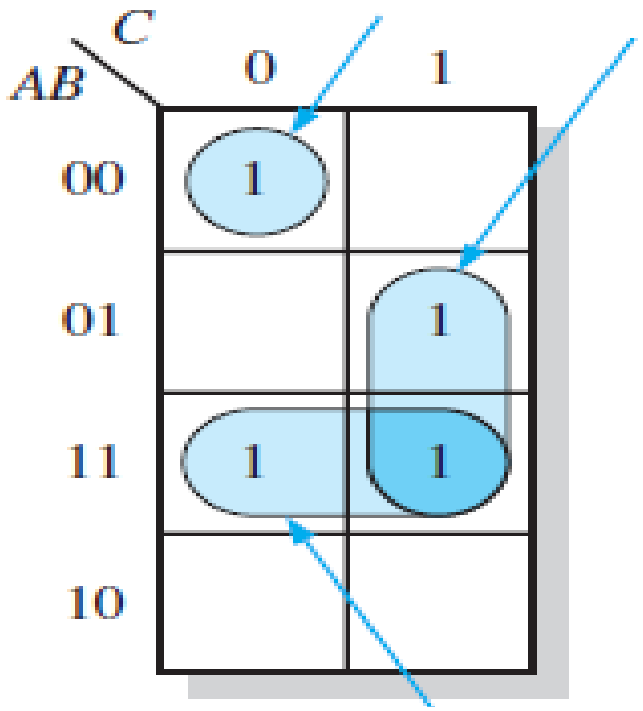
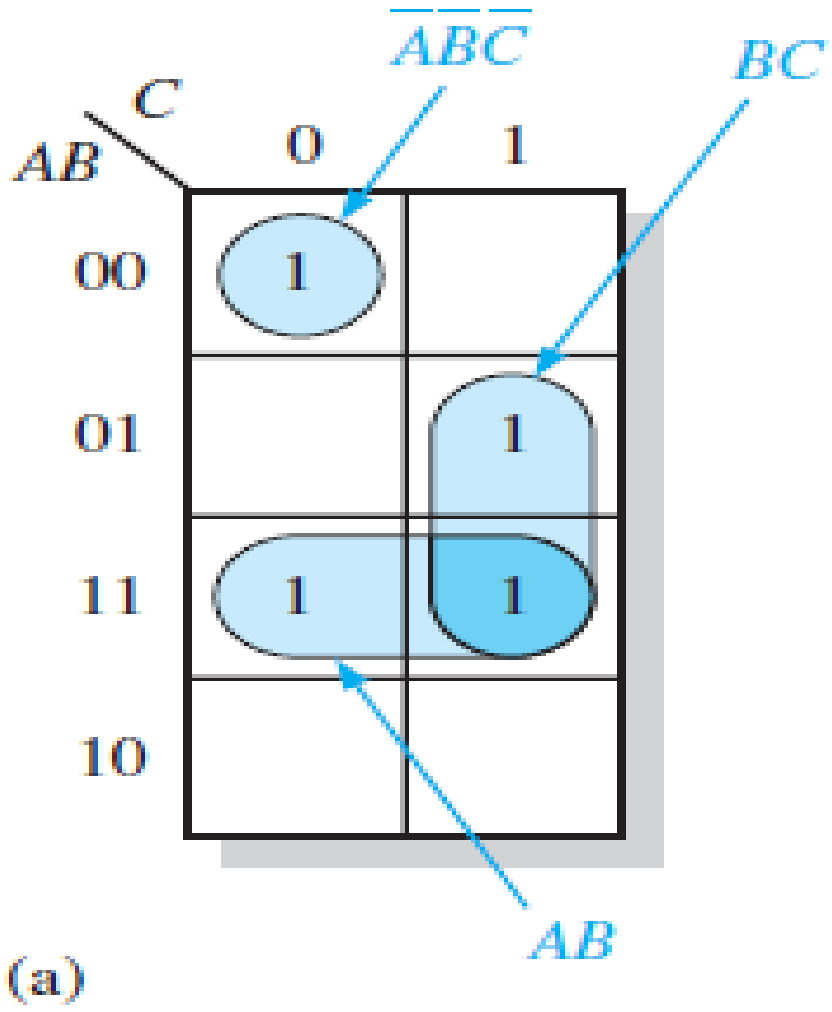
1
$\overline{A}C$
B
$A\overline{C}D$

Determine the product terms for each of the Karnaugh maps in Figure 4–36 and write the resulting minimum SOP expression.



(a)

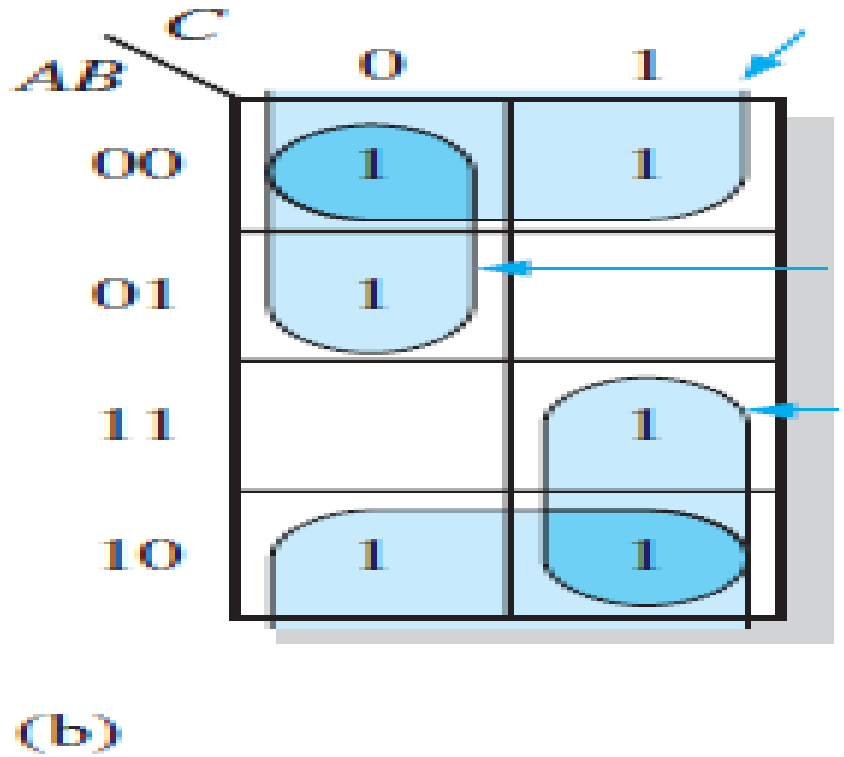
FIGURE 4–36



(a)

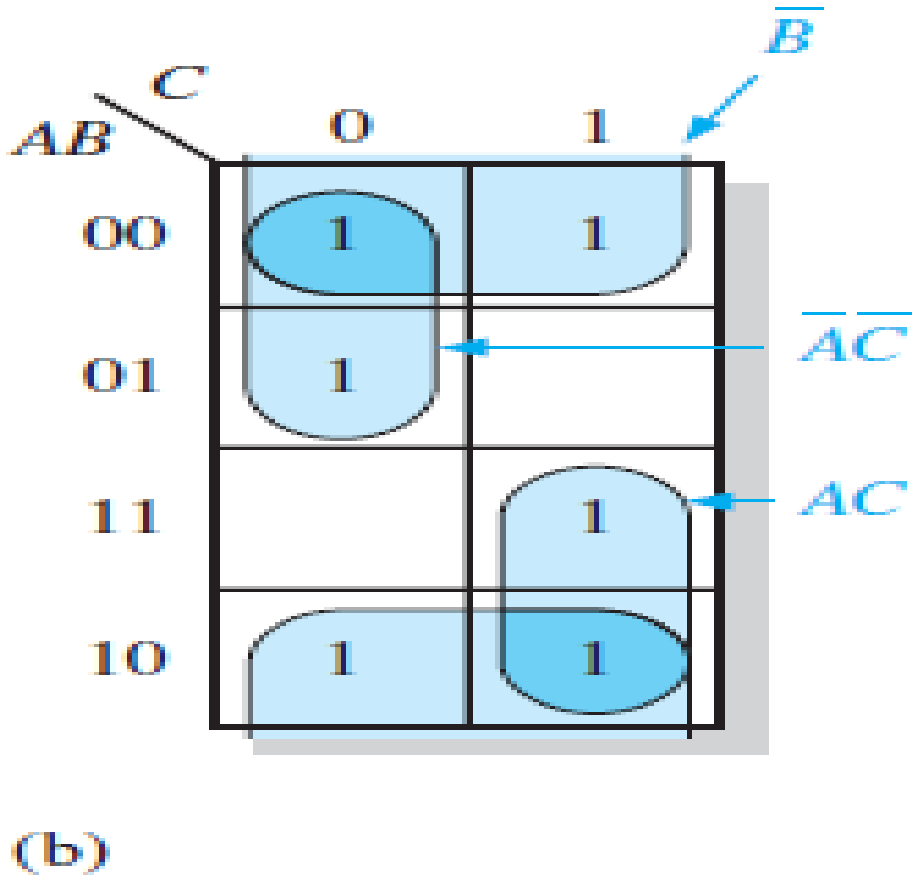
FIGURE 4–36

Questions



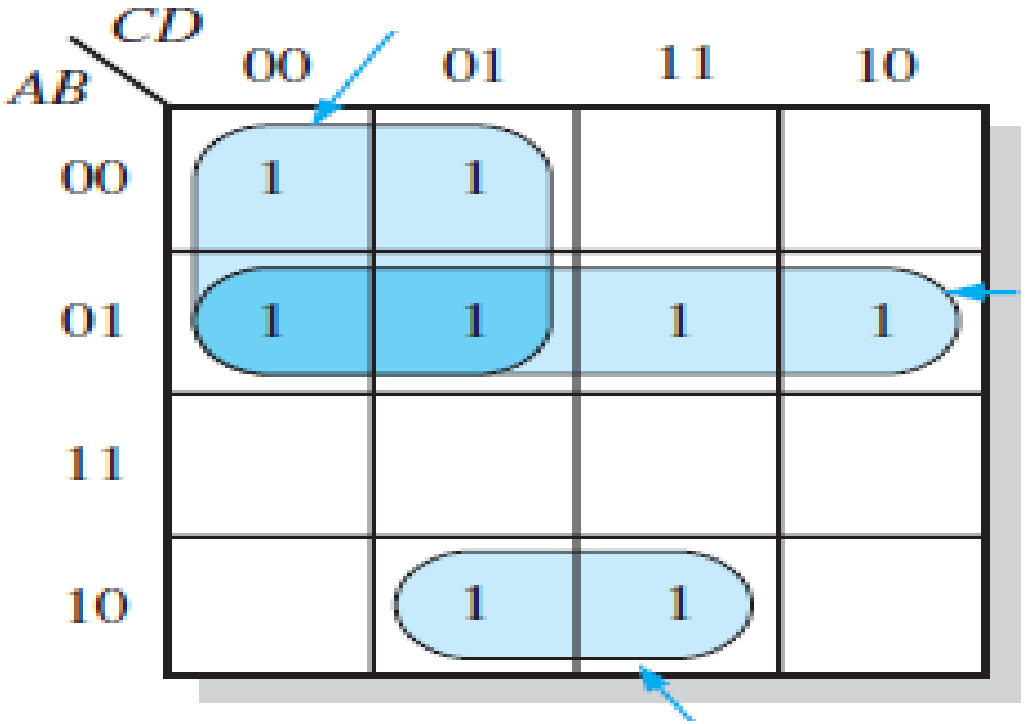
(b)

Solutions

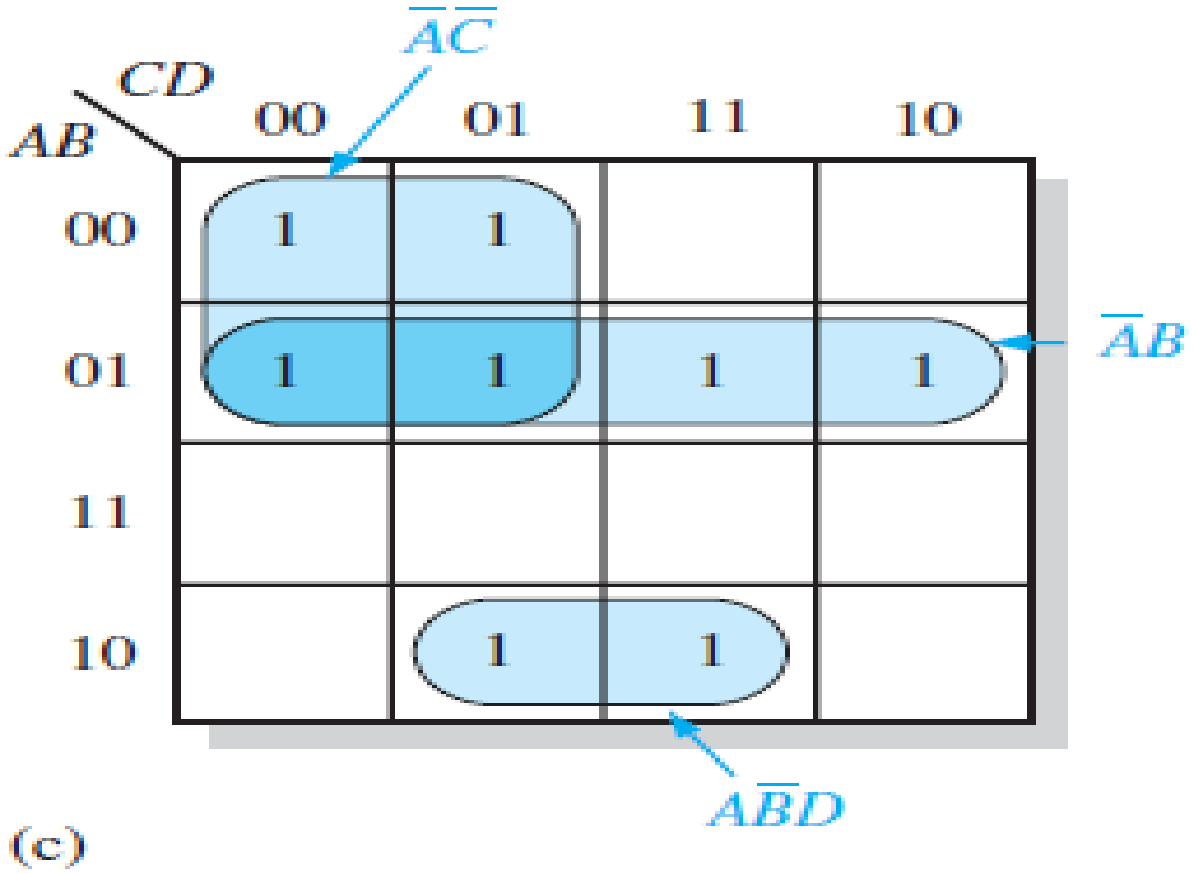


(b)

Questions



(c)


$\overline{A}\,\overline{C}$
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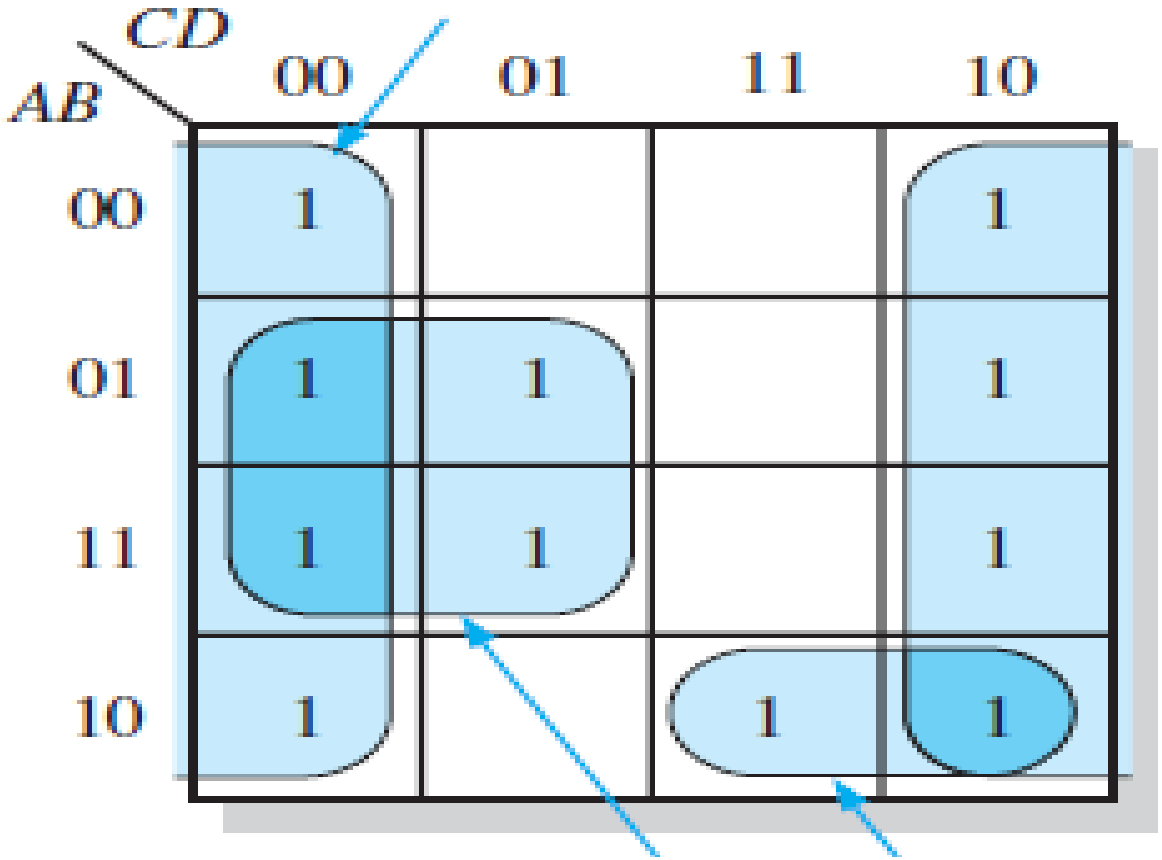
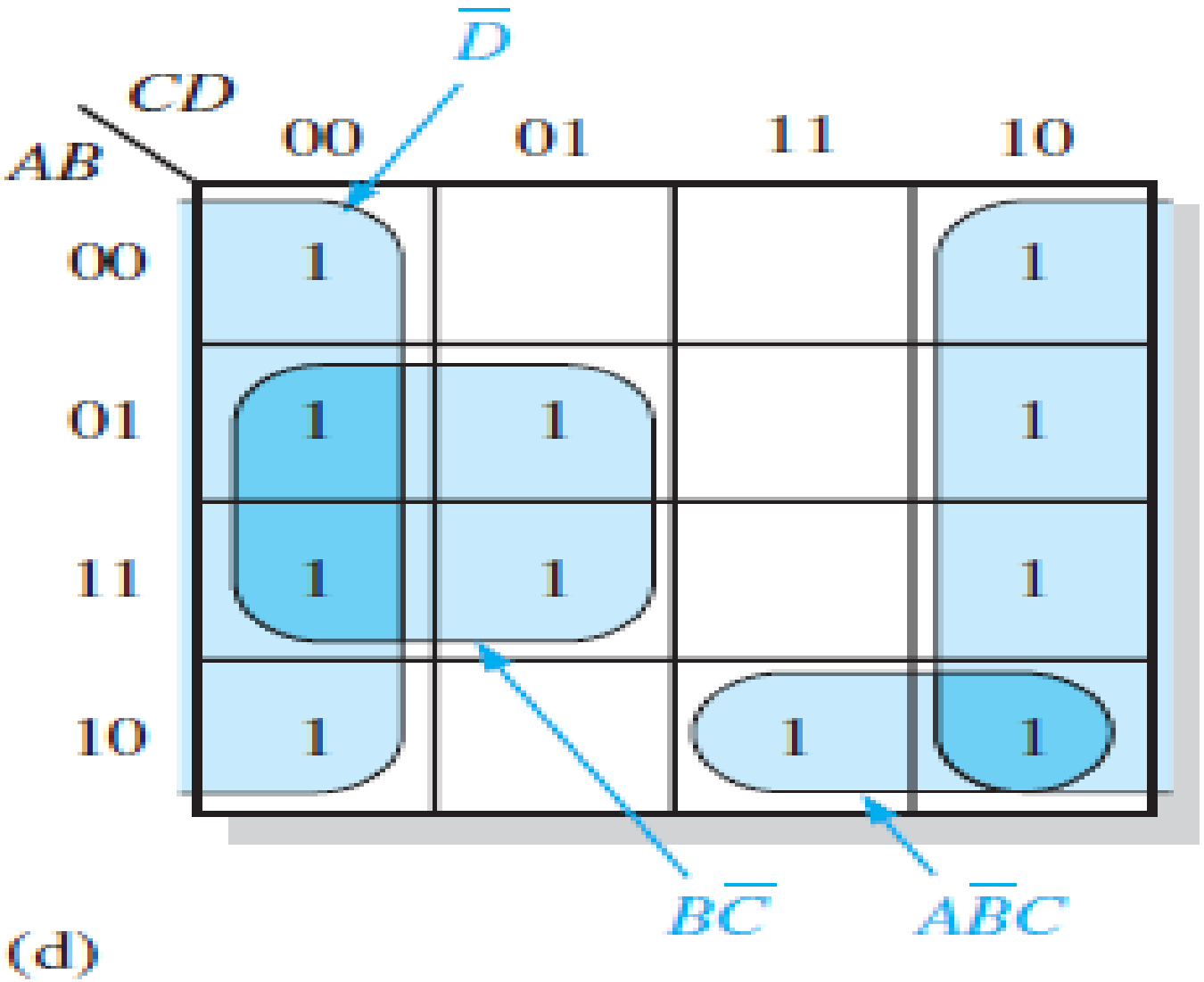
$\overline{A}B$
$A\overline{B}D$

(c)



(d)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$B\overline{C}$
$A\overline{B}C$

(d)