# Assignment 11

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## 1 PROBLEM

If $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices , show that

$$\mathbf{AB} - \mathbf{BA} = \mathbf{I} \tag{1.0.1}$$

is impossible.

## 2 SOLUTION

$$trace(\mathbf{AB}) = \sum_{i=1}^{n} (\mathbf{AB})_{ii} \tag{2.0.1}$$

$$= \sum_{i=1}^{n} \sum_{k=1}^{n} \mathbf{A}_{ik}\mathbf{B}_{ki} \tag{2.0.2}$$

$$= \sum_{i=1}^{n} \sum_{k=1}^{n} \mathbf{B}_{ki}\mathbf{A}_{ik} \tag{2.0.3}$$

$$= \sum_{i=1}^{n} (\mathbf{BA})_{kk} \tag{2.0.4}$$

$$= trace(\mathbf{BA}) \tag{2.0.5}$$

$$trace(\mathbf{AB}) = trace(\mathbf{BA}) \tag{2.0.6}$$

$$trace(\mathbf{I}) = \sum_{j=1}^{n} I_{jj} \tag{2.0.7}$$

$$\implies \sum_{j=1}^{n} 1 = n \tag{2.0.8}$$

Taking Trace on both sides (1.0.1)

$$trace(\mathbf{AB} - \mathbf{BA}) = trace(\mathbf{I}) \tag{2.0.9}$$

$$\implies trace(\mathbf{AB}) - trace(\mathbf{BA}) = trace(\mathbf{I}) \tag{2.0.10}$$

From (2.0.6) and (2.0.8)

$$trace(\mathbf{AB}) - trace(\mathbf{BA}) = 0 \tag{2.0.11}$$

$$trace(\mathbf{I}) = n \tag{2.0.12}$$

$$= 0 \neq n \tag{2.0.13}$$

Thus:-

$$trace(\mathbf{AB} - \mathbf{BA}) \neq trace(\mathbf{I}) \tag{2.0.14}$$

hence no solution possible.