How The Go Stone Moves

Simulating the motion of go stones as rigid bodies

Posted by Glenn Fiedler (http://web.archive.org/web/20181107181454/https://gafferongames.com/about) on Thursday, February 21, 2013

Introduction

Hi, I'm Glenn Fiedler. Welcome to **Virtual Go** (http://web.archive.org/web/20181107181454/https://gafferongames.com/categories/virtual-go/), my project to create a physically accurate computer simulation of a Go board and stones.

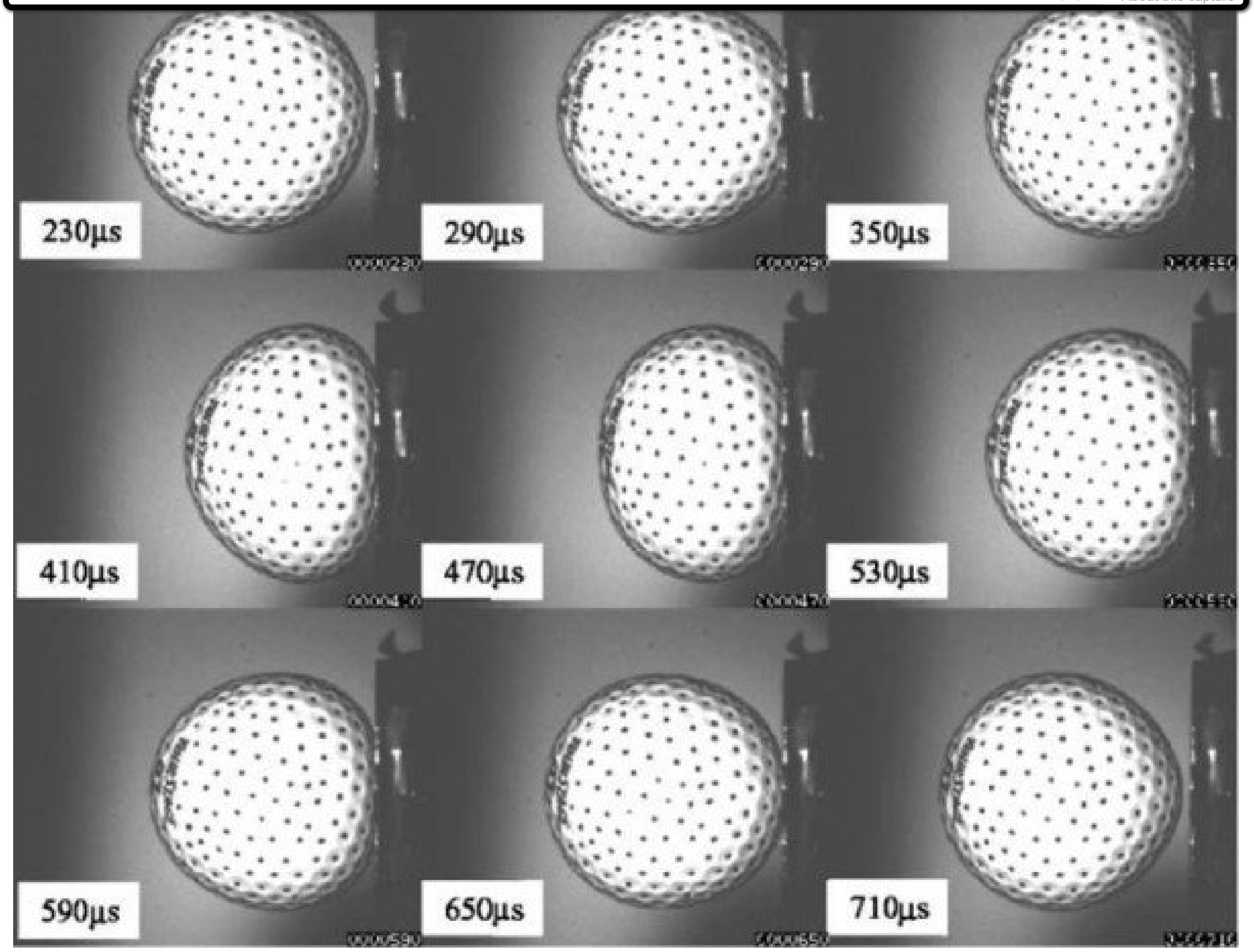
In previous articles we mathematically defined the shape of a go stone and tessellated its shape so it can be drawn with 3D graphics hardware.

Now we want to make the go stone move, obeying Newton's laws of motion so the simulation is physically accurate. The stone should be accelerated by gravity and fall downwards. I also want the stone to rotate so it tumbles as it falls through the air.

The Rigid Body Assumption

Try biting down on a go stone and you'll agree: go stones are very, very hard.

Golf balls are pretty hard too, but if you look at a golf ball being hit by a club in super-slow motion, you'll see that it deforms considerably during impact.



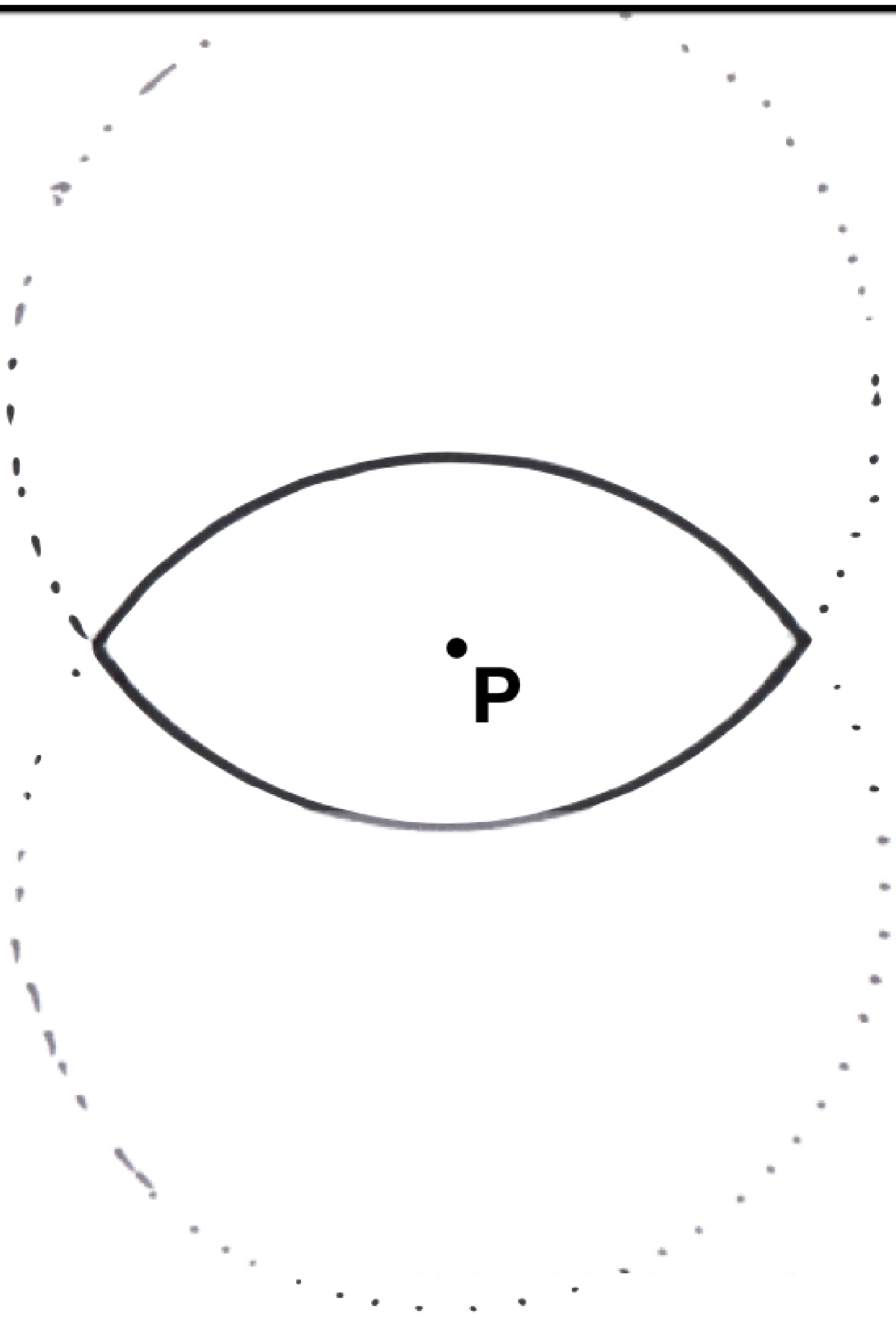
The same thing happens to all objects in the real world to some degree. Nothing is truly rigid. No real material is so hard that it never deforms.

But this is not the real world. This is Virtual Go :) It's a simulation and here we are free to make whatever assumptions we want. And the smartest simplification we can make at this point is to assume that the go stone is perfectly rigid and does not deform under any circumstance.

This is known as the rigid body (http://web.archive.org/web/20181107181454/https://en.wikipedia.org/wiki/Rigid_body) assumption.

Working in Three Dimensions

Because the go stones are rigid, all we need to represent their current position is the position of the center. As the center moves, so does the rest of the stone.

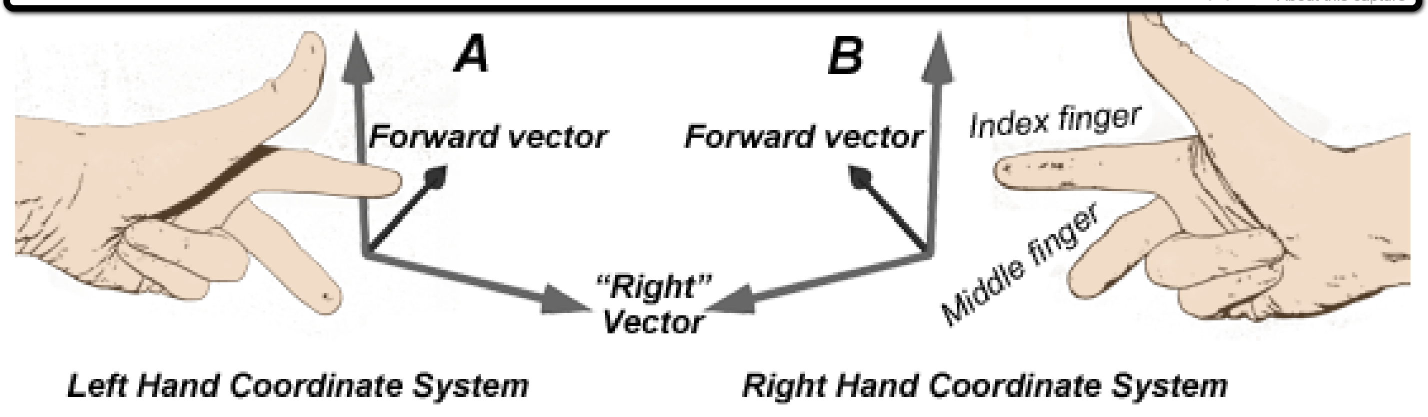


We'll represent this position using a three dimensional vector **P**.

Let's define the axes so we know what the x,y,z components of P mean:

- Positive x is to the right
- Positive y is up
- Positive z is into the screen

This is what is known as a left-handed coordinate system. So called because I can use the fingers on my left hand to point out each positive axis direction without breaking them.



I've chosen a left-handed coordinate system purely on personal preference. Also, I'm left-handed and I like my fingers :)

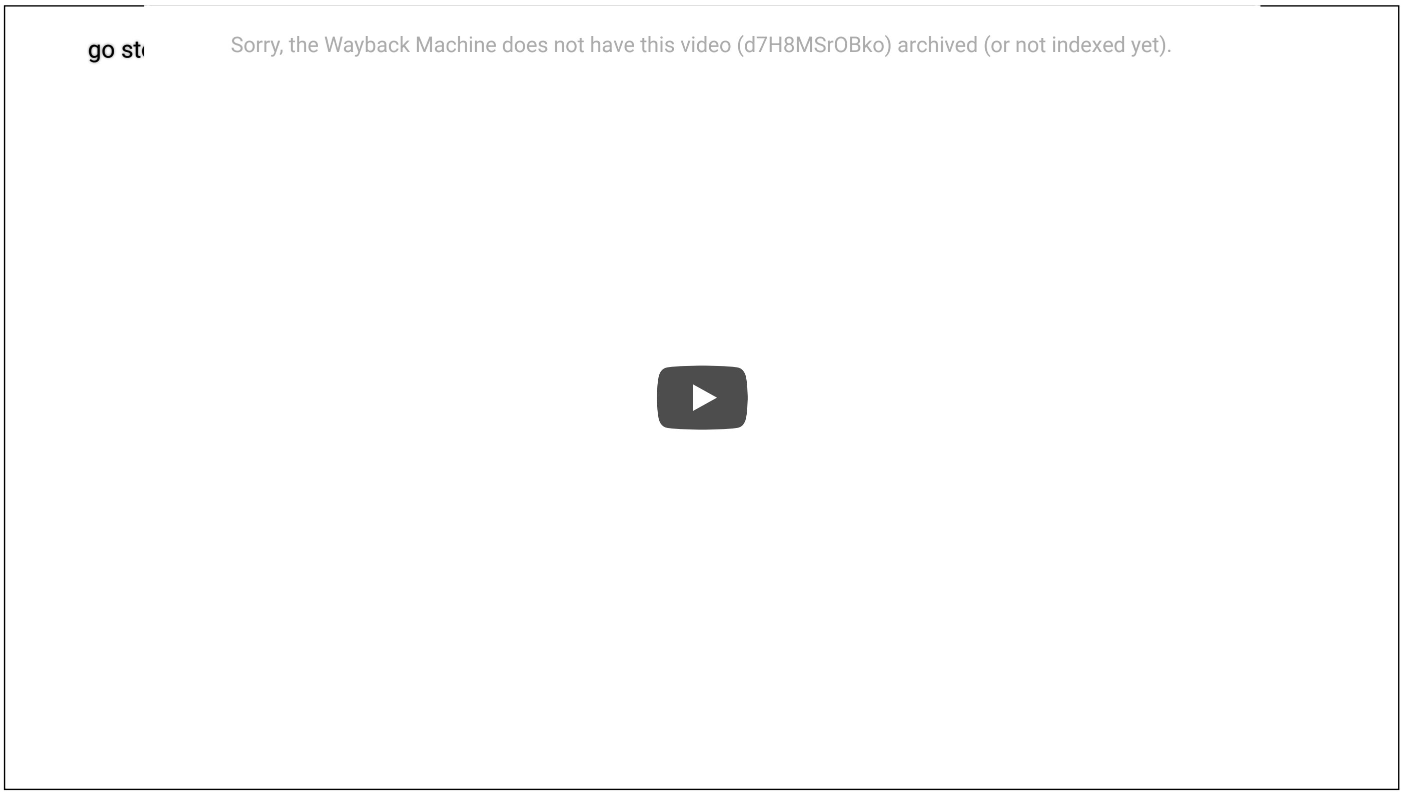
Linear Motion

Now we want to make the stone move.

To do this we need the concept of velocity. Velocity is also a vector but it's not a point like P. Think of it more like a direction and a length. The direction of the velocity vector is the direction the stone is moving and the length is the speed it's moving in some unit per-second. Here I'll use centimeters per-second because go stones are small.

For example, if we the stone to move to the right at a rate of 5 centimeters per-second then the velocity vector is (5,0,0).

To make the stone move, all we have to do is add the velocity to the position once per-second: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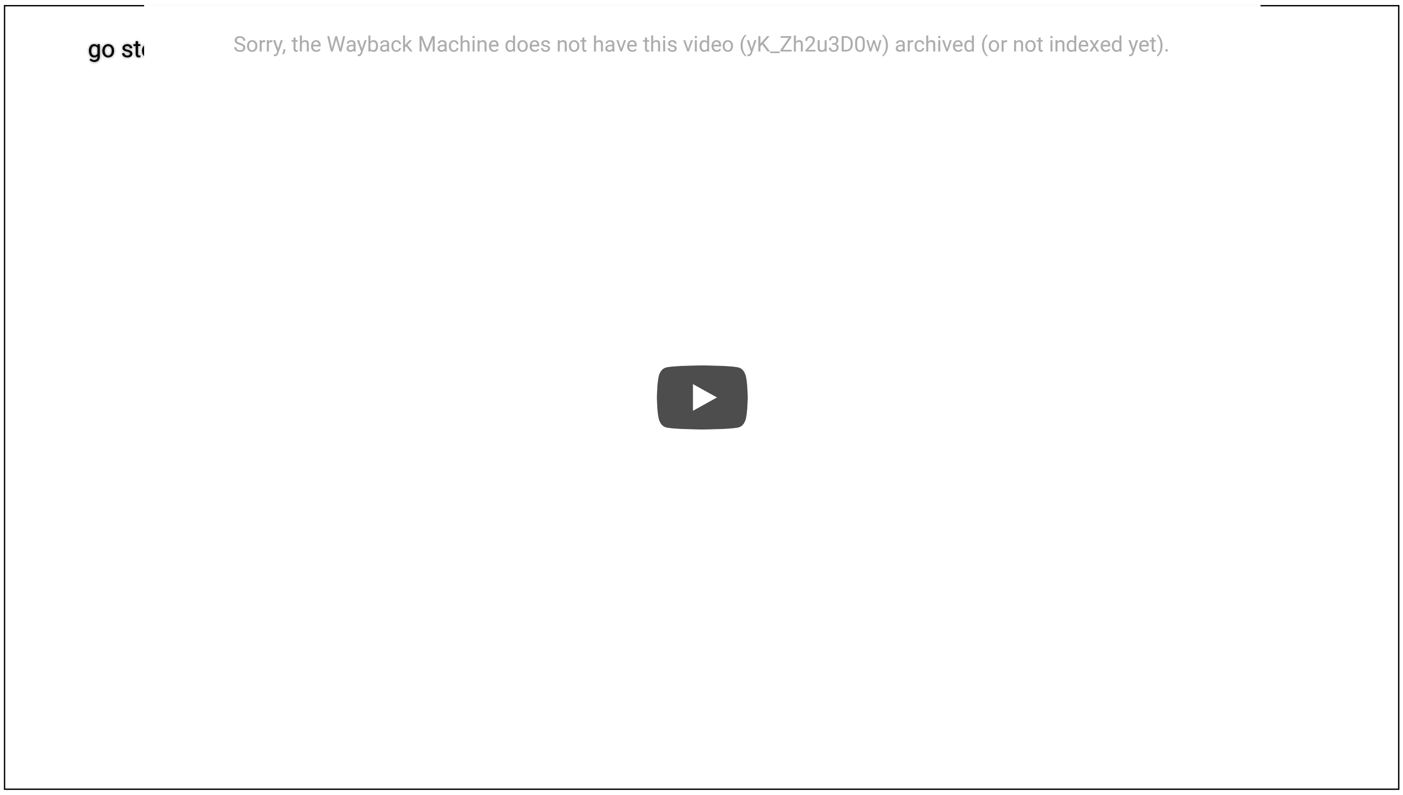
one big step, we'll take 60 smaller steps per-second, each step being $^1/_{60}$ of the velocity.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You can generalize this to any framerate with the concept of delta time or "dt". To calculate delta time invert frames per second: dt = 1/fps and you have the amount of time per-frame in seconds. Next, multiply velocity by delta time and you have the change in position per-frame.

```
const float fps = 60.0f;
const float dt = 1 / fps;
while ( !quit )
{
    stone.rigidBody.position += stone.rigidBody.velocity * dt;
    RenderStone( stone );
    UpdateDisplay();
}
```

This is actually a very simple type of numerical integration (http://web.archive.org/web/20181107181454/https://en.wikipedia.org/wiki/Euler_method).

Gravitational Acceleration

Next we want to add gravity.

To do this we need to change velocity each frame by some amount downwards due to gravity. Change in velocity is known as acceleration. Gravity provides a constant acceleration of 9.8 meters per-second, per-second, or in our case, 98 centimeters per-second, per-second because we're working in centimeters.

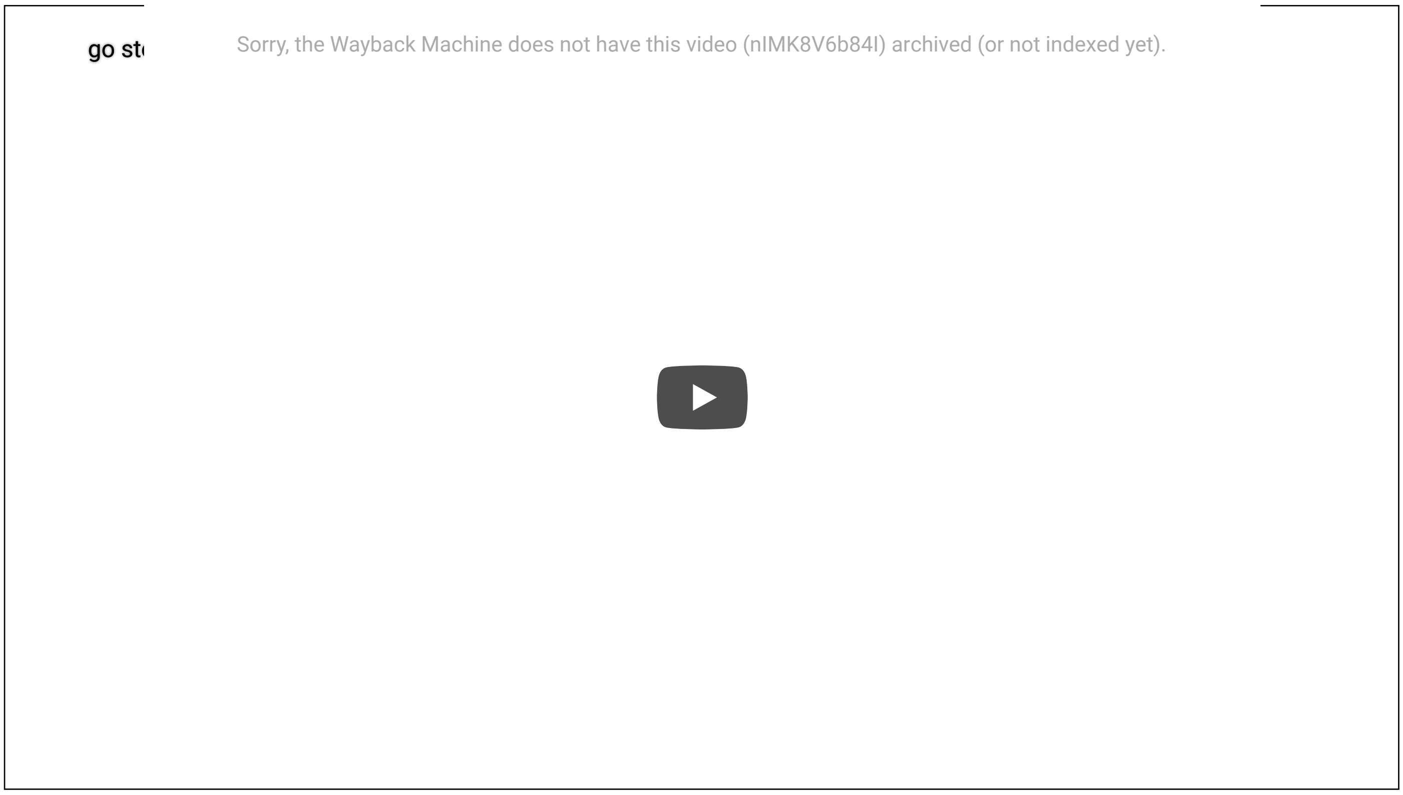
Acceleration due to gravity is also a vector. Since gravity pulls objects down, the acceleration vector is (0,-98,0). Remember, +y axis is up, so -y is down.

So how much does gravity accelerate the go stone in 1/60th of a second? Well, 98 * $^1/_{60}$ = 1.633… Hey wait. This is exactly what we did with velocity to get position!

Here's the code:

```
    float gravity = 9.8f * 10;
    float fps = 60.0f;
    float dt = 1 / fps;
    while ( !quit )
    {
        stone.rigidBody.velocity += vec3f( 0, -gravity, 0 ) * dt;
        stone.rigidBody.position += stone.rigidBody.velocity * dt;
        RenderStone( stone );
        UpdateDisplay();
    }
```

And here's the result:



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As you can see, now that we've added acceleration due to gravity the go stone moves in a parabola just like it does in the real world when it's thrown.

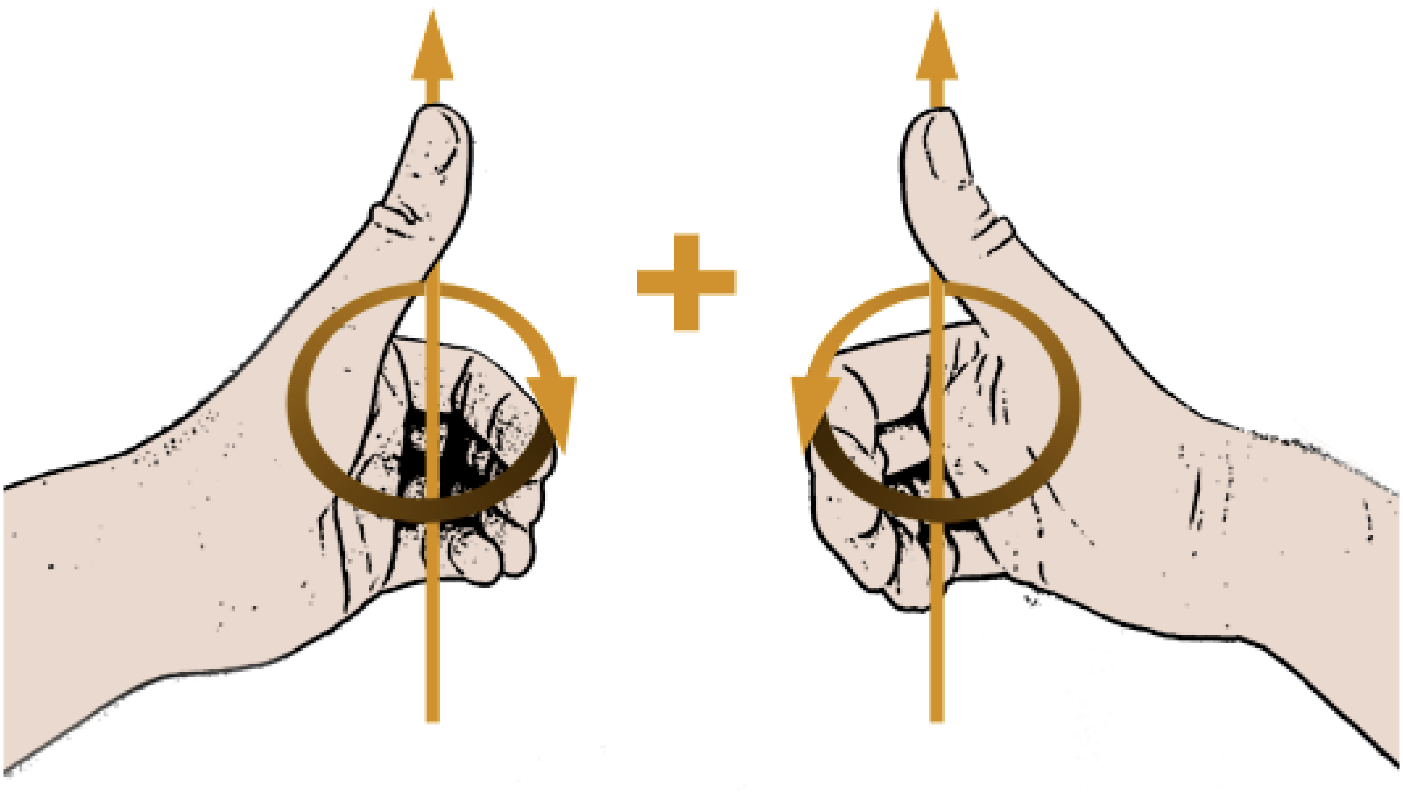
Angular Motion

Now let's make the stone rotate!

First we have to define how we represent the orientation of the stone. For this we'll use a quaternion (http://web.archive.org/web/20181107181454/https://en.wikipedia.org/wiki/Quaternions_and_spatial_rotation).

Next we need the angular equivalent of velocity known as… wait for it… angular velocity. This too is a vector aka a direction and a length. It's direction is the axis of rotation and the length is the rate of rotation in radians per-second. One full rotation is 2*pi radians or 360 degrees so if the length of the angular velocity vector is 2*pi the object rotates around the axis once per-second.

curling your fingers. The direction your fingers curl is the direction of rotation. Notice if you do the same thing with your right hand the rotation is the other way.



How do we integrate orientation from angular velocity? Orientation is a quaternion and angular velocity is a vector. We can't just add them together.

The solution requires a reasonably solid understanding of quaternion math and how it relates to complex numbers. Long story short, we need to convert our angular velocity into a quaternion form and then we can integrate that just like we integrate any other vector. For a full derivation of this result please refer to this excellent article (http://web.archive.org/web/20181107181454/https://fgiesen.wordpress.com/2012/08/24/quaternion-differentiation/).

Here is the code I use to convert angular velocity into quaternion form:

```
inline quat4f AngularVelocityToSpin( quat4f orientation, vec3f angularVelocity )
{
    const float x = angularVelocity.x();
    const float y = angularVelocity.y();
    const float z = angularVelocity.z();
    return 0.5f * quat4f( 0, x, y, z ) *
        orientation;
}
```

And once I have this spin quaternion, I can integrate it to find the change in the orientation quaternion just like any other vector.

```
    {
        quat4f spin = AngularVelocityToSpin(
            stone.rigidBody.orientation,
            stone.rigidBody.angularVelocity );
        stone.rigidBody.orientation += spin * iteration_dt;
        stone.rigidBody.orientation = normalize( stone.rigidBody.orientation );
        RenderStone( stone );
        UpdateDisplay();
    }
```

The only difference is that after integration I renormalize the quaternion to ensure it doesn't drift from unit length, otherwise it stops representing a rotation.

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Yep. That go stone is definitely rotating.

Why Quaternions?

Graphics cards typically represent rotations with matrices, so why are we using quaternions when calculating physics instead of 4x4 matrices? Aren't we bucking the trend a bit here?

Not really. There are many good reasons to work with quaternions:

- It's easier to integrate angular velocity using a quaternion than a 3x3 matrix
- Normalizing a quaternion is faster than orthonormalizing a 3x3 matrix
- It's really easy to interpolate between two quaternions

We'll still use matrices but as a secondary quantity. This means that each frame after we integrate we convert the quaternion into a 3x3 rotation matrix and combine it with the position into a 4x4 rigid body matrix and its inverse like this:

```
        mat4f matrix;
        rotation.toMatrix( matrix );
        matrix.value.w = simd4f_create( position.x(),
                                        position.y(),
                                        position.z(),
                                        1 );
        return matrix;
    }

    mat4f RigidBodyInverse( const mat4f & matrix )
    {
        mat4f inverse = matrix;
        vec4f translation = matrix.value.w;
        inverse.value.w = simd4f_create(0,0,0,1);
        simd4x4f_transpose_inplace( &inverse.value );
        vec4f x = matrix.value.x;
        vec4f y = matrix.value.y;
        vec4f z = matrix.value.z;
        inverse.value.w =
            simd4f_create( -dot( x, translation ),
                           -dot( y, translation ),
                           -dot( z, translation ),
                           1.0f );
        return inverse;
    }
```

Now whenever we transform vectors want to go in/out of stone body space we'll use this matrix and its inverse. It's the best of both worlds.

Bringing It All Together

The best thing about rigid body motion is that you can calculate linear and angular motion separately and combine them together and it just works.

Here's the final code with linear and angular motion combined:

```
const float gravity = 9.8f * 10;
const float fps = 60.0f;
const float dt = 1 / fps;

while ( !quit )
{
    stone.rigidBody.velocity += vec3f( 0, -gravity, 0 ) * dt;

    stone.rigidBody.position += stone.rigidBody.velocity * dt;

    quat4f spin =
        AngularVelocityToSpin(
            stone.rigidBody.orientation,
            stone.rigidBody.angularVelocity );

    stone.rigidBody.orientation += spin * dt;
    stone.rigidBody.orientation = normalize( stone.rigidBody.orientation );

    RenderStone( stone );

    UpdateDisplay();
}
```

And here is the end result:

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I think this is fairly convincing. The go stone is moving quite realistically!

NEXT ARTICLE: Go Stone vs. Go Board (http://web.archive.org/web/20181107181454/https://gafferongames.com/post/go_stone_vs_go_board/)

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