Computer Science & DA

Calculus and Optimization

Functions

Lecture No. 02

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Recap of previous lecture

Topic Function

Topic Graphs

Topics to be Covered

Topic Types of Functions

Topic :Types of Functions

Properties of log function

1. $\log_e a + \log_e b = \log(ab)$
2. $\log a - \log b = \log\left(\frac{a}{b}\right)$
3. $\log_b a = \frac{\log_e a}{\log_e b}$
4. $\log_b a = \frac{1}{\log_a b}$
5. $\log_a x = b \Rightarrow \boxed{x = a^b}$
6. $a^{\log_a x} = x$; $x > 0$

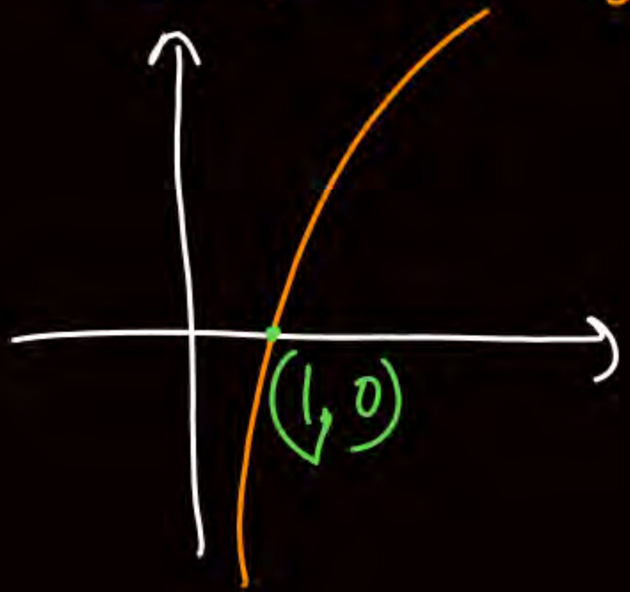
   eg $e^{\log_e x} = x$
7. $\log_1 x = \frac{\log_e x}{\log_e 1} = \frac{\log_e x}{0} = N.D$
8. $\log_e(a^b) = b \log_e a$
9. $\log_{(a^b)} x = \frac{1}{b} \log_a x$

(10) $\log_e 1 = 0$

$\log_e \infty = \infty \simeq ND$

$\log_e 0 = -\infty \simeq ND$

(11) $y = \log x \Rightarrow$ Domain $(0, \infty)$ or $x > 0$

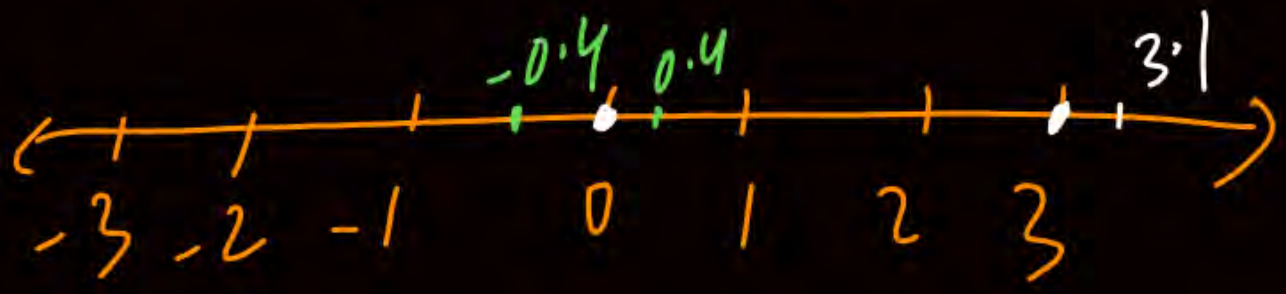


Greatest Integer function →

$[2.3] = 2$, $[4.7] = 4$, $[3.1] = 3$,

$[-2.3] = -3$, $[-4.7] = -5$, $[-3.1] = -4$

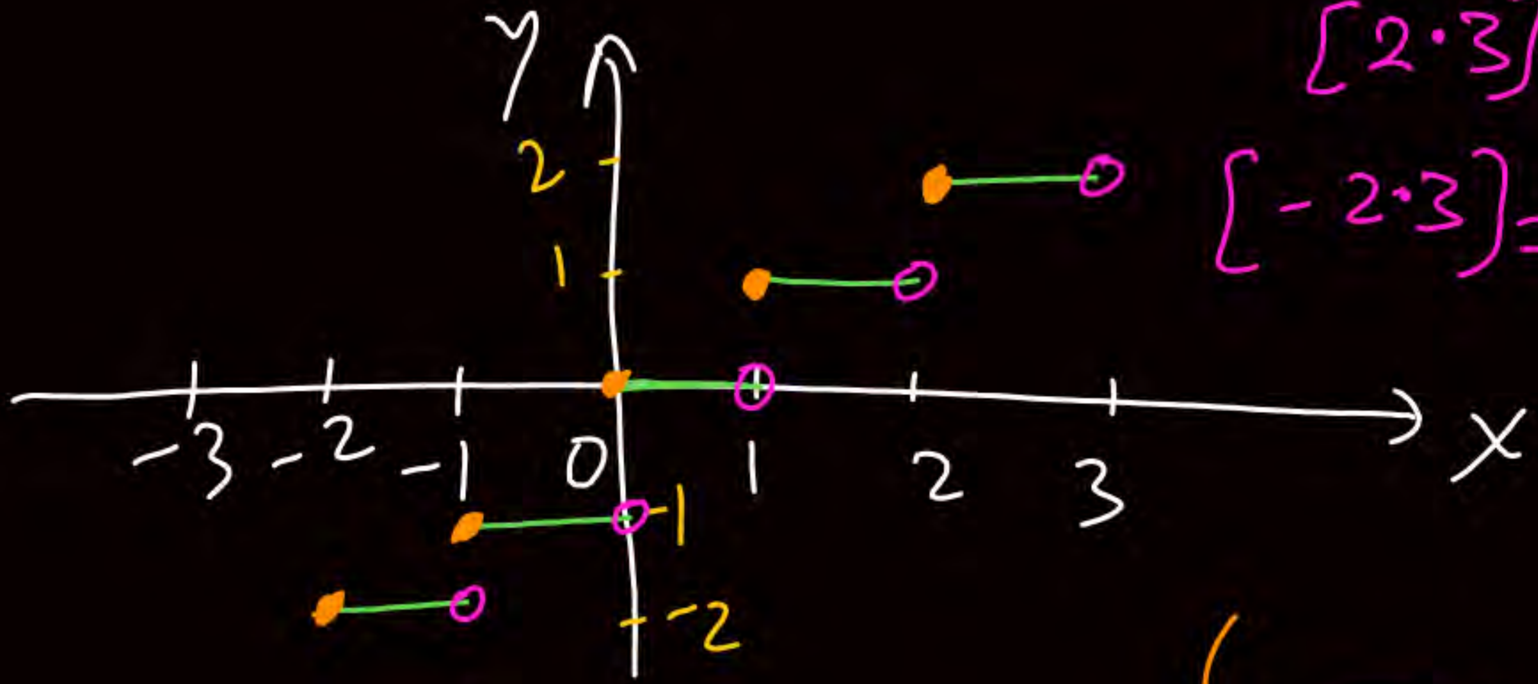
$[0.4] = 0$, $[-0.4] = -1$



$[2] = 2$, $[3] = 3$

$[-3] = -3$

$$y = [x] = \begin{cases} -2 & , -2 \le x < -1 \\ -1 & , -1 \le x < 0 \\ 0 & , 0 \le x < 1 \\ 1 & , 1 \le x < 2 \\ 2 & , 2 \le x < 3 \\ \vdots & \vdots \end{cases}$$



$[2\cdot3] = 2\cdot3 - 2 = 0\cdot3$

$[-2\cdot3] = x+3 = 0\cdot7$

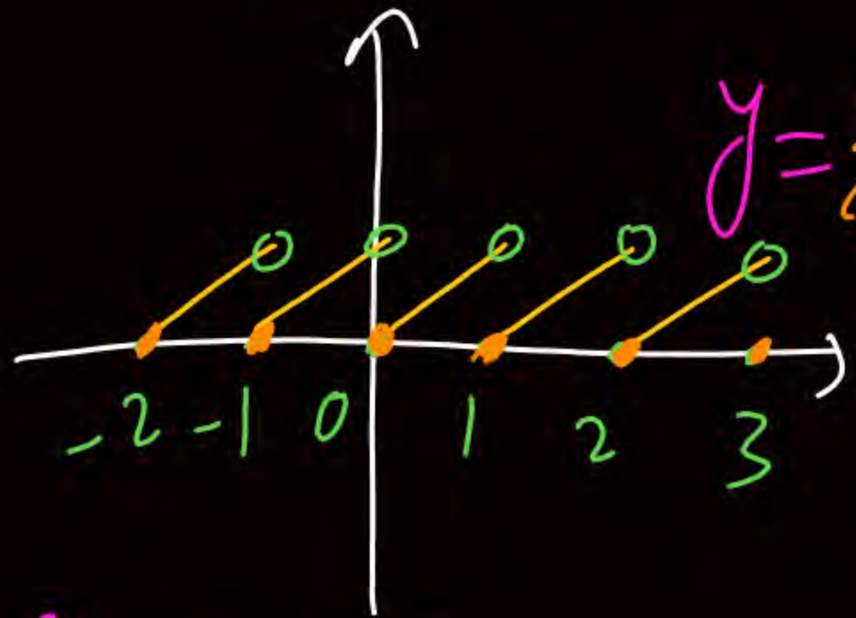
Fractional part function →

$\{2\cdot3\} = 0\cdot3$

$\{1\cdot5\} = 0\cdot5$

$\{1\cdot9\} = 0\cdot9$

$$\boxed{\{x\} = x - [x]}$$



$$y = \{x\} = \begin{cases} x+2 & , -2 \le x < -1 \\ x+1 & , -1 \le x < 0 \\ x & , 0 \le x < 1 \\ x-1 & , 1 \le x < 2 \\ x-2 & , 2 \le x < 3 \\ \vdots & \end{cases}$$

$\{x\} = x - [x]$

$= x - (-2) \quad -2 \le x < -1$

$= x + 2$

$$f(x) = |x| = \begin{cases} -x, & x < 0 \\ x, & x > 0 \end{cases}$$

Even function → $y = f(x)$ is said to be an Even function if "it's graph is symmetrical about Y axis" i.e $\boxed{f(-x) = f(x)}$

odd function → $y = f(x)$ is said to be an odd function if "it's graph is symmetrical about origin i.e symmetrical in opposite Quadrants.

$(\mathrm{I} \longleftrightarrow \mathrm{III}), (\mathrm{II} \longleftrightarrow \mathrm{IV})$

i.e $\boxed{f(-x) = -f(x)}$

Neither Even Nor odd → if $f(-x) \neq f(x)$ or $\neq -f(x)$ then $f(x)$ is NENO funcn

it's graph is neither symmetrical about Y axis nor about origin.

Q. Check the nature of the following functions :-

① $y = x^2$

even funⁿ

$$f(-x) = (-x)^2$$
$$= x^2$$
$$= f(x)$$

② $y = x^3$

odd funⁿ

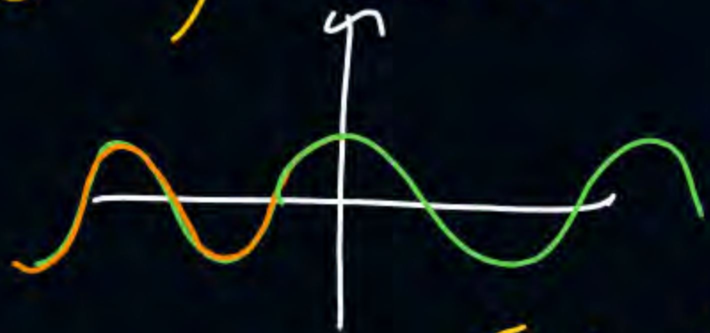
$$f(-x) = (-x)^3$$
$$= -x^3$$
$$= -f(x)$$

③ $y = \sin x$

odd funⁿ.

$$f(-x) = \sin(-x)$$
$$= -\sin x$$
$$= -f(x)$$

④ $y = \cos x$



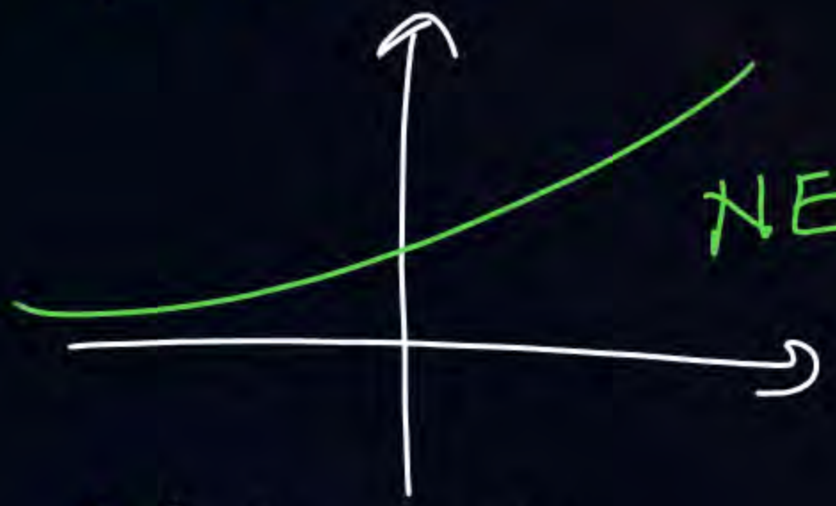
Even funcⁿ.

$$f(-x) = \cos(-x)$$
$$= \cos x$$
$$= f(x)$$

(5) $f(x) = \dfrac{x^2 \sin x}{x^2 + \cos x} \Rightarrow f(-x) = \dfrac{(-x)^2 \sin(-x)}{(-x)^2 + \cos(-x)} = -\dfrac{x^2 \sin x}{x^2 + \cos x} = -f(x)$

odd func^n.

(6) $f(x) = e^x$

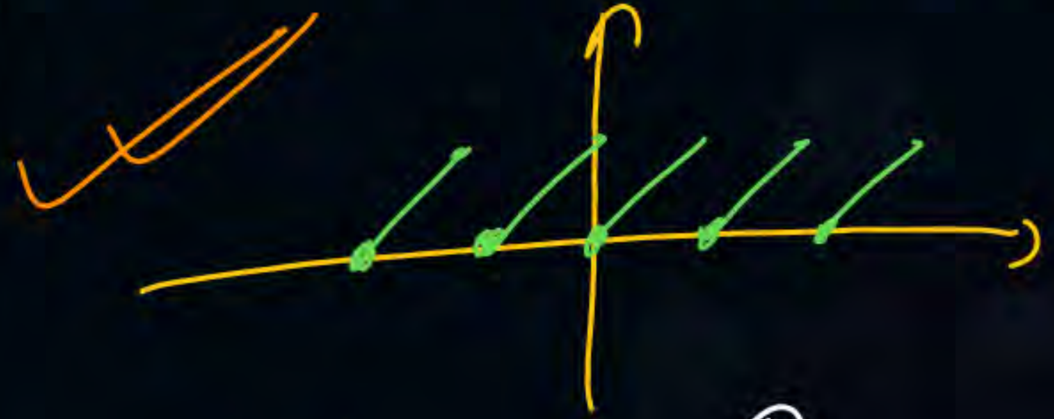


NENO.

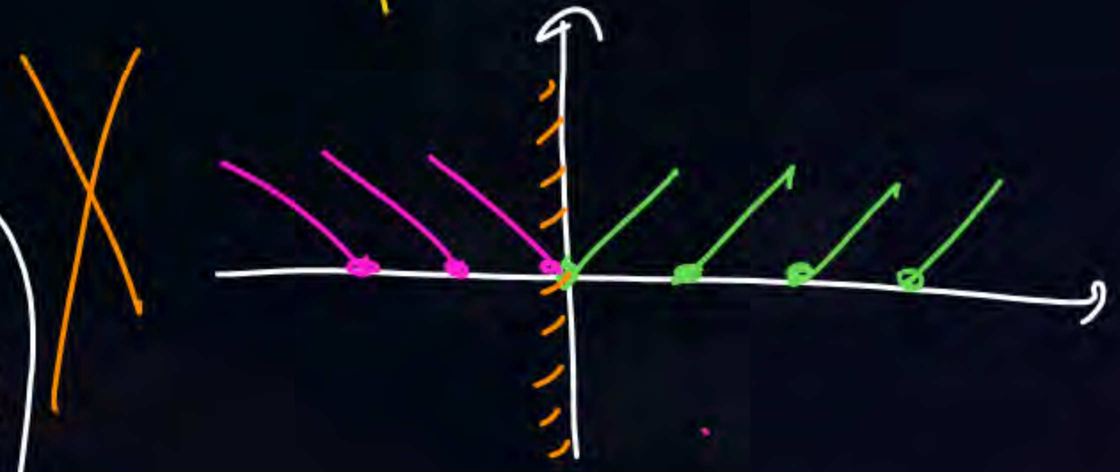
$\because f(-x) = e^{-x} = \dfrac{1}{e^x} = \dfrac{1}{f(x)}$

$\because f(-x) \neq f(x)$ or $-f(x)$ ie it is NENO

(7) $y = f(x) = \{x\} = x - [x]$



NENO



PERIODIC function → if Graph of func^n repeats after certain duration T, then it is called Periodic func^n with period T i.e. $\boxed{f(x+T) = f(x)}$

eg $f(x) = \sin x \Rightarrow f(x+2\pi) = \sin(2\pi + x) = \sin x = f(x)$ so Periodic with Period $T = 2\pi$

eg $f(x) = \cos x$ Periodic with Period $T = 2\pi$

eg $f(x) = \tan x$ " " " $T = \pi$ $\because \tan(\pi + x) = \tan x$

eg $f(x) = e^x$ Non Periodic. $\because f(x+T) \neq f(x)$

eg: which of the following is a polynomial fun^{c}.

(a) $f(x) = \frac{x-1}{2x+3}$

Rational fun^{n}

$D = R - \{-3/2\}$

(b) $f(x) = 4x^{1/3} - 5$

Irrational fun^{n}.

$D = (-\infty, \infty)$

(c) $f(x) = 2x - 1$

Linear poly ; $(-\infty, \infty)$

(d) $f(x) = |x|$

$D_f = (-\infty, \infty)$

Piecewise $func^{n}$

(e) $f(x) = 5x^3 + 4x^2 - 3$

Cubic poly ; $(-\infty, \infty)$

(f) $f(x) = e^x$

Transcedental $func^{n}$.

$D = (-\infty, \infty)$

(g) $f(x) = x$ (Linear poly)

$D = R$

(h) $f(x) = \frac{x^2}{x} = x$; $R - \{0\}$.

Polynomial fn^n : Degree $\in W$ & Dom $= (-\infty, \infty)$, Def^n should be same throughout the domain

$f(x) = K$ (Constant poly) & degree $= 0$

$f(x) = ax + b$ (Linear) & degree $= 1$

$f(x) = ax^2 + bx + c$ (Quadratic) & degree $= 2$

$f(x) = ax^3 + bx^2 + cx + d$ (Cubic) & degree $= 3$

⋮

eg. Why $f(x) = |x|$ is not poly.

$$|x| = \begin{cases} -x, & x < 0 \\ +x, & x \geq 0 \end{cases}$$

Domain $= (-\infty, \infty)$ But def^n is not same for all 'x'

- Functions
  - Algebraic func^n
    - Poly fun^n
    - Rational func^n
    - Irrational func^n
    - Piecewise func^n
      - Modulus fun^n
      - Signum fun^n
      - G.I. function
      - Fractional Part func^n
  - Transcedental func^n
    - Trig. func^n
    - Inverse func^n
    - Exponential
    - log func^n

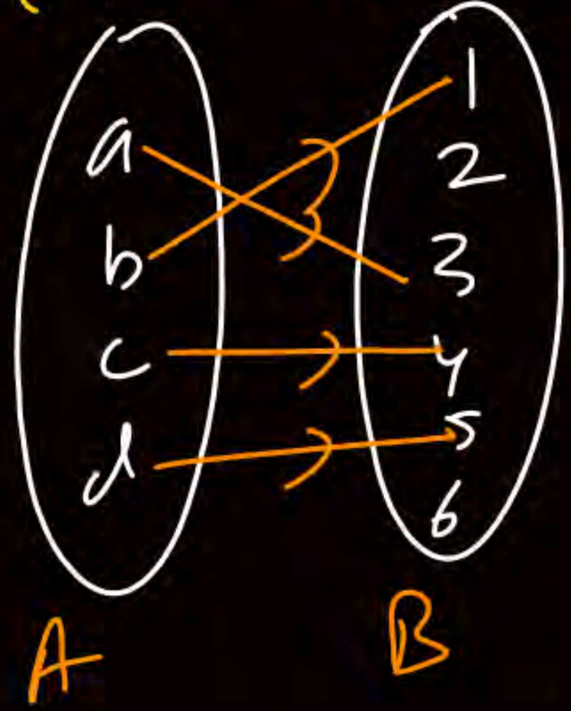
Piecewise func^n – if $f(x)$ is defined by Multiple subfunc^n s.t domain of each sub func^n is different then $f(x)$ is called P. func^n.

Elementary func^n → eg: All poly. func^n, e^x, $\log x$, Trig func^n, Inverse Trig. func^n are called E-func^n.

(*) All E-func^n are Continuous as well as Differential in their Domain

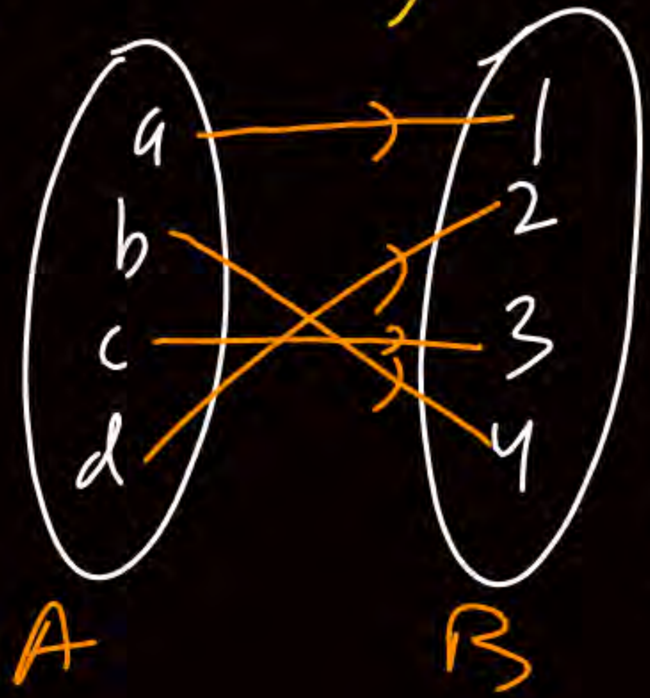
Range $= \{1, 3, 4, 5\}$, Codomain $= \{1, 2, 3, 4, 5, 6\}$

(*) 1



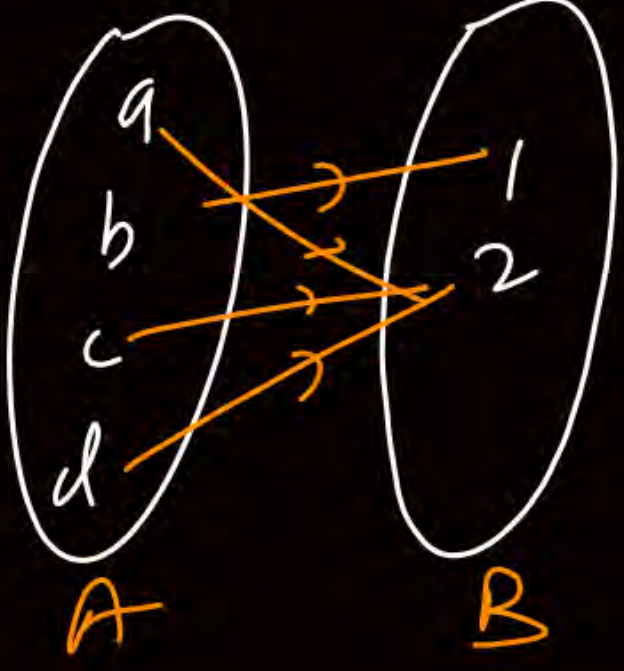
one one / INTO

2



one one / ONTO

3



Many one / ONTO

Many one / INTO

Range $= \{1, 2\}$

Codomain $= \{1, 2, 3\}$

(*) if Range f = Codomain then f is ONTO.

& if Range $f \subset$ Codomain " " " INTO.

(*) if $x_1 \neq x_2 \Longrightarrow f(x_1) = f(x_2)$ then f is Many one

if $\underbrace{x_1 \neq x_2 \Longrightarrow f(x_1) \neq f(x_2)}$ " " one-one

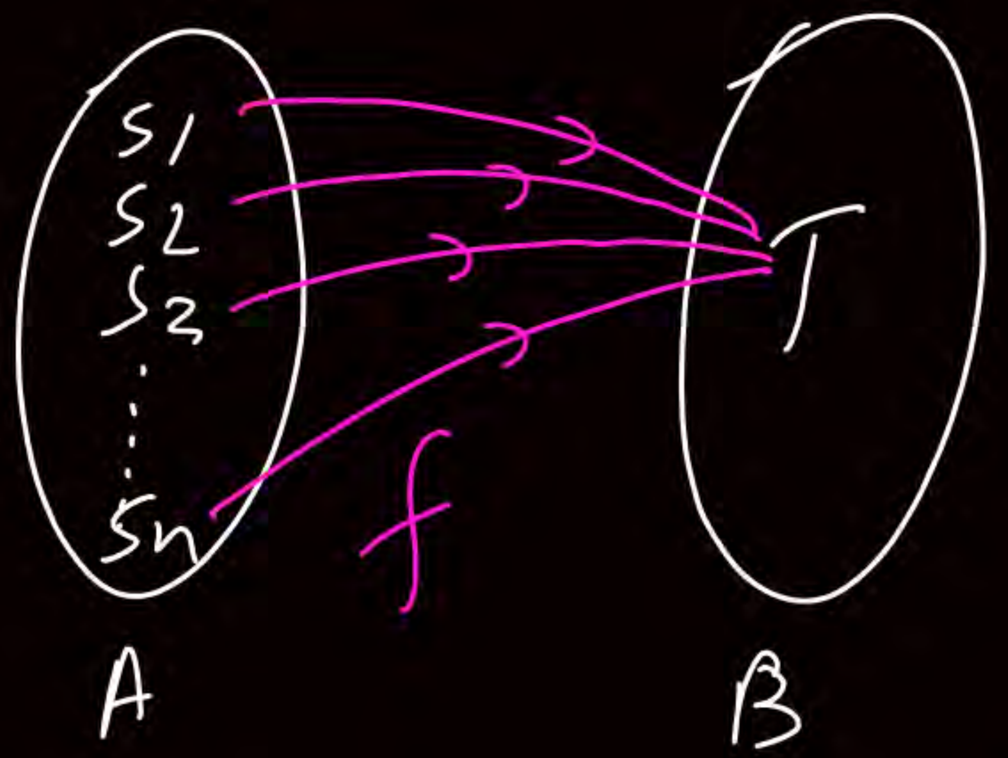
i.e Different elements have different Images.

BIJECTION ⟶ one one / ONTO.
(one-one Correspondence)

(*) one-one Mapping
$\Longrightarrow$ Injective Mapping.

(*) if $y = f(x)$ is given func^n then $f^{-1}(x)$ exist only when f is one one / ONTO.

eg



Here f is Many one/ONTO

$\frac{10}{2} = 5$

Some Important Points →

① **Infinity** → It is not a very large number. It is the presentation of that thought which is beyond Imagination.

and it is not unique

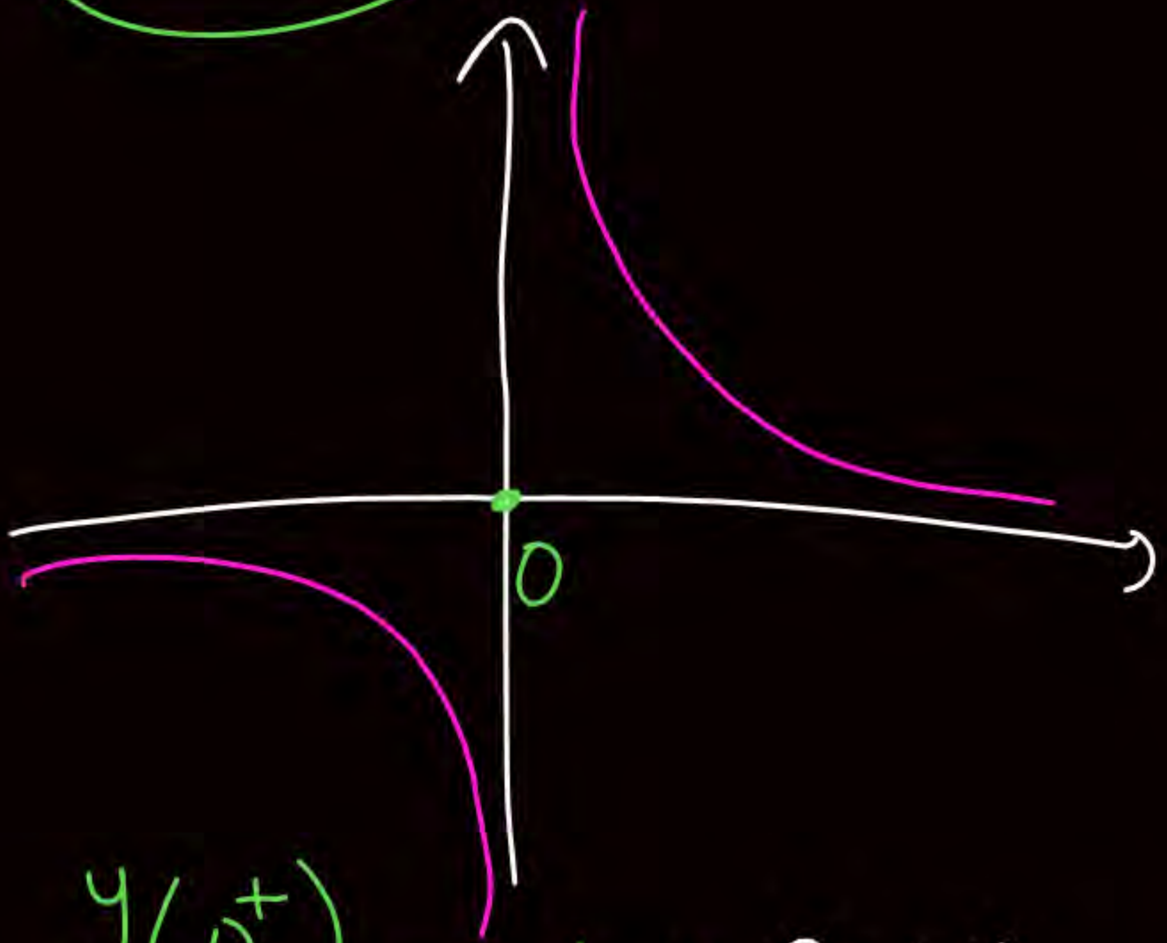
$\infty + \infty = \infty \simeq ND$

$\infty \times \infty = \infty \simeq ND$

$\frac{\text{Something}}{\infty} \simeq 0$ (Assumption)

$\frac{\text{Something}}{0} = ND \simeq \left(\begin{array}{l}\text{Either } \infty \\ \text{or } -\infty\end{array}\right)$

$y = \frac{1}{x}$



$y(0^+) = +\infty \simeq ND$

$y(0^-) = -\infty \simeq ND$

$\frac{\text{something}}{0} = ND$ But

$\frac{0}{0} = \underbrace{1, 2, 3, -4, \frac{1}{3}, \sqrt{7}, \frac{5}{4}, \dots}_{\text{Infinite answers are possible.}}$

$\frac{\infty}{\infty} = \frac{1/0}{1/0} = \frac{0}{0}$ = IND form.

$0 \times \infty = 0 \times \frac{1}{0} = \frac{0}{0}$ = IND form

INDETERMINATE form – If an expression has Infinite number of answers then that expression is said to be in IND form.

there are exactly seven IND forms as follows;

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \times \infty$, $\infty - \infty$, 0^0, ∞^0, 1^∞

(*) Let $0^0 = K$

or $K = 0^0$

$$\log K = \log(0^0)$$

$$= 0 \log 0$$

$$= 0 \times (-\infty)$$

$$= -0 \times \infty$$

$$= -0 \times \frac{1}{0}$$

$$\log_e K = \frac{0}{0}$$

$$K = e^{\frac{0}{0}} = IND$$

(2) Let $\infty^0 = K$

or $K = \infty^0$

$$\log K = \log(\infty^0)$$

$$= 0 \times \log \infty$$

$$= 0 \times \infty$$

$$= 0 \times \frac{1}{0}$$

$$\log_e K = \frac{0}{0}$$

$$K = e^{\frac{0}{0}} = IND$$

(3) Let $1^\infty = K$

or $K = 1^\infty$

$$\log K = \log(1^\infty)$$

$$= \infty \times \log(1)$$

$$= \infty \times 0$$

$$\log_e K = \frac{1}{0} \times 0 = \frac{0}{0}$$

$$K = e^{\frac{0}{0}} = IND$$

$1^{20} = 1$

$1^{50} = 1$

$1^{\infty} \neq 1$

Eg Check the Nature of 0^{∞}

Sol: Let $K = 0^{\infty}$

$\log_e K = \log(0^{\infty}) = \infty \times \log_e 0 = \infty \times (-\infty)$

$= -(\infty \times \infty)$

$\log_e K = -\infty$

$K = e^{-\infty} = 0$ i.e. Determinate.

2 mins Summary

Topic

Topic

Topic

Topic

Topic

THANK - YOU