

Assignment 12

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1 PROBLEM UGCJUNE2017 Q75

Which of the following 3x3 matrices are diagonalizable over \mathbb{R} ?

$$1. \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix} \quad 2. \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad 3. \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{pmatrix} \quad 4. \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

2 EXPLANATION

Test for diagonalizability	<p>Let \mathbf{W}_i be the eigenspace corresponding to eigenvalue λ_i of \mathbf{A}</p> <p>1) \mathbf{A} is diagonalizable</p> <p>2) characteristic polynomial of \mathbf{A} is</p> <p>$f = (\mathbf{x} - \lambda_1)^{d_1} \dots (\mathbf{x} - \lambda_k)^{d_k}$ and $\dim(\mathbf{W}_i) = d_i$</p> <p>3) $\sum_{i=1}^k \dim(\mathbf{W}_i) = n$</p>
Concept for diagonalization	<p>A linear operator \mathbf{A} on a n-dimensional space \mathbb{V} is</p> <p>diagonalizable, if and only if \mathbf{A} has n distinct</p> <p>characteristic vectors or null spaces corresponding to the characteristic values</p>

TABLE 1: Illustration of theorem.

3 SOLUTION

Option A	<p>Given matrix is</p> $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$
Finding Characteristics polynomial	<p>Characteristics polynomial of the matrix \mathbf{A} is $\det(x\mathbf{I} - \mathbf{A})$</p> $\det(x\mathbf{I} - \mathbf{A}) = \begin{vmatrix} (x-1) & -3 & -2 \\ 0 & (x-4) & -5 \\ 0 & 0 & x-6 \end{vmatrix}$ <p>Characteristic Polynomial = $(x-1)(x-4)(x-6)$</p>
Testing diagonalizability over \mathbb{R}	<p>1) As the characteristics polynomial is product of linear factors over \mathbb{R}.</p> <p>2) To find characteristic values of the operator $\det(xI - A) = 0$ which gives $\lambda_1 = 1, \lambda_2 = 4, \lambda_3 = 6$</p> <p>Thus over \mathbb{R} matrix \mathbf{A} has three distinct characteristic values. There will be atleast one characteristics vector i.e., one dimension with each characteristics value . Thus $\dim \mathbf{W}_i = d_i$</p> <p>3) $\sum_i \mathbf{W}_i = n = 3$, which is equal to \dim of \mathbf{A}.</p>
Conclusion on Option A	Option A satisfy all three condition of Diagonalizability over \mathbb{R} .
Option B	<p>Given matrix is</p> $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Finding Characteristics polynomial	<p>Characteristics polynomial of the matrix $\det(x\mathbf{I} - \mathbf{A})$</p> $\det(x\mathbf{I} - \mathbf{A}) = \begin{vmatrix} x & -1 & 0 \\ 1 & x & 0 \\ 0 & 0 & x-1 \end{vmatrix}$ <p>Characteristic Polynomial = $(x-1)(x+i)(x-i)$</p>

Testing diagonalizability over \mathbb{R}	1) As the characteristics polynomial is not the product of linear factors over \mathbb{R} . Thus \mathbf{A} is not diagonalizable over \mathbb{R} .
Conclusion on Option B	Option B does not satisfy condition 1.
Option C	<p>Given matrix is</p> $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{pmatrix}$
Finding Characteristics polynomial	<p>Characteristics polynomial of the matrix \mathbf{A} is $\det(x\mathbf{I} - \mathbf{A})$</p> $\det(x\mathbf{I} - \mathbf{A}) = \begin{vmatrix} (x-1) & -2 & -3 \\ -2 & (x-1) & -4 \\ -3 & -4 & x-1 \end{vmatrix}$ <p>Characteristic Polynomial = $(x + 3.19)(x + 0.877)(x - 7.07)$</p>
Testing diagonalizability over \mathbb{R}	<p>1) As the characteristics polynomial are product of linear factors over \mathbb{R} .</p> <p>2) To find characteristic values of the operator $\det(x\mathbf{I} - \mathbf{A}) = 0$ which gives $\lambda_1 = -3.19, \lambda_2 = -0.887, \lambda_3 = 7.07$</p> <p>Thus over \mathbb{R} matrix \mathbf{A} has three distinct characteristic values. There will be atleast one characteristics vector i.e., one dimension with each characteristics value . Thus $\dim \mathbf{W}_i = d_i$</p> <p>3) $\sum_i \mathbf{W}_i = n = 3$, which is equal to \dim of \mathbf{A}.</p>
Conclusion on Option C	Option C satisfy all three condition of Diagonalizability over \mathbb{R} .

Option D	<p>Given matrix is</p> $\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
Finding Characteristics polynomial	<p>Characteristics polynomial of the matrix \mathbf{A} is $\det(x\mathbf{I} - \mathbf{A})$</p> $\det(x\mathbf{I} - \mathbf{A}) = \begin{vmatrix} x & -1 & -2 \\ 0 & x & -1 \\ 0 & 0 & x \end{vmatrix}$ <p>Characteristic Polynomial = $(x)(x)(x) = x^3$</p>
Testing diagonalizability over \mathbb{R}	<p>1) As the characteristics polynomial is product of linear factors over \mathbb{R}.</p> <p>2) To find characteristic values of the operator $\det(x\mathbf{I} - \mathbf{A}) = 0$ $\lambda_1 = 0$ $d_1 = 3$</p> $\mathbf{W}_1 = \mathbf{A} - \lambda_1\mathbf{I} \Rightarrow \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} - 0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ <p>$\dim \mathbf{W}_1 = 2$ $\dim \mathbf{W}_i \neq d_i$ Algebraic Multiplicity is not equal to Geometric Multiplicity.</p>
Conclusion on Option D	Option D does not satisfy second condition of Diagonalizability.
Answer	Option A and Option C are Diagonalizable over \mathbb{R} .

TABLE 2: Option Checking Table