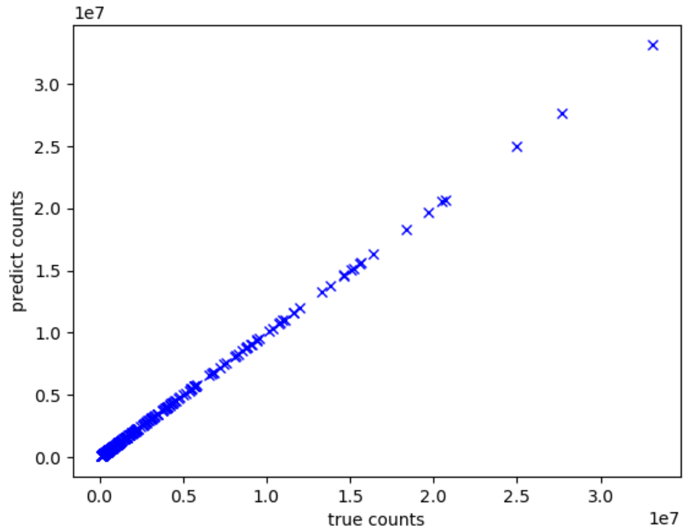

4.

(a)

$$\frac{\partial}{\partial \eta}\int p(y;\eta)dy = 0$$

$$\begin{aligned}
\frac{\partial}{\partial \eta}\int p(y;\eta)dy &= \int \frac{\partial}{\partial \eta} p(y;\eta)dy \\
&= \int b(y)\exp\{\eta y - a(\eta)\}(y - \frac{\partial a(\eta)}{\partial \eta})dy \\
&= \int p(y;\eta)(y - \frac{\partial a(\eta)}{\partial \eta})dy \\
&= \int yp(y;\eta)dy - \frac{\partial a(\eta)}{\partial \eta}\int p(y;\eta)dy \\
&= E[Y;\eta] - \frac{\partial a(\eta)}{\partial \eta}
\end{aligned}$$

$$E[Y;\eta] = E[Y|X;\theta] = \frac{\partial a(\eta)}{\partial \eta}$$

(b)

$$\frac{\partial}{\partial \eta}\int yp(y;\eta)dy = \frac{\partial^2 a(\eta)}{\partial \eta^2}$$

$$\begin{aligned}
\frac{\partial}{\partial \eta}\int yp(y;\eta)dy &= \int y\frac{\partial}{\partial \eta} p(y;\eta)dy \\
&= \int yp(y;\eta)(y - \frac{\partial a(\eta)}{\partial \eta})dy \\
&= \int y^2 p(y;\eta)dy - \frac{\partial a(\eta)}{\partial \eta}\int yp(y;\eta)dy \\
&= E[Y^2;\eta] - E^2[Y;\eta] \\
&= Var[Y;\eta]
\end{aligned}$$

$$Var[Y;\eta] = Var[Y|X;\theta] = \frac{\partial^2 a(\eta)}{\partial \eta^2}$$

(c)

$$
\begin{aligned}
\ell(\theta) &= -\sum_{i=1}^{m} \log p(y^{(i)}|x^{(i)};\theta) \\
&= \sum_{i=1}^{m} -\log b(y^{(i)}) - \theta^T x^{(i)} y^{(i)} + a(\theta^T x^{(i)})
\end{aligned}
$$

$$
\frac{\partial \ell(\theta)}{\partial \theta_j} = \sum_{i=1}^{m} [a'(\theta^T x^{(i)}) - y^{(i)}] x_j^{(i)}
$$

$$
H_{jk} = \frac{\partial^2 \ell(\theta)}{\partial \theta_j \theta_k} = \sum_{i=1}^{m} a''(\theta^T x^{(i)}) x_j^{(i)} x_k^{(i)}
$$

$$
\begin{aligned}
z^T H z &= \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{n} a''(\theta^T x^{(i)}) x_j^{(i)} x_k^{(i)} z_j z_k \\
&= \sum_{i=1}^{m} a''(\theta^T x^{(i)}) [(x^{(i)})^T z]^2
\end{aligned}
$$

$$
a''(\theta^T x) = Var[Y|X;\theta] \geq 0 \ \Rightarrow \ z^T H z \geq 0
$$

5.