

Assignment 1

Pulkit Saxena

Download all python codes from

<https://github.com/pulkitsaxena92/IITH-EE5609/tree/master/assignment%201>

and latex-tikz codes from

<https://github.com/pulkitsaxena92/IITH-EE5609/tree/master/assignment%201>

1 QUESTION NO. 42

Find the equation of lines through the point $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ which make an angle of 45° to the line

$$(1 \ -2)\mathbf{x} = 3 \quad (1.0.1)$$

2 EXPLANATION

2.1 Formulae Used

1) Equation of Line Passing through $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ with a normal Vector is given by

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \quad (2.1.1)$$

where A is the point satisfying the Equation.

2) Inner Product of two vectors is given by the formulae

$$\cos \theta = \frac{\mathbf{n}_1^T \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (2.1.2)$$

where θ is the angle between two lines

3) Normal Vector of a line is given by

$$\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \quad (2.1.3)$$

Where m is the slope of the line

2.2 Solution

On comparing $(1 \ -2)\mathbf{x} = 3$ with Formulae 1 we get

$$\mathbf{n}_1^T = (1 \ -2) \quad (2.2.1)$$

Let the normal vector of the other line is

$$\mathbf{n}_2 = \begin{pmatrix} -m \\ 1 \end{pmatrix} \quad (2.2.2)$$

Angle between these two lines is 45°

$$\theta = 45^\circ \implies \cos 45^\circ = \frac{1}{\sqrt{2}} \quad (2.2.3)$$

$$(2.2.4)$$

Substituting \mathbf{n}_1^T , \mathbf{n}_2 and $\cos \theta$ in Formulae 2 we get

$$\frac{1}{\sqrt{2}} = \frac{(1 \ -2) \times \begin{pmatrix} -m \\ 1 \end{pmatrix}}{\sqrt{5} \times \sqrt{m^2 + 1}} \quad (2.2.5)$$

$$3m^2 - 4m - 3 = 0 \quad (2.2.6)$$

Solving this equation we get two roots

$$m_1 = 3 \quad (2.2.7)$$

$$m_2 = -\frac{1}{3} \quad (2.2.8)$$

2.2.1 Case 1: Equations of Line with Slope $m_1 = 3$ and passing through point $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is given as

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \quad (2.2.9)$$

$$\mathbf{n}^T = (-3 \ 1) \quad (2.2.10)$$

Substituting Values we get

$$(-3 \ 1) \left(x - \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right) = 0 \quad (2.2.11)$$

$$(-3 \ 1)x = -7 \quad (2.2.12)$$

2.2.2 Case 2: Equations of Line with Slope $m_2 = -\frac{1}{3}$ and passing through point $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is given as

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \quad (2.2.13)$$

$$\mathbf{n}^T = \left(-\frac{1}{3} \ 1\right) \quad (2.2.14)$$

Substituting Values we get

$$\begin{pmatrix} \frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} x - \frac{3}{2} \end{pmatrix} = 0 \quad (2.2.15)$$

$$\begin{pmatrix} \frac{1}{3} & 1 \end{pmatrix} x = 3 \quad (2.2.16)$$

2.3 Answers

the equation of lines through the point $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ which make an angle of 45° to the line

$$\begin{pmatrix} 1 & -2 \end{pmatrix} \mathbf{x} = 3 \quad (2.3.1)$$

are

$$\begin{pmatrix} -3 & 1 \end{pmatrix} x = -7 \quad (2.3.2)$$

$$\begin{pmatrix} \frac{1}{3} & 1 \end{pmatrix} x = 3 \quad (2.3.3)$$

The Figure below shows the plot of all three lines

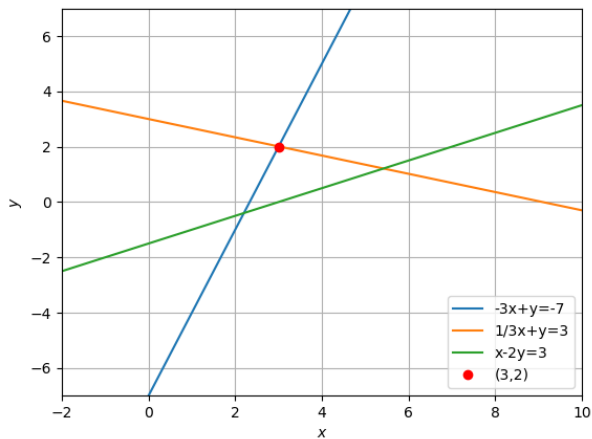


Fig. 0: Plotting these Equation