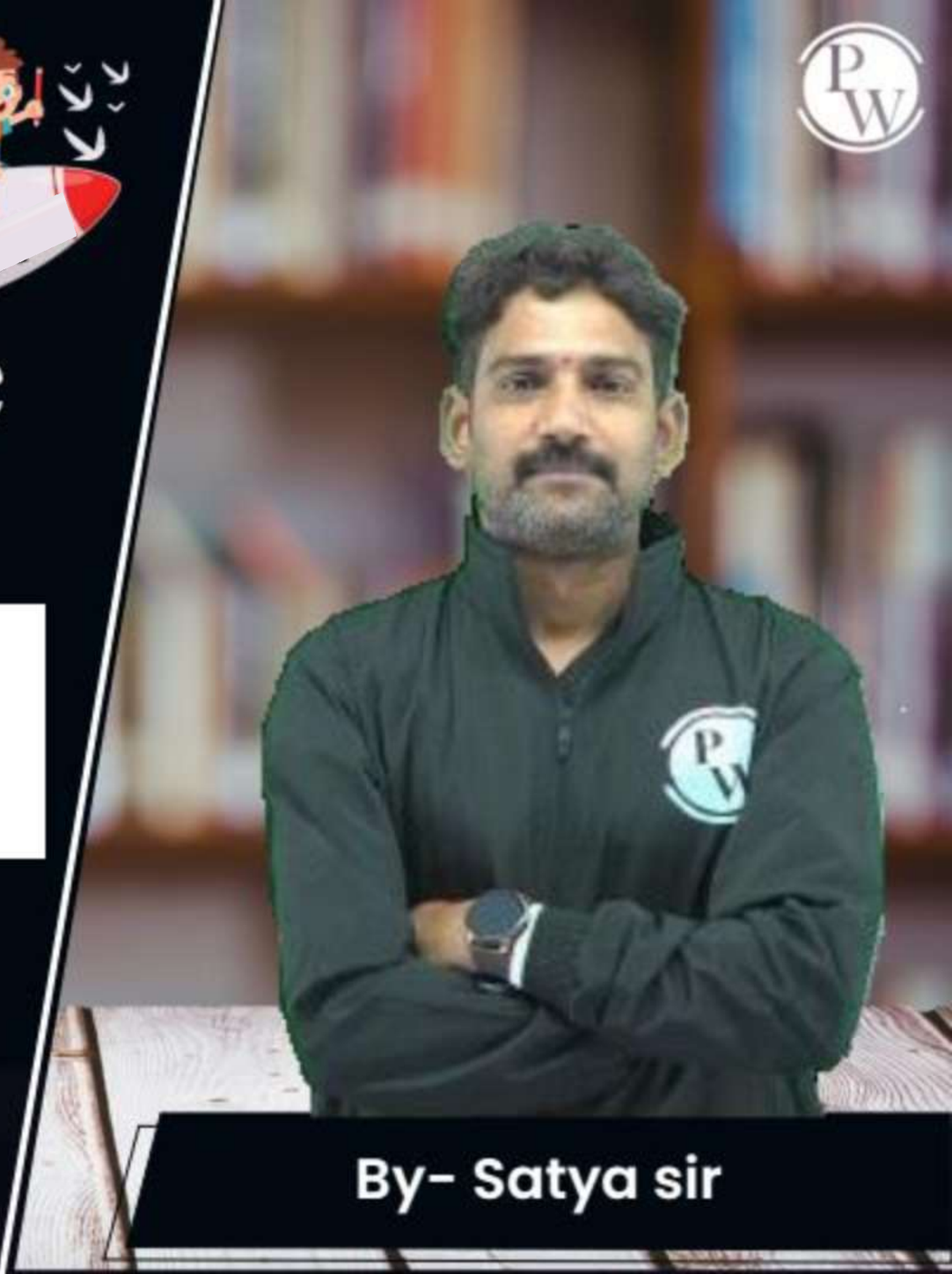
# Data Science & Artificial Intelligence

## Data Structures Through Python

## TREES

Lecture No.- 02



By- Satya sir

# Recap of Previous Lecture

- Trees ?
- Types of Binary Trees
  - Full Binary Tree
  - Complete Binary Tree
  - Perfect Binary Tree
  - Skewed Binary Tree
  - Degenerated Binary Tree

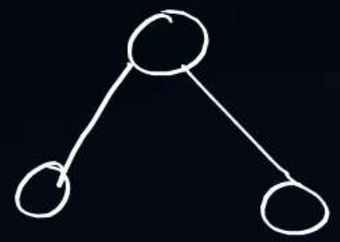
# Topics to be Covered

Formulae on Trees, Binary Trees

# Topic : Formulae Of Binary Trees

1) - In FBT/PBT The Total No. of Nodes $2x+1$ — $x$ internal Nodes, $x+1$ External Nodes

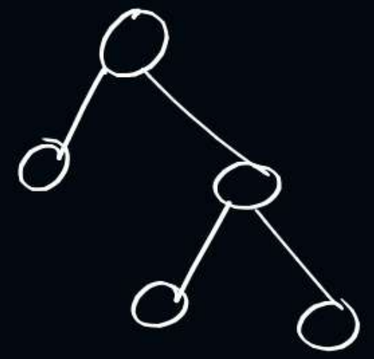
Ex: ①



Internal Nodes, $x = 1$

External Nodes, $x+1 = 2$

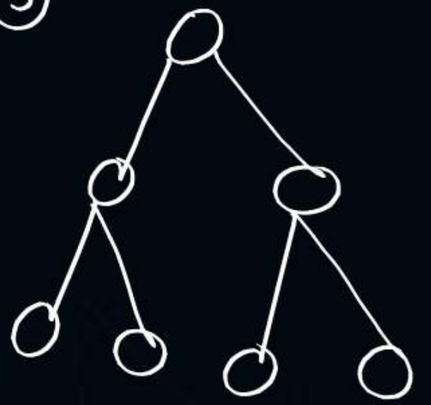
②



$x = 2$

$x+1 = 3$

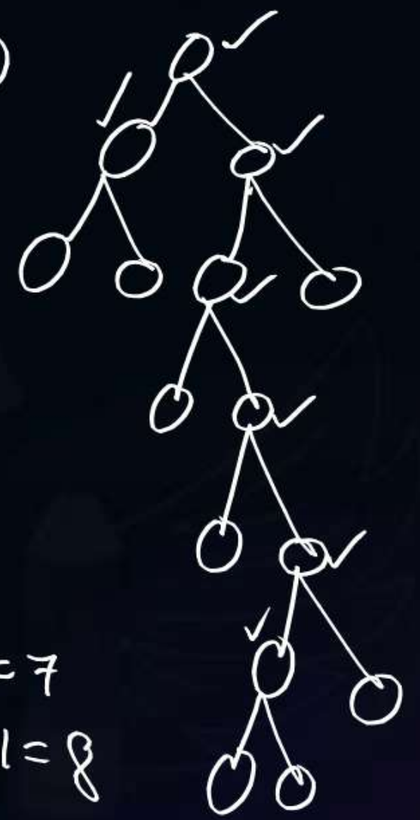
③



$x = 3$

$x+1 = 4$

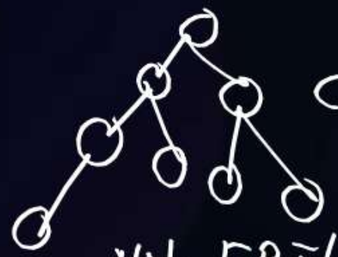
④



$x = 7$

$x+1 = 8$

⑤



CBT

Not FBT/PBT

# Topic : Formulae Of Binary Trees

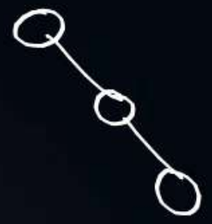
2) In a Binary Tree, The Number of Unlabelled Binary Trees Possible with 'n' nodes

$$= \frac{2^n C_n}{(n+1)}$$

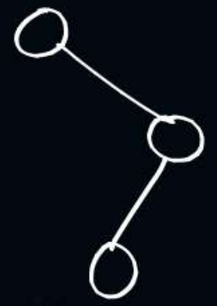
Ex: n=3

1) 

2) 

3) 

4) 

5) 

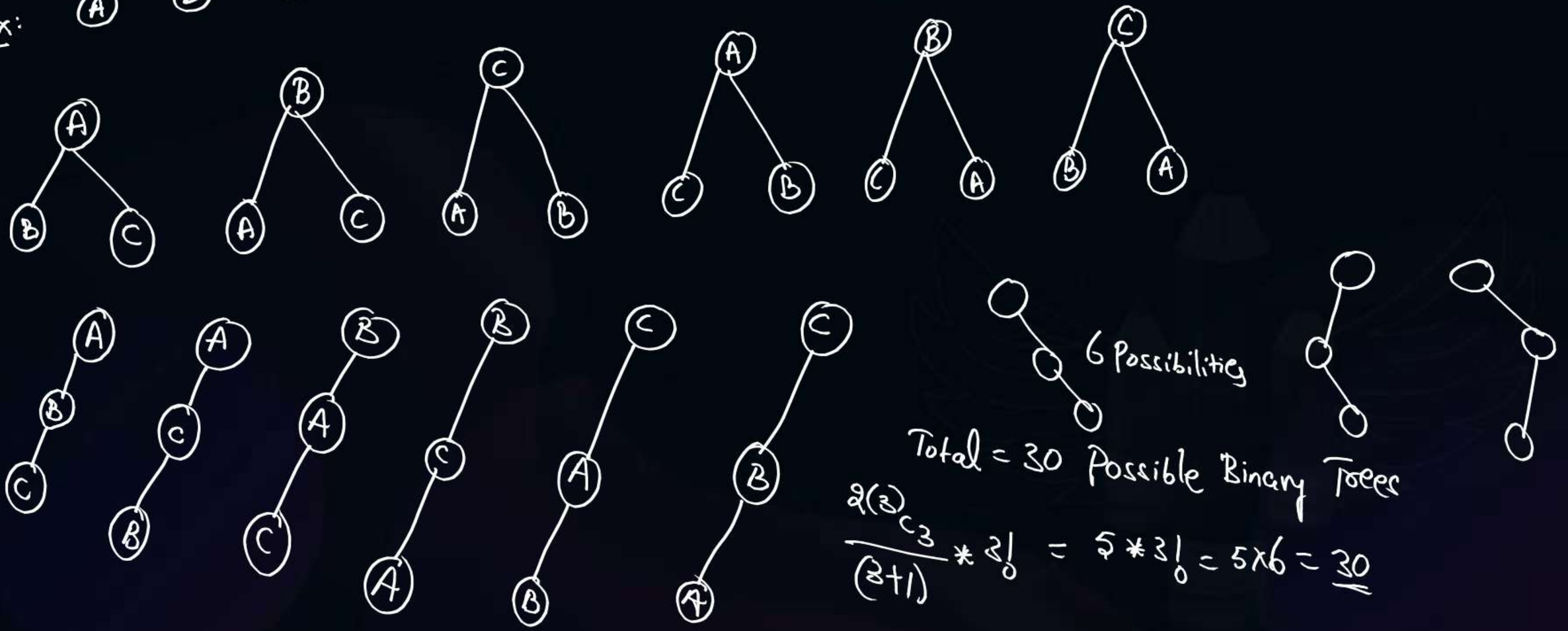
$$n=3 \quad \frac{2(3)C_3}{(3+1)} = \frac{6C_3}{(3+1)} = \frac{\frac{6!}{(6-3)!\,3!}}{4} = \frac{\cancel{6}\times 5\times \cancel{4}\times \cancel{3!}}{\cancel{3!}\times \cancel{4}} * \frac{1}{\cancel{4}} = \underline{5}$$

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

# Topic : Formulae Of Binary Trees

3) The Number of Labelled Binary Trees Possible with 'n' nodes $= \left(\frac{{}^{2n}C_n}{n+1}\right) * n!$

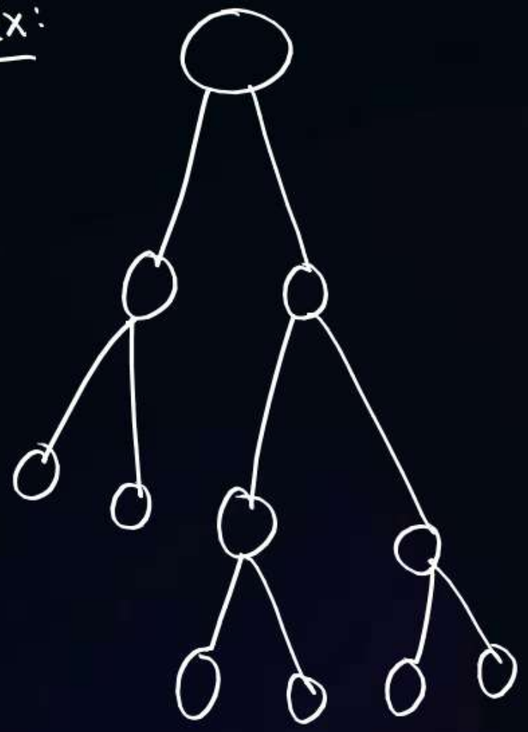
Ex: (A) (B) (C) Nodes



# Topic : Formulae Of Binary Trees

④ In Binary Tree, if '$i$' Leaf Nodes, Then The Number of Nodes with Exactly 2 children $= (i-1)$

Ex:



$i = 6$

$i-1$ = Nodes with Exactly 2 children $= (6-1) = 5$

# Topic : Formulae Of Binary Trees

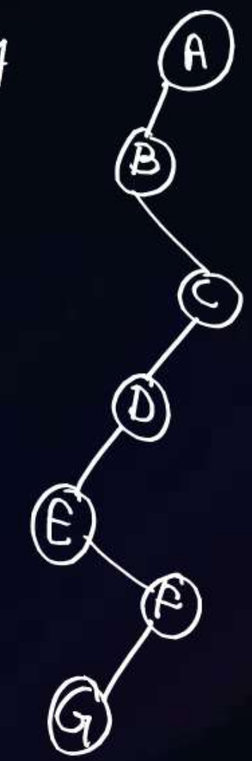
4) In a Binary Tree, with 'N' Nodes,

The maximum height Possible = $(N-1)$ [SBT/DBT]

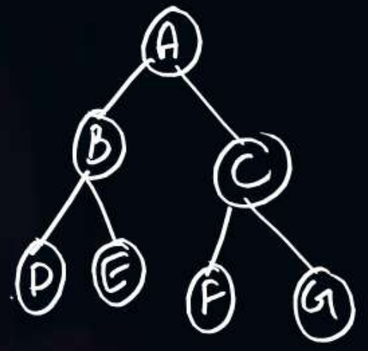
The minimum height Possible = $\lceil \log_2^{(N+1)} \rceil - 1$ [PBT/CBT]

Ex:

N = 7

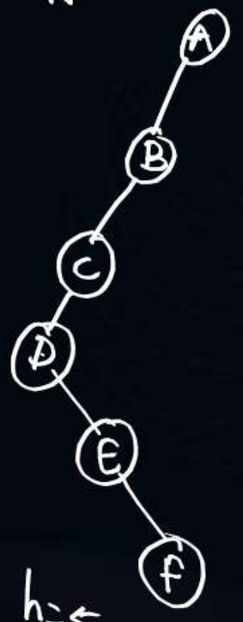



h = 6

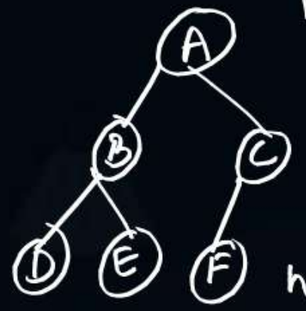



h = 2

N = 6




h = 5




h = 2

h = 1

h = 0

$\lceil \log_2^{(6+1)} \rceil - 1$

$= \lceil \log_2^{7} \rceil - 1 \quad = 3-1$

$= 2$

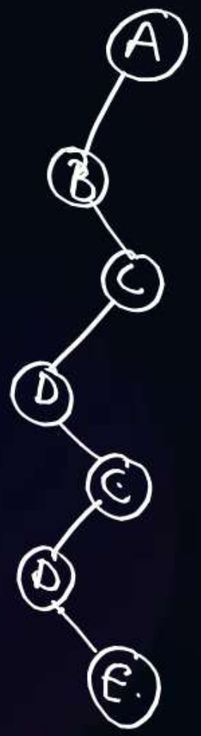
# Topic : Formulae Of Binary Trees

6) In a Binary Tree with height 'H',

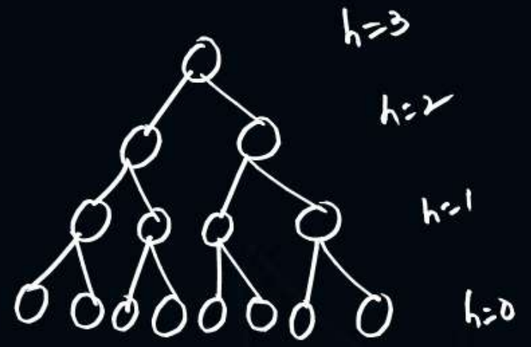
Maximum Number of Nodes Needed = $2^{(H+1)} - 1$

minimum Number of Nodes Needed = $(H+1)$

$H = 6$




$H = 3$




height(Tree) == 3

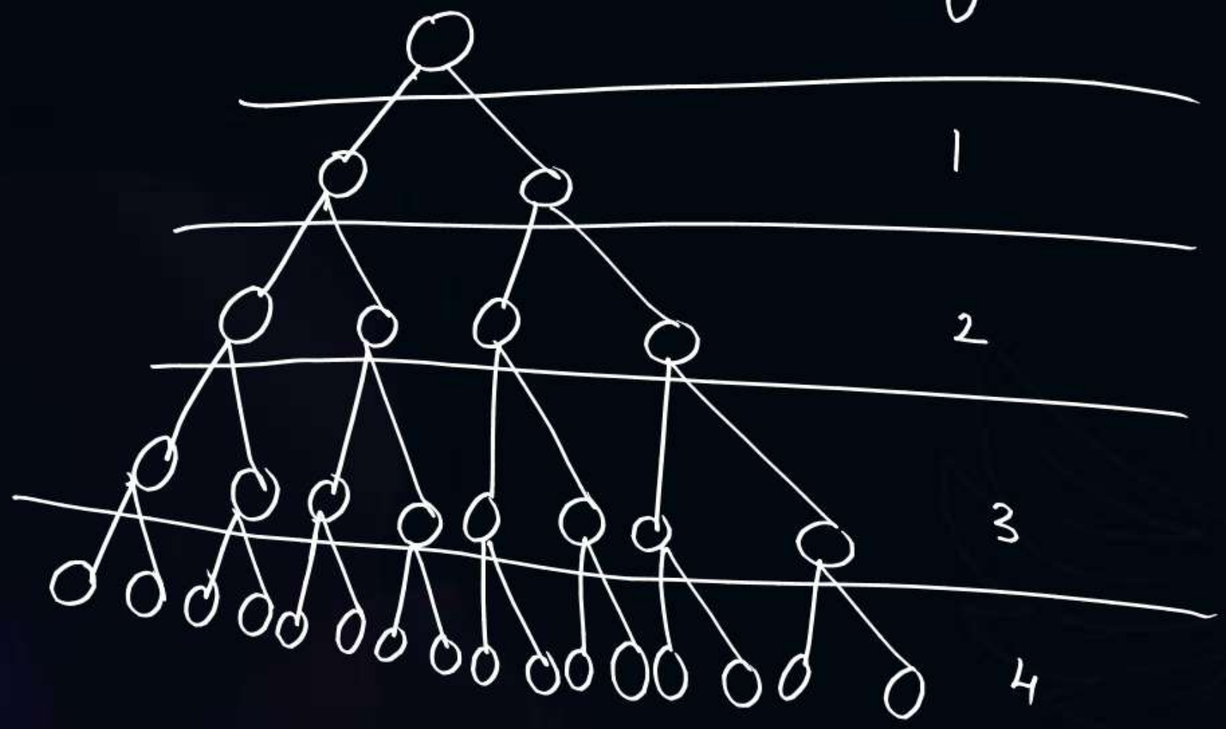
$2^{(3+1)} - 1$

$= 16 - 1 = 15$

# Topic : Formulae Of Binary Trees

⑦ In PBT, The Number of Nodes at Level 'L' will be $2^L$ (Level numbering from 0)

Ex:




| Level | No. of Nodes at Level 'L' |
|---|---|
| 0 | $1 = 2^0$ |
| 1 | $2 = 2^1$ |
| 2 | $2^2 = 4$ |
| 3 | $8 = 2^3$ |
| 4 | $16 = 2^4$ |

# 2 mins Summary

(8) The Total Number of Binary Heap orderings Possible with 'N' Nodes,

$$T(N) = {}^{(N-1)}C_{L} * T(L) * T(R)$$

# THANK - YOU