VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test II - OCTOBER 2023

Programme : B.Tech.		Semester :	Fall 2022-23
Course :	Discrete Mathematics and Graph Theory	Code :	BMAT205L
		Slot :	D1+TD1+TDD1
Faculty :	Dr. Berin Greeni, Dr. Jayagopal R, Prof. Aarthy B, Prof. Vignesh R, Prof. Anitha G, Prof. Sumathi S, Prof. Sakthidevi K, Prof. Gnanaprasanna K	Class ID :	CH2023240101195, 1047, 1191, 1192, 1193, 1195, 1196, 1197, 1198
Time :	90 Minutes	Max.Marks :	50

Answer ALL the questions $[5 \times 10 = 50]$

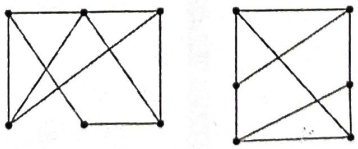
1. Suppose that there are two goats on an island initially. The number of goats on the island doubles every year by natural reproduction and some goats are either added or removed each year.

   a) Construct a recurrence relation for number of goats on the island at the start of the n^{th} year, assuming that during each year an extra 100 goats are put on the island. **(3 Marks)**

   b) Solve the recurrence relation from part (a) to find the number of goats on the island at the start of the n^{th} year. **(7 Marks)**

2. a) Let N be the set all positive integers, and R_1, R_2 be two relations on N defined as follows:
   Definition of R_1: For all $a, b \in N, (a, b) \in R_1$ if and only if $(a + b)$ is even.
   Definition of R_2: For all $a, b \in N, (a, b) \in R_2$ if and only if $\frac{a}{b} = 2^i$ for some integer $i \geq 0$.
   Are relations R_1 and R_2 symmetric, anti-symmetric and transitive. **(5 Marks)**

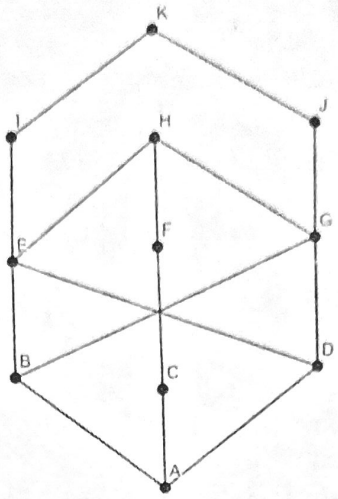
   b) Consider the two graphs given below. **(5 Marks)**



   i) Determine whether the graphs are planar or not, if yes draw the graph.

ii) Write the incidence matrix for the above two graphs.

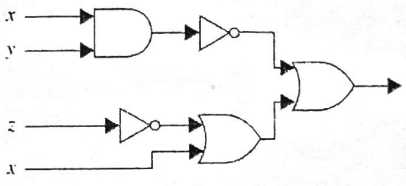
3. a) For the Poset given in the figure below find the maximal, minimal, greatest and least elements if it exists.



b) Determine whether the Poset is a lattice. If not, make minimum changes in the Hasse diagram, so that it becomes a lattice.

c) Find the least upper bound and the greatest lower bound of the subsets $\{B,F,G\}$ and $\{E,C,D\}$.

4. a) Find the output of the following circuit and find it's sum of product. **(3 marks)**



b) Simplify the following Boolean function using Karnaugh map. **(7 marks)**

$$f(a,b,c,d)=\sum(1,2,4,6,10,11,12,13,14,15)$$

5. a) Construct two simple graphs G and H by considering the factors of 12 and 18 as vertices $V(G)$ and $V(H)$, respectively, and the set of edges defined as "two distinct vertices are connected by an edge if and only if the one divides the other or vice versa". **(5 Marks)**

i) Is G isomorphic to H? Justify.

ii) Verify Hand Shaking theorem for these two graphs.

b) Construct a simple graph G by considering the set of numbers starting from 2 to 10 as vertices V and the set of edges defined by the following "two vertices are connected by an edge if and only if the corresponding numbers are co-prime". **(5 Marks)**

i) Find the maximum degree, minimum degree, total number of edges in the graph G.

ii) Is the constructed graph regular? Justify.