

**VIT**

Vellore Institute of Technology

Reg. No.:
Name:**Continuous Assessment Test – September 2023**

Programme	: B.Tech.	Semester	: Fall 2023-24
Course	: Discrete Mathematics and Graph Theory	Code	: BMAT205L
Faculty	: Dr. Berin Greeni A Dr. Jayagopal R Prof. Aarthi B Prof. Vignesh R Prof. Anitha G Prof. Sumathi S Prof. Sakthidevi K Prof. Gnanaprasanna K	Class ID's	: CH2023240101195 CH2023240101047 CH2023240101191 CH2023240101192 CH2023240101193 CH2023240101195 CH2023240101196 CH2023240101197 CH2023240101198
Duration	: 90 minutes	Slot	: D1-TD1-TDD1
		Max. Marks	: 50

Answer all the questions ($5 \times 10 = 50$ Marks)

- | Q. No. | Question Description | Marks |
|--------|--|--------|
| 1. | a) Show that $\forall x(P(x) \vee Q(x))$ implies the conclusion $\forall xP(x) \vee \exists xQ(x)$ using the method of contradiction.
b) Find the generator matrix and parity check matrix corresponding to the encoding function $e: B^3 \rightarrow B^6$ given by $e(000) = 000000, e(001) = 001101, e(010) = 010011, e(100) = 100110, e(011) = 011110, e(101) = 101011, e(110) = 110101$ and $e(111) = 111000$. | 5
5 |
| | b) Identify the bound variable, free variable and the scope of the following expression $\forall x \exists y(P(x, y) \wedge Q(x, y)) \vee \forall y(R(x, y) \rightarrow S(x, y)) \wedge M(x, y)$. | 3 |
| 2. | a) Show that the following argument is valid. If today is Wednesday, I have a test in Mathematics or Economics. If my Economics Professor is sick, I will not have a test in Economics. Today is Wednesday and my Economics Professor being sick. Therefore, I have a test in Mathematics.
b) Without using the truth table find the PCNF of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$. | 7
5 |
| | b) Without using the truth table prove that the premises $P \rightarrow Q, P \rightarrow R, Q \rightarrow \neg R$ and P are inconsistent. | 5 |
| 3. | a) Let $G = \{(a, b) a, b \in \mathbb{R}, a \neq 0\}$ and \star be a binary operation defined on G such that $(a, b) \star (c, d) = (ac, bc + d)$ for all $(a, b), (c, d) \in G$. Examine if (G, \star) is a commutative group.
b) Let $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$ be a group with respect to matrix addition and $(\mathbb{C}, +)$ | 6 |

be another group. Check whether the following mapping $f: (G, +) \rightarrow (\mathbb{C}, +)$, where \mathbb{C} is the set of complex numbers, defined by $f\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = a + ib$, is a group homomorphism or not.

5. a) If 20 processors are interconnected and every processor is connected to at least one other, show that at least two processors are directly connected to the same number of processors.
- b) In how many ways can 7 people be arranged about a circular table? If two of them insist on sitting next to each other, how many arrangements are possible?



Course Code & Name : BMAT205L - Discrete Mathematics and Graph Theory
Exam Duration : 90 Minutes

Fall Semester - 2023~2024
Continuous Assessment Test - I
Programme Name & Branch : B.Tech

Slot : A1 + TA1 + TAA1
Maximum Marks : 50

Answer ALL the Questions

Each question carries equal marks ($5 \times 10 = 50$ Marks)

1. Obtain the principal disjunctive and conjunctive normal forms of the statement $(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$ [10 M]
2. Derive $p \rightarrow (q \rightarrow s)$ using the CP-rule from the premises $p \rightarrow (q \rightarrow r)$ and $q \rightarrow (r \rightarrow s)$ [10 M]
3. Show that $\forall(x) (p(x) \vee q(x)) \Rightarrow (\exists x) p(x) \vee (\exists x) q(x)$ by indirect method of proof. [10 M]
4. Show that a non-empty subset S of G is a subgroup of (G, \cdot) iff for any pair of elements $a, b \in S$, $a * b^{-1} \in S$ [10 M]
5. Consider the group (\mathbb{Z}_6, \oplus_6) . (i) Construct the Cayley's table (ii) Find the order of each element (iii) Find the inverse of each element (iv) Write all possible non-trivial subgroups (v) Obtain the left cosets of $H = \{0, 2, 4\}$ in \mathbb{Z}_6 . [10 M]



VIT

Vellore Institute of Technology

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS

SLOT A2

CONTINUOUS ASSESSMENT TEST-I (September 2023)

FALL SEMESTER 2023-24

Programme Name & Branch: B.Tech

Course Code: BMAT205L

Course Name: Discrete Mathematics and Graph theory

Exam Duration: 90 minutes

Maximum Marks: 50

General instruction(s): Answer all questions $5 \times 10 = 50$

Sl.No.	Question	Marks
✓1.	Obtain the PCNF and PDNF of $(p \wedge \neg q) \vee (q \wedge \neg p) \vee (r \wedge p)$ by using <i>(i)</i> Truth table technique <i>(ii)</i> Algebraic method	10
✓2.	Prove that the following premises are inconsistent. $a \rightarrow (b \rightarrow c)$, $d \rightarrow (b \wedge \neg c)$ and $\underline{(a \wedge d)}$	10
✓3.	<i>(i)</i> Show that "All flowers are plants. Sunflower is a flower". "Therefore, sunflower is a plant". <i>(ii)</i> Show that $(\forall x)[(B(x) \vee C(x)) \rightarrow A(x)] \Rightarrow (\forall x)[B(x) \rightarrow A(x)]$ by using the rules of inference for predicate calculus.	5+5
✓4.	Let $(Z_5, +_5)$ be the group with respect to addition module 5. (i) Form the Cayley's table (ii) find the order of the group (iii) find the identity element of the group (iv) find the order of each element in the group (v) find the inverse of each element in the group.	10
✓5.	State and Prove Lagrange theorem on groups.	10

**VIT**Vellore Institute of Technology
Established by the Government of Tamil Nadu in 1984

Vellore - 632014, Tamil Nadu, India.

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
WINTER SEMESTER - 2023~2024Continuous Assessment Test - I

Programme Name & Branch : B.Tech. (All Branches)

Course Code : BMAT205L

Course Name : Discrete Mathematics and Graph Theory

Duration : 90 Minutes

Maximum Marks : 50

Slot : C1+TC1+TCC1

General Instruction(s) : —

Answer ALL the Questions(5 × 10 = 50 Marks)

1. Show that the following statements are inconsistent.
 If Jack gets his UG degree, then he will go for a job.
 If Jack goes for a job, then he will get married soon.
 If Jack goes for higher study, then he will not get married.
 Jack gets his UG degree and goes for higher study.

[10 M]

2. Obtain the principal disjunctive and conjunctive normal forms of the following statement:

$$(P \rightarrow (Q \wedge R)) \vee (\neg P \rightarrow (\neg Q \wedge \neg R)).$$

[10 M]

3. Show that the conclusion $(\forall x)(F(x) \rightarrow \neg S(x))$ follows logically from the premises $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y))$ and $(\exists y)(M(y) \wedge \neg W(y))$.

[10 M]

4. (i) For any commutative monoid $(M, *)$, show that the set of idempotent elements of M forms a submonoid.
 (ii) Give an example of a semigroup but not a monoid. Also give an example of a monoid but not a group. Justify the answers.

[10 M]

5. Show that $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0 \text{ and } a \in \mathbb{R} \right\}$ is a group under usual matrix multiplication.

[10 M]



Continuous Assessment Test I – September 2022

Programme	: B.Tech.	Semester	: FALLSEM 2022-23
Course Title	: Discrete Mathematics and Graph Theory	Code	: BMAT205L
Faculty(s)	: Dr.Balamurugan BJ, Dr.V.Vidhya, Dr.Kalyan, Dr.Umamaheswari, Dr.N.Nathiya, Dr.Somnath Bera, Dr.S.Devi Yamini, Dr.Durga, Dr.Prasannalakshmi, Dr.P.Dhivya, Dr.Saurabh Chandra, Dr.Kamalesh and Dr.Amith kumar Rahul	Slot	: D1+TD1+TDD1
Time	: 90 Minutes	Class Nos.	: C1H2022231021458
		Max. Marks	: 50

Answer ALL the Questions

Q.No.	Sub. Sec.	Question Description	Marks
1.	a.	Use inference theory to check the validity of the following argument: “There are people who are writers. All writers eat healthy food. All writers are happy. Hence, some people who eat healthy food are happy”.	5
	b.	Show that the following using CP rule $P \rightarrow (Q \rightarrow R), Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S)$	5
2.		Without using truth table, find the PCNF and PDNF of $(p \wedge (\neg q \wedge \neg r)) \vee (((p \wedge q) \vee \neg r) \vee p)$	10
3.	a.	Check whether the following argument is valid or not : (Justification required) “If Lavanya meets her friends then she will buy a gift or cake. Lavanya buys a gift only if she does not meet her friends. Lavanya does not buy a cake whenever she visits temple. She meets her friends. Hence, Lavanya does not visit temple.”	5
	b.	Use the rules of inference to show that if $\forall x (P(x) \vee Q(x)), \forall x (\neg Q(x) \vee S(x)), \forall x (R(x) \rightarrow \neg S(x)), \exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.	5
4.	a.	Write the following as a logical expression using quantifiers and find its negation assuming the domain as real numbers: “The product of a positive real number and a negative real number is always a negative real number”.	5
	b.	Enumerate all the subgroups of $(\mathbb{Z}_8, +_8)$. Let H be a subgroup of $(\mathbb{Z}_8, +_8)$ such that $O(H) = 4$. Find all the distinct left cosets of H in \mathbb{Z}_8 .	5
5.		a) Let $G = \{a + b\sqrt{2} \in \mathbb{R} \mid a, b \in \mathbb{Q}\}$. Prove that the non-zero elements of G forms an abelian group under usual multiplication. (5 marks)	10
		b) Consider the following Cayley's table for the set G under an operation *: $\begin{array}{c ccccc} & e & a & b & c & d \\ \hline e & e & a & b & c & d \\ a & a & e & d & b & c \\ b & b & d & e & a & c \\ c & c & b & a & d & e \\ d & d & c & b & e & a \end{array}$	

*	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	e	d	f	b	e
b	b	c	e	a	f	d
c	c	b	f	d	e	a
d	d	f	a	e	c	b
f	f	d	c	b	a	e

(i) Check whether G is a group or not. (3 marks)

(ii) Is G abelian? (1 mark)

(iii) Find an element $x \in G$ such that $d * x * c = f$. (1 mark)



Reg. No.:

Name :

**VIT[®]**Vellore Institute of Technology
(Deemed to be University under section 2(f) of UGC Act, 1956)**Continuous Assessment Test I – September 2022**

Programme	: B.Tech.	Semester	: FALLSEM 2022-23
Course Title	: Discrete Mathematics and Graph Theory	Code	: BMAT205L
Faculty(s)	: Dr. Balamurugan, Dr. Kalyan, Dr. Uma Maheshwari, Dr. Berin Greeni, Dr. Nathiya, Dr. Somnath Bera, Dr. Devi Yamini, Dr. Durga, Dr. Prasannalakshmi, Dr. Dhivya, Dr. Pavithra, Dr. Karan Kumar Pradhan, Dr. Kamalesh, Dr. Amit Kumar Rahul	Slot	: D2+TD2+TDD2
Time	: 90 Minutes	Class Nos.	: CH2022231001488; CH2022231001464; CH2022231001466; CH2022231001468; CH2022231001470; CH2022231001477; CH2022231001480; CH2022231001482; CH2022231001484; CH2022231001490; CH2022231001493; CH2022231001495; CH2022231001497; CH2022231001500

Answer ALL the Questions (5 X 10 = 50 Marks)

Q.No.	Sub. Sec.	Question Description	Marks						
1.	a.	Show that the conclusion $C:r$ follows from the premises $H_1:p \vee q, H_2:p \rightarrow r, H_3:q \rightarrow r$ using the truth table technique.	5						
	b.	(i) Consider the statement “Given any positive integer, there is a greater positive integer”. Symbolize this statement with and without using set of positive integers as the universe of discourse. (2 Marks) (ii) Consider these propositional functions: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>p(n)</td> <td>n is prime</td> </tr> <tr> <td>q(n)</td> <td>n is even</td> </tr> <tr> <td>r(n)</td> <td>$n > 2$</td> </tr> </table> Express these formulas in proper English sentences: (a) $\exists n \in Z(p(n) \wedge q(n))$ (b) $\forall n \in Z(r(n) \Rightarrow (p(n) \vee q(n)))$ (c) $\exists n \in Z(p(n) \wedge q(n) \vee r(n))$ (3Marks)	p(n)	n is prime	q(n)	n is even	r(n)	$n > 2$	5
p(n)	n is prime								
q(n)	n is even								
r(n)	$n > 2$								
2.		Without constructing the truth table, (i) Find the principal conjunctive normal form of $(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$ (5 Marks) (ii) Find the principal disjunctive normal form of $(\neg a \rightarrow b) \wedge (b \leftrightarrow a)$ (5 Marks)	10						
3.	a.	Show that the premises “There exist some students in this class and they know how to write programs in Python”, “Everyone who knows how to write programs in Python gets a high-paying job” imply the conclusion “Some students in the class can get a high-paying job”	5						

	b.	Show that the premises "If Claghorn has wide support, then he will be asked to run for the senate. If Claghorn yells "Eureka" in Iowa, he will not be asked to run for the senate. Claghorn yells "Eureka" in Iowa" imply the conclusion "Claghorn does not have wide support".	5
4.	a.	Show that b can be derived from the premises $a \rightarrow b, c \rightarrow b, d \rightarrow (a \vee c), d$ by the indirect method.	5
	b.	(i) If α and β are elements of the symmetric group S_4 , given by $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ Find $\beta\alpha, \alpha\beta$ and α^{-1} . (ii) Obtain all the distinct left cosets of $\{(0), (3)\}$ in the group $(Z_6, +_6)$. (2 Marks)	5
5.	a.	Let $*$ be an operation defined as $x * y = x + y + 2xy, \forall x, y \in \mathbb{R}$ (real numbers). (i) Check whether $(\mathbb{R}, *)$ is a monoid or not. (ii) Is it commutative? (iii) Find the inverse of each element in \mathbb{R} if it exists.	3
	b.	(i) Assume that G is a finite group with subgroups H of order 12 and K of order 30. If the order of G is less than 200, what are the possible values for the order of G . (3 Marks) (ii) Let $S = \{a \in \mathbb{Q} / a \neq 0\}$ be closed under the commutative binary operation $*$ defined by $a * b = \frac{ab}{3}$. Prove that the set S with the binary operation is an abelian group. (4 Marks)	7

↔↔↔

Answer all questions

Q. No	Sub Sec	Description	Marks
1.	(a)	Without constructing truth table, find the principal conjunctive normal form of $(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$.	(5)
	(b)	Derive $a \rightarrow (b \rightarrow d)$ using the Rule of Conditional Proof (CP-rule) from the premises $a \rightarrow (\neg b \vee c)$ and $b \rightarrow (c \rightarrow d)$	(5)
2.	(a)	Show that the hypotheses “If you send me an e-mail message, then I will finish writing the program”, “If you do not send me an e-mail message, then I will go to sleep early”, and “If I go to sleep early, then I will wake up feeling refreshed” lead to the conclusion “If I do not finish writing the program, then I will wake up feeling refreshed.”	(5)
	(b)	Determine whether these system specifications are consistent: “The diagnostic message is stored in the buffer or it is retransmitted”, “The diagnostic message is not stored in the buffer”, “If the diagnostic message is stored in the buffer, then it is retransmitted”.	(5)
3.	(a)	Show that the premises “A student in this class has not read the book,” and “Everyone in this class passed the first exam” imply the conclusion “Someone who passed the first exam has not read the book”	(6)
	(b)	Show that q can be derived from the premises using indirect method $p \rightarrow q, r \rightarrow q, s \rightarrow (p \vee r)$.	(4)
4.	(a)	If α and β are elements of the symmetric group S_4 , given by $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$. Find $\alpha\beta, \beta\alpha$ and α^{-1} .	(3)
	(b)	Obtain all the distinct left cosets of $\{[0], [3]\}$ in the group $(Z_6, +_6)$.	(2)

	(c)	If $S = N \times N$, the set of ordered pairs of positive integers with operation * defined by $(a, b) * (c, d) = (ad + bc, bd)$. Show that $(S, *)$ is a semigroup. And, if $f: (S, *) \rightarrow (Q, +)$ is a function defined by $f(a, b) = \frac{a}{b}$, show that f is homomorphism.	(5)
5	(a)	Find the code words generated by the encoding function $e: B^2 \rightarrow B^5$ with respect to the parity check matrix $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.	(6)
	(b)	Prove that the set $G = \{1, 2, 3, 4\}$ is abelian group with respect to ordinary multiplication modulo 5.	(4)

***** All the best *****



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Vellore Institute of Technology
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CHENNAI

Reg. Number:

22BCE1344

Continuous Assessment Test (CAT) – I August 2024

Programme	:	B.Tech.	Semester	:	FALL 2024-2025
Course Code & Course Title	:	BMAT205L Discrete Mathematics and Graph Theory	Slot	:	C2+TC2+TCC2
Faculty	:	Prof. Aarthy B Dr. Amit Kumar Rahul Prof. Anitha G Dr. Ankit Kumar Dr. Padmaja N Dr. Poulomi De Dr. Surath Ghosh	Class Number	:	CH2024250102066 CH2024250102265 CH2024250102267 CH2024250102069 CH2024250102266 CH2024250102068 CH2024250102268
Duration	:	90 Minutes	Max. Mark	:	50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Use statistical tables supplied from the exam cell as necessary
- Use graph sheets supplied from the exam cell as necessary
- Only non-programmable calculator without storage is permitted

**Answer all questions
(5×10=50)**

Q. No	Su b Sec	Description	Marks
1.	(a)	Without using truth table, find PDNF of $\neg(p \vee (\neg p \wedge \neg q \wedge r))$.	(5)
	(b)	Identify the bound variable, free variable and the scope of the following expression: $\forall x(P(x) \wedge Q(x)) \vee \forall yR(y)$. Also, write the converse, contrapositive and inverse of the following proposition symbolically and in words “If the weather is nice, then I’ll wash the car”.	(2+3)
2.	(a)	Prove that $\neg p \leftrightarrow q, q \rightarrow r, \neg r \Rightarrow p$ is valid.	(5)
	(b)	Show that the premises “An employee in my office has not completed his daily work” and “Everyone in my office completed his monthly files” imply the conclusion “Someone who completed his monthly files has not completed his daily work”.	(5)
3.	(a)	Prove the following equivalences by proving the equivalences of the dual without using truth table: $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \equiv p \wedge q$	(5)

	(b)	Let $P(x)$ be the statement “ x has visited universal studios” where the universe consists of the students at UCF. Express each of the following statements using quantifiers (i) Some students at UCF have not visited universal studios. (ii) Not all students at UCF have visited universal studios. (iii) No student at UCF has not visited universal studios.	(2+2+1)
4.	(a)	If $*$ is the binary operation on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a, b) * (c, d) = (ac, ad + b)$, i) Prove $(S, *)$ is a semi group. Is it commutative? ii) Find the identity element of S iii) Which elements, if any, have inverses, and what are they?	(6)
	(b)	Let $\mathbb{R} - \{0\}$ represents set of all nonzero real numbers and M denotes the set of all 2×2 invertible matrices over \mathbb{R} . Determine whether the following map is a homomorphism. If so, what is its kernel? Given the map $f: \mathbb{R} - \{0\} \rightarrow M$ defined by $f(a) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$.	(4)
5	(a)	Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ corresponding the encoding function $e: B^3 \rightarrow B^6$ find the parity check matrix and use it to decode the following received words and hence find the original message. Are all the words decoded uniquely? (i) 111101 (ii) 100100 (iii) 111100 (iv) 010100	(5)
	(b)	In the group S_6 , a permutation group over $\{1, 2, 3, 4, 5, 6\}$ $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 5 & 3 & 6 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 6 & 1 & 2 & 4 \end{pmatrix}$ determine α^{-2} and $x \in S_6$ such that $\alpha x = \beta$.	(1+2)
	(c)	Provide a justification for why $U(8) = \{1, 3, 5, 7\}$, under multiplication modulo 8, is not a cyclic group.	(2)

***** All the best *****



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Vellore Institute of Technology
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CHENNAI

Reg. Number:	22 B(E) 1351
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Continuous Assessment Test (CAT) – I August 2024

Programme	:	B.Tech.	Semester	:	FALL 2024-2025
Course Code & Course Title	:	BMAT205L Discrete Mathematics and Graph Theory	Slot	:	C2+TC2+TCC2
Faculty	:	Prof. Aarthy B Dr. Amit Kumar Rahul Prof. Anitha G Dr. Ankit Kumar Dr. Padmaja N Dr. Poulomi De Dr. Surath Ghosh	Class Number	:	CH2024250102066 CH2024250102265 CH2024250102267 CH2024250102069 CH2024250102266 CH2024250102068 CH2024250102268
Duration	:	90 Minutes	Max. Mark	:	50

General Instructions:

- Write only your registration number on the question paper in the box provided and do not write other information.
- Use statistical tables supplied from the exam cell as necessary
- Use graph sheets supplied from the exam cell as necessary
- Only non-programmable calculator without storage is permitted

**Answer all questions
(5×10=50)**

Q. No	Sub Sec	Description	Marks
1.	(a)	Without using truth table, find PDNF of $\neg(p \vee (\neg p \wedge \neg q \wedge r))$.	(5)
	(b)	Identify the bound variable, free variable and the scope of the following expression: $\forall x(P(x) \wedge Q(x)) \vee \forall yR(y)$. Also, write the converse, contrapositive and inverse of the following proposition symbolically and in words “If the weather is nice, then I’ll wash the car”.	(2+3)
2.	(a)	Prove that $\neg p \leftrightarrow q, q \rightarrow r, \neg r \Rightarrow p$ is valid.	(5)
	(b)	Show that the premises “An employee in my office has not completed his daily work” and “Everyone in my office completed his monthly files” imply the conclusion “Someone who completed his monthly files has not completed his daily work”.	(5)
3.	(a)	Prove the following equivalences by proving the equivalences of the dual without using truth table: $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \equiv p \wedge q$	(5)

	(b)	<p>Let $P(x)$ be the statement “x has visited universal studios” where the universe consists of the students at UCF. Express each of the following statements using quantifiers</p> <ul style="list-style-type: none"> (i) Some students at UCF have not visited universal studios, (ii) Not all students at UCF have visited universal studios. (iii) No student at UCF has not visited universal studios. 	(2+2+1)
4.	(a)	<p>If $*$ is the binary operation on $S = Q \times Q$, the set of ordered pairs of rational numbers and given by $(a, b) * (c, d) = (ac, ad + b)$,</p> <ul style="list-style-type: none"> i) Prove $(S, *)$ is a semi group. Is it commutative? ii) Find the identity element of S iii) Which elements, if any, have inverses, and what are they? 	(6)
	(b)	<p>Let $\mathbb{R} - \{0\}$ represents set of all nonzero real numbers and M denotes the set of all 2×2 invertible matrices over \mathbb{R}. Determine whether the following map is a homomorphism. If so, what is its kernel? Given the map $f: \mathbb{R} - \{0\} \rightarrow M$ defined by $f(a) = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$.</p>	(4)
5	(a)	<p>Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ corresponding the encoding function $e: B^3 \rightarrow B^6$ find the parity check matrix and use it to decode the following received words and hence find the original message. Are all the words decoded uniquely?</p> <ul style="list-style-type: none"> (i) 111101 (ii) 100100 (iii) 111100 (iv) 010100 	(5)
	(b)	<p>In the group S_6, a permutation group over $\{1, 2, 3, 4, 5, 6\}$</p> $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 5 & 3 & 6 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 6 & 1 & 2 & 4 \end{pmatrix}$ <p>determine α^{-2} and $x \in S_6$ such that $\alpha x = \beta$.</p>	(1+2)
	(c)	<p>Provide a justification for why $U(8) = \{1, 3, 5, 7\}$, under multiplication modulo 8, is not a cyclic group.</p>	(2)

***** All the best *****



VIT[®]

Vellore Institute of Technology
(Approved by the University Grants Commission (UGC) Act, 1956)

Continuous Assessment Test - I

Programme Name & Branch: B.Tech

Course Name & Code: Discrete Mathematics and Graph Theory - MAT1014

Slot: A2+TAA2+TA2+V3

Exam Duration: 90 minutes

Maximum Marks: 50

Answer ALL the Questions ($5 \times 10 = 50$ marks)

S.No	Questions	Marks
1.a)	Using P: I will study Mathematics, Q: I will watch TV, R: I am happy, write the statement corresponding to the formula $(\neg P \wedge Q) \rightarrow R$.	(2)
b)	Obtain the principal disjunctive and principal conjunctive normal forms of $S \Leftrightarrow (p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$	(8)
2.a)	Verify the equivalence of $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.	(5)
b)	Test whether the premises $P \rightarrow Q$, $Q \rightarrow R$, $S \rightarrow R$ and $P \wedge S$ are consistent	(5)
3.a)	If $S(x)$: x is a spy novel, $L(x)$: x is long and $M(x)$: x is a mystery then write the well-formed formula for the statements (i) Not every mystery is a spy novel (ii) Every long spy novel is a mystery.	(2)
b)	Use indirect method of proof to verify the following $(x) (P(x) \vee Q(x)) \Rightarrow [(\forall x) P(x)] \vee [(\exists x) Q(x)]$.	(8)
4.	Is the following argument valid? All lecturers are determined, Anyone who is determined and intelligent will give satisfactory service, Clara is an intelligent lecturer. Therefore Clara will give satisfactory service.	(10)
5.a)	Let Z be the set of integers and $*$ be a binary operation defined as $a * b = a + b - ab$. Determine whether $(Z, *)$ is a Monoid.	(5)
b)	Find whether $(Z_4, +_4)$ is a group.	(5)

Course Name: Discrete Mathematics and Graph Theory
Slot : A1+TA1+TAA1

Course Code : MAT1014
Exam Duration: 90 minutes

Answer All the Questions ($5 \times 10 = 50$)

1. (a) Write down the contrapositive, the converse and the inverse of the statement, "If it is raining, then the home team wins." (2)

(b) Obtain PDNF and PCNF of the statement formula $(P \rightarrow (Q \wedge R)) \wedge ((\neg P \rightarrow (\neg Q \wedge \neg R)))$. (3)

2. Construct an argument to show that the following premises imply the conclusion "it rained."

- (i) If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on.
- (ii) If the sports day is held then trophy will be awarded.
- (iii) The trophy was not awarded.

(10)

3. (a) Let $P(m, n)$ be "n is greater than or equal to m" where the domain (universe of discourse) is the set of nonnegative integers. What are the truth values of

$$(i) (\exists n)(\forall m)P(m, n) \quad (ii) (\exists m)(\forall n)P(m, n).$$

(b) Show that the premises "A student in this class has not read the book" and

"Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." (8)

4. Show that $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y))$ and $(\exists y)(M(y) \rightarrow W(y)) \text{ imply } (\forall x)(F(x) \rightarrow \neg S(x))$. (10)

5. (a) Define Semigroup and Monoid. What is the relationship between them? Justify your answer.

(b) Prove that the set of four functions f_1, f_2, f_3 and f_4 on the set of non-zero complex numbers $\mathbb{C} - \{0\}$ defined by

$f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}, \forall z \in \mathbb{C} - \{0\}$ forms an abelian group with respect to the composition of functions. (8)

VIT

Vellore Institute of Technology

Winter Semester 2019-2020

Continuous Assessment Test - I

Programme Name & Branch: B. Tech.

Slot: A2+TA2+TAA2

Maximum Marks: 50

Answer All the Questions ($5 \times 10 = 50$)

1. Find the PDNF and PCNF of $(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$.

(i) With truth table method (ii) without truth table method.

2. (i) Prove that $\neg(p \wedge q) \rightarrow (\neg p \vee (\neg p \vee q)) \Leftrightarrow (\neg p \vee q)$.

(ii) Test the validity of the following arguments:

If milk is black then every crow is white.

If every crow is white then it has 4 legs.

If every crow has 4 legs then every Buffalo is white and brisk.

The milk is black.

So, every Buffalo is white.

3. (i) Show that the following statement is valid.

All men are mortal

Socrates is a man

Therefore Socrates is a mortal.

(ii) Prove that $(\exists x)(p(x) \wedge q(x)) \Rightarrow (\exists x)p(x) \wedge (\exists x)q(x)$

4. Prove that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)(P(x)) \vee (\exists x)(Q(x))$ using indirect method.

5. Show that the set of all real (2×2) matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}, ad - bc \neq 0$ is a group under matrix multiplication as binary operation.



VIT

Vellore Institute of Technology

Winter Semester 2019-2020
Continuous Assessment Test - I
Programme Name: B. Tech.

Course Name: Discrete Mathematics and Graph Theory
Slot : A1+TA1+TAA1

Course Code : MAT1014
Exam Duration: 90 minutes

Answer All the Questions ($5 \times 10 = 50$)

1. (a) Write down the contrapositive, the converse and the inverse of the statement, "If it is raining, then the home team wins." (2)

(b) Obtain PDNF and PCNF of the statement formula $(P \rightarrow (Q \wedge R)) \wedge ((\neg P \rightarrow (\neg Q \wedge \neg R)))$. (3)

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(iii) The trophy was not awarded. (10)

3. (a) Let $P(m, n)$ be " n is greater than or equal to m " where the domain (universe of discourse) is the set of nonnegative integers. What are the truth values of

(i) $(\exists n)(\forall m)P(m, n)$ (ii) $(\exists m)(\forall n)P(m, n)$. (2)

(b) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." (8)

4. Show that $(\exists x)(F(x) \wedge S(x)) \rightarrow (\exists y)(M(y) \rightarrow W(y))$ and $(\exists y)(M(y) \rightarrow W(y))$ imply $(\exists x)(F(x) \rightarrow \neg S(x))$. (10)

5. (a) Define Semigroup and Monoid. What is the relationship between them? Justify your answer. (2)

(b) Prove that the set of four functions f_1, f_2, f_3 and f_4 on the set of non-zero complex numbers $\mathbb{C} - \{0\}$ defined by

$f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}$ and $f_4(z) = -\frac{1}{z}, \forall z \in \mathbb{C} - \{0\}$ forms an abelian group with respect to the composition of functions. (8)

**VIT**Vellore Institute of Technology
(Approved by UGC & affiliated under Sec 2(f) of UGC Act, 1956)

Course Name & Code: Discrete Mathematics and Graph Theory & MAT1014

Exam Duration: 90 Minutes

Winter Semester 2018-19

Continuous Assessment Test – 1

Programme Name & Branch: B. Tech.

Slot : A1+TA1

Maximum Marks: 50

Answer All the Questions ($5 \times 10 = 50$)

S.No	Questions	
1.	<p>Test the consistency of the following Statements.</p> <ul style="list-style-type: none"> i. If Jack studies well he will pass in exams. ii. If Jack studies well he will get a job. iii. Succeeding in exam and getting a job simultaneously are not possible for him. iv. Jack either enjoys or studies well. v. Finally, Jack enjoys. 	[10 Marks]
2.	Obtain the PCNF and PDNF of the following formula and hence conclude whether it is a tautology $(P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$.	[10 Marks]
3.	<p>(i) Symbolize the arguments and check the validity?</p> <p>Every person likes ice cream. Billy is a person. Therefore, Billy likes ice cream.</p> <p>(ii) What is the negation of the following statements?</p> <ul style="list-style-type: none"> (a) Some subsets of N have 5 elements. (b) All people like ice cream. 	[6+4 Marks]
4.	<p>Show that from</p> <ul style="list-style-type: none"> (a) $(\exists x)(F(x) \wedge S(x)) \rightarrow (\forall y)(M(y) \rightarrow W(y))$ (b) $(\exists y)(M(y) \wedge \neg W(y))$ <p>the conclusion $(\forall x)(F(x) \rightarrow \neg S(x))$.</p>	[10 Marks]
5	<p>(i) Prove that the set of idempotent elements of a commutative monoid $\langle M, * \rangle$ forms a submonoid.</p> <p>(ii) Show that the set Q^* of all positive rational numbers forms an abelian group under the operation \bullet defined by $a * b = \frac{1}{2}(a.b); \forall a, b \in Q^*$.</p>	[10 Marks]