# Assignment 4

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## 1 Problem(loney pg 98 Q7)

Find the value of k so that following equation may represent pairs of straight lines,

$$12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0 \tag{1.0.1}$$

## 2 Solution

The general equation of second degree is given by,

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.1}$$

In vector from the equation (2.0.1) can be expressed as,

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

where,

$$\mathbf{V} = \mathbf{V}^T = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u} = \begin{pmatrix} d \\ e \end{pmatrix} \tag{2.0.4}$$

Now, comparing (2.0.1) to (1.0.1) we get, a =12, b=-5, c = 2, d = $\frac{11}{2}$,e = -$\frac{5}{2}$, f = k. Hence, substituting these values in (2.0.3) and (2.0.4) we get,

$$\mathbf{V} = \begin{pmatrix} 12 & -5 \\ -5 & 2 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{u} = \begin{pmatrix} \frac{11}{2} \\ -\frac{5}{2} \end{pmatrix} \tag{2.0.6}$$

(1.0.1) represents pair of straight lines if,

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \tag{2.0.7}$$

$$\begin{vmatrix} 12 & -5 & \frac{11}{2} \\ -5 & 2 & -\frac{5}{2} \\ \frac{11}{2} & -\frac{5}{2} & k \end{vmatrix} = 0 \tag{2.0.8}$$

$$\implies k = 2 \tag{2.0.9}$$

Lines Intercept if

$$|\mathbf{V}| < 0 \tag{2.0.10}$$

$$|\mathbf{V}| = -1 < 0 \tag{2.0.11}$$

Hence Line intercept.
Let $(\alpha,\beta)$ be their point of intersection, then

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -d \\ -e \end{pmatrix} \tag{2.0.12}$$

Substituting in (2.0.12)

$$\begin{pmatrix} 12 & -5 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -\frac{11}{2} \\ \frac{5}{2} \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} \\ -\frac{5}{2} \end{pmatrix} \tag{2.0.14}$$

Spectral Decomposition of $\mathbf{V}$ is given as

$$\mathbf{V} = \mathbf{P}\mathbf{D}\mathbf{P}^T \tag{2.0.15}$$

$$\mathbf{V} = \begin{pmatrix} 12 & -5 \\ -5 & 2 \end{pmatrix} \tag{2.0.16}$$

$$\mathbf{P} = \begin{pmatrix} -1 - \sqrt{2} & -1 + \sqrt{2} \\ 1 & 1 \end{pmatrix} \tag{2.0.17}$$

$$\mathbf{D} = \begin{pmatrix} 7 + 5\sqrt{2} & 0 \\ 0 & 7 - 5\sqrt{2} \end{pmatrix} \tag{2.0.18}$$

Using Spectral decomposition concept and substution

$$u_1(x-\alpha) + u_2(y-\beta) = \pm\sqrt{-\frac{\lambda_2}{\lambda_1}}(v_1(x-\alpha) + v_2(y-\beta)) \tag{2.0.19}$$

Substituting (2.0.14), (2.0.17) and (2.0.18) in (2.0.19)

$$\left(-1 - \sqrt{2}\right)\left(x - \frac{-3}{2}\right) + \left(y - \frac{-5}{2}\right)$$
$$= \pm\sqrt{-\frac{7+5\sqrt{2}}{7-5\sqrt{2}}}\left(\left(-1 + \sqrt{2}\right)\left(x - \frac{-3}{2}\right) + \left(y - \frac{-5}{2}\right)\right) \tag{2.0.20}$$

Simplifying (2.0.20),

$$-6x + 2y - 4 = 0 \text{ and } -2x + y - \frac{1}{2} = 0 \quad (2.0.21)$$

$$\implies (-6x + 2y - 4)\left(-2x + y - \frac{1}{2}\right) = 0 \quad (2.0.22)$$

Thus the equation of lines are

$$\begin{pmatrix} -6 & 2 \end{pmatrix} \mathbf{x} = 4 \quad (2.0.23)$$

$$\begin{pmatrix} -2 & 1 \end{pmatrix} \mathbf{x} = \frac{1}{2} \quad (2.0.24)$$

Hence, Plot is shown below

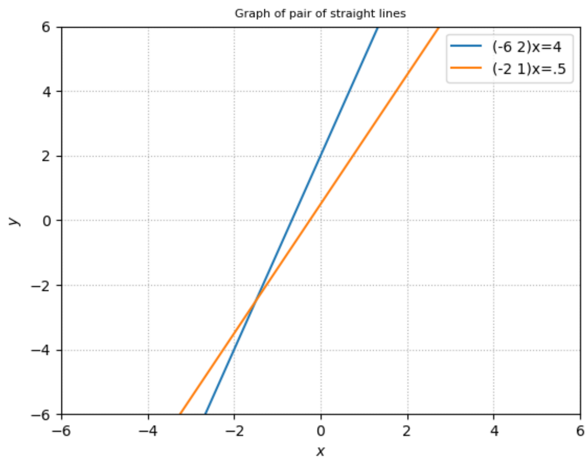


Fig. 0: Pair of lines