CS 229, Fall 2018
Problem Set #0: Linear Algebra and Multivariable Calculus

Notes: (1) These questions require thought, but do not require long answers. Please be as concise as possible. (2) If you have a question about this homework, we encourage you to post your question on our Piazza forum, at https://piazza.com/stanford/fall2018/cs229 (3) If you missed the first lecture or are unfamiliar with the collaboration or honor code policy, please read the policy on Handout #1 (available from the course website) before starting work. (4) This specific homework is *not graded*, but we encourage you to solve each of the problems to brush up on your linear algebra. Some of them may even be useful for subsequent problem sets. It also serves as your introduction to using Gradescope for submissions.

2. **[0 points] Positive definite matrices**

   A matrix $A \in \mathbb{R}^{n \times n}$ is *positive semi-definite* (PSD), denoted $A \succeq 0$, if $A = A^T$ and $x^T A x \geq 0$ for all $x \in \mathbb{R}^n$. A matrix A is *positive definite*, denoted $A \succ 0$, if $A = A^T$ and $x^T A x > 0$ for all $x \neq 0$, that is, all non-zero vectors x. The simplest example of a positive definite matrix is the identity I (the diagonal matrix with 1s on the diagonal and 0s elsewhere), which satisfies $x^T I x = \|x\|_2^2 = \sum_{i=1}^{n} x_i^2$.

(a) Let $z \in \mathbb{R}^n$ be an n-vector. Show that $A = zz^T$ is positive semidefinite.

(b) Let $z \in \mathbb{R}^n$ be a *non-zero* n-vector. Let $A = zz^T$. What is the null-space of A? What is the rank of A?

(c) Let $A \in \mathbb{R}^{n \times n}$ be positive semidefinite and $B \in \mathbb{R}^{m \times n}$ be arbitrary, where $m, n \in \mathbb{N}$. Is BAB^T PSD? If so, prove it. If not, give a counterexample with explicit A, B.