VIT
Vellore Institute of Technology

Final Assessment Test (FAT) - November/December 2023

Programme	B.Tech.	Semester	FALL SEMESTER 2023 - 24
Course Title	DISCRETE MATHEMATICS AND GRAPH THEORY	Course Code	BMAT205L
Faculty Name	Prof. Berin Greeni A	Slot	D1+TD1+TDD1
		Class Nbr	CH2023240101195
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

01. (a) Using algebra of logics, find the $PCNF$ and $PDNF$ of the following proposition: [10]
$[\neg(Q \to P) \wedge (Q \vee \neg R)] \vee (Q \wedge R)$. [7 Marks]
(b) Write the negation of the following proposition symbolically and in words: "Ravi takes a healthy diet whenever he feels tired or he is advised by a doctor". [3 Marks]

02. (a) Test the validity of the following argument: "If Ravi is hungry, then he gets angry. Ravi does [10] not get angry if he is not mad. If Ravi is mad, then his clothes are dirty. So, if Ravi is hungry, then his clothes are dirty". [7 Marks]
(b) Express the following statement using quantifiers: "Some student of VIT has asked every faculty member a question". [3 Marks]

03. (a) Check whether the following sets is a group or not: [5 Marks] [10]
(i) Set of all polynomials of degree less than or equal to 2 such that the polynomials are positive at 0.
(ii) Set of all 2×2 matrices with real entries such that the trace of a matrix is equal to the sum of the first row.
(b) Check whether the following is a group homomorphism or not: [5 Marks]
(i) $T : R^2 \to R^2$ such that $T(x, y) = (2x, 0)$
(ii) $T : R^2 \to R^2$ such that $T(x, y) = (x, y + 1)$

04. (a) Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ corresponding to the encoding [10]
function $e : B^3 \to B^6$, find the corresponding parity check matrix and use it to decode the following received words $(1\,1\,0\,1\,0\,1)$, $(0\,0\,1\,1\,1\,1)$, $(1\,1\,0\,0\,0\,1)$ and $(1\,1\,1\,1\,1\,1)$. Also, find the original message. Are all the words decoded uniquely? [5 Marks]
(b) (i) Using the Pigeonhole principle, prove that if 11 integers are selected from among $1, 2, \ldots, 20$, then the selection includes integer a and b such that $a - b = 2$, where a and b are selected uniquely. [3 Marks]
(ii) How many ways a 7-digit number can be selected from $0 - 9$, which does not change by reversing the order of digits? [2 Marks]

05. Solve the recurrence relation $a_{n+2} = 4a_{n+1} - 4a_n + n^2 - 5n + 2$ if $a_0 = 0$ and $a_1 = 2$. [10]

06. (a) Let C be the ordered set consisting of the mathematics courses and their prerequisites [10] appearing in the following table: [5 Marks]

Courses	Prerequisites
BMAT101L	None
BMAT201L	BMAT101L
BMAT250L	BMAT101L
BMAT251L	BMAT250L
BMAT340L	BMAT201L
BMAT341L	BMAT340L
BMAT450L	BMAT201L & BMAT250L
BMAT500L	BMAT450L & BMAT251L

If R is a relation such that $(Course\ X, Course\ Y) \in R$ if and only if the course $Course\ X$ is a prerequisite of the $Course\ Y$ then

(i) Prove that the realtion R is a POSET.

(ii) Draw the Hasse diagram for the POSET.

(iii) Find the greatest and least elements of the POSET.

(b) Simplify the following Boolean function $f(a, b, c, d) = \pi(0, 1, 2, 4, 5, 6, 8, 9, 15)$ using Karnuagh map. [5 *Marks*]

07. (a) Let X be the set of all VIT students with their own YouTube channel. For any $x, y \in X$, define xRy if and only if x commented on $y's$ YouTube channel. Check whether R is reflexive, symmetric, and transitive relation. [5 *Marks*] [10]

(b) Find at least four different perfect matchings and the minimum edge cover for the following graph. [5 *Marks*]
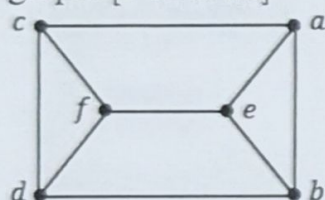
08. The following graph illustrates the distance between two cities for some of the major roads in Tamil Nadu, with the weights on the edges corresponding to the distance between the cities. Find the shortest route along with the total distance between Chennai and all other cities. [10]
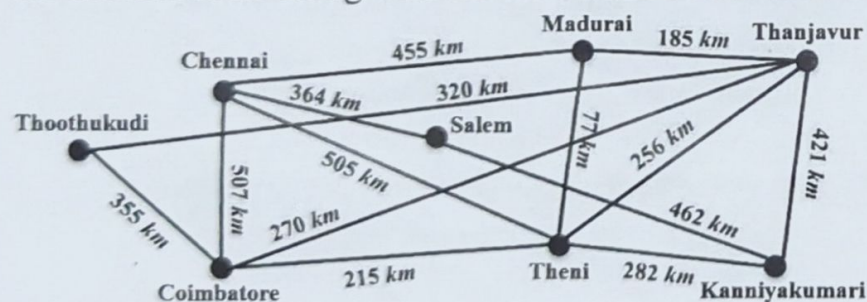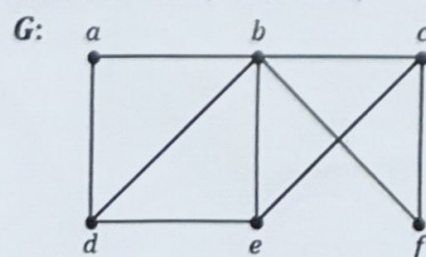

09. (a) Determine whether the following sequences are graphical or not. If it is graphical, construct a graph and if it is not graphical, explain why it is not graphical. [6 *Marks*] [10]
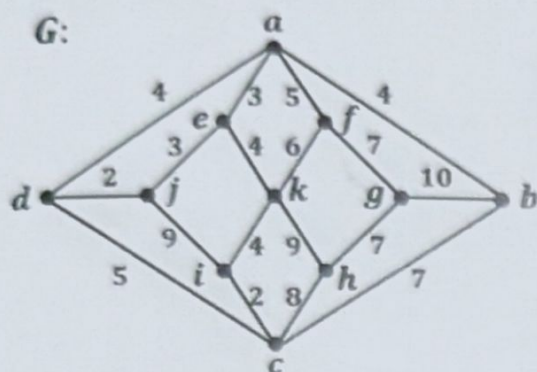
(i) $(6, 6, 6, 6, 3, 3, 2, 2)$

(ii) $(4, 4, 3, 3, 2, 2, 2, 2)$

(iii) $(9, 9, 7, 7, 7, 7, 6, 6, 5, 5)$

(b) Find a walk of length three from the vertex b to itself by finding the powers of the adjacency matrix of G. [4 *Marks*]

10. (a) Find a minimum spanning tree T for the weighted graph G given below and obtain the minimum weight of T using Kruskal's algorithm. [6 Marks] **[10]**

G:

(b) Obtain the fundamental cycles of G by removing the weights from the minimum spanning tree T, which is constructed in part (a). [4 Marks]

11. (a) In the given graph G, consider the spanning trees T_1, T_2 and T_3 with edge sets **[10]**
$E(T_1) = \{e_1, e_2, e_3, e_6, e_8\}$, $E(T_2) = \{e_3, e_4, e_5, e_7, e_8\}$, and $E(T_3) = \{e_1, e_3, e_4, e_8, e_9\}$
respectively. Find the fundamental cut sets for these spanning trees. [5 Marks]
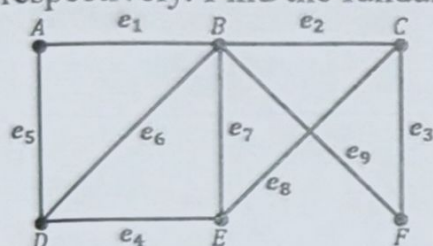
(b) (i) What is the maximum number of non-leaf vertices in a binary tree of height 5?
[2 Marks]

(ii) In a tree, there are $2n$ vertices of degree 1, $3n$ vertices of degree 2, and n vertices of degree 3. Determine the number of vertices and edges in the tree. [3 Marks]

12. Find the chromatic number and the chromatic polynomial for the following graph G. **[10]**

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