Assignment 2

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Download all python codes from

https://github.com/pulkitsaxena92/EE20MTECH14016_MatrixEE5609/tree/master/Assignment2

and python codes from

https://github.com/pulkitsaxena92/EE20MTECH14016_MatrixEE5609/tree/master/Assignment2/code

1 Question

If $\mathbf{A} = \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and $\mathbf{I}$ is identity matrix of order 2 , show that

$$\mathbf{I}+\mathbf{A} = \left(I - A\right)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & cos\alpha \end{pmatrix} \tag{1.0.1}$$

2 Solution

Since

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{A} = \tan\frac{\alpha}{2}\begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{I}-\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \tan\frac{\alpha}{2}\begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix} \tag{2.0.3}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix} \cos\frac{\alpha}{2} & 0 \\ 0 & \cos\frac{\alpha}{2} \end{pmatrix} - \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}\begin{pmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{pmatrix} \tag{2.0.4}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix} \cos\frac{\alpha}{2} & 0 \\ 0 & \cos\frac{\alpha}{2} \end{pmatrix} - \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix} 0 & -\sin\frac{\alpha}{2} \\ \sin\frac{\alpha}{2} & 0 \end{pmatrix} \tag{2.0.5}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix} \cos\frac{\alpha}{2} & \sin\frac{\alpha}{2} \\ -\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix} \tag{2.0.6}$$

The matrix ($\mathbf{I}-\mathbf{A}$ is a rotational Matrix with rotation $-\frac{\alpha}{2}$

The Matrix $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ is also a rotational Matrix with an angle $+\alpha$.

Multiplying two rotational matrices gives the resultant rotational matrix $+\alpha - \frac{\alpha}{2} = +\frac{\alpha}{2}$

$$RHS = \left(I - A\right)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & cos\alpha\end{pmatrix} \tag{2.0.7}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix}\cos\frac{\alpha}{2} & \sin\frac{\alpha}{2}\\ -\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2}\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & cos\alpha\end{pmatrix} \tag{2.0.8}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix}\cos\frac{\alpha}{2} & -\sin\frac{\alpha}{2}\\ \sin\frac{\alpha}{2} & \cos\frac{\alpha}{2}\end{pmatrix} \tag{2.0.9}$$

$$\tag{2.0.10}$$

Solving LHS= $\mathbf{I} + \mathbf{A}$

$$\mathbf{I} + \mathbf{A} = \begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix} \tag{2.0.11}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix}\cos\frac{\alpha}{2} & -\sin\frac{\alpha}{2}\\ \sin\frac{\alpha}{2} & \cos\frac{\alpha}{2}\end{pmatrix} \tag{2.0.12}$$

This term is a rotational Matrix with angle $+\frac{\alpha}{2}$.Hence both sides evaluates to be a rotational matrix with angle $+\frac{\alpha}{2}$.