

M	T	W	T	F	S	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Assignment - Parameter Estimation

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Q.1)
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$f(x_i | \theta_1, \theta_2)$ is probability density function of a normal distribution with mean θ_1 and variance θ_2 .

$$f(x_i | \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

Taking natural log of Likelihood function :-

$$\ln L(\theta_1, \theta_2 | x_1, x_2, \dots, x_n) = \sum_{i=1}^n \ln f(x_i | \theta_1, \theta_2)$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

Maximum Likelihood estimators for θ_1 and θ_2 are

$$\hat{\theta}_1 = \bar{x}$$

$$\hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

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M	T	W	T	F	S
		1	2	3	4
6	7	8	9	10	11
13	14	15	16	17	18
20	21	22	23	24	25
27	28	29	30	31	

Q.2)

In Binomial distribution :-

m is known

 θ is unknown

PMF :-

$$P(X=x) = \binom{m}{x} \theta^x (1-\theta)^{m-x}$$

The Likelihood function,

$$L(\theta) = \prod_{i=1}^n \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking natural log of $L(\theta)$

$$\ln L(\theta) = \sum_{i=1}^n \left[\ln \binom{m}{x_i} + x_i \ln \theta + (m-x_i) \ln (1-\theta) \right]$$

To find maximum, differentiate $\ln L(\theta)$

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^n \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^n \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^n \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = \sum_{i=1}^n \left(\frac{x_i(1-\theta) - \theta(m-x_i)}{\theta(1-\theta)} \right)$$

$$\sum_{i=1}^n \left(\frac{x_i - m\theta}{\theta(1-\theta)} \right) = 0$$

JUNE 2024

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					1	2
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MAY

07

$$\sum_{i=1}^n \left(\frac{x_i}{\theta(1-\theta)} \right) - \sum_{i=1}^n \left(\frac{n}{1-\theta} \right) = 0$$

$$\frac{\sum_{i=1}^n x_i}{\theta(1-\theta)} = \frac{nm}{1-\theta}$$

$$\frac{\sum_{i=1}^n x_i}{n} = \frac{m\theta}{1-\theta}$$

$$\frac{\sum_{i=1}^n x_i}{n} (1-\theta) = m\theta$$

$$\frac{\sum_{i=1}^n x_i}{n} = m\theta + \frac{\sum_{i=1}^n x_i \theta}{n}$$

$$\theta = \frac{\sum_{i=1}^n x_i}{nm + \sum_{i=1}^n x_i}$$

Maximum Likelihood estimator of θ is $\frac{\sum_{i=1}^n x_i}{nm + \sum_{i=1}^n x_i}$