Assignment 2

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Download all python codes from

https://github.com/pulkitsaxena92/EE20MTECH14016_MatrixEE5609/tree/master/Assignment2

and python codes from

https://github.com/pulkitsaxena92/EE20MTECH14016_MatrixEE5609/tree/master/Assignment2/code

1 Question

If $\mathbf{A} = \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix}$ and $\mathbf{I}$ is identity matrix of order 2 , show that

$$\mathbf{I} + \mathbf{A} = \left(I - A\right)\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & cos\alpha \end{pmatrix} \tag{1.0.1}$$

2 Solution

We will solve both LHS and RHS and equate them
Since

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.1}$$

2.0.1 Solving LHS:

$$\mathbf{I} + \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix} \tag{2.0.2}$$

$$LHS = \begin{pmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{pmatrix} \tag{2.0.3}$$

2.1 Solving RHS

$$\mathbf{I} - \mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{pmatrix} \tag{2.1.1}$$

$$= \begin{pmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{pmatrix} \tag{2.1.2}$$

$$= \begin{pmatrix} 1 & \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ -\frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & 1 \end{pmatrix} \tag{2.1.3}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix} \cos\frac{\alpha}{2} & \sin\frac{\alpha}{2} \\ -\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix} \tag{2.1.4}$$

Since :-

$$RHS = \left(I - A\right)\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & cos\alpha\end{pmatrix} \tag{2.1.5}$$

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix}\cos\frac{\alpha}{2} & \sin\frac{\alpha}{2}\\ -\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2}\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & cos\alpha\end{pmatrix} \tag{2.1.6}$$

Since left equation is rotated by $-\frac{\alpha}{2}$ and right is rotated by $+\alpha$ so the overall matrix is rotated by $+\frac{\alpha}{2}$ and we get the following resultant Matrix

$$= \frac{1}{\cos\frac{\alpha}{2}}\begin{pmatrix}\cos\frac{\alpha}{2} & -\sin\frac{\alpha}{2}\\ \sin\frac{\alpha}{2} & \cos\frac{\alpha}{2}\end{pmatrix} \tag{2.1.7}$$

$$RHS = \begin{pmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{pmatrix} \tag{2.1.8}$$

Since LHS=RHS Hence Proved.