Computer Science & DA

Calculus and Optimization

Function

Lecture No. 01

By- Dr. Puneet Sharma Sir

Topics to be Covered

Topic Function

Topic Graphs

Topic :Function and their Graphs

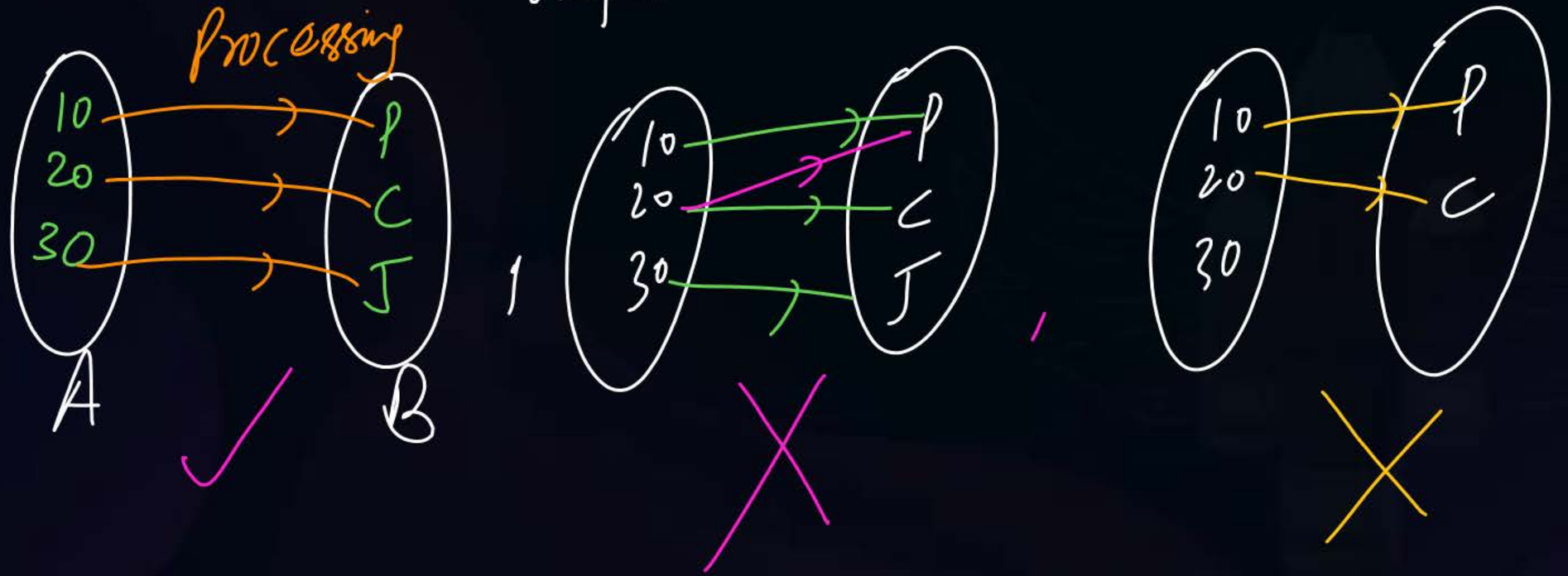
Function :- f is said be a function from A to B if

$\forall x \in A$ $\exists$ unique $y \in B$ s.t $y = f(x)$

Domain, Codomain

x : Input

y : output

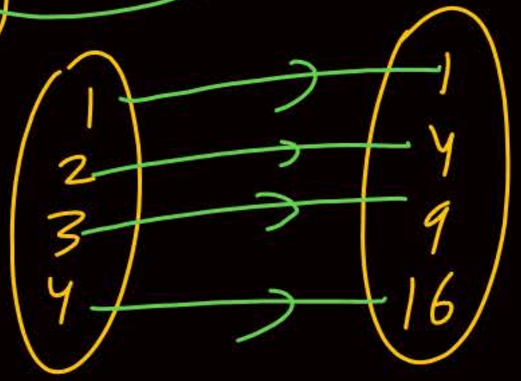


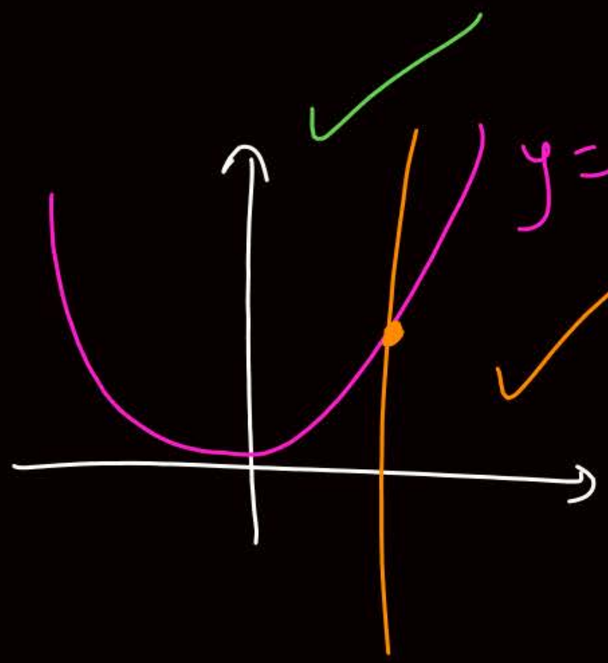
Domain: set of Permissible Values of x for which $y=f(x)$ is defined is called Domain

Range: set of Permissible Values of y for which $y=f(x)$ is defined is called Range & generally Range $f \subseteq$ Codomain

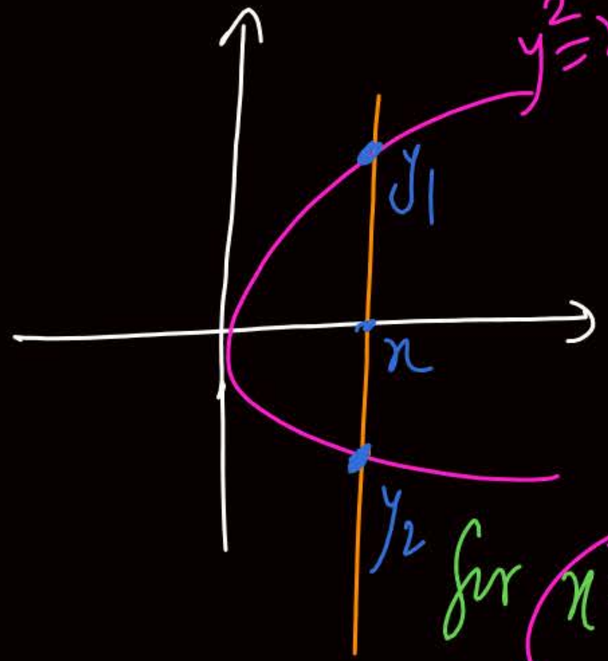
Note- there is no proper Method of finding Domain and Codomain it is Calculated only with the help of Common Sense or By previous knowledge.

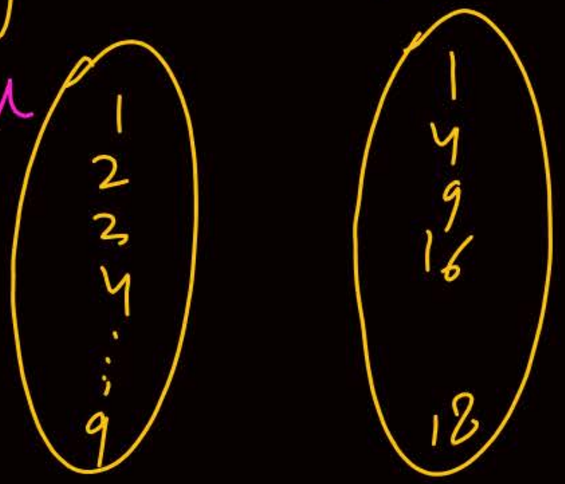
eg $y = x^2$





$y^2 = x \Rightarrow y = \pm\sqrt{x}$





for $x = 9$

$y = \sqrt{9} = 3$

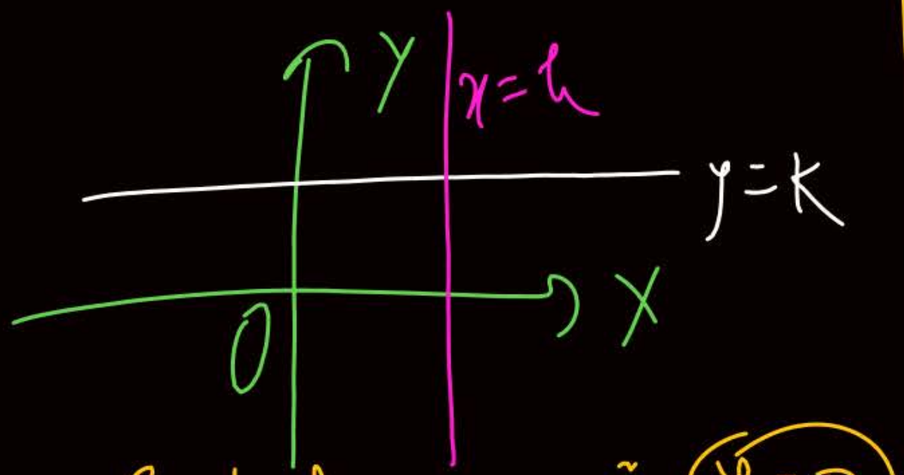
$y = -\sqrt{9} = -(3) = -3$

Vertical line test:

if line ⊥ᵍ to y axis cuts the graph only at one point

then it is function.

Basic Graphs: ①



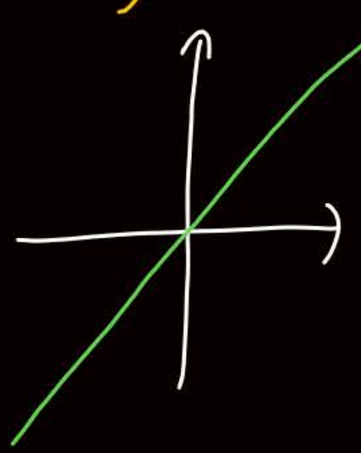
eqⁿ of X axis : $y = 0$

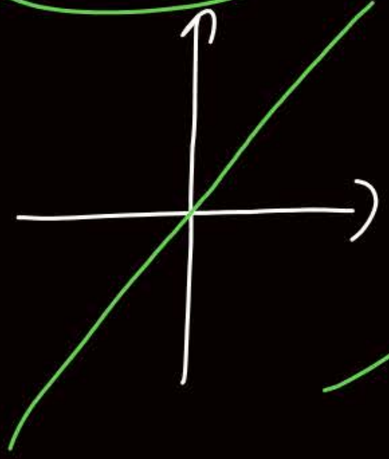
" " Y axis : $x = 0$

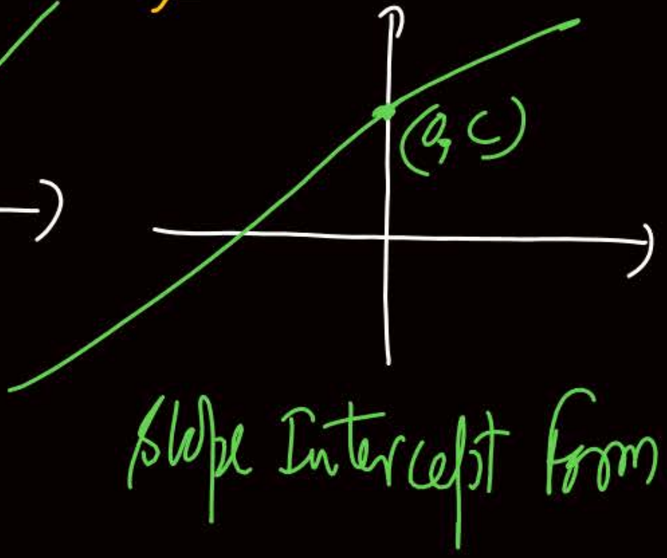
② Eqⁿ of line ⊥ᵍ to X axis $y = K$

" " " " Y axis $x = h$

③ $y = x$; $y = mx$, $y = mx + c$

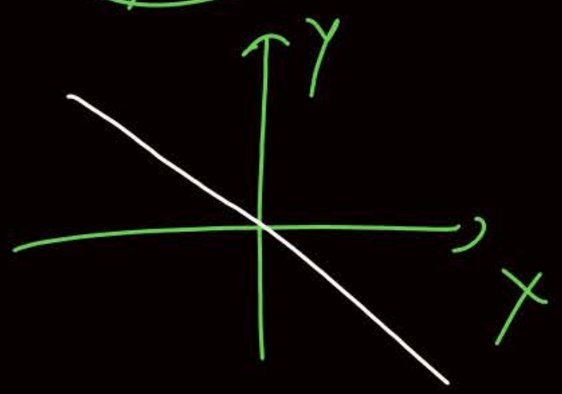




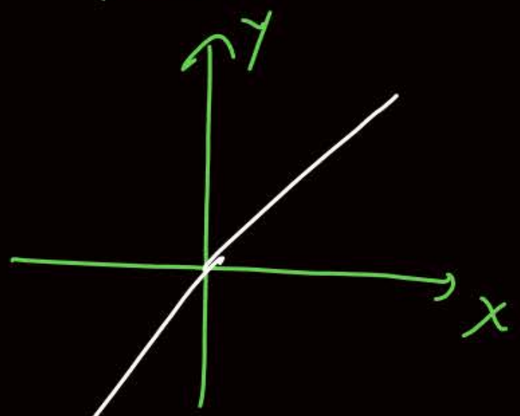


Slope Intercept form

$y=-x$

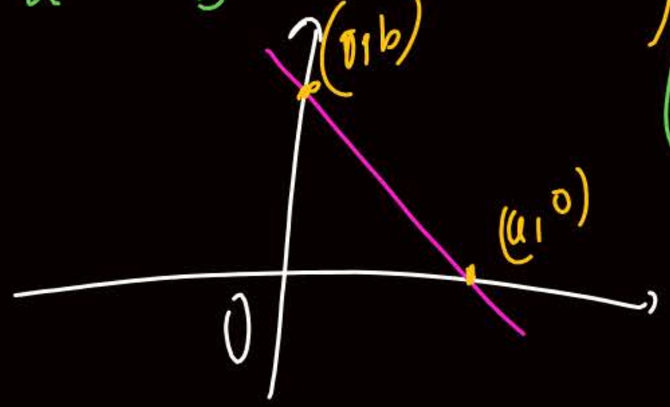


$y=x$



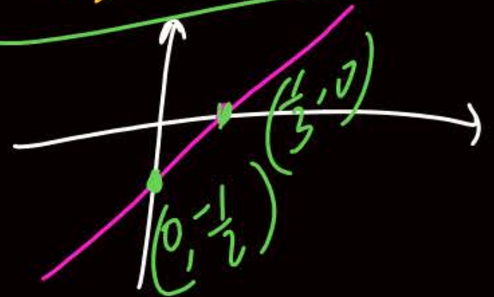
⊗ Intercept form of line:

$\frac{x}{a}+\frac{y}{b}=1$

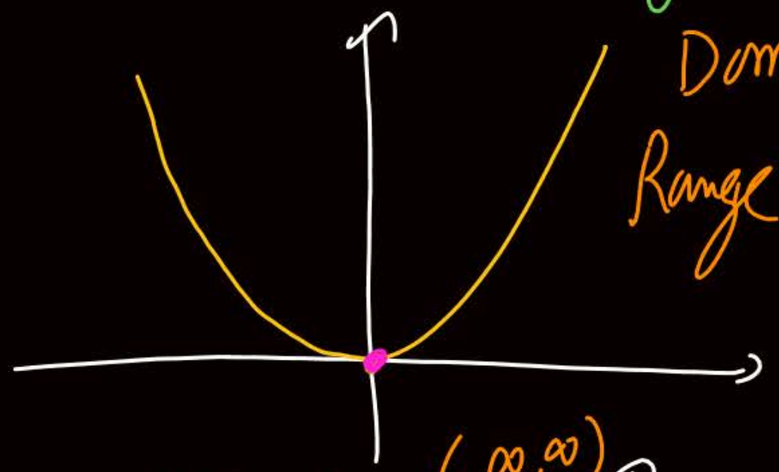


eg $3x-2y=1$

$\frac{x}{1/3}+\frac{y}{-1/2}=1$



⊗ $y=x^2$, $y=x^4$, $y=x^6$ .....



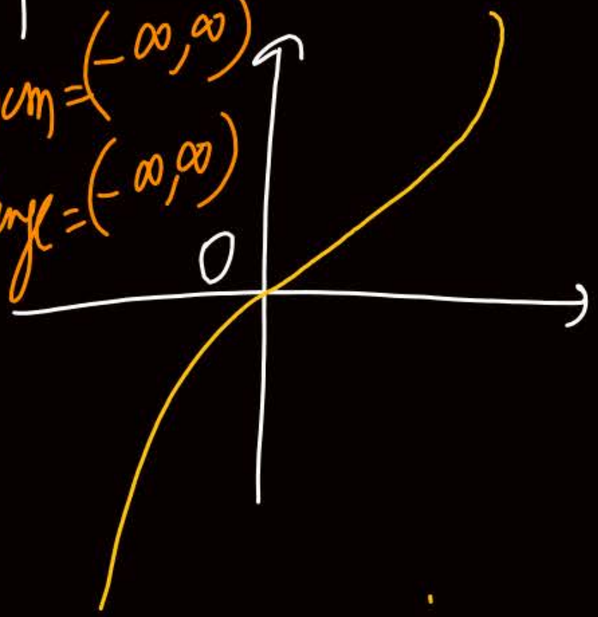
Dom $=(-\infty,\infty)$

Range $=[0,\infty)$

⊗ $y=x^3$

$y=x^5$

$y=x^7$



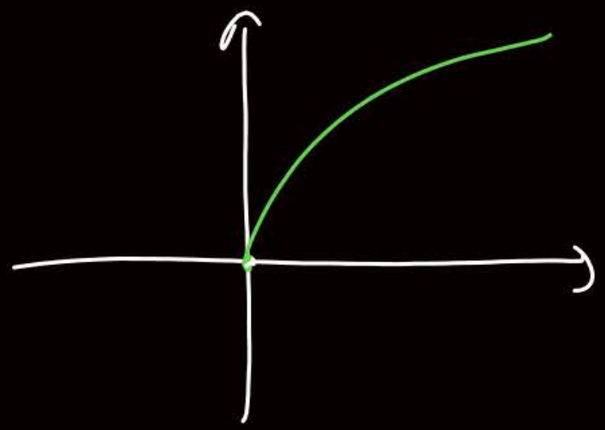
Dom $=(-\infty,\infty)$

Range $=(-\infty,\infty)$

① $y^2 = x$ ← Domain = ?
← Range = ?

Senseless Question.
!!
Bcoz it is not a func^n

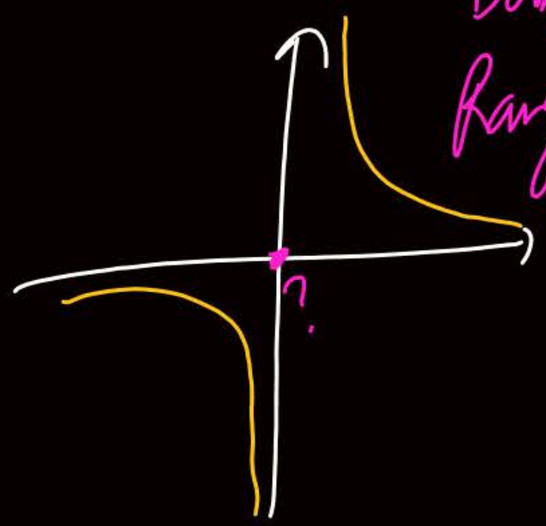
② $y = \sqrt{x}$



Dom $= [0, \infty)$

Range $= [0, \infty)$

(*) $y = \frac{1}{x}$



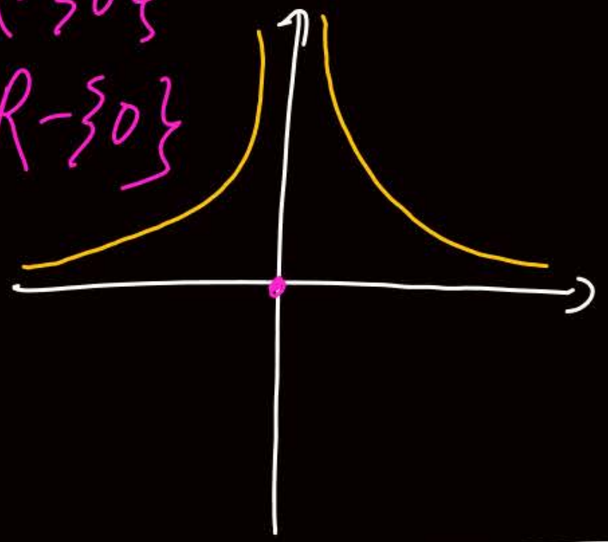
Dom = $R - \{0\}$

Range = $R - \{0\}$

Reciprocal Function

(or Rectangular Hyperbola)

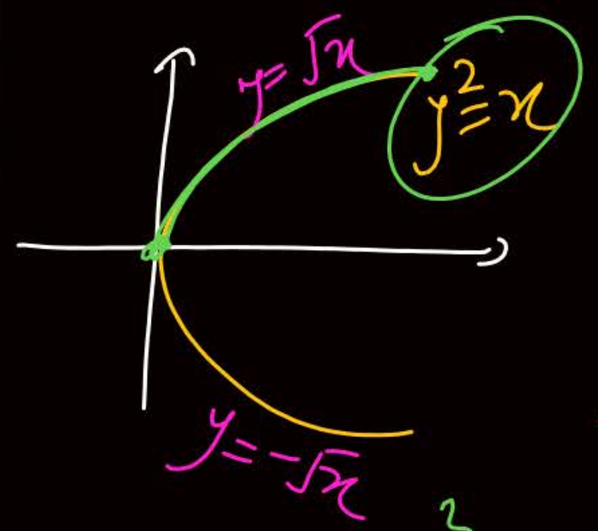
(*) $y = \frac{1}{x^2}$



Dom = $R - \{0\}$

Range = $(0, \infty)$

(*) $y = x^{1/2}$



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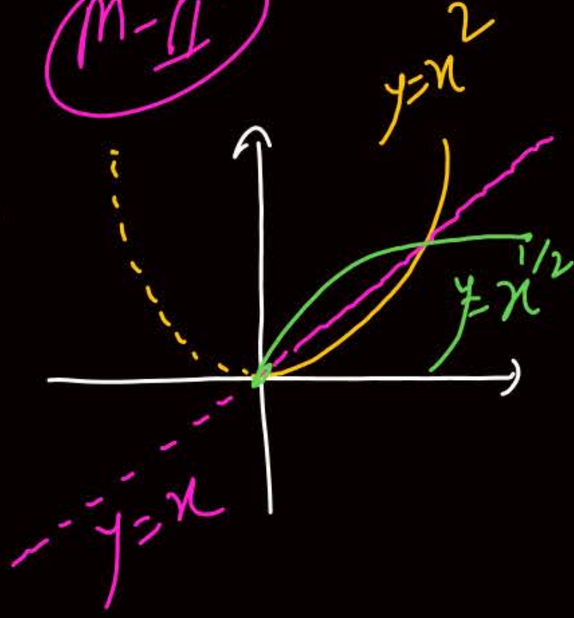
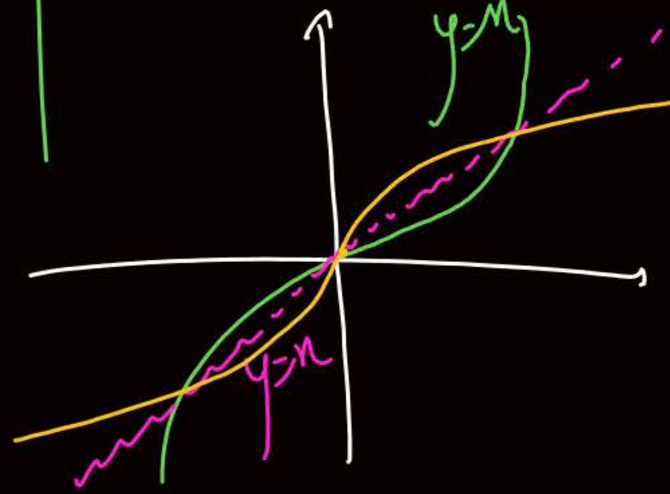




① $x^2+y^2=r^2$ (circle) → Centre $=(0,0)$, rad $=r$

② $x^2+y^2+2gx+2fy+c=0$ → Centre $=(-g,-f)$, rad $=\sqrt{g^2+f^2-c}$

$(x^2+2gx+g^2)+(y^2+2fy+f^2)-g^2-f^2+c=0$

$(x+g)^2+(y+f)^2=g^2+f^2-c$

$(x+g)^2+(y+f)^2=\left(\sqrt{g^2+f^2-c}\right)^2$

Centre $=(-g,-f)$, $r=\sqrt{g^2+f^2-c}$

or $(x-h)^2+(y-k)^2=r^2$

Centre $=(h,k)$, rad $=r$

eg $x^2+y^2+3x+6y-4=0$

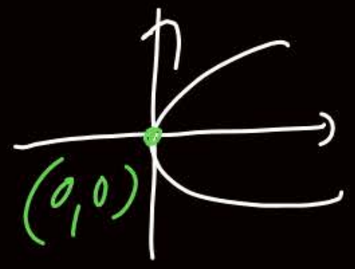
on comparision: $g=\frac{3}{2}$, $f=3$, $c=-4$

Centre $=\left(-\frac{3}{2},-3\right)$

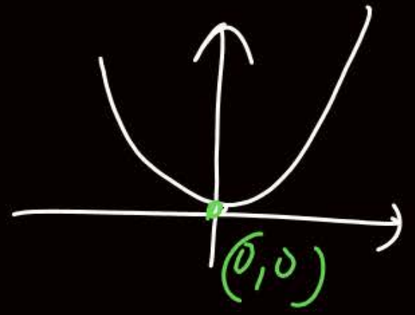
$r=\sqrt{\frac{9}{4}+9+4}=?$

⊛ Parabola :

- $y^2 = 4ax$



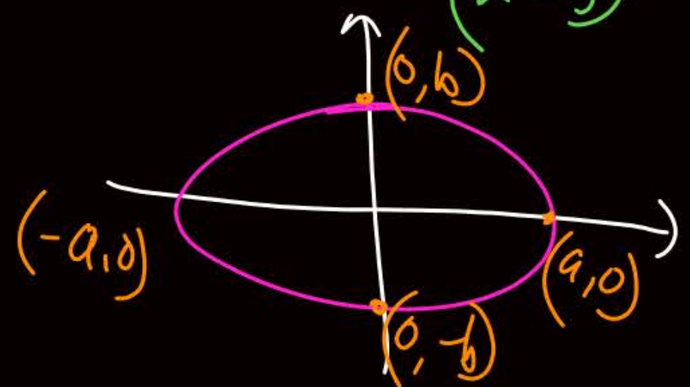
- $x^2 = 4ay$

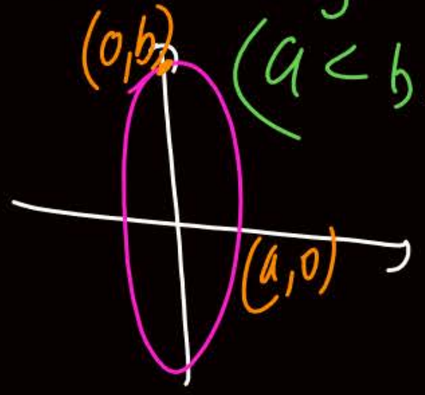


⊛ Ellipse :-

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

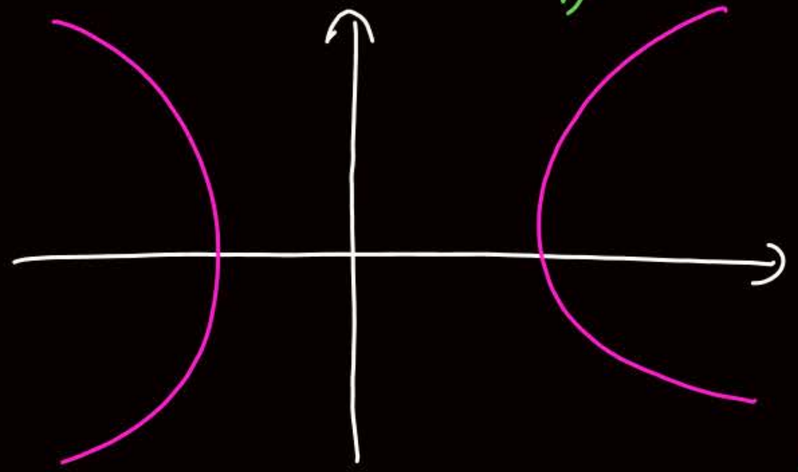
$(a > b)$ $(a < b)$



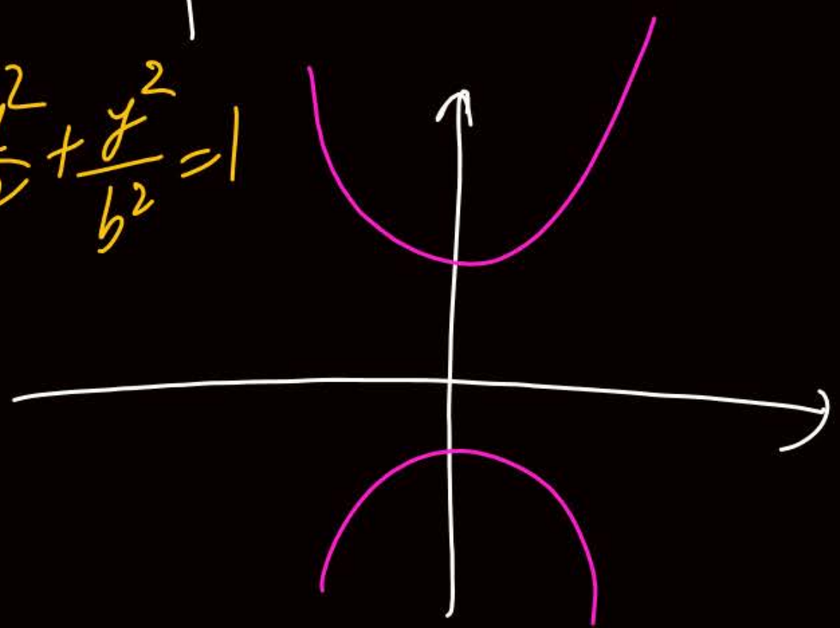


Hyperbola :-

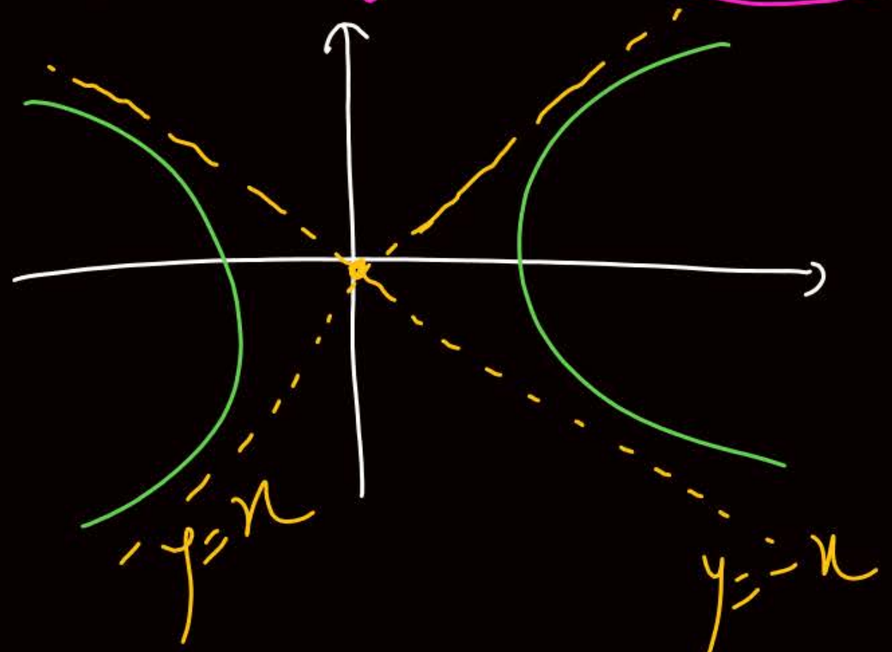
① $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



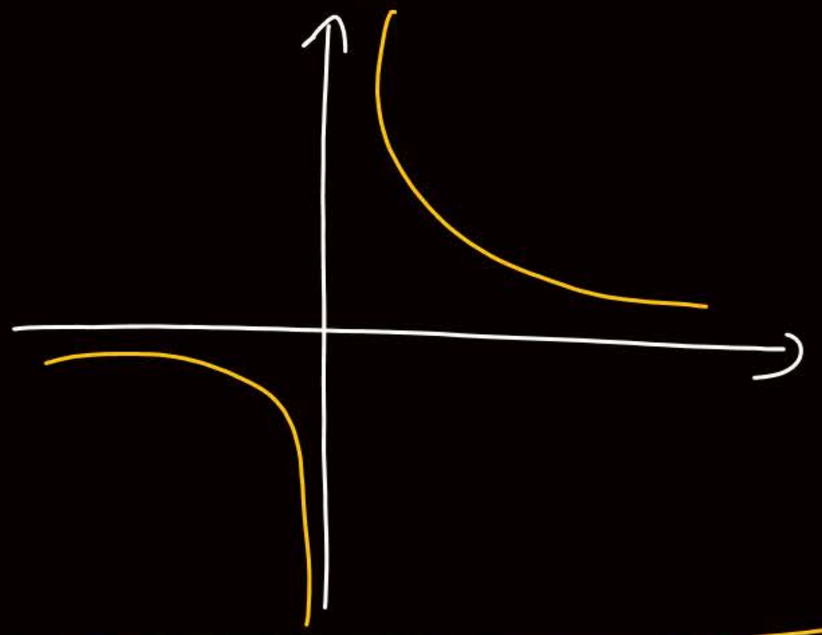
② $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



(*) $x^2 - y^2 = a^2$ when $a = b$

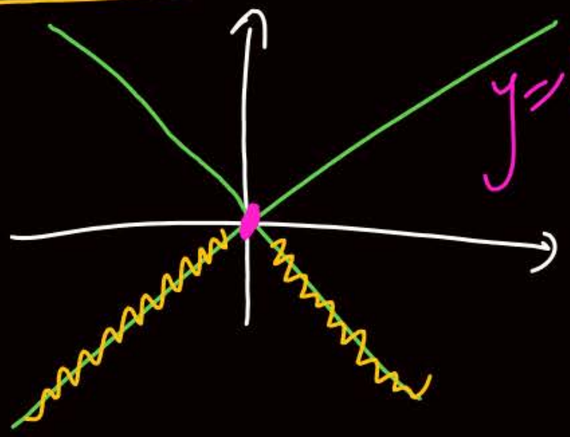


(*) $xy = c^2$



(*) Modulus func^n

$$y = |x| = \begin{cases} -x, & x < 0 \\ +x, & x \geq 0 \end{cases}$$



$y = |x|$ Dom $= R$

Range $= [0, \infty)$

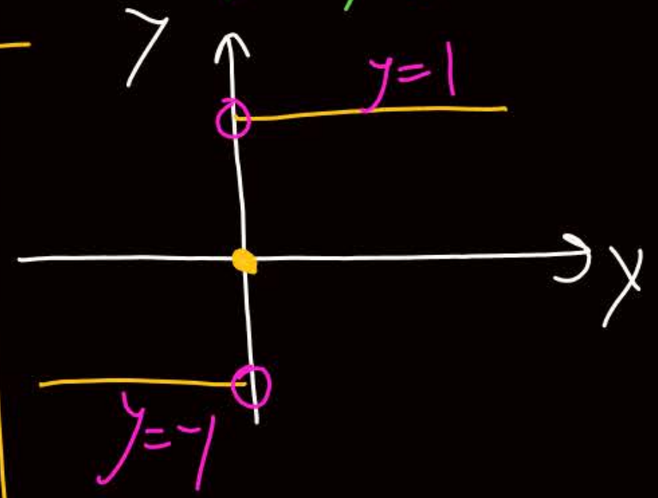
Dom $= R$

Range $= \{-1, 0, 1\}$ ⇒ three points

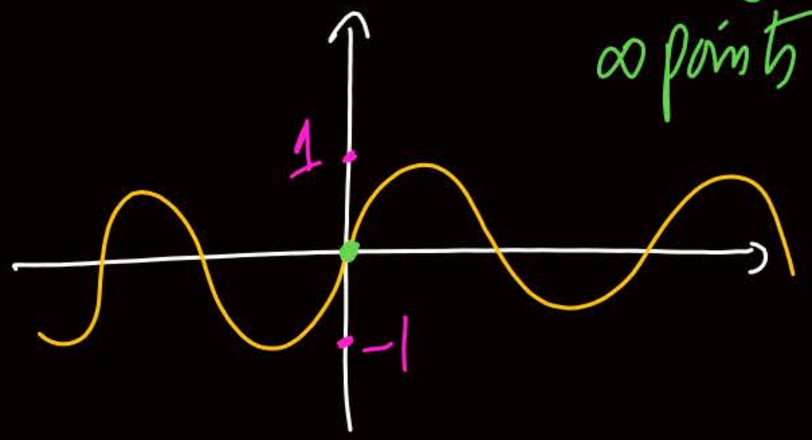
Signum function

$$Sgn(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

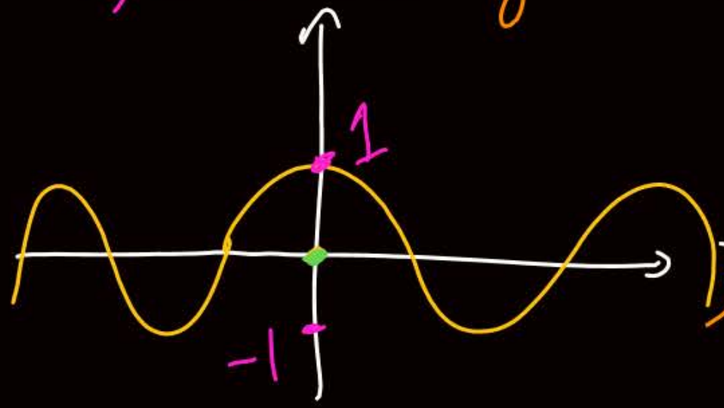
$$y = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$$



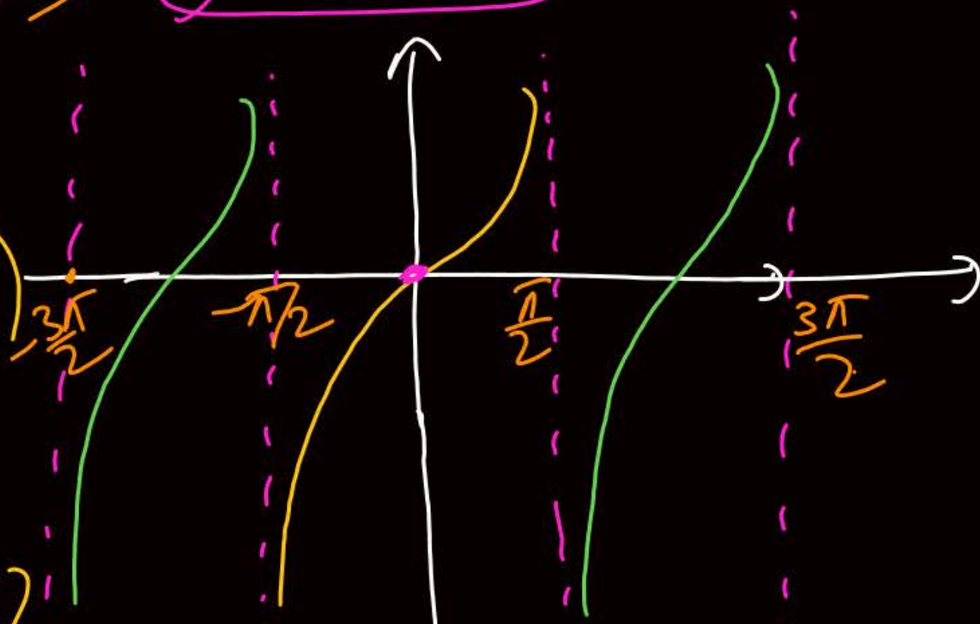
⊛ $y = \sin x$ Dom $= R$ Range $= [-1, 1]$ ⇓ ∞ points



$y = \cos x$ Dom $= R$ Range $= [-1, 1]$



$y = \tan x$



Q one of the possible solution of $\tan x = x$ may be?

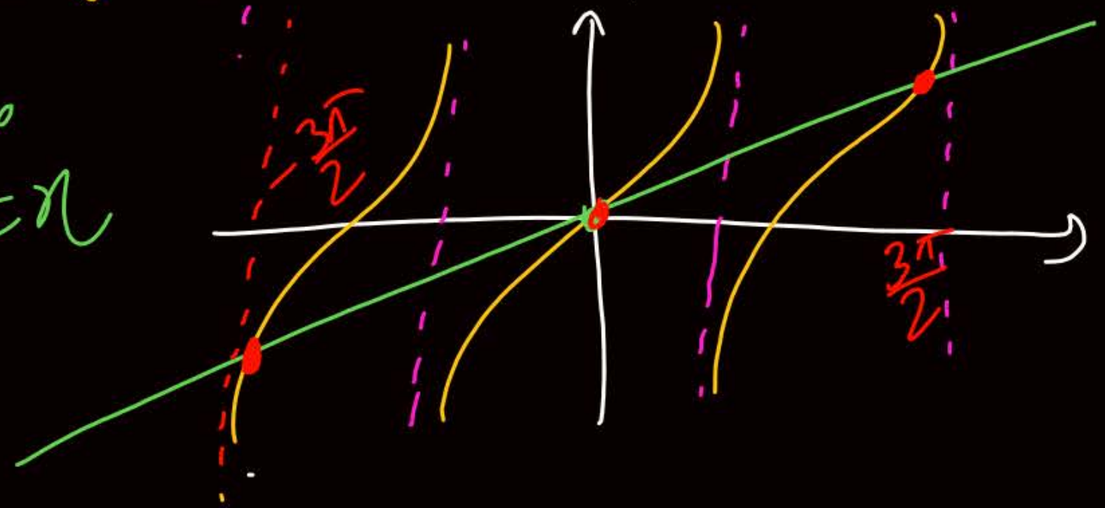
(a) $1.5 \approx \pi/2$

(b) $3.14 \approx \pi$

(c) $4.5 \approx \frac{3\pi}{2}$

(d) None

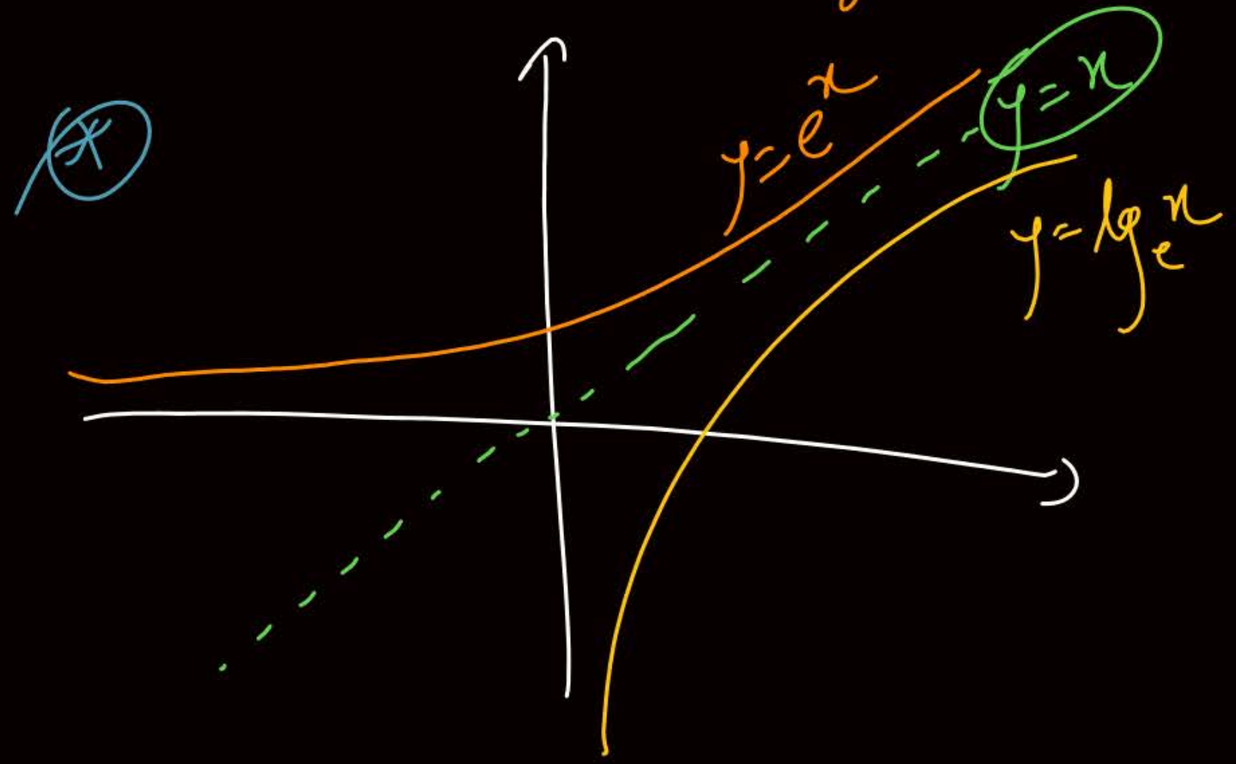
$y = \tan x$ $y = x$



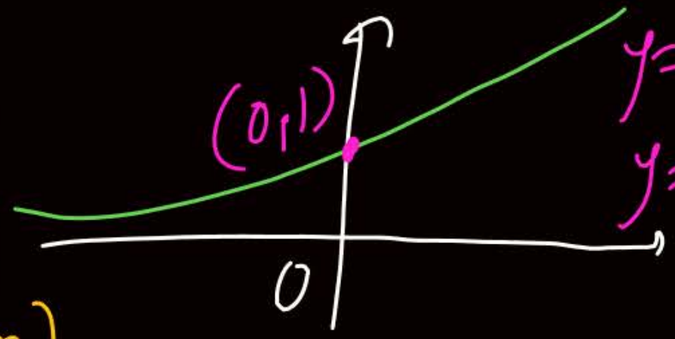
(*) $y = \tan x$

- Domain $= R - (2n+1)\frac{\pi}{2}$
- Range $= (-\infty, \infty)$

(*)



(*) $y = e^x$



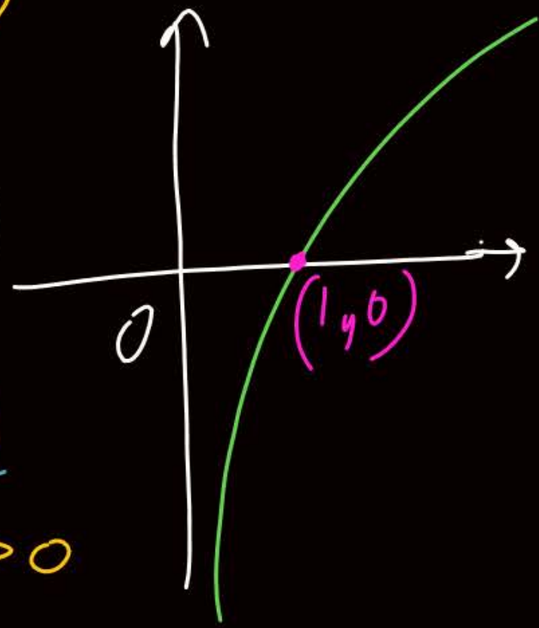
$y = e^{\infty} = \infty$

$y = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$ i.e $e^{-\infty} = 0$

Dom $= (-\infty, \infty)$

Range $= (0, \infty)$

(*) $y = \log_e x$



$y(1) = \log_e 1 = 0$

$y(\infty) = \log_e \infty = \infty \approx$ N.D

$y(0) = \log_e 0 = -\infty \approx$ N.D

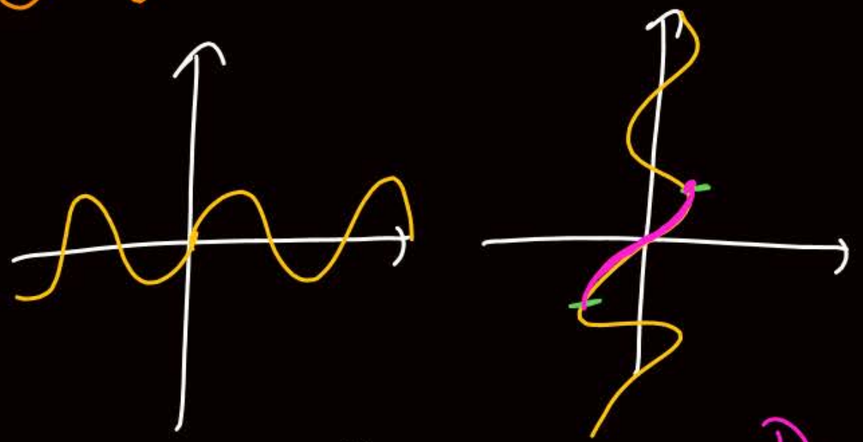
Domain $= (0, \infty)$ or $x > 0$

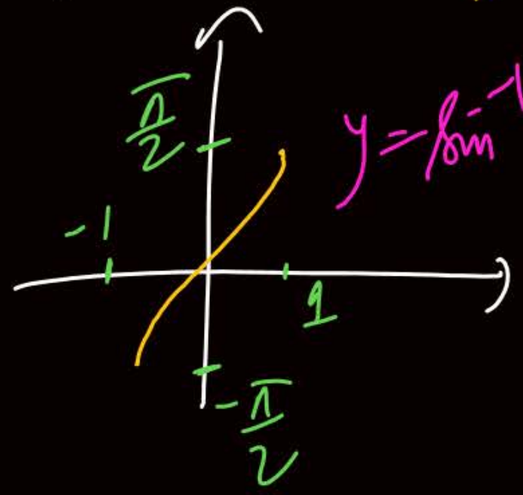
Range $= (-\infty, \infty)$

$y(-\infty) = \log(-\infty) =$ senseless Question

i.e $y = \log x$ is defined only for +ve values of x.

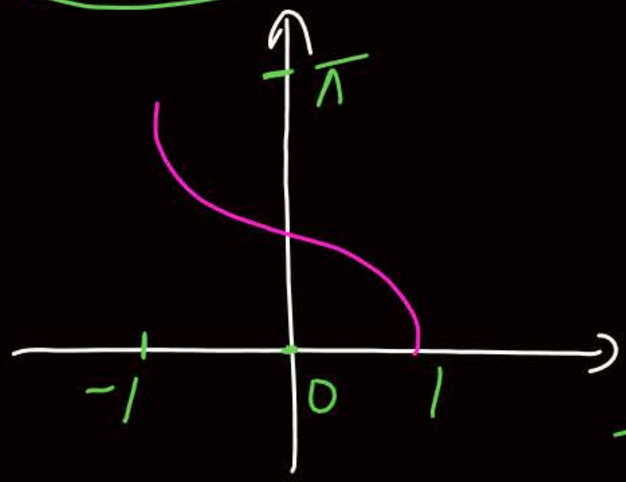
① $y = \sin^{-1} x$





$y = \sin^{-1} x$ $\begin{cases} D = [-1, 1] \\ R = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \end{cases}$

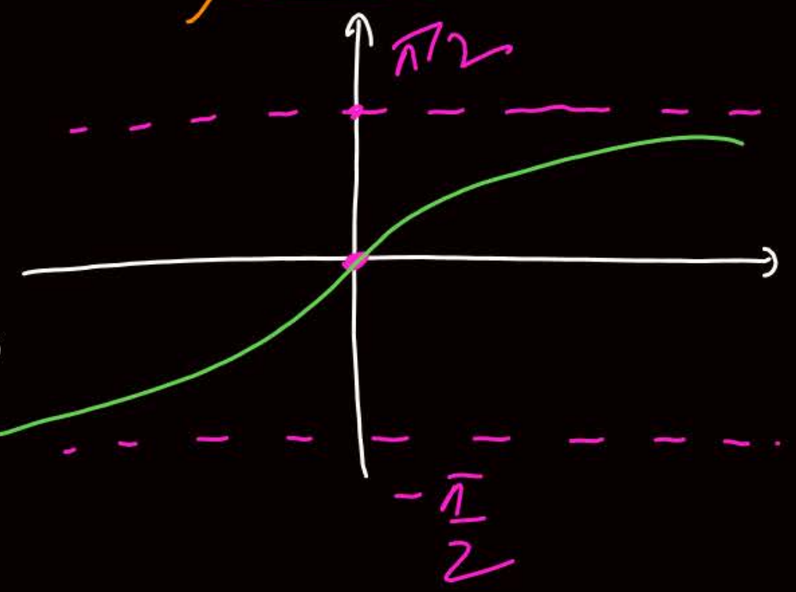
② $y = \cos^{-1} x$



$Dom = [-1, 1]$

$Range = [0, \pi]$

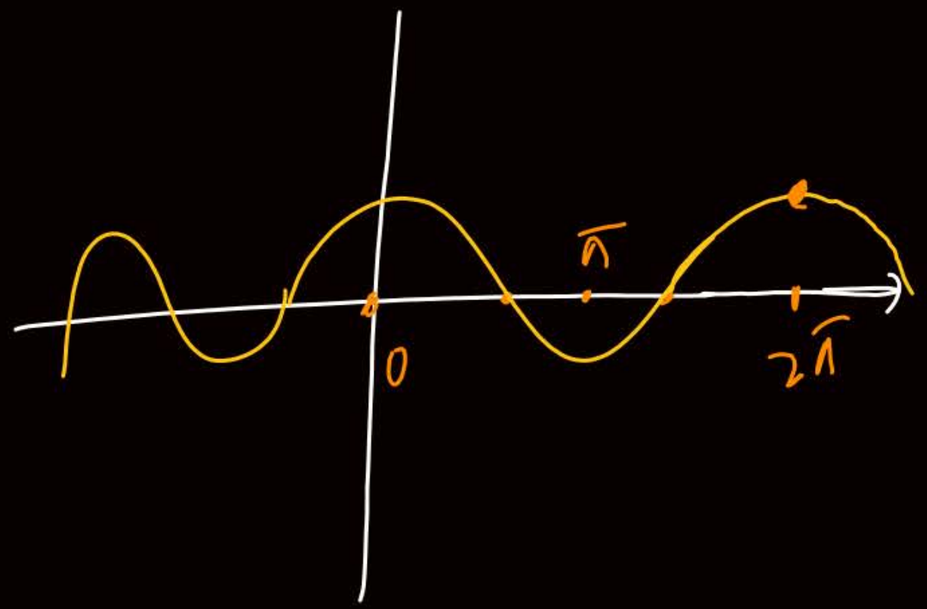
③ $y = \tan^{-1} x$



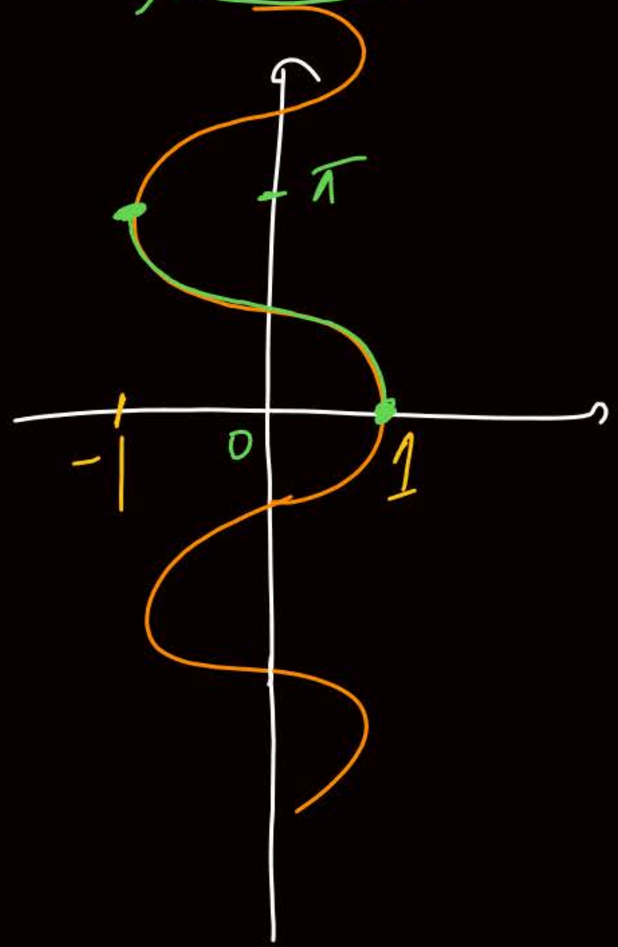
$Dom = (-\infty, \infty)$

$Range = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

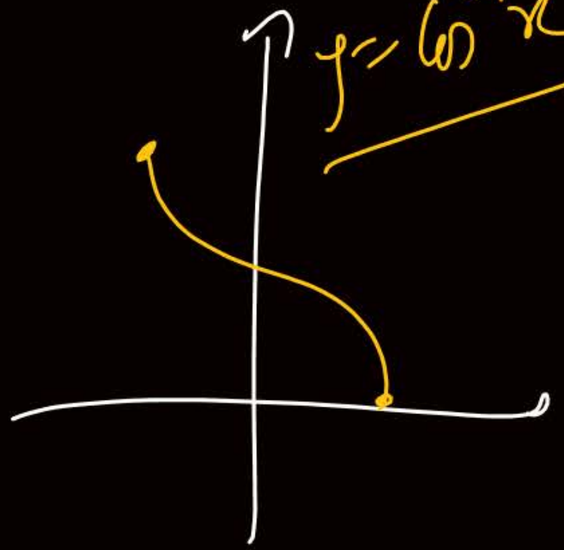
$y = \cos x$

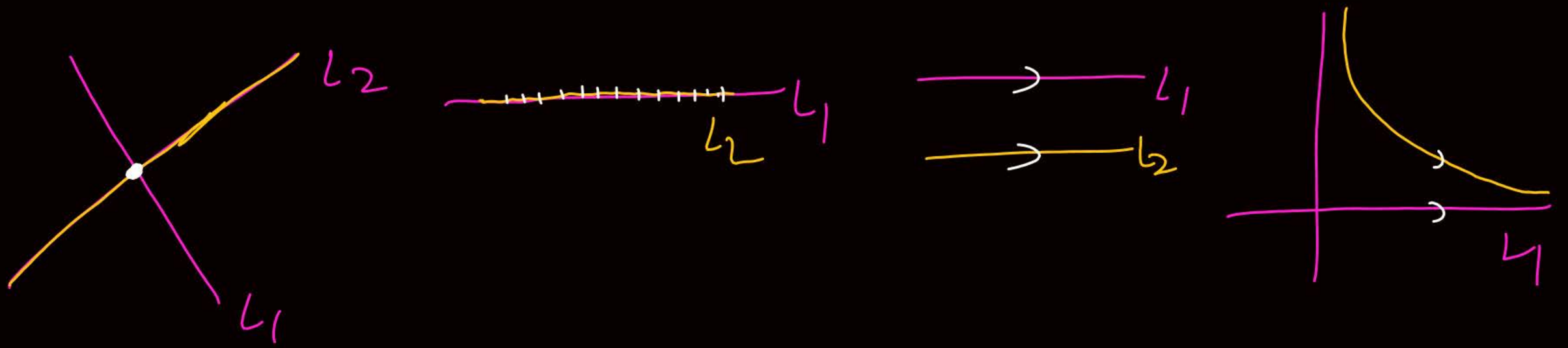


$y = \cos^{-1} x$



$y = \cos^{-1} x$




L_2
L_1
L_1
L_2
L_1
L_2
L_1

2 mins Summary

- Topic
- Topic
- Topic
- Topic
- Topic


THANK - YOU