Assignment 1

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Download all python codes from

https://github.com/pulkitsaxena92/IITH−EE5609/tree/master/assignment%201

and latex-tikz codes from

https://github.com/pulkitsaxena92/IITH−EE5609/tree/master/assignment%201

1 Question No. 42

Find the equation of lines through the point $\begin{pmatrix}3\\2\end{pmatrix}$ which make an angle of 45°to the line

$$\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{x} = 3 \tag{1.0.1}$$

2 Explanation

2.1 Formulae Used

1) Equation of Line Passing through $\begin{pmatrix}3\\2\end{pmatrix}$ with a normal Vector is given by

$$\mathbf{n}^T(\mathbf{x}-\mathbf{A}) = 0 \tag{2.1.1}$$

where A is the point satisfying the Equation.
2) Inner Product of two vectors is given by the formulae

$$\cos\theta = \frac{\mathbf{n}_1^T\mathbf{n}_2}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} \tag{2.1.2}$$

where is the angle between two lines
3)Normal Vector of a line is given by

$$\mathbf{n} = \begin{pmatrix}-m\\1\end{pmatrix} \tag{2.1.3}$$

Where m is the slope of the line

2.2 Solution

On comparing $\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{x} = 3$ with Formulae 1 we get

$$\mathbf{n}_1^T = \begin{pmatrix}1 & -2\end{pmatrix} \tag{2.2.1}$$

Let the normal vector of the other line is

$$\mathbf{n}_2 = \begin{pmatrix}-m\\1\end{pmatrix} \tag{2.2.2}$$

Angle between these two lines is 45°

$$\theta = 45^\circ \implies \cos 45^\circ = \frac{1}{\sqrt{2}} \tag{2.2.3}$$

$$\tag{2.2.4}$$

Substituting $\mathbf{n}_1^T\mathbf{n}_2$ and $\cos\theta$ in Formulae 2 we get

$$\frac{1}{\sqrt{2}} = \frac{\begin{pmatrix}1 & -2\end{pmatrix}\times\begin{pmatrix}-m\\1\end{pmatrix}}{\sqrt{5}\times\sqrt{m^2+1}} \tag{2.2.5}$$

$$3m^2 - 4m - 3 = 0 \tag{2.2.6}$$

Solving this equation we get two roots

$$m_1 = 3 \tag{2.2.7}$$

$$m_2 = -\frac{1}{3} \tag{2.2.8}$$

2.2.1 Case 1: Equations of Line with Slope $m_1 = 3$ and passing through point $\begin{pmatrix}3\\2\end{pmatrix}$ is given as

$$\mathbf{n}^T(\mathbf{x}-\mathbf{A}) = 0 \tag{2.2.9}$$

$$\mathbf{n}^T = \begin{pmatrix}-3 & 1\end{pmatrix} \tag{2.2.10}$$

Substituting Values we get

$$\begin{pmatrix}-3 & 1\end{pmatrix}\left(\mathbf{x} - \begin{pmatrix}3\\2\end{pmatrix}\right) = 0 \tag{2.2.11}$$

$$\begin{pmatrix}-3 & 1\end{pmatrix}\mathbf{x} = -7 \tag{2.2.12}$$

2.2.2 Case 2: Equations of Line with Slope $m_2 = -\frac{1}{3}$ and passing through point $\begin{pmatrix}3\\2\end{pmatrix}$ is given as

$$\mathbf{n}^T(\mathbf{x}-\mathbf{A}) = 0 \tag{2.2.13}$$

$$\mathbf{n}^T = \begin{pmatrix}-\frac{1}{3} & 1\end{pmatrix} \tag{2.2.14}$$

Substituting Values we get

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}\left(\mathbf{x}-\begin{pmatrix}3\\2\end{pmatrix}\right)=0 \tag{2.2.15}$$

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}\mathbf{x}=3 \tag{2.2.16}$$

2.3 Answers

the equation of lines through the point $\begin{pmatrix}3\\2\end{pmatrix}$ which make an angle of 45°to the line

$$\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{x}=3 \tag{2.3.1}$$

are

$$\begin{pmatrix}-3 & 1\end{pmatrix}\mathbf{x}=-7 \tag{2.3.2}$$

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}\mathbf{x}=3 \tag{2.3.3}$$

The Figure below shows the plot of all three lines

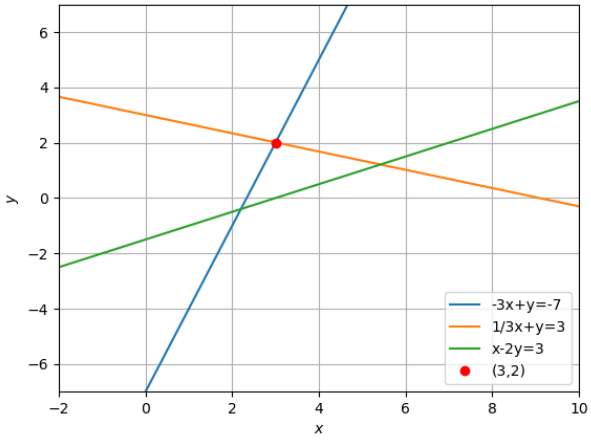

Fig. 0: Plotting these Equation