

CS 229, Fall 2018

Problem Set #0 Solutions: Linear Algebra and Multivariable Calculus

1.

(a)

$$\nabla f(x) = \nabla\left(\frac{1}{2}x^T A x + b^T x\right) = A x + b$$

(b)

$$\frac{\partial g(h(x))}{\partial x_i} = \frac{\partial g(h(x))}{\partial h(x)} \frac{\partial h(x)}{\partial x_i} = g'(h(x)) \frac{\partial h(x)}{\partial x_i}$$

$$\nabla f(x) = \nabla g(h(x)) = g'(h(x)) \nabla h(x)$$

(c)

$$\begin{aligned} \nabla^2 f(x) &= \begin{bmatrix} \frac{\partial \nabla f(x)}{\partial x_1} & \frac{\partial \nabla f(x)}{\partial x_2} & \dots & \frac{\partial \nabla f(x)}{\partial x_n} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial \nabla(Ax+b)}{\partial x_1} & \frac{\partial \nabla(Ax+b)}{\partial x_2} & \dots & \frac{\partial \nabla(Ax+b)}{\partial x_n} \end{bmatrix} \\ &= \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix} = A \end{aligned}$$

(d)

$$\nabla f(x) = \nabla g(a^T x) = g'(a^T x) \nabla(a^T x) = g'(a^T x) a$$

$$\frac{\partial^2 g(h(x))}{\partial x_i \partial x_j} = \frac{\partial^2 g(h(x))}{\partial (h(x))^2} \frac{\partial h(x)}{\partial x_i} \frac{\partial h(x)}{\partial x_j} = g''(h(x)) \frac{\partial h(x)}{\partial x_i} \frac{\partial h(x)}{\partial x_j}$$

$$\frac{\partial^2 g(a^T x)}{\partial x_i \partial x_j} = g''(a^T x) \frac{\partial(a^T x)}{\partial x_i} \frac{\partial(a^T x)}{\partial x_j} = g''(a^T x) a_i a_j$$

$$\nabla^2 f(x) = \nabla^2 g(a^T x) = g''(a^T x) \begin{bmatrix} a_1 a_1 & a_1 a_2 & \dots & a_1 a_n \\ a_2 a_1 & a_2 a_2 & \dots & a_2 a_n \\ \vdots & \vdots & \ddots & \vdots \\ a_n a_1 & a_n a_2 & \dots & a_n a_n \end{bmatrix} = g''(a^T x) a a^T$$
