Assignment 2

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Download all python codes from

https://github.com/pulkitsaxena92/EE20MTECH14016_MatrixEE5609/tree/master/Assignment2

and python codes from

https://github.com/pulkitsaxena92/EE20MTECH14016_MatrixEE5609/tree/master/Assignment2/code

1 Question

If $\mathbf{A} = \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and $\mathbf{I}$ is identity matrix of order 2 , show that

$$\mathbf{I} + \mathbf{A} = \left(I - A\right)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & cos\alpha \end{pmatrix} \tag{1.0.1}$$

2 Formulae Used

1) Half Angle formulae are

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \tag{2.0.1}$$

$$\tag{2.0.2}$$

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \tag{2.0.3}$$

2) In general,the complex number $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ has matrix representation as

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.4}$$

3 Solution

We will solve both LHS and RHS and equate them
Since Identity and $\mathbf{A}$ in Complex form

$$\mathbf{I} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{A} = \begin{pmatrix} 0 \\ \tan\frac{\alpha}{2} \end{pmatrix} = \tan\frac{\alpha}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3.0.2}$$

3.0.1 Solving LHS:

$$\mathbf{I} + \mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \tan\frac{\alpha}{2} \end{pmatrix} \tag{3.0.3}$$

$$LHS = \begin{pmatrix} 1 \\ \tan\frac{\alpha}{2} \end{pmatrix} \tag{3.0.4}$$

3.1 Solving RHS

$$\mathbf{I}-\mathbf{A}=\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}0\\ \tan\frac{\alpha}{2}\end{pmatrix} \tag{3.1.1}$$
$$=\begin{pmatrix}1\\ -\tan\frac{\alpha}{2}\end{pmatrix} \tag{3.1.2}$$

$$RHS=\left(I-A\right)\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{3.1.3}$$
$$=\begin{pmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & cos\alpha\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{3.1.4}$$
$$=\begin{pmatrix}\cos\alpha+\tan\frac{\alpha}{2}\sin\alpha & -\sin\alpha+\tan\frac{\alpha}{2}\cos\alpha\\ -\tan\frac{\alpha}{2}\cos\alpha+\sin\alpha & \tan\frac{\alpha}{2}\sin\alpha+\cos\alpha\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{3.1.5}$$

Using Half angle formula's and substuting sin and cos in terms of tan

$$\begin{pmatrix}\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}+\tan\frac{\alpha}{2}\cdot\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{-2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}+\tan\frac{\alpha}{2}\cdot\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\\ -\tan\frac{\alpha}{2}\cdot\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}+\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \tan\frac{\alpha}{2}\cdot\frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}+\frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{3.1.6}$$

On Solving we get

$$\begin{pmatrix}\frac{1-\tan^2\frac{\alpha}{2}+2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{-2\tan\frac{\alpha}{2}+\tan\frac{\alpha}{2}-\tan^3\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\\ \frac{-\tan\frac{\alpha}{2}+2\tan\frac{\alpha}{2}+\tan^3\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{2\tan^2\frac{\alpha}{2}+1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{3.1.7}$$

$$=\begin{pmatrix}\frac{1+\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \frac{-\tan\frac{\alpha}{2}\left(1+\tan^2\frac{\alpha}{2}\right)}{1+\tan^2\frac{\alpha}{2}}\\ \frac{\tan\frac{\alpha}{2}\left(1+\tan^2\frac{\alpha}{2}\right)}{1+\tan^2\frac{\alpha}{2}} & \frac{1+\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{3.1.8}$$

$$=\begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{3.1.9}$$
$$RHS=\begin{pmatrix}1\\ \tan\frac{\alpha}{2}\end{pmatrix} \tag{3.1.10}$$

Since LHS=RHS Hence Proved.