



Continuous Assessment Test (CAT) – I : FEB 2024

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| Programme | : | B. Tech. | Semester | : | Winter |
| Course Code & Course Title | : | BMAT201L & Complex Variables and Linear Algebra | Slot | : | A1+TA1+TAA1 |
| Faculty | : | Dr. Amit Kumar Rahul, Dr. Ashish Kumar Nandi, Dr. Jaganathan B, Dr. Kalyan Manna, Dr. Manivannan A, Dr. Sagithya, Dr. Somnath Bera. | Class Number | : | CH2023240500810 CH2023240500806 CH2023240500801 CH2023240500798 CH2023240500816 CH2023240500804 CH2023240500812 |
| Duration | : | 90 Minutes | Max. Mark | : | 50 |

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Use statistical tables supplied from the exam cell as necessary.
- Use graph sheets supplied from the exam cell as necessary.
- Only non-programmable calculator without storage is permitted.

Answer all questions.

| Q. No | Sub Sec. | Description | Marks |
|-------|----------|--|-------|
| 1. | | Determine the analytic function $f(z) = u + iv$ given that $u - v = \frac{\cos x + \sin x - e^{-y}}{2 \cos x - e^y - e^{-y}}$ and $f\left(\frac{\pi}{2}\right) = 0$. | 10 |
| 2. | (a) | If $\phi(x, y) = x^2 - y^2 - 2xy - 2x - y - 1$ is the velocity potential of an incompressible fluid flow through a channel, then calculate the complex potential $w = \phi(x, y) + i\psi(x, y)$. | 6 |
| | (b) | Check the condition for orthogonality for the family of curves $u(x, y) = c_1$ and $u(x, y) = c_2$, when $f(z) = u + iv = (x^4 - 6x^2y^2 + y^4) + i(4x^3y - 4xy^3)$. | 4 |
| 3. | (a) | Find the points where the mapping $w = e^{-\sinh z} + 2$ is not conformal. | 4 |
| | (b) | Find the image of the region $z\bar{z} < 1$ under the transformation $w = \left(\frac{1}{2}e^{\frac{i\pi}{2}}\right)z$. | 6 |
| 4. | | Determine the bilinear mapping which maps the points $z = 2i, -i, 0$ onto the points $w = \frac{-i}{3}, \infty, 5i$ respectively. Find and sketch the image of the region $ z - i \leq 1$ under this transformation. List the fixed points of this mapping. | 10 |
| 5. | (a) | Expand the function $f(z) = \frac{z-1}{z+1}$ as a Taylor series about the point $z = 1$. Determine the region of convergence. | 4 |
| | (b) | Expand the function $f(z) = \frac{1}{z(z-1)}$ in a Laurent series which is valid for $1 < z - 2 < 2$. | 6 |

*****All the best *****