K-space Sampling Trajectories

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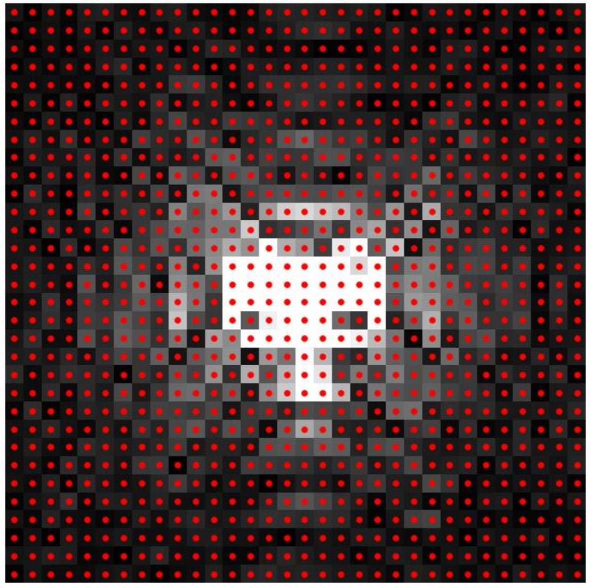
Slides adapted from

Frank Zijlstra (2024): "K-space sampling trajectories", ***German Chapter of ISMRM*****, Freiburg.** **https://github.com/pulseq/MR-Physics-with-Pulseq/blob/main/slides/D1-1350-kspace_trajectories_Frank_Zijlstra.pptx**

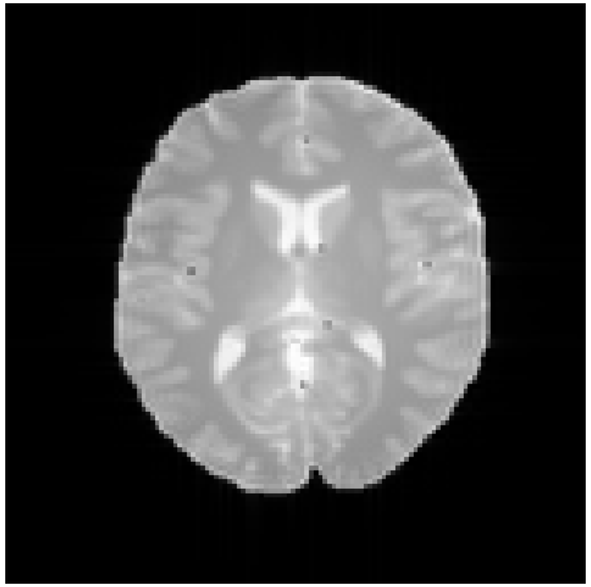
Example: 2D Cartesian trajectory

- K-space trajectory encodes image in the frequency domain
  - Cartesian: Aligned to rectilinear 2D/3D grid
    - Fast and easy reconstruction using inverse FFT
- Main parameters
  - Field of view (FOV) $(= \frac{1}{\Delta k})$
  - Resolution $(= \frac{1}{2}\frac{1}{k_{max}})$
  - Nyquist limit
    - Simply put: To encode a 100x100 image with a given FOV, we need to measure a 100x100 k-space, at Δk intervals. Anything less will give aliasing artifacts.

K-space

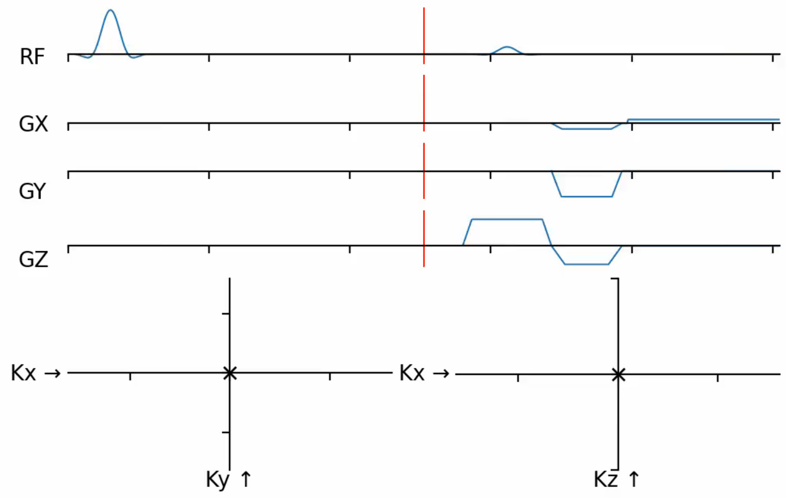


Inverse FFT



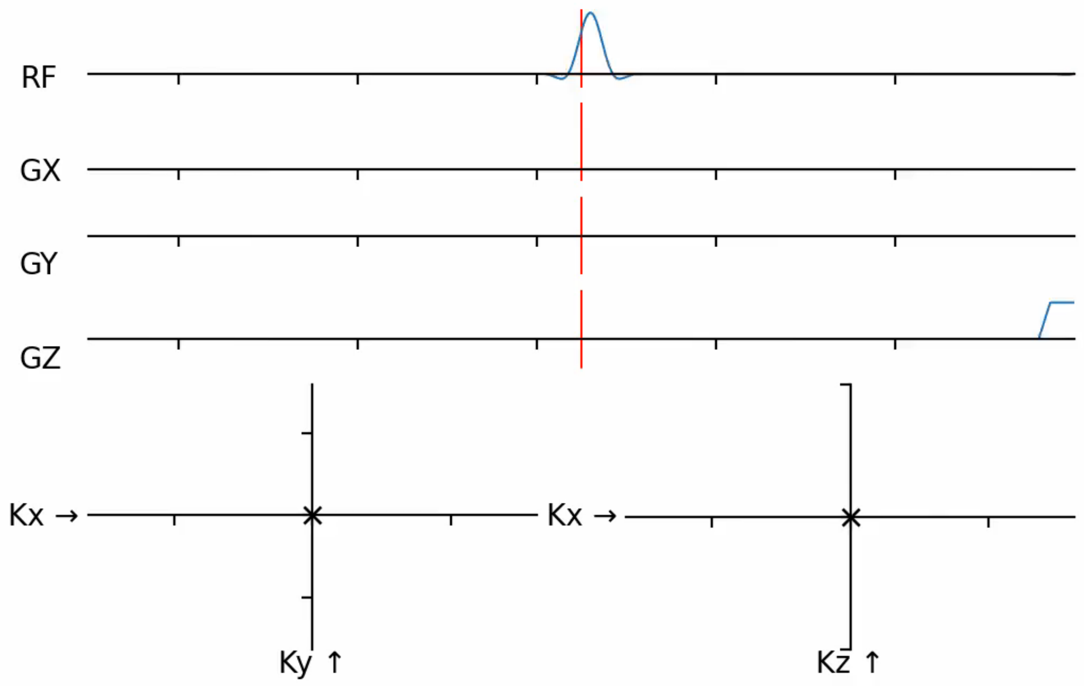
Example: 2D Cartesian trajectory

- Other choices which affect the trajectory
  - Echo time / Repetition time
  - Readout bandwidth
  - Phase/frequency encoding direction
  - Oversampling in frequency encoding direction
    - Encode larger FOV by measuring more samples, same measurement time
    - Suppress aliasing artifacts, and artifacts from filters applied in receive coil hardware
    - Often removed in first step of reconstruction
      - 1D IFFT → Crop → (1D FFT) on each line of raw data



Example: 2D Cartesian trajectory

- Full trajectory:

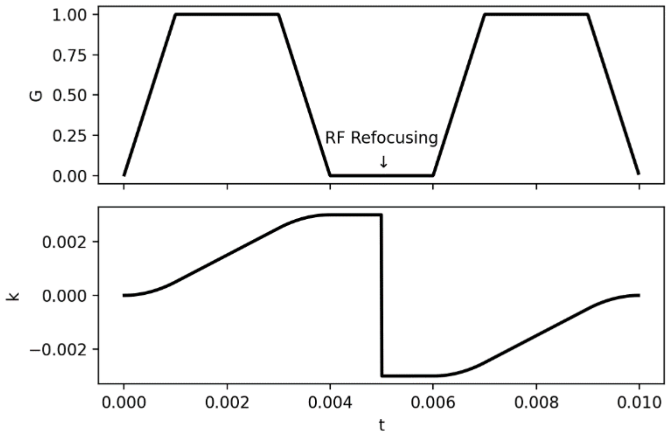


What defines the k-space trajectory?

- An MRI pulse sequence traverses k-space using gradients and RF pulses

- Area under gradients after excitation pulse
  - Reset to center of k-space at an RF excitation pulse
  - Mirrors in center of k-space at a RF refocusing pulse

$$k(t) = \int_0^t G(t)dt$$



- Simplified view of reality
  - Effects of RF pulses are more complicated
  - Imperfections
    - e.g. eddy currents, gradient non-linearity

Calculating the k-space trajectory in Pulseq

- Pulseq in Matlab provides `` `seq.calculate_kspacePP` `` function
  - `[ktraj_adc, t_adc, ktraj, t_ktraj, t_excitation, t_refocusing] = seq.calculateKspacePP() ;`
- Calculates both the full k-space trajectory (`ktraj`), and k-space trajectory during ADC events (`ktraj_adc`)
  - Also outputs RF excitation and refocusing times, and ADC times
- Note: In MATLAB, use `seq.calculate_kspace`
  - `k_traj_adc, k_traj, t_excitation, t_refocusing, t_adc = seq.calculate_kspace()`

Designing a k-space trajectory

- Using Pulseq we can design almost any gradient waveforms
  - Therefore: We can design almost any k-space trajectory
  - Important: What part of k-space is measured when?

- Limited by hardware
  - Maximum gradient strength
    - Maximum speed of k-space traversal
  - Maximum slew rate
    - Maximum change in k-space traversal speed/direction
  - Gradient duty cycle

- Physiological limits
  - Peripheral nerve stimulation (PNS)
  - Sometimes: Specific absorption rate (SAR)

Why change the k-space trajectory?

- Imaging speed
  - Faster k-space traversal can lower total scan time
  - Scan efficiency
    - Time spent receiving signal vs total scan time
  - Tradeoff SNR/resolution/scan time

- Sensitivity to artifacts
  - Trajectory affects distortion due to B_0-inhomogeneity (e.g. around metal implants)
  - Appearance of motion/flow artifacts
  - Artifacts from signal inconsistencies
    - E.g. relaxation between multiple acquisitions

- Contrast
  - Echo time (TE) typically defined by the time when k-space trajectory passes through center of k-space
  - Non-Cartesian trajectories can achieve very low echo times

MRI signal is not constant over time!

- Many contributing factors
  - Relaxation effects
    - T2/T2* relaxation
    - T1 relaxation/saturation
    - Diffusion
    - Magnetization transfer
  - Off-resonance
  - Physiology
    - Flow, motion
    - Injection (and wash out) of contrast agent
  - Sequence
    - Imaging in transient-state vs steady-state
    - Use of preparation gradients/RF pulses
    - Cardiac/respiratory triggering
  - Etc.

- Some factors affect mainly short term (e.g. frequency encoding), others affect longer term (e.g. phase encoding)

- Assuming constant signal is a common mistake in retrospective image reconstruction experiments
  - i.e. measured signal = FFT(image)

Overview

- Variations on Cartesian trajectories
  - Multiple echoes
  - K-space ordering
  - K-space undersampling

- Non-Cartesian trajectories
  - Radial
  - Spiral

Multiple echoes

- We are not limited to having one readout per TR!
  - You can have as many ADC events as you want (within reason..)
  - They do not even need to have the same number of samples!

- We are not limited to measuring a k-space location only once
  - Add measurements together
    - Averaging for better SNR, or to mitigate artifacts (e.g. motion)
  - Acquire multiple contrasts in one sequence

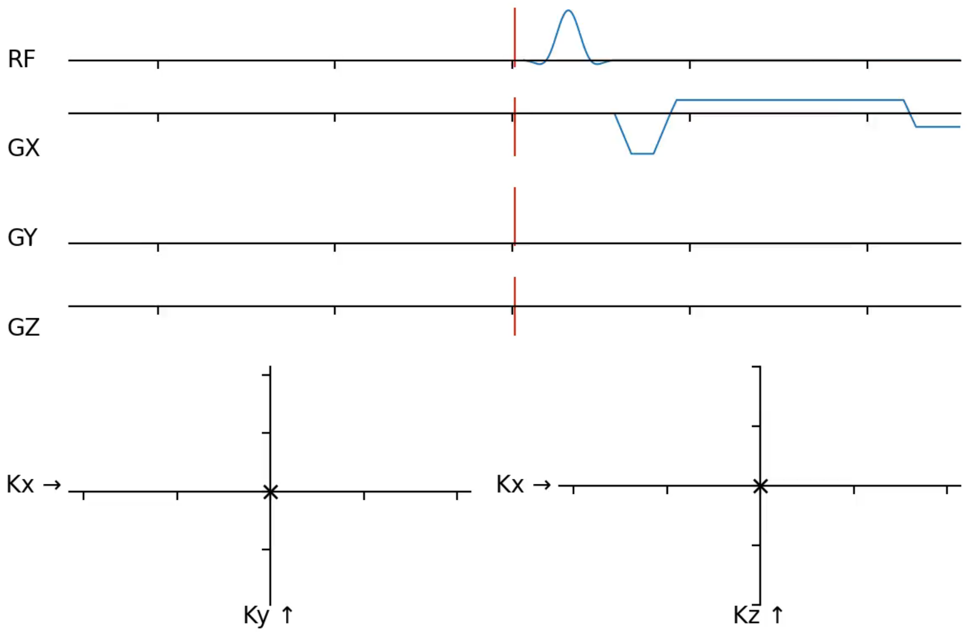
Multiple echoes

- Bi-polar readouts: Going back and forth
  - Acquires same k-space line twice (or more)
  - Multiple echo times
  - Opposite readout directions
    - Opposite direction for ΔB_0-induced distortion

- Applications:
  - T2* mapping
  - Quantitative susceptibility mapping
  - Dixon water-fat separation

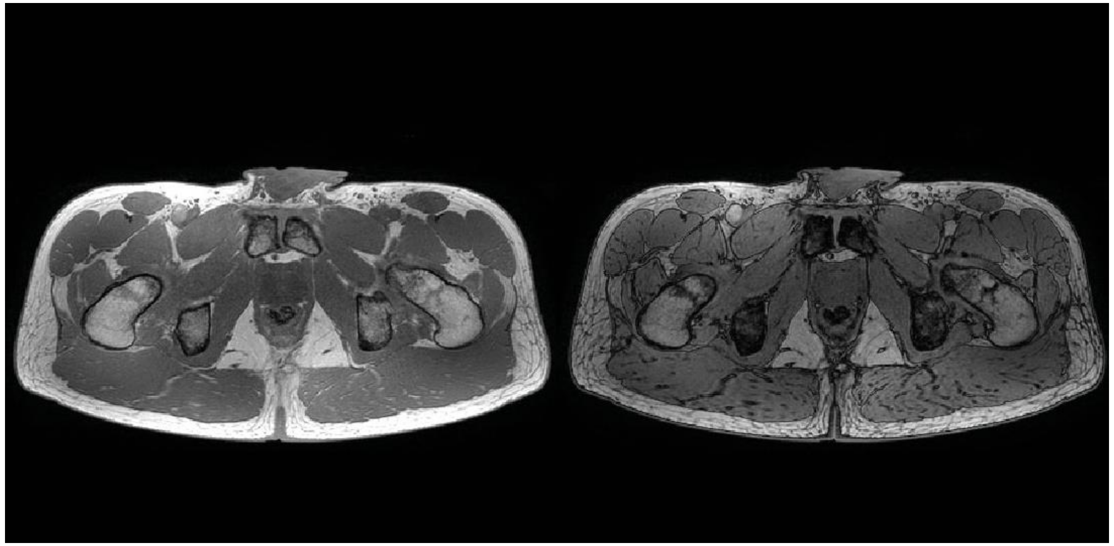
Implementation:

```
seq.add_block(gx_pre, gy_pre)
seq.add_block(gx, adc)
seq.add_block(pp.scale_grad(gx, -1), adc)
```



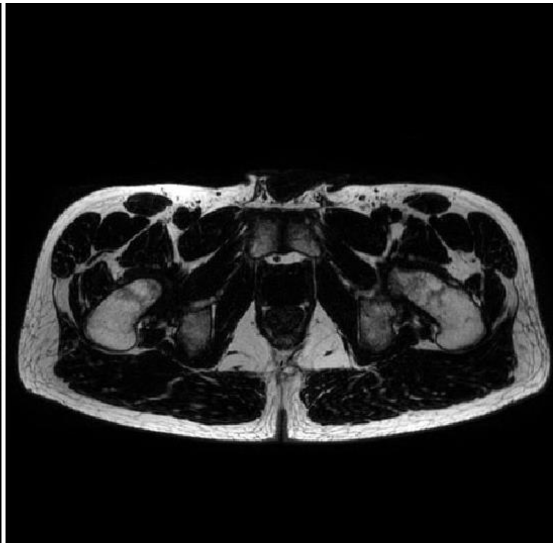
Multiple echoes: Water-fat separation

Acquired images (2 echoes, complex-valued)



Water

Fat

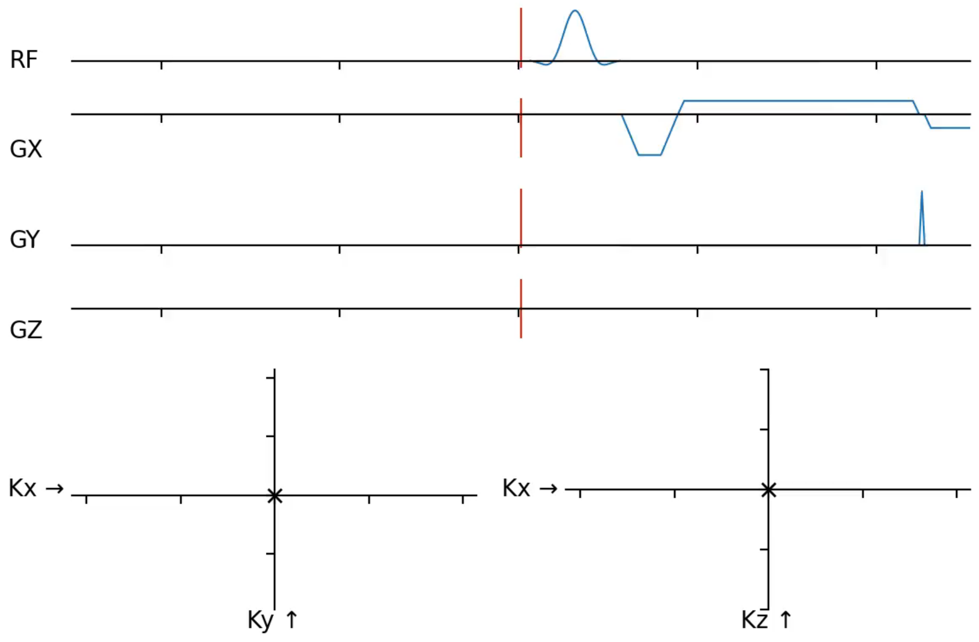


Multiple echoes: EPI

- Alternatively, we can add phase encoding to encode multiple k-space lines
  - Echo planar imaging (EPI)
  - Bi-polar readout, with small phase encoding gradient ("blip") in between
- Applications:
  - Fast imaging
  - Diffusion-weighted imaging

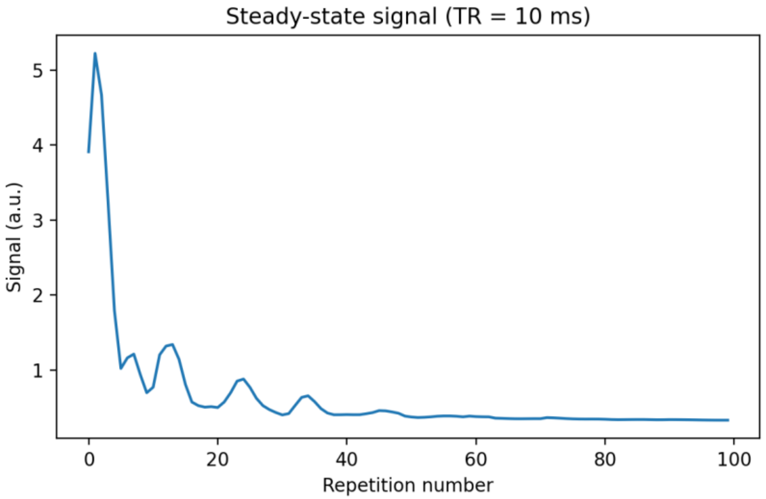
Implementation:

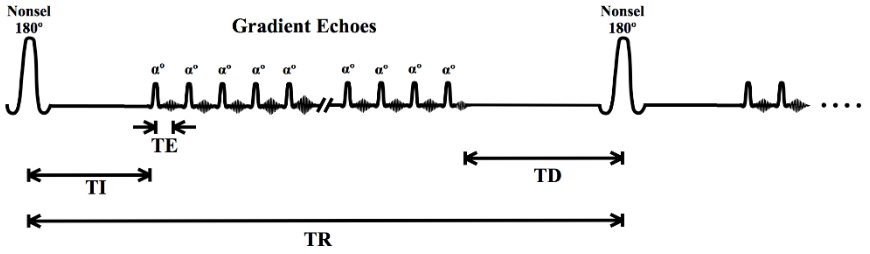
```
seq.addBlock(gxPre, gyPre) ;
seq.addBlock(gx, adc) ;
seq.addBlock(gyBlip) ;
seq.addBlock(mr.scaleGrad(gx, -1), adc) ;
```

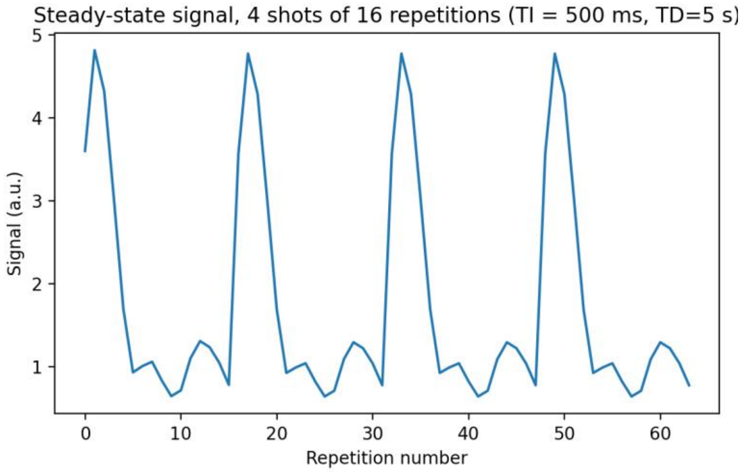


K-space ordering

- Ordering matters!
  - Signal changes can occur over repetitions
    - E.g. patient motion, flow
    - Contrast injection
    - Triggered scans
- In many sequences, the MR signal changes by design
  - Imaging in transient-state (e.g. start of gradient echo acquisitions)
  - Sequences with pre-pulses
    - Example: IR-GRE (Siemens: MPRAGE)

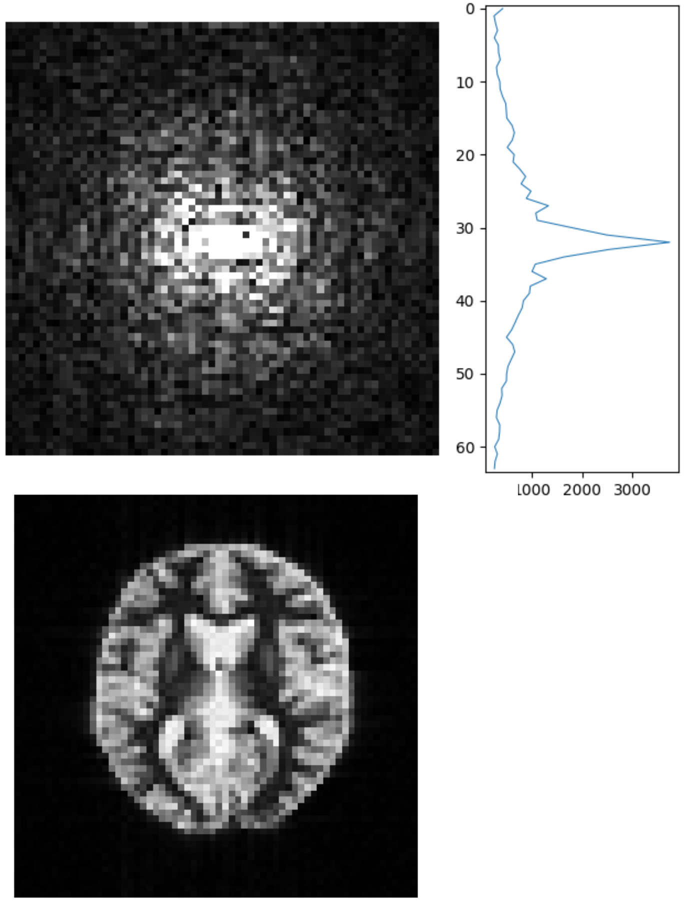




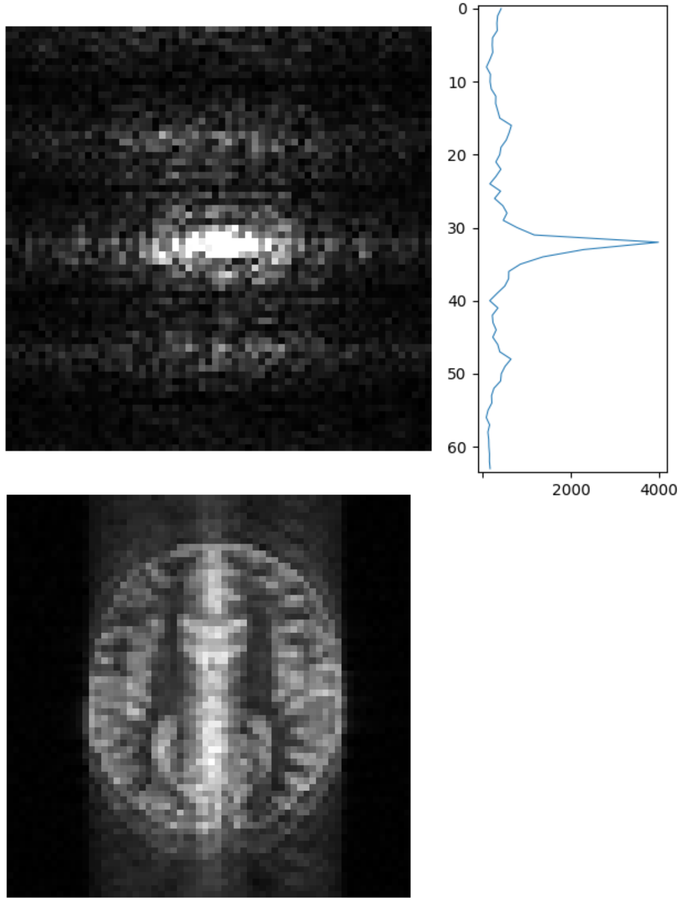


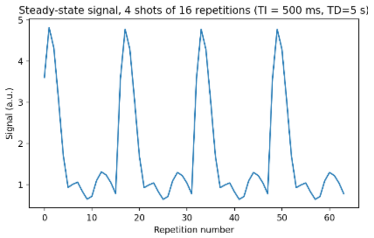
K-space ordering: IR-GRE, linear order

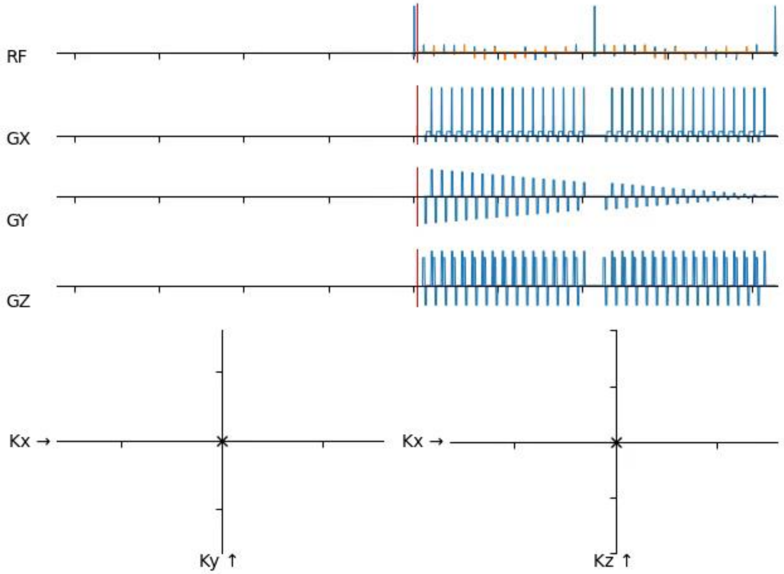
Expectation



Reality

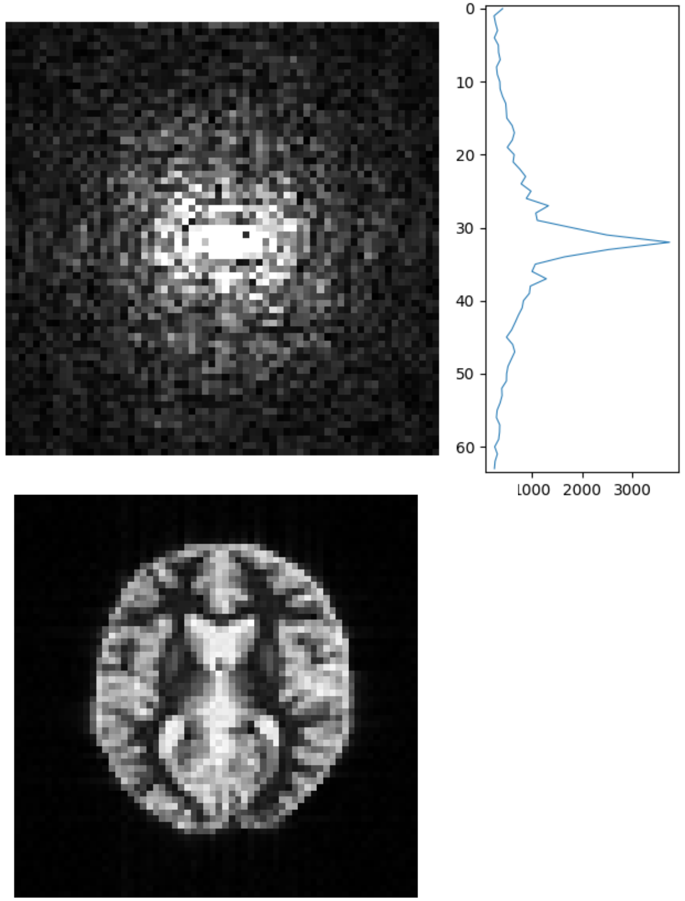




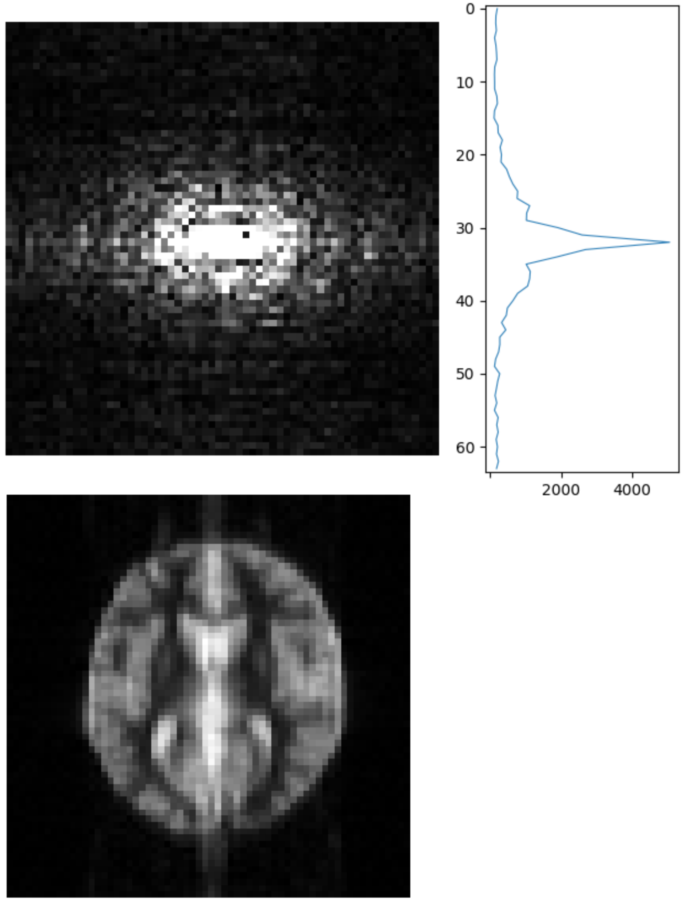


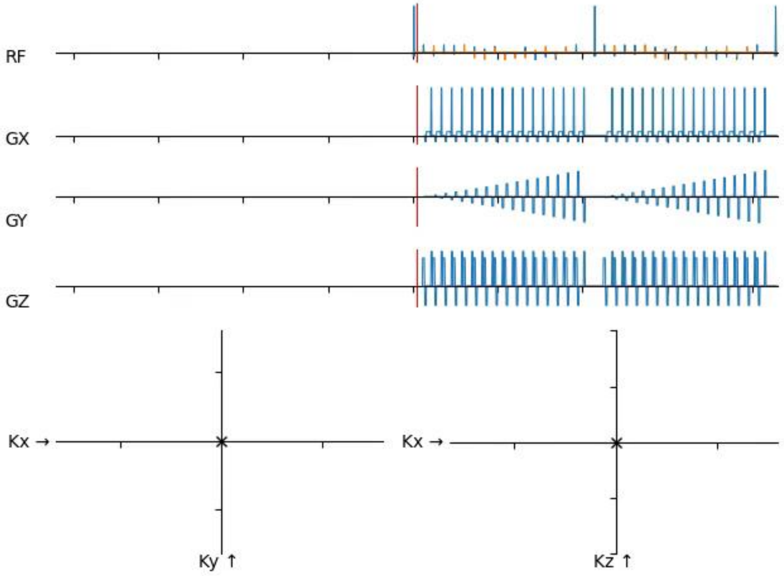
K-space ordering: IR-GRE, centric order

Expectation



Reality





K-space ordering

- Common types of ordering in Cartesian acquisition
  - Linear (high to low, or low to high)
    - In 3D: YZ vs ZY
  - Center-out (or reversed)
  - Random

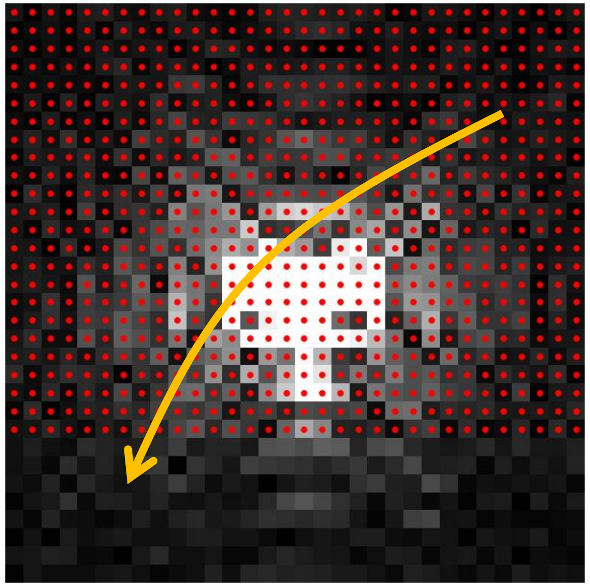
- Implementation: DIY in exercise session!

Undersampling

- Break the Nyquist limit to scan faster
  - Acquire fewer phase encoding lines
- Common undersampling methods
  - Partial Fourier
  - Parallel imaging
  - Compressed sensing
- Special reconstruction needed to fill in missing data
  - Note: It may not always be apparent, but SNR almost always decreases with undersampling!
- Implementation
  - Special case of k-space ordering: also decide which k-space lines we acquire

Partial Fourier

$k_{\vec{r}} \cong \overline{k_{-\vec{r}}}$

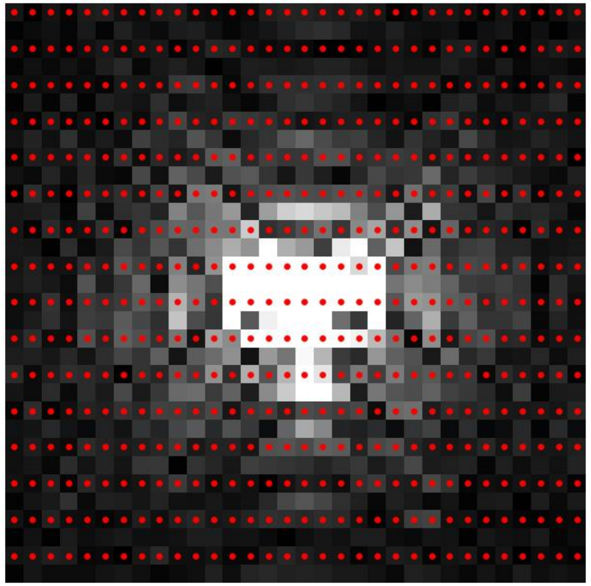


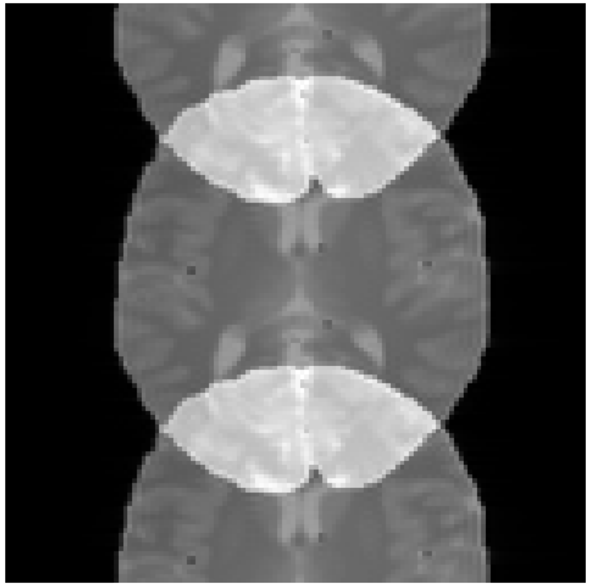
- If phase information in an image is smooth, the k-space is approximately Hermitian symmetric
- Accelerate by acquiring asymmetric k-space
- Homodyne reconstruction

McGibney G, Smith MR, Nichols ST, Crawley A. Quantitative evaluation of several partial Fourier reconstruction algorithms used in MRI. Magn Reson Med 1993;30:51-59

Parallel imaging

- Information from multiple receive coils (often) gives information on spatial encoding
- Encode smaller FOV by skipping lines
- Reconstruction
  - SENSE: External calibration scan to get coil sensitivities
    - Reconstruction in image space
  - GRAPPA: Fully sampled k-space center (calibration lines)
    - Reconstruction in k-space
  - CG-SENSE: Generalized solution
    - Linear solver for: $MFSx = y$



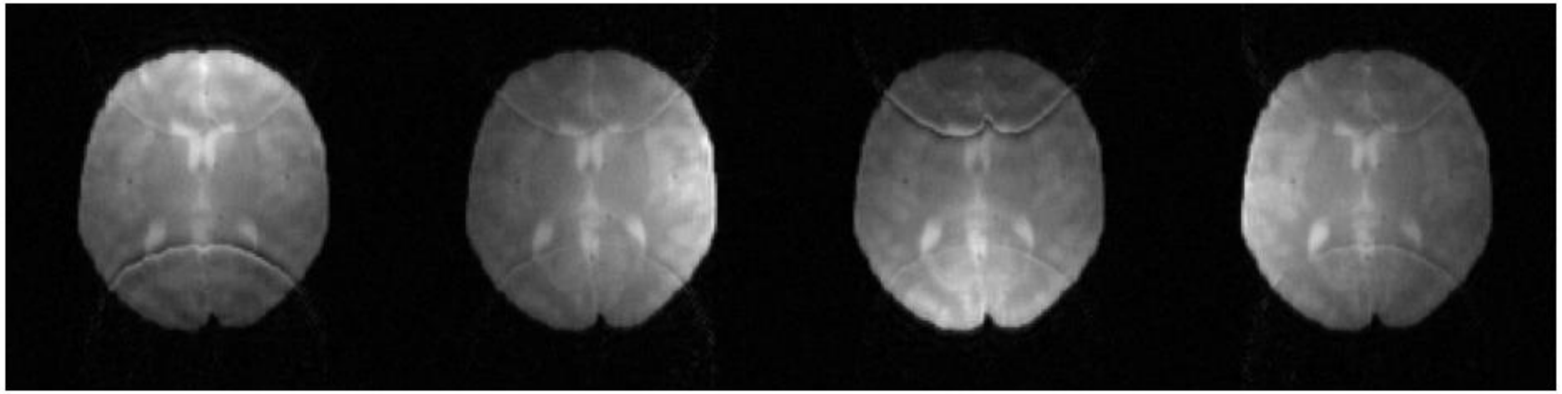


Pruessmann KP, Weiger M, Scheidegger MB, Boesiger P. SENSE: Sensitivity encoding for fast MRI. Magn Reson Med 1999; 42:952-962.

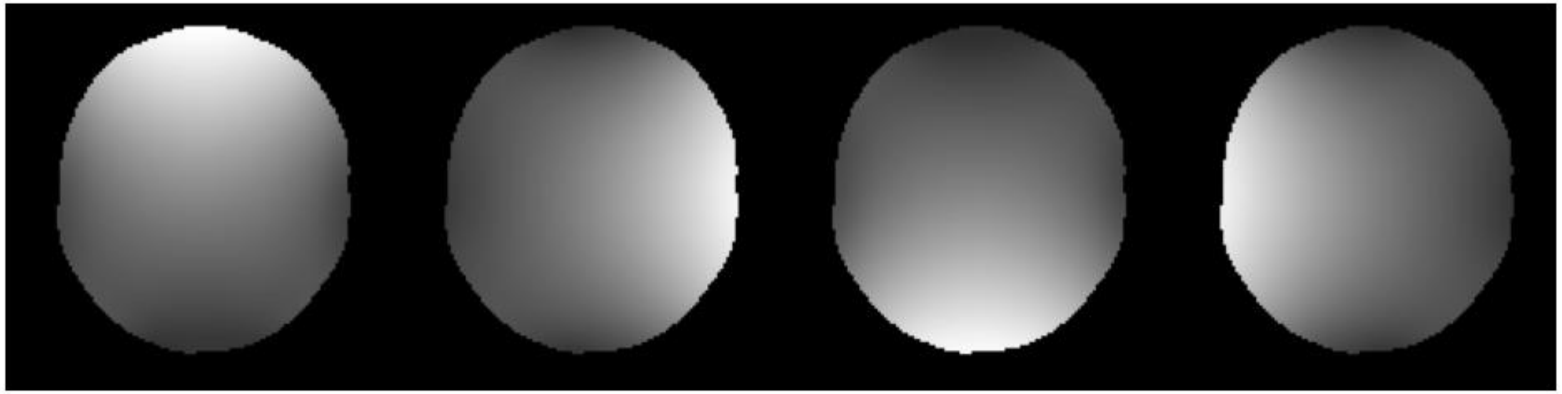
Griswold MA, Jakob PM, Heidemann RM, et al. Generalized autocalibrating partially parallel acquisitions (GRAPPA). Magn Reson Med 2002; 47:1202-1210

Parallel imaging

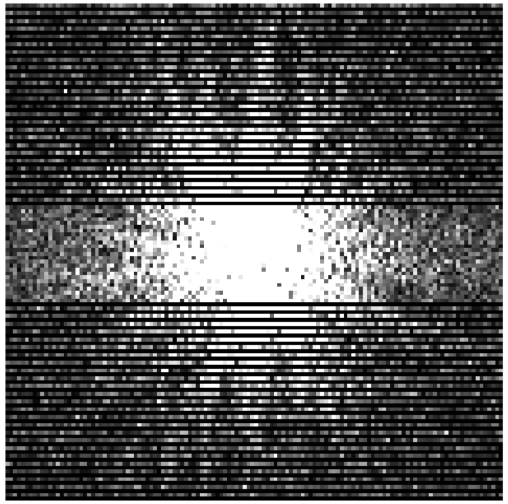
Coil images



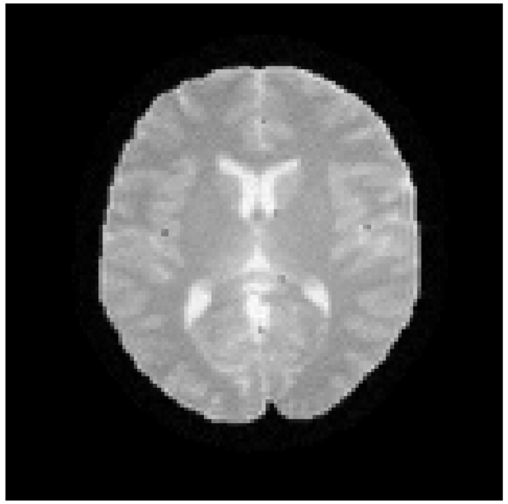
Coil sensitivities



K-space (first coil)



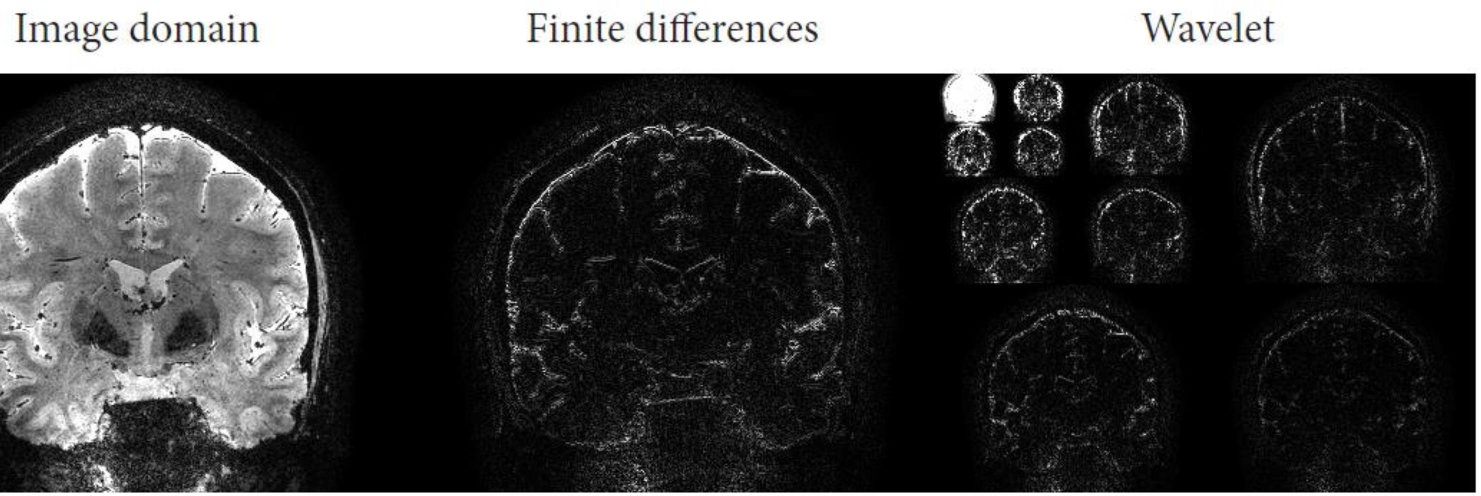
Reconstruction



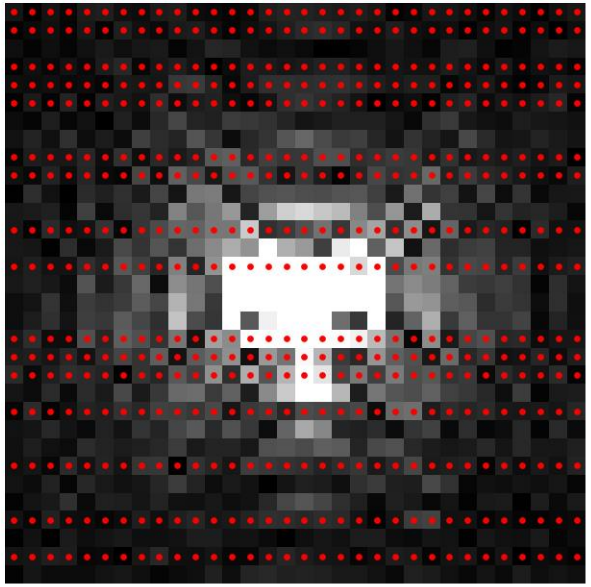
Compressed sensing

- Medical images are often compressible
  - Information content is less than number of voxels
  - Sparsity
    - Many zeroes/low signals in image
    - Smooth or piecewise constant signal
    - Many zeroes after a given transform (e.g. wavelet transform)

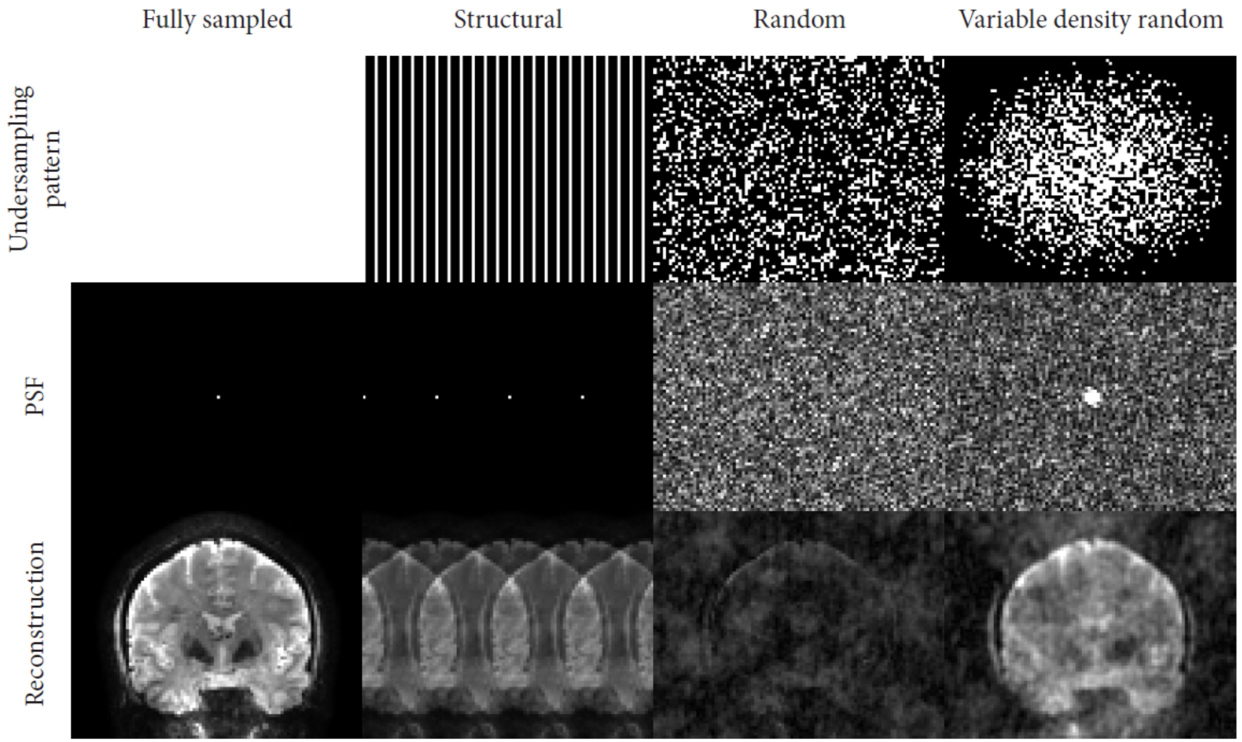
- Sub-Nyquist sampled images can be reconstructed by assuming the reconstructed image is sparse in some fashion



Compressed sensing



- Instead of dropping k-space lines at regular intervals, sample (pseudo-)randomly
  - Random aliasing pattern



Compressed sensing: Reconstruction

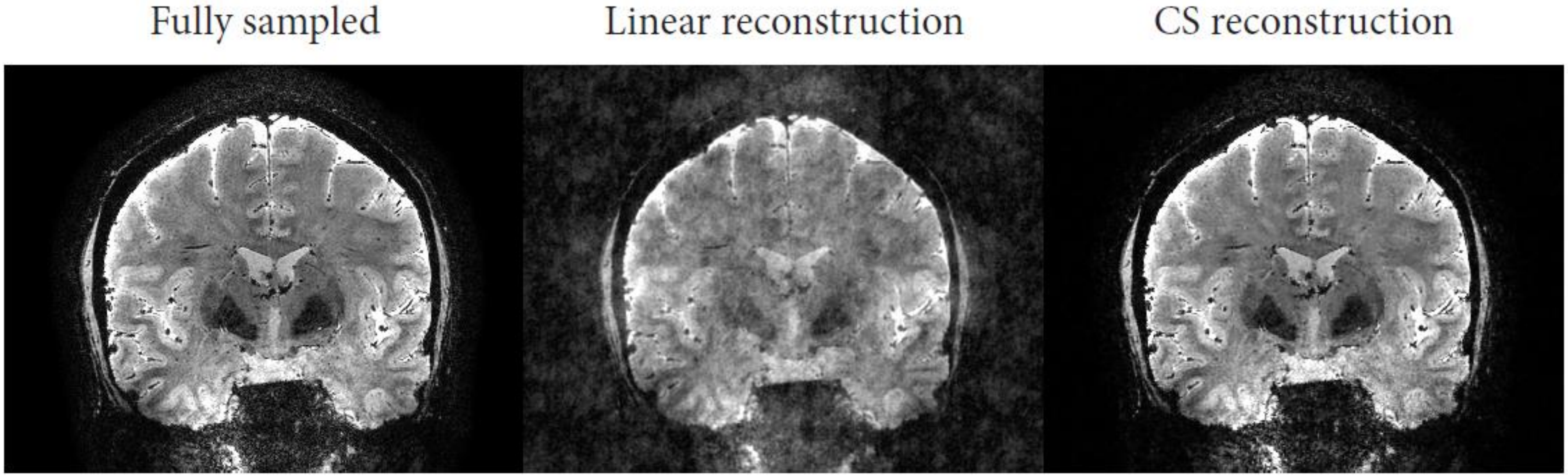
- Iterative reconstruction
  - Enforce data consistency
  - Enforce sparsity

  - Minimize: $||MFSx - y||_2^2 + \lambda||Wx||_1$

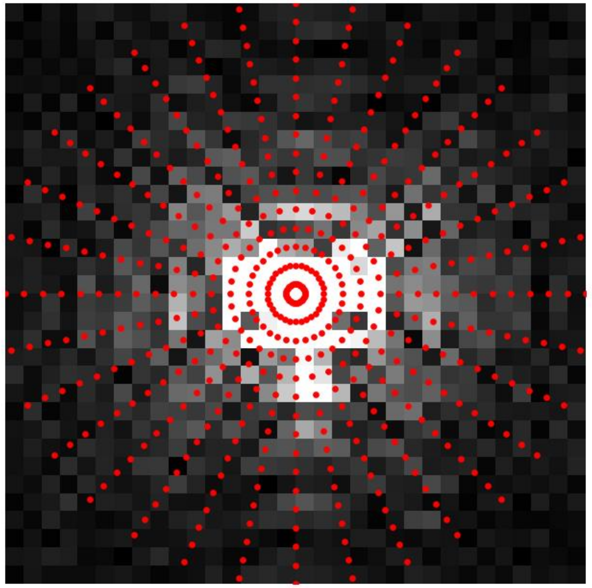
- In practice almost always combined with parallel imaging
- More recently: AI, deep neural networks

- CS also combines really well with non-Cartesian sampling

Compressed sensing: Reconstruction

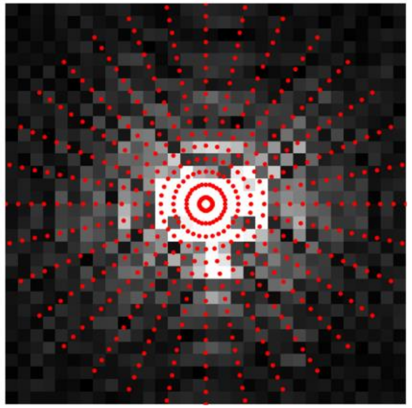


Non-Cartesian sampling

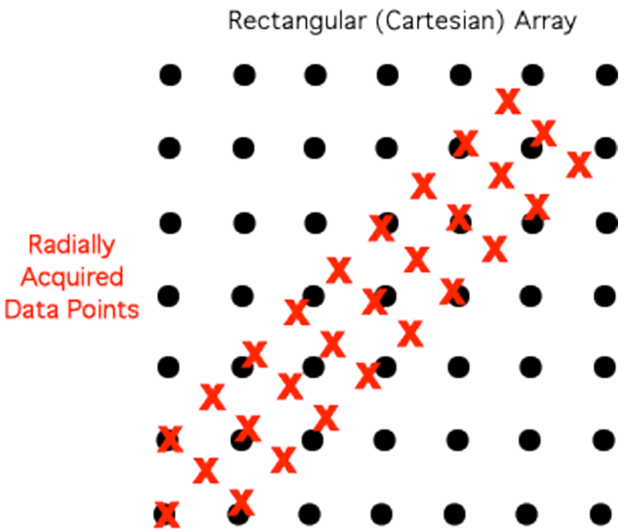


- Sampling on a Cartesian grid makes reconstruction easy
  - But we are not restricted to this grid
- Why?
  - Artifacts for motion and other inconsistencies can be mitigated
  - Scan efficiency
  - Dynamic imaging
    - E.g. free-breathing abdominal/cardiac imaging (collect data first, then separate motion phases and reconstruct)
  - Ultimate freedom in k-space trajectory
- Challenges
  - Implementation
  - Reconstruction
  - Artifacts (e.g. B_0-inhomogeneity)
  - Typically not available in clinical applications

Non-Cartesian reconstruction



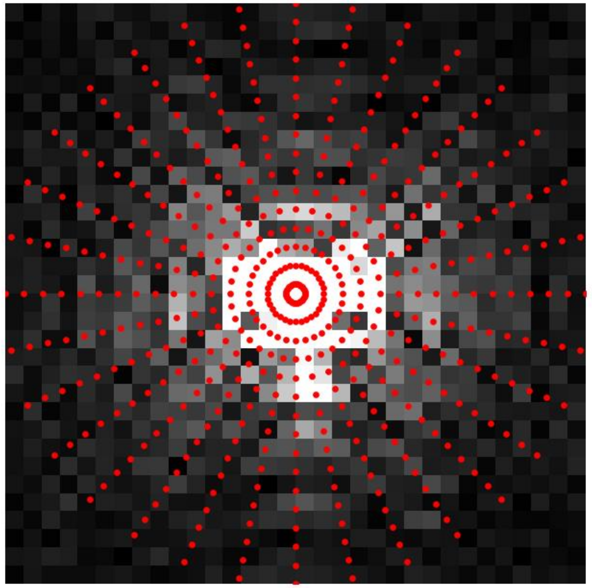
- Adjoint Non-uniform Fast Fourier Transform (NUFFT)
  - Gridding: Interpolate data at known non-Cartesian locations to Cartesian grid
  - Then perform 2D/3D IFFT
  - Many implementations available!
  - Note: Adjoint NUFFT is not an inverse NUFFT



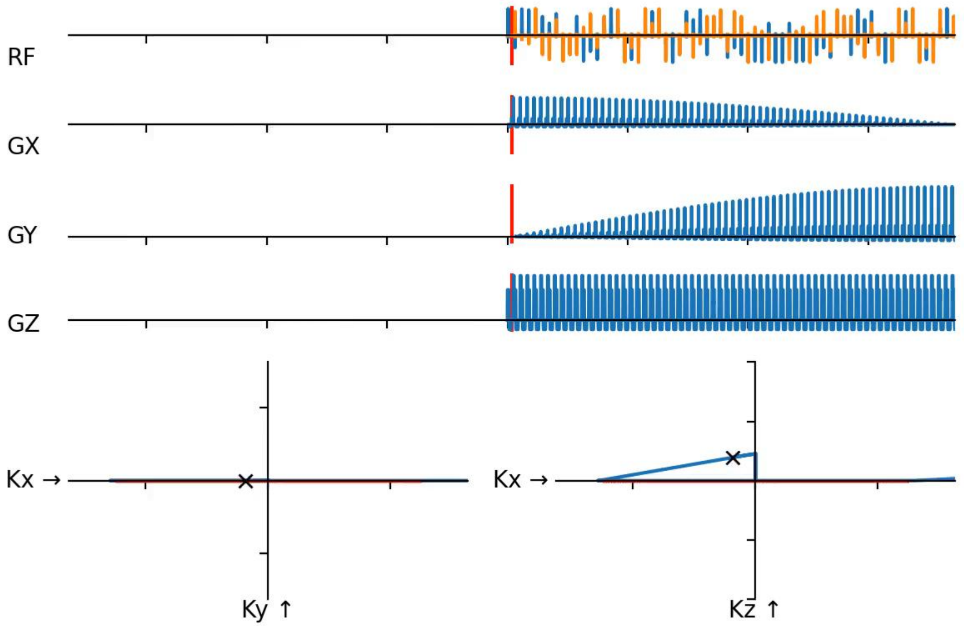
- Approximate inverse NUFFT
  - Perform sampling density compensation
    - Areas with many samples need to be weighted less
  - Then adjoint NUFFT
  - Alternatively: Iterative linear least squares reconstruction

- K-space coordinates easily calculated with Pulseq!
  - `seq.calculateKspacePP()`

Radial



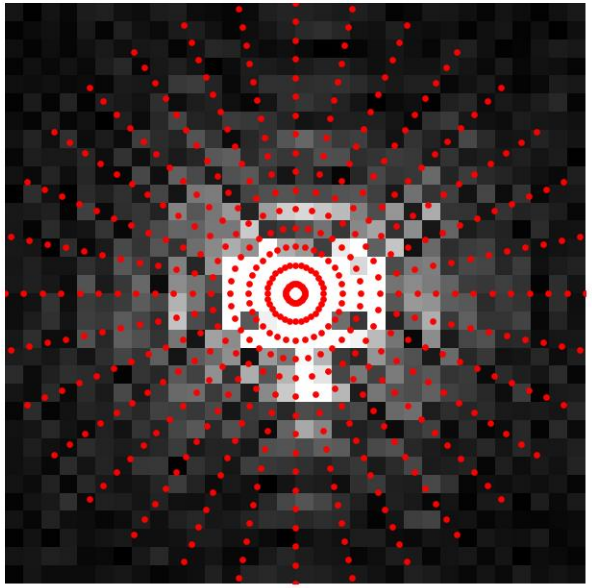
- "Star" pattern
  - Every acquisition is a line through center of k-space ("spoke")
    - Projection of all signal perpendicular to direction
  - No phase encoding
  - Sampling density: Very dense sampling in k-space center



Implementation:

```
seq.addBlock(mr.rotate('z', phi, gxPre, gyPre)) ;
seq.addBlock(mr.rotate('z', phi, gx), adc) ;
```

Radial



- Advantages
  - Averaging in k-space center
    - Signal inconsistencies average out
    - More benign motion artifacts (blurring)
  - Frequency encoding in all directions
    - Easy to reduce artifacts from signal outside FOV with readout oversampling
  - Benign undersampling artifacts

- Disadvantages
  - Nyquist sampling requires more repetitions because low sampling density on k-space boundary
  - Streaking artifacts can spread through entire image

Radial

- Example: Free-breathing imaging

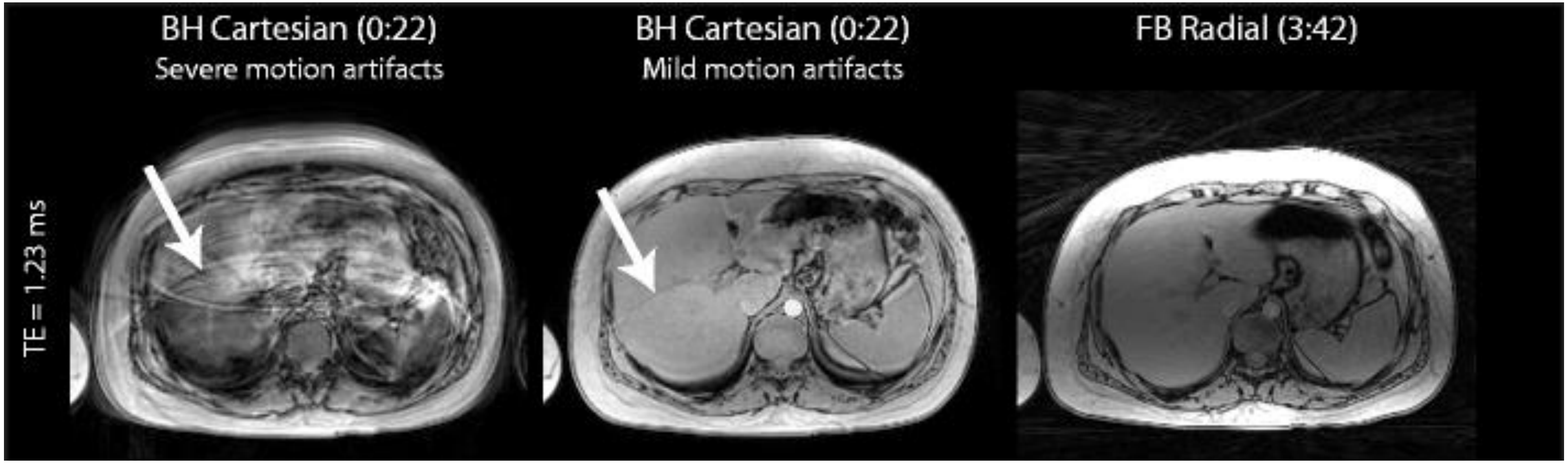
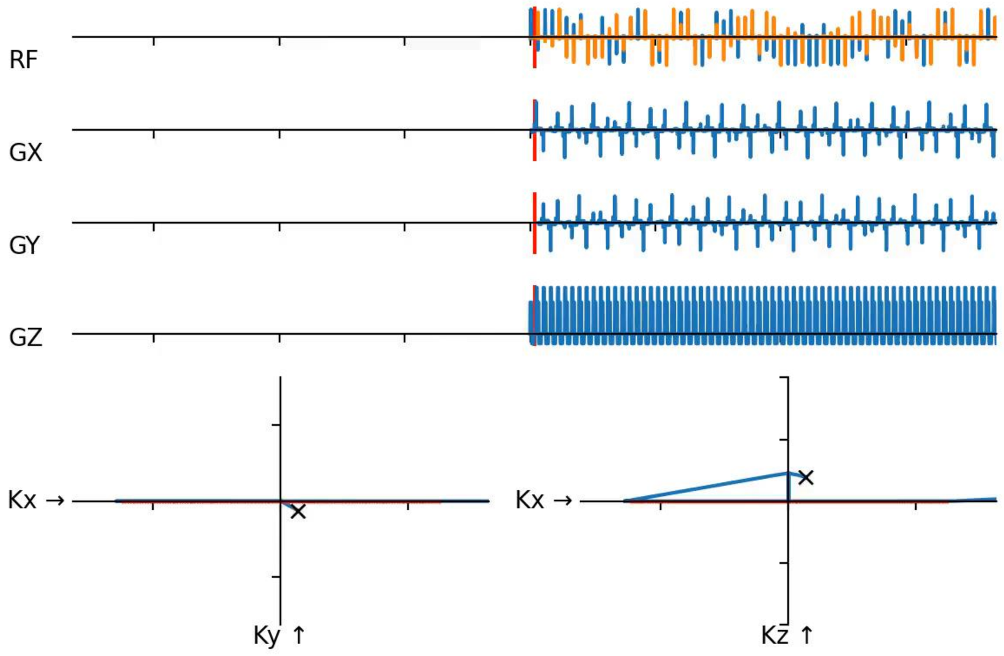


Image source:
Armstrong, T., Ly, K.V., Murthy, S. et al. Free-breathing quantification of hepatic fat in healthy children and children with nonalcoholic fatty liver disease using a multi-echo 3-D stack-of-radial MRI technique. Pediatr Radiol 48, 941–953 (2018). https://doi.org/10.1007/s00247-018-4127-7

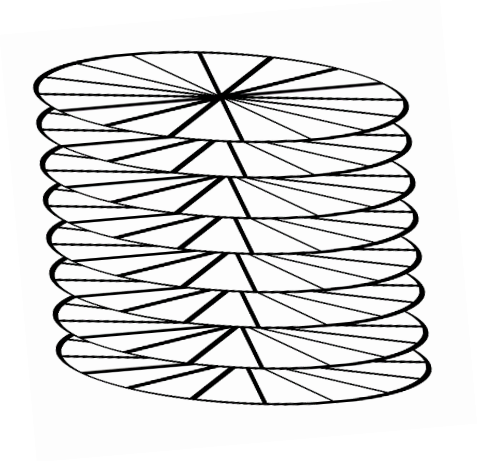
Golden angle radial

- Golden-angle increments in spoke angle
  - More or less uniform sampling of k-space over any sub-segment of scan
  - Often used for dynamic/real-time imaging



3D Radial

- Stack of stars
  - In-plane radial combined with Cartesian phase-encoding in slice direction



- Kooshball
  - Radial encoding in all 3 dimensions

Center-out radial

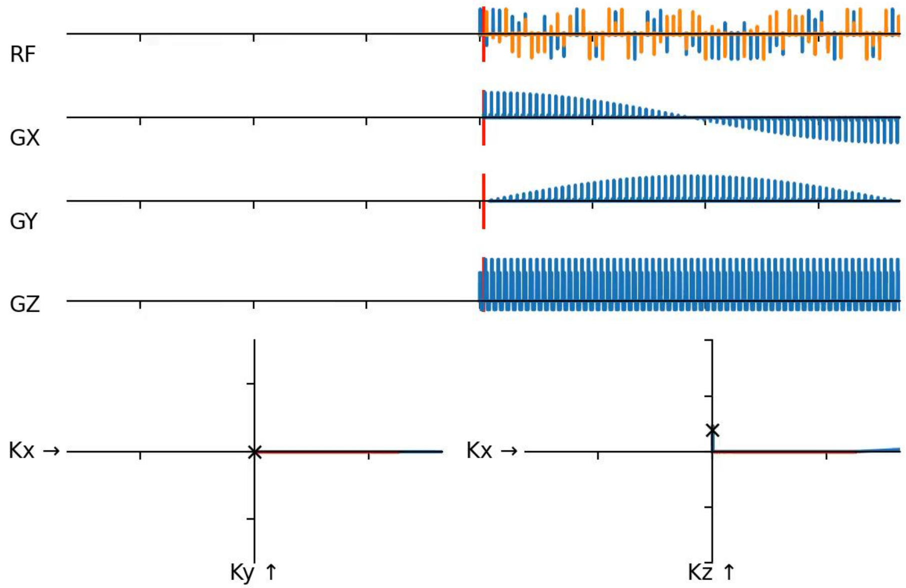
- Immediately start acquisition from the center of k-space

- Ultra-short echotime (UTE)
  - In the order of ~10+ µs
  - Sample during the ramp-up of the readout gradient

- Often non-selective 3D kooshball to avoid time spent in phase encoding and slice selection

Implementation:

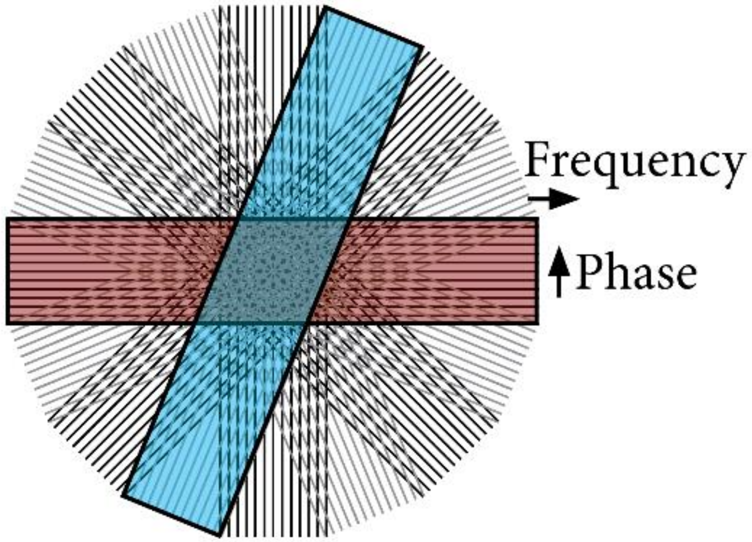
```
seq.addBlock(mr.rotate('z', phi, gxPre, gyPre)) ;
seq.addBlock(mr.rotate('z', phi, gx), adc) ;
```



Radial: PROPELLOR/BLADE

- Rotate a 2D Cartesian sampling pattern in a radial pattern
  - Each blade encodes small Cartesian k-space

- Each blade can be reconstructed individually
  - Low resolution perpendicular to radial direction
  - Enables retrospective motion correction

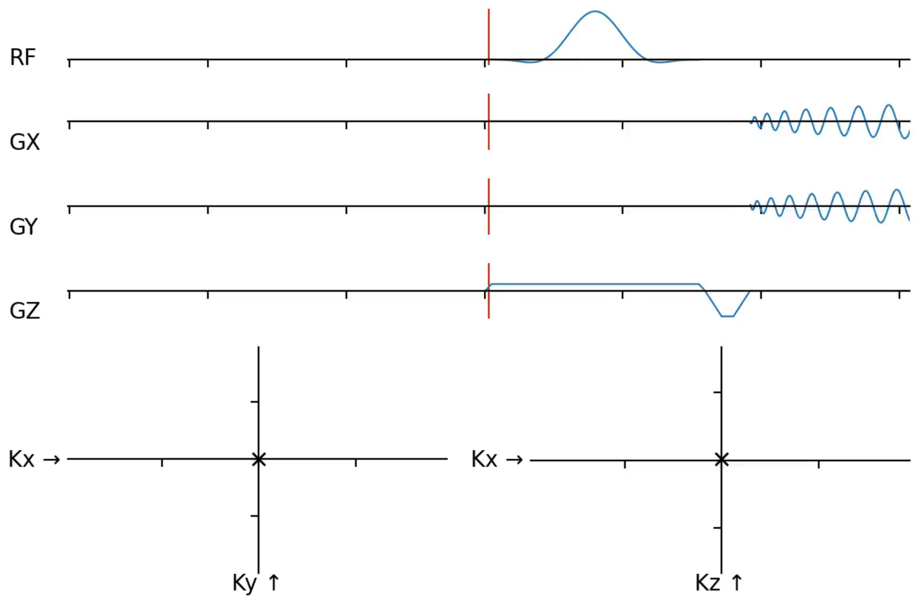


Spiral

- Trajectory start in k-space center and spirals outward
  - Sinusoidal gradient waveforms

- Single-shot
  - Encode whole k-space in one repetition
  - Real-time dynamic imaging
  - Not suitable for 3D

- Multi-shot
  - Same gradient waveforms, but each repetition uses a different rotation (similar to radial)
  - 3D: e.g. spiral cones



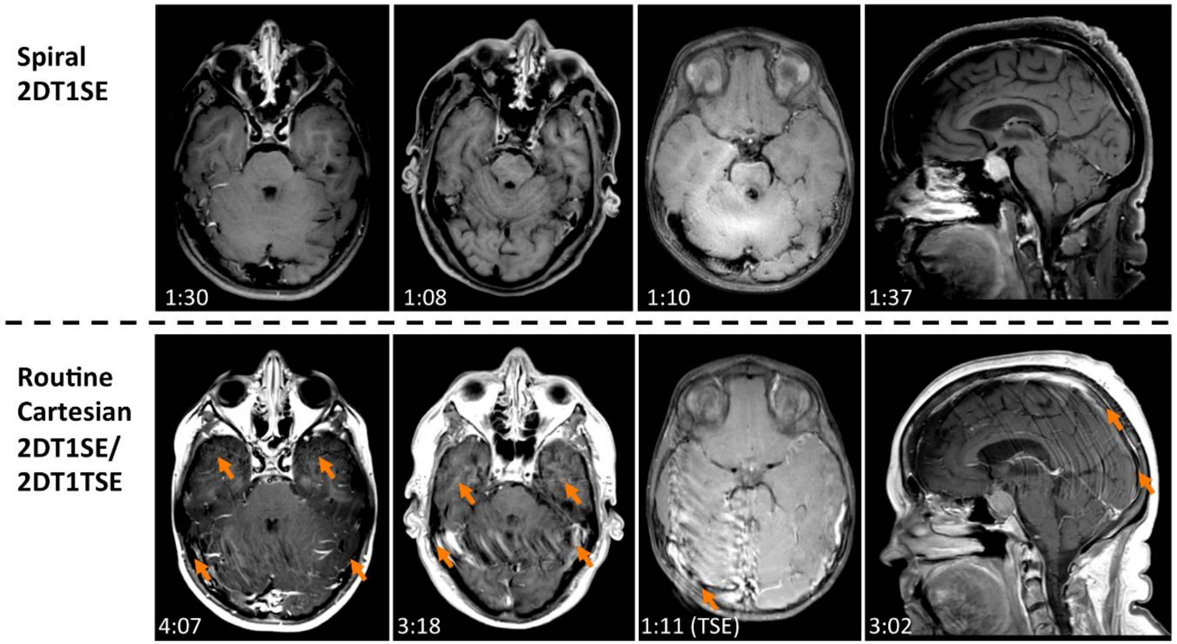
Spiral

- Advantages
  - Very efficient
    - High speed of k-space traversal
    - Almost no dead-time
  - Low TE
  - Robust to motion
  - SNR efficiency
    - Incoherent aliasing artifacts
      - Also appropriate for CS reconstruction and MR fingerprinting
    - Low T2* decay in center of k-space
    - Uniform sampling everywhere but the center

- Disadvantages
  - Susceptible to off-resonance artifacts (e.g. susceptibility, fat, bad shimming)
  - Reconstruction

Spiral: Artifacts

Flow artifacts



Susceptibility artifacts

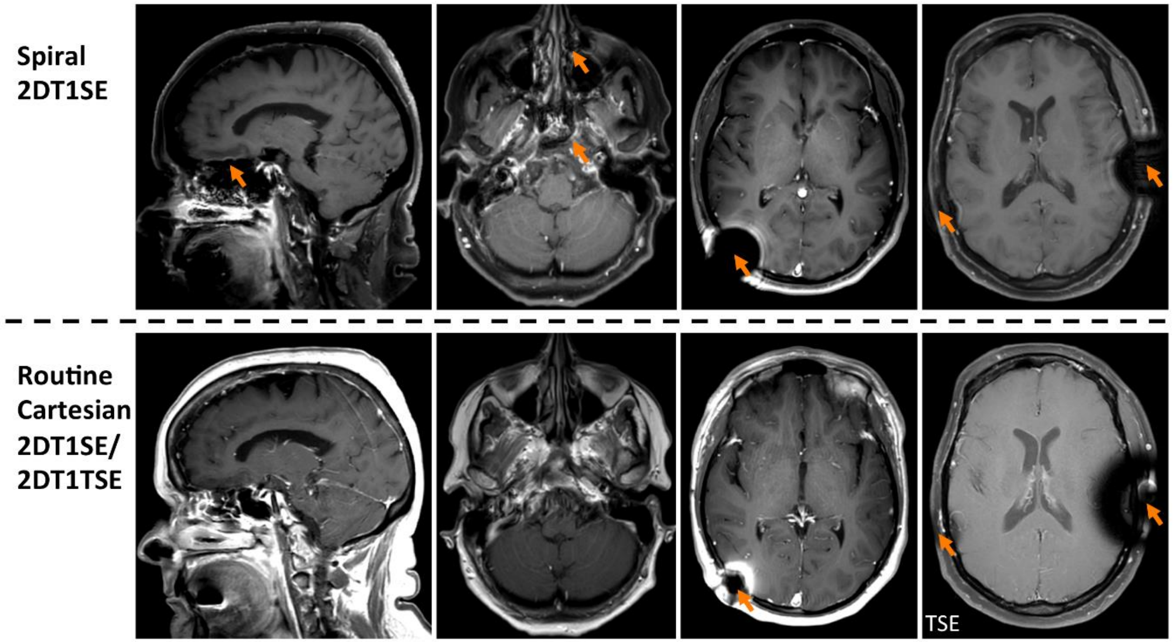


Image source:
M. Ooi et al, Benefits and Challenges of Spiral MRI in Routine Clinical Brain Imaging: Early Results, in Proceedings of the 26th Annual Meeting of the ISMRM, nr. 0349, 2018

Spiral: Implementation notes

- Rapid switching of gradients
  - Easy to hit PNS limits
  - If you have access to SAFE PNS model parameters:
    - `seq.calcPNS(…)`
  - Acoustic resonances: check “forbidden frequencies” of your scanner

- `traj2grad` function converts trajectory specification to gradient waveforms
  - Does not check maximum gradient amplitude, slew rate and PNS limits

- Using a long ADC event
  - Siemens-specific:
    - `seq.setDefinition('MaxAdcSegmentLength', samples_per_segment)`
      - Tells interpreter to split long ADC into segments of specified length
    - Each segment start/end needs to be aligned to gradient raster time
    - Maximum of 128 segments

Final remarks

- Which trajectory works for you depends on the application
  - Nothing comes for free
- Do not be afraid to experiment and combine different elements of different trajectories

- Questions?