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Vellore Institute of Technology

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Reg. No. : 22BAI1266

Final Assessment Test (FAT) - May 2024

Programme	**B.Tech.**	Semester	**WINTER SEMESTER 2023 - 24**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. AMIT KUMAR RAHUL**	Slot	**A1+TA1+TAA1**
		Class Nbr	**CH2023240500810**
Time	**3 Hours**	Max. Marks	**100**

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer any 10 questions (10 X 10 Marks = 100 Marks)

01. If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, then find $f(z)$ in terms of z by the Milne-Thomson method. **[10]**

02. (a) If $w = \phi + i\psi$ represents the complex potential of an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$, then determine ϕ. [5 Marks] **[10]**
(b) Evaluate the integral $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$. [5 Marks]

03. Find the bilinear mapping which maps the points $z = -2i, i, \infty$ onto the points $w = 0, -3, \frac{1}{3}$ respectively. Then, find the image of the region $|z| \leq 1$ under this bilinear mapping and sketch both the regions in z- and w-planes respectively. **[10]**

04. Using the contour integration, evaluate $\int_0^\infty \frac{dx}{x^4+1}$. **[10]**

05. (a) Show that the bilinear mapping $w = \frac{az+b}{cz+d}$ transforms a straight line in the z-plane onto a unit circle in the w-plane if $|a| = |c|$. [5 Marks] **[10]**
(b) Let $W = \{A \in \mathbb{M}_{2\times2}(\mathbb{R}) | \text{trace}(A) = 0\}$. Verify whether W is a subspace or not. If so, then find its basis and dimension. [5 Marks]

06. **[10]**

$$\text{Let } A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & s & 4 \\ 1 & 2 & 3 & 4 & r \end{pmatrix}.$$

(a) For what values of r and s, the nullity of the matrix A is 1? Hence, find the null-space of the obtained matrix A. [8 Marks]
(b) For what values of r and s, the matrix A is of full rank? [2 Marks]

07. Check whether the mapping $T : \mathbb{R}^3 \to \mathbb{R}^2$, given by $T(x, y, z) = (x - y, y - z)$, is linear. Find the kernel and range of T, and their dimensions. Further, verify the rank-nullity theorem. Also, determine whether T is one-to-one and/or onto. **[10]**

08. Find the matrix of change of basis P from the basis $\alpha = \{x^2 + x + 1, x^2 + 1, x - 1\}$ to the basis $\beta = \{2x^2 + 3x + 1, 2x^2 + 2x + 1, -x^2 - 2\}$ for $\mathcal{P}_2(\mathbb{R})$, the set of all polynomials of degree at most 2. Hence, find the matrix of change of basis from β to α using the matrix P. Further, assume that the polynomial $p(x) \in \mathcal{P}_2(\mathbb{R})$ has coordinates $(1, 2, 3)$ with respect to the basis β. What are the coordinates of $p(x)$ with respect to the basis α? [10]

09. Using the Gram-Schmidt process, obtain an orthonormal basis from the basis set $\{(1, 0, 1), (1, 1, 1), (1, 3, 4)\}$ of the Euclidean space $\mathbb{R}^3$ with the standard inner product. [10]

10. (a) Let $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$, and define $\langle x, y\rangle = x_1y_1 + 3x_2y_2 - x_3y_3$. Does $\langle\, ,\, \rangle$ represent an inner product in $\mathbb{R}^3$? Justify your answer. [5 Marks] [10]

(b) Find the angle between the vectors $(1, 1, 0)$ and $(1, 1, 1)$ under the inner product $\langle(x_1, x_2, x_3), (y_1, y_2, y_3)\rangle = x_1y_1 + 2x_2y_2 + \frac{1}{3}x_3y_3$ defined on $\mathbb{R}^3$. [5 Marks]

11. Find the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{pmatrix}$. [10]

12. Using the Gauss-Jordan elimination method, solve the following system of equations: [10]

$x + y + z = 5,$

$2x + 3y + 5z = 8,$

$4x + 5z = 2.$