

Assignment 11

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1 PROBLEM

If \mathbf{A} and \mathbf{B} are $n \times n$ matrices, show that

$$\mathbf{AB} - \mathbf{BA} = \mathbf{I} \quad (1.0.1)$$

is impossible.

2 SOLUTION

$$\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA}) \quad (2.0.1)$$

$$\text{trace}(\mathbf{I}) = \sum_{j=1}^n I_{jj} \quad (2.0.2)$$

$$\Rightarrow \sum_{j=1}^n 1 = n \quad (2.0.3)$$

Taking Trace on both sides (1.0.1)

$$\text{trace}(\mathbf{AB} - \mathbf{BA}) = \text{trace}(\mathbf{I}) \quad (2.0.4)$$

$$\Rightarrow \text{trace}(\mathbf{AB}) - \text{trace}(\mathbf{BA}) = \text{trace}(\mathbf{I}) \quad (2.0.5)$$

From (2.0.1) and (2.0.3)

$$\text{trace}(\mathbf{AB}) - \text{trace}(\mathbf{BA}) = 0 \quad (2.0.6)$$

$$\text{trace}(\mathbf{I}) = n \quad (2.0.7)$$

$$= 0 \neq n \quad (2.0.8)$$

Thus:-

$$\text{trace}(\mathbf{AB} - \mathbf{BA}) \neq \text{trace}(\mathbf{I}) \quad (2.0.9)$$

hence no solution possible.