Assignment - Parameter estimation

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Q.1)
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$f(x_i \mid \theta_1, \theta_2)$ is probability density function of a normal distribution with mean θ_1 and variance θ_2.

$$f(x_i \mid \theta_1, \theta_2) = \frac{1}{\sqrt{2\pi\theta_2}} e^{\left(-\frac{(x_i-\theta_1)^2}{2\theta_2}\right)}$$

Taking natural log of Likelihood function :-

$$\ln L(\theta_1, \theta_2 \mid x_1, x_2, ---, x_n) = \sum_{i=1}^{n} \ln f(x_i \mid \theta_1, \theta_2)$$

$$\bar{x} = 1/n \sum_{i=1}^{n} x_i$$
$$S^2 = 1/n \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Maximum Likehood estimators for θ_1 and θ_2 are

$$\hat{\theta_1} = \bar{x}$$
$$\hat{\theta_2} = 1/n \sum_{i=1}^{n} (x_i - \bar{x})^2.$$

Q.2)

In Binomial distribution :-

m is known

θ is unknown

PMF :-

$$P(X=x) = \binom{m}{x} \theta^{x} (1-\theta)^{m-x}$$

The likelihood function,

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking natural log of $L(\theta)$

$$\ln L(\theta) = \sum_{i=1}^{n} \left[\ln \binom{m}{x_i} + x_i \ln\theta + (m-x_i) \ln(1-\theta) \right]$$

To find maximum, differentiate $\ln L(\theta)$

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = \sum_{i=1}^{n} \left(\frac{x_i(1-\theta) - \theta(m-x_i)}{\theta(1-\theta)} \right)$$

$$\sum_{i=1}^{n} \left(\frac{x_i - m\theta}{\theta(1-\theta)} \right) = 0$$

$$\sum_{i=1}^{n}\left(\frac{x_i}{\theta(1-\theta)}\right)-\sum_{i=1}^{n}\left(\frac{m}{1-\theta}\right)=0$$

$$\frac{\sum_{i-1}^{n} x_i}{\theta(1-\theta)}=\frac{mn}{1-\theta}$$

$$\frac{\sum_{i-1}^{n} x_i}{n}=\frac{m\theta}{1-\theta}$$

$$\frac{\sum_{i-1}^{n} x_i}{n}(1-\theta)=m\theta$$

$$\frac{\sum_{i-1}^{n} x_i}{n}=m\theta+\frac{\sum_{i=1}^{n} x_i\theta}{n}$$

$$\theta=\frac{\sum_{i=1}^{n} x_i}{nm+\sum_{i=}^{n} x_i}$$

Maximum Likelihood estimator of θ is $\dfrac{\sum_{i=1}^{n} x_i}{nm+\sum_{i=1} x_i}$