VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test II – May 2023

Programme	:	**B.Tech.**	Semester	:	Winter 2022-2023
Course Title	:	**Differential Equations and Transforms**	Code	:	BMAT 102L
			Slot	:	A2+TA2+TAA2
Faculty	:	Dr. Saroj Kumar Dash, Dr. Lakshmanan Shanmugham, Dr. Abhishek Kumar Singh, Dr. Harshavarthini Shanmugam, Dr. P.T. Sowndarrajan, Dr. Soumendu Roy, Dr. Manimaran J.	Class No.	:	CH2022232300442, CH2022232300445, CH2022232300446, CH2022232300447, CH2022232300448, CH2022232300449, CH2022232300450
Time	:	**90 Minutes**	Max. Marks	:	**50**

Answer ALL the Questions (5x10 = 50)

Q.No.	Sub. Sec.	Question Description	Marks
1.	a)	Find the Laplace transform of $\lvert \sin 5t \rvert$.	5
	b)	Find the inverse Laplace transform of $\frac{1}{\sqrt[3]{8s-27}}$.	5
2.	a)	Find the Laplace transform of $\int_0^t u\, e^{-u} \sin 4u \; du$.	5
	b)	Find the inverse Laplace transform of $\frac{(1+e^{-2s})^2}{2s+1}$.	5
3.		Solve the following system of Initial Value Problems: $3x' + y' + 2x = H(t-1)$, $x' + 4y' + 3y = 0$ such that $x(0) = 0 = y(0)$. **Note:** $H(t-a)$ is the **Heaviside function**, which is also known as **unit step function** at the point a.	10
4.		Solve the following initial and boundary value problem (by using Laplace Transform): $u_t(x,t) + u_x(x,t) = x$, such that $u(0,t) = e^{3t}$, for all $t \geq 0$ and $u(x,0) = 0$, for all $x \geq 0$.	10
5.		Consider the 2π-periodic function $f(x) = \begin{cases} (x-\pi)^2, & 0 \leq x < \pi \\ \pi^2, & \pi \leq x < 2\pi \end{cases}$ Find the Fourier series expansion of $f(x)$ and hence deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ by using the Fourier series of the given $f(x)$.	10

⇔⇔⇔

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test II - May 2023

Programme	:	B.Tech.	Semester	:	Win 2022-23
Course	:	Differential Equations and Transforms	Code	:	BMAT102L
			Slot	:	A1+TA1+TAA1
Faculty	:	Dr. Soumendu, Dr. Hannah, Dr. Radha, Dr. Abhishek, Dr. David, Dr. P. Vijay Kumar, Dr. Sowndarrajan	Class ID	:	CH20222310004 29,430, 433, 434, 438, 436, 439
Time	:	90 Minutes	Max.Marks	:	50

Answer ALL the questions

1. Find $L\left\{t\int_0^{\infty} e^{-2t}t\cos 3t dt\right\}$. [05]

2. Find the Fourier series of the given function: [05]

$$f(x)=\begin{cases}\frac{\pi}{3} & 0\le x\le\frac{\pi}{3}\\ 0 & \frac{\pi}{3}\le x\le\frac{2\pi}{3}\\ \frac{-\pi}{3} & \frac{2\pi}{3}\le x\le\pi\end{cases}$$

3. Using the Convolution theorem, find the inverse Laplace transform of $\frac{1}{(s^2+1)^3}$. [10]

4. Find the Fourier series expansion of $f(x)=\begin{cases}0 & -\pi\le x<a\\ 1 & a\le x\le b\\ 0 & b<x\le 2\pi\end{cases}$. Hence, find the sum of the series for $x=4\pi+a$ and deduce that [10]

$$\sum_{n=1}^{\infty}\frac{\sin n(b-a)}{n}=\frac{\pi-b+a}{2}.$$

5. Solve the PDE $\frac{\partial y}{\partial x}-\frac{\partial y}{\partial t}=0$, $y(x,0)=e^{-2x}$, $y(0,t)=e^{-2t}$ using Laplace transform method. [10]

6. Solve $y''+3y'-28y=u(t-2)$, $y(0)=2$, $y'(0)=-3$. [10]

VIT

Vellore Institute of Technology

Continuous Assessment Test (CAT)-II– May 2023

Programme	: B. Tech	Semester	: Winter Semester I year 2022-2023
Course Title	: Differential Equations and Transforms	Code	: BMAT1021L
Faculty	: Dr. A. Berin Greeni, Dr. G.K. Revathi, Dr. Sandip Saha, Dr. Saurabh Chandra Maury, Dr. Ankit Kumar, Dr. Ashis	Slot	: C2+TC2+TCC2
Duration	: 1 ½ Hours	Class No.	: CH2022232300619, 675, 679, 620, 621,7
		Max. Marks	: 50

Answer all the Questions (50 marks)

Q. No.	Question Description	Marks
1.	Find the inverse Laplace transform of the function $F(s)$. $$F(s) = \frac{s(1 + e^{-1.5s} + e^{-2.2s}) + e^{-1.5s}}{s(s+2)}$$	[10]
2.	A) Find $L(te^{-2t}\sin 2t \sin 3t)$ B) Obtain the Fourier series expansion of the periodic function $f(x) = \frac{\pi}{2}x^2, -\pi \le x \le \pi$.	[5+5]
3.	Obtain the Fourier series expansion of the periodic function $f(x) = \frac{3}{2}(\pi - x), 0 < x < 2$ of period 2. Hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$	[10]
4.	Solve the following differential equation using Laplace transform $$y'' + 4y = \sin t - u(t-\pi)\cos(t-\pi), \quad y(0) = 1, \quad y'(0) = 0$$ where $u(t)$ denotes the unit step function	[10]
5.	Solve the following partial differential equation using Laplace transform $$u_x - u_t = 1 - e^{-t}, \quad u(x,0) = 6e^{-3x}, \quad u(0,t) = -t$$	[10]

Reg. No.:

Name :

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act 1956)

Continuous Assessment Test II – May 2023

Programme	:	B.Tech.	Semester	:	WIN SEM 2022-23
Course Title	:	Differential Equations and Transforms	Code	:	BMAT102L
			Slot	:	C1+TC1+TCC1
Faculty(s)	:	Dr. Saroj Kumar Dash; Dr. Srutha Keerthi B; Dr. Somnath Bera; Dr. Ashish Bera; Dr. Kriti Arya	Class Nos.		CH2022232300616; CH2022232300617; CH2022232300673; CH2022232300618; CH2022232300682
Time	:	90 Minutes	Max. Marks	:	50

Answer ALL the Questions

Q.No.	Question Description	Marks
1.	Find the Laplace transform of the following functions (i) $f(t) = \frac{\sin^2 t}{t}$, $t > 0$. (ii) $f(t) = (t^2 - 2t + 1)e^{2-2t}u(t-1)$, where $u(t)$ denotes the unit step function.	10
2.	Find the inverse Laplace transform of the following function $\frac{s}{s^4+5s^2+9}$.	10
3.	Using Laplace transform solve the following differential equation. $y'' + 3y' - 16y = 12\,\delta(t-13)$, with $y(0) = 2$, and $y'(0) = -2$, where $\delta(t)$ denotes the unit impulse function.	10
4.	Solve the following partial differential equation using Laplace transform $$\frac{\partial u(x,t)}{\partial x} = 2\frac{\partial u(x,t)}{\partial t} + u(x,t)$$ with $0 \le t < \infty$, $u(x,0) = 6e^{-3x}$ and $u(x,t)$ is bounded for $x > 0$ and $t > 0$	10
5.	Find the Fourier series expansion of $f(x) = \pi^2 - x^2$, $x \in (-\pi, \pi)$ and hence obtain the following identities. $$(i)\ \frac{\pi^2}{12} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$ $$(ii)\ \frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$	10

⇔⇔⇔

VIT

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Continuous Assessment Test II - April 2024

Programme	:	B.Tech.	Semester	:	Win 2023-24
Course	:	Differential Equations and Transforms	Code	:	BMAT102L
			Slot	:	C1+TC1+TCC1
Faculty	:	Dr. Balaji S, Dr. Jayagopal R, Dr. David Raj Michael, Dr. Surat Ghosh, Dr. Dhivya M, Dr. P. Vijay Kumar Dr. Sowndarrajan, Dr. Radha S Dr. Manimaran, Dr. Sethukumaraswamy Dr. Durgaprasad, Dr. Amit Kumar Rahul Dr. Tharasi Dilleswar	Class ID	:	CH2023240501574, 1575,1579, 1573, 1576,1577, 1578, 1408, 0789, 0790,0791, 0792,0793
Time	:	90 Minutes	Max.Marks	:	50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information
- Only non-programmable calculator without storage is permitted

Part – A ($5 \times 10 = 50$)

Answer all the Questions

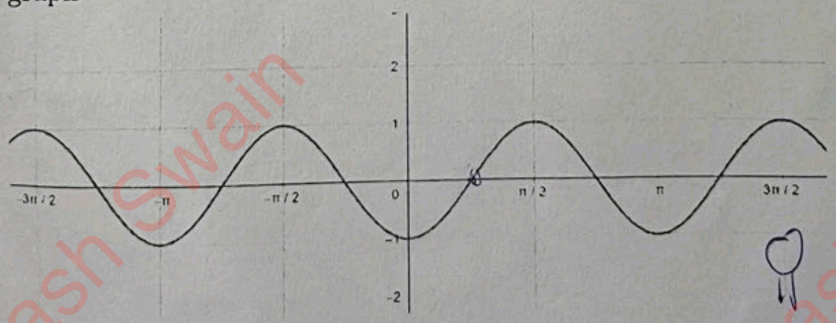
1. Find inverse Laplace transform of $\dfrac{e^{-3s}+s}{(s+2)(s^2+2s+2)}$. [10]

2. Solve the following initial value problem using Laplace transform: [10]

$$y''+5y'+6y=\delta\left(t-\frac{\pi}{2}\right)+\begin{cases}\cos t, & t\geq\pi;\\ 0, & t<\pi,\end{cases} \quad y(0)=0, \quad y'(0)=0\,,$$

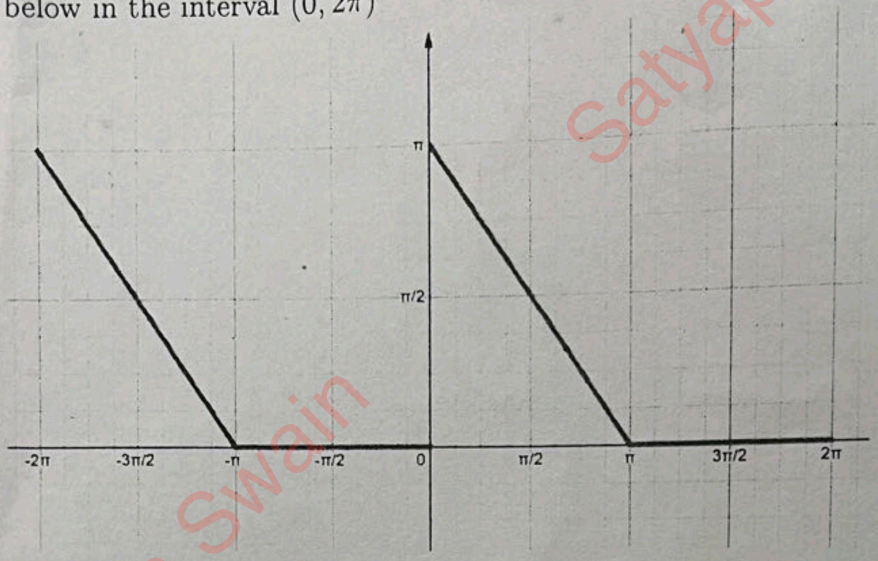
where $\delta\left(t-\frac{\pi}{2}\right)$ is the impulse function.

(b) Solve $\dfrac{\partial u(x,t)}{\partial t}+\dfrac{\partial u(x,t)}{\partial x}+2u(x,t)=0$, with the initial and boundary conditions $u(0,t)=-\cos 2t$ and $u(x,0)=e^{-2x}f(x)+1$, where $f(x)$ is given by the following graph [10]



4. Find the Half range Fourier Cosine and Sine series expansion of $f(x) = 12x^2 - 16x + 8$ in $(0, 1)$. [10]

5. Find the Fourier series expansion of the periodic function shown by the graph given below in the interval $(0, 2\pi)$ [10]



and hence find the sum of the series $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)
CHENNAI

Reg. Number:	

Continuous Assessment Test (CAT) – II - APRIL 2024

Programme	:	B.Tech.	**Semester**	:	Winter 2023-2024
Course Code & Course Title	:	BMAT102L & Differential Equations and Transforms	**Slot**	:	D2+TD2+TDD2
Faculty	:	Dr. SOUMENDU ROY Dr. THARASI DILLESWAR Dr. POULOMI DE Dr. MOHANA N Dr. KRITI ARYA Dr. SUDIP DEBNATH Dr. SAURABH CHANDRA MAURY Dr. OM NAMHA SHIVAY Dr. SANKARSAN TARAI Dr. SAGITHYA	**Class Number**	:	CH2023240500779 CH2023240500780 CH2023240500781 CH2023240500782 CH2023240500783 CH2023240500784 CH2023240500785 CH2023240500786 CH2023240500787 CH2023240503429
Duration	:	1 ½ Hours	**Max. Mark**		50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Only non-programmable calculator without storage is permitted.

Answer all questions

Q. No	Sub Sec.	Description	Marks
1	(a)	Find the Laplace transform of $f(t) = \frac{\cos 4t \sin 2t}{t}$.	5
	(b)	Find the inverse Laplace transform of $F(s) = \frac{1-3s}{s^2+8s+21}$.	5
2		Using Laplace transforms, find the solution of the initial value problem $y'' + 9y = f(t) + \sin(3t+2),\ y(0) = y'(0) = 0$ where $f(t) = \begin{cases} 0, & 0 < t < 3 \\ 9, & 3 \leq t < \infty \end{cases}$.	10
3		Using Laplace transforms, solve the following pair of simultaneous differential equations $3\frac{dy}{dt} + 5x + 3\frac{dx}{dt} - 25\,cost = 0$ $2\frac{dx}{dt} - 3\frac{dy}{dt} - 5\,sint = 0$, given that at $t = 0, x = 2$ and $y = 3$.	10
4		Find the half range cosine series expansion for the function $f(x) = (x-1)^2$ in the interval $0 < x < 1$ and hence deduce the value of $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$ from the half range cosine series expansion of $f(x)$.	10

[P.T.O]

5		Find the Fourier series for the function $f(x)$ given by $f(x) = \begin{cases} x, -\pi < x < 0 \\ 0, \ 0 \le x < \frac{\pi}{2} \\ x - \frac{\pi}{2}, \ \frac{\pi}{2} < x < \pi \end{cases}$ and hence deduce the value of $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ from the Fourier series expansion of $f(x)$.	**10**
		**********All the best **********	

VIT
Vellore Institute of Technology
CHENNAI

Reg. Number:	23BAI1038

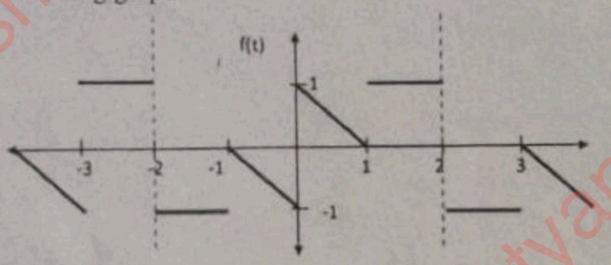
Continuous Assessment Test (CAT) – II - APRIL 2024

Programme	:	B.Tech.	Semester	:	Winter 2023-2024
Course Code & Course Title	:	BMAT102L - Differential Equations and Transforms	Slot	:	D1+TD1+TDD1
Faculty	:	Dr. Jayagopal R, Dr. Mini Ghosh Dr. Poulomi De, Dr. Mohana N Dr. Kriti Arya, Dr. Sudip Debnath Dr. Srutha Keerthi B, Dr. Radha S Dr. Dhivya M	Class Number	:	CH2023240500770, 71, 72, 73, 74, 75, 76, 77, 78
Duration	:	1 ½ Hours	Max. Mark		50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Only non-programmable calculator without storage is permitted

Answer all questions

Q. No	Sub Sec	Description	Marks
1.		Find the Fourier series for the function $f(t)$ defined by the following graph: 	10
2.		Using Laplace transformation, solve the differential equation: $\frac{d^2x}{dt^2} + 9x = u(t-2)\cos 2(t-2)$, with $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$ where $u(t-2)$ represents an unit step function.	10
3.		Using Laplace transform, obtain a solution of the PDE $\frac{\partial u}{\partial x} + 2x\frac{\partial u}{\partial t} = xt$ for $x > 0, t > 0$, with $u(x,0) = 0$ and $u(0,t) = t$.	10
4.	(a)	Find the inverse Laplace transform of $H(s) = \frac{se^{-4s}}{(3s+2)(s-2)}$	5
	(b)	Find the Laplace transform of $e^{-2t}\int_0^t \frac{1-\cos t}{t}dt$.	5
5.		Verify whether the given function $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ in the interval $[-\pi, \pi]$ is even or odd. Also, find the Fourier series of the function and hence deduce the value of $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$.	10

**********All the best **********