Assignment5

Pulkit Saxena

1 Problem

Find QR decomposition of matrix

$$\mathbf{V} = \begin{pmatrix} 12 & -5 \\ -5 & 2 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Let $\mathbf{x}$ and $\mathbf{y}$ be the column vectors of the given matrix.

$$\mathbf{x} = \begin{pmatrix} 12 \\ -5 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{y} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \tag{2.0.2}$$

The column vectors can be expressed as follows,

$$\mathbf{x} = k_1 \mathbf{u}_1 \tag{2.0.3}$$

$$\mathbf{y} = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.0.4}$$

$$k_1 = \|\mathbf{x}\| \tag{2.0.5}$$

$$\mathbf{u}_1 = \frac{\mathbf{x}}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \mathbf{y}}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u}_2 = \frac{\mathbf{y} - r_1 \mathbf{u}_1}{\|\mathbf{y} - r_1 \mathbf{u}_1\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u}_2^T \mathbf{y} \tag{2.0.9}$$

The (2.0.3) and (2.0.4) can be written as,

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

Now, $\mathbf{R}$ is an upper triangular matrix and also,

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{2.0.12}$$

Now using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{12^2 + 5^2} = 13 \tag{2.0.13}$$

$$\mathbf{u}_1 = \begin{pmatrix} \frac{12}{13} \\ \frac{-5}{13} \end{pmatrix} \tag{2.0.14}$$

$$r_1 = \begin{pmatrix} \frac{12}{13} & \frac{-5}{13} \end{pmatrix} \begin{pmatrix} -5 \\ 2 \end{pmatrix} = -\frac{70}{13} \tag{2.0.15}$$

$$\mathbf{u}_2 = \begin{pmatrix} -\frac{5}{13} \\ -\frac{12}{13} \end{pmatrix} \tag{2.0.16}$$

$$k_2 = \begin{pmatrix} -\frac{5}{13} & -\frac{12}{13} \end{pmatrix} \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \frac{1}{13} \tag{2.0.17}$$

Thus putting the values from (2.0.13) to (2.0.17) in (2.0.10) we obtain QR decomposition,

$$\begin{pmatrix} 12 & -5 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} \frac{12}{13} & -\frac{5}{13} \\ -\frac{5}{13} & -\frac{12}{13} \end{pmatrix} \begin{pmatrix} 13 & -\frac{70}{13} \\ 0 & \frac{1}{13} \end{pmatrix} \tag{2.0.18}$$

which can also be written as,

$$\begin{pmatrix} 12 & -5 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix} \begin{pmatrix} -13 & \frac{70}{13} \\ 0 & -\frac{1}{13} \end{pmatrix} \tag{2.0.19}$$