Assignment 1

Pulkit Saxena

Download all python codes from

https://github.com/pulkitsaxena92/IITH−EE5609/tree/master/assignment%201/code

and latex-tikz codes from

https://github.com/pulkitsaxena92/IITH−EE5609/tree/master/assignment%201

1 Question No. 42

Find the equation of lines through the point $\begin{pmatrix}3\\2\end{pmatrix}$ which make an angle of 45° to the line

$$\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{x} = 3 \tag{1.0.1}$$

2 Explanation

2.1 Assumption

Let m_2 be the slope of the other Line

2.2 Formulae Used

1) Equation of Line Passing through $\begin{pmatrix}3\\2\end{pmatrix}$ with a normal Vector is given by

$$n^T(X - A) = 0 \tag{2.2.1}$$

where A is the point satisfying the Equation.
2) Inner Product of two vectors is given by the formulae

$$\cos\theta = \frac{\mathbf{n_1}^T\mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{2.2.2}$$

$$\tag{2.2.3}$$

where is the angle between two lines
3)Normal Vector For Second Line is given by

$$n_2 = \begin{pmatrix}-m_2\\1\end{pmatrix} \tag{2.2.4}$$

Where m_2 is the slope of second line

2.3 Solution

On comparing $\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{x} = 3$ with Formulae 1 we get

$$n_1^T = \begin{pmatrix}1 & -2\end{pmatrix} \tag{2.3.1}$$

$$n_2 = \begin{pmatrix}-m_2\\1\end{pmatrix} \tag{2.3.2}$$

$$\theta = 45^\circ \implies \cos 45^\circ = \frac{1}{\sqrt{2}} \tag{2.3.3}$$

$$\tag{2.3.4}$$

Substuting $n_1^T\ n_2$ and θ in Formulae 2 we get

$$\frac{1}{\sqrt{2}} = \frac{\begin{pmatrix}1 & -2\end{pmatrix}\times\begin{pmatrix}-m_2\\1\end{pmatrix}}{\sqrt{5}\times\sqrt{m_2^2+1}} \tag{2.3.5}$$

$$3m_2^2 - 4m_2 - 3 = 0 \implies m_2 = 3, -\frac{1}{3} \tag{2.3.6}$$

2.3.1 Case 1: Equations of Line with Slope $m_2 = 3$ and passing through point $\begin{pmatrix}3\\2\end{pmatrix}$ is given as

$$n^T(X - A) = 0 \tag{2.3.7}$$

Substuting Values we get

$$\begin{pmatrix}-3 & 1\end{pmatrix}\left(X - \begin{pmatrix}3\\2\end{pmatrix}\right) = 0 \tag{2.3.8}$$

$$\begin{pmatrix}-3 & 1\end{pmatrix}x = -7 \tag{2.3.9}$$

2.3.2 Case 2: Equations of Line with Slope $m_2 = -\frac{1}{3}$ and passing through point $\begin{pmatrix}3\\2\end{pmatrix}$ is given as

$$n^T(X - A) = 0 \tag{2.3.10}$$

Substuting Values we get

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}\left(X-\begin{pmatrix}3\\2\end{pmatrix}\right)=0 \tag{2.3.11}$$

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}x=3 \tag{2.3.12}$$

2.4 Answers

the equation of lines through the point $\begin{pmatrix}3\\2\end{pmatrix}$ which make an angle of 45° to the line

$$\begin{pmatrix}1 & -2\end{pmatrix}\mathbf{x}=3 \tag{2.4.1}$$

are

$$\begin{pmatrix}-3 & 1\end{pmatrix}x=-7 \tag{2.4.2}$$

$$\begin{pmatrix}\frac{1}{3} & 1\end{pmatrix}x=3 \tag{2.4.3}$$

The Figure below shows the plot of all three lines

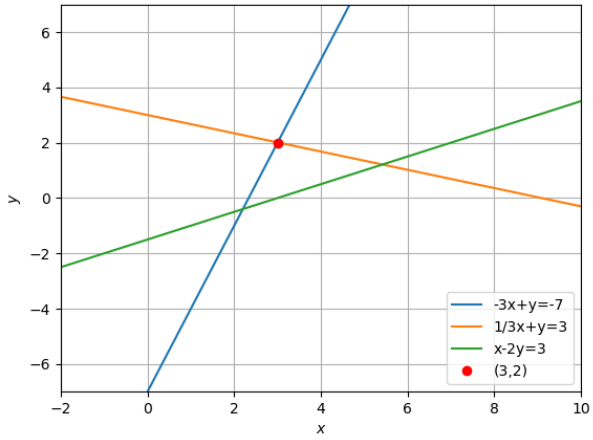

Fig. 0: Plotting these Equation