# Assignment 12

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## 1 Problem ugcjune2017 Q75

Which of the following 3x3 matrices are diagonizable over $\mathbb{R}$?

1) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$
2) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
3) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{pmatrix}$
4) $\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

## 2 Explanation

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| Test for diagonalizability | Let $\mathbf{W}_i$ be the eigenspace corresponding to eigenvalue $\lambda_i$ of $\mathbf{A}$<br>1)$\mathbf{A}$ is diagonalizable<br>2) characteristic polynomial of $\mathbf{A}$ is<br>f = $(\mathbf{x}-\lambda_1)^{d_1}....(\mathbf{x}-\lambda_k)^{d_k}$ and $dim(\mathbf{W}_i) = d_i$<br>3) $\sum_{i=1}^{k} \mathbf{W_i} = n$ |
| Concept<br>for diagonalization | A linear operator $\mathbf{A}$ on a $n$-dimensional space $\mathbb{V}$ is<br>diagonalizable , if and only if $\mathbf{A}$ has $n$ distinct<br>characteristic vectors or null spaces corresponding to the characteristic values |

TABLE 1: Illustration of theorem.

## 3 Solution

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| Option A | Given matrix is<br><br>$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$ |
| Finding Characteristics<br><br>polynomial | Characteristics polynomial of the matrix $\mathbf{A}$ is $det(x\mathbf{I} - \mathbf{A})$<br>$\det(x\mathbf{I} - \mathbf{A}) = \begin{vmatrix} (x-1) & -3 & -2 \\ 0 & (x-4) & -5 \\ 0 & 0 & x-6 \end{vmatrix}$<br><br>Characteristic Polynomial $= (x-1)(x-4)(x-6)$ |
| Testing diagonalizability over $\mathbb{R}$ | 1) As the characteristics polynomial is product of linear factors over $\mathbb{R}$ .<br><br>2) To find characteristic values of the operator $\det(xI - A) = 0$ which gives $\lambda_1 = 1, \lambda_2 = 4, \lambda_3 = 6$<br><br>Thus over $\mathbb{R}$ matrix $\mathbf{A}$ has three distinct characteristic values.<br>There will be atleast one characteristics vector i.e., one dimension with each characteristics value .<br>Thus $dim\mathbf{W}_i = d_i$<br><br>3) $\sum_i \mathbf{W_i} = n = 3$ , which is equal to $dim$ of $\mathbf{A}$. |
| Conclusion on Option A | Option A satisfy all three condition of Diagonalizability over $\mathbb{R}$. |

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| Option B | Given matrix is<br><br>$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ |
| Finding Characteristics<br><br>polynomial | Characteristics polynomial of the matrix $det(x\mathbf{I} - \mathbf{A})$<br>$det(x\mathbf{I} - \mathbf{A}) = \begin{vmatrix} x & -1 & 0 \\ 1 & x & 0 \\ 0 & 0 & x-1 \end{vmatrix}$<br><br>Characteristic Polynomial $= (x-1)(x+i)(x-i)$ |

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| Testing diagonalizability over $\mathbb{R}$ | 1) As the characteristics polynomial is not the product of linear factors over $\mathbb{R}$ beacuse roots of characteristic eq are complex .<br>Thus $\mathbf{A}$ is not diagonalizable over $\mathbb{R}$. |
| Conclusion on Option B | Option B does not satisfy condition 1. |
| Option C | Given matrix is<br>$\mathbf{A}=\begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 4\\ 3 & 4 & 1\end{pmatrix}$ |
| Finding Characteristics<br><br>polynomial | Characteristics polynomial of the matrix $\mathbf{A}$ is $det(x\mathbf{I}-\mathbf{A})$<br>$det(x\mathbf{I}-\mathbf{A})=\begin{vmatrix}(x-1) & -2 & -3\\ -2 & (x-1) & -4\\ -3 & -4 & x-1\end{vmatrix}$<br><br>Characteristic Polynomial $=(x+3.19)(x+0.877)(x-7.07)$ |
| Testing diagonalizability over $\mathbb{R}$ | 1) As the characteristics polynomial are product of linear factors over $\mathbb{R}$ .<br><br>2) To find characteristic values of the operator $det(x\mathbf{I}-\mathbf{A})=0$ which gives $\lambda_1=-3.19, \lambda_2=-0.887, \lambda_3=7.07$<br><br>Thus over $\mathbb{R}$ matrix $\mathbf{A}$ has three distinct characteristic values.<br>There will be atleast one characteristics vector i.e., one dimension with each characteristics value .<br>Thus $dim\mathbf{W}_i = d_i$<br><br>3) $\sum_i \mathbf{W_i} = n = 3$ , which is equal to $dim$ of $\mathbf{A}$. |
| Conclusion on Option C | Option C satisfy all three condition of Diagonalizability over $\mathbb{R}$. |

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| Option D | Given matrix is <br> $\mathbf{A}=\begin{pmatrix}0 & 1 & 2\\0 & 0 & 1\\0 & 0 & 0\end{pmatrix}$ |
| Finding Characteristics polynomial | Characteristics polynomial of the matrix $\mathbf{A}$ is $det(x\mathbf{I}-\mathbf{A})$ <br> $det(x\mathbf{I}-\mathbf{A})=\begin{vmatrix}x & -1 & -2\\0 & x & -1\\0 & 0 & x\end{vmatrix}$ <br> Characteristic Polynomial = $(x)(x)(x) = x^3$ |
| Testing diagonalizability over $\mathbb{R}$ | 1) As the characteristics polynomial is product of linear factors over $\mathbb{R}$ . <br> 2) To find characteristic values of the operator $\det(x\mathbf{I}-\mathbf{A}) = 0$ <br> $\lambda_1 = 0$ <br> $d_1 = 3$ <br> $\mathbf{W}_1 = \mathbf{A} - \lambda_1\mathbf{I} \implies \begin{pmatrix}0 & 1 & 2\\0 & 0 & 1\\0 & 0 & 0\end{pmatrix} - 0\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} = \begin{pmatrix}0 & 1 & 2\\0 & 0 & 1\\0 & 0 & 0\end{pmatrix}$ <br> $dim\mathbf{W}_1 = 2$ <br> $dim\mathbf{W}_i \neq d_i$ <br> Algebric Multiplicity is not equal to Geometric Multiplicity. |
| Conclusion on Option D | Option D does not satisfy second condition of Diagonalizability. |
| Answer | Option A and Option C are Diagonalizable over $\mathbb{R}$. |

TABLE 2: Option Checking Table