

# Data Science and Artificial Intelligence

## Machine Learning

Support Vector Machine

Lecture No. 2



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# Recap of Previous Lecture



Topic

Generative & Discriminative

Topic

Naïve Bayes Advantage & Disadvantage

Topic

Svm

Topic

Topic

Turn on Slide map



# Topics to be Covered



Topic

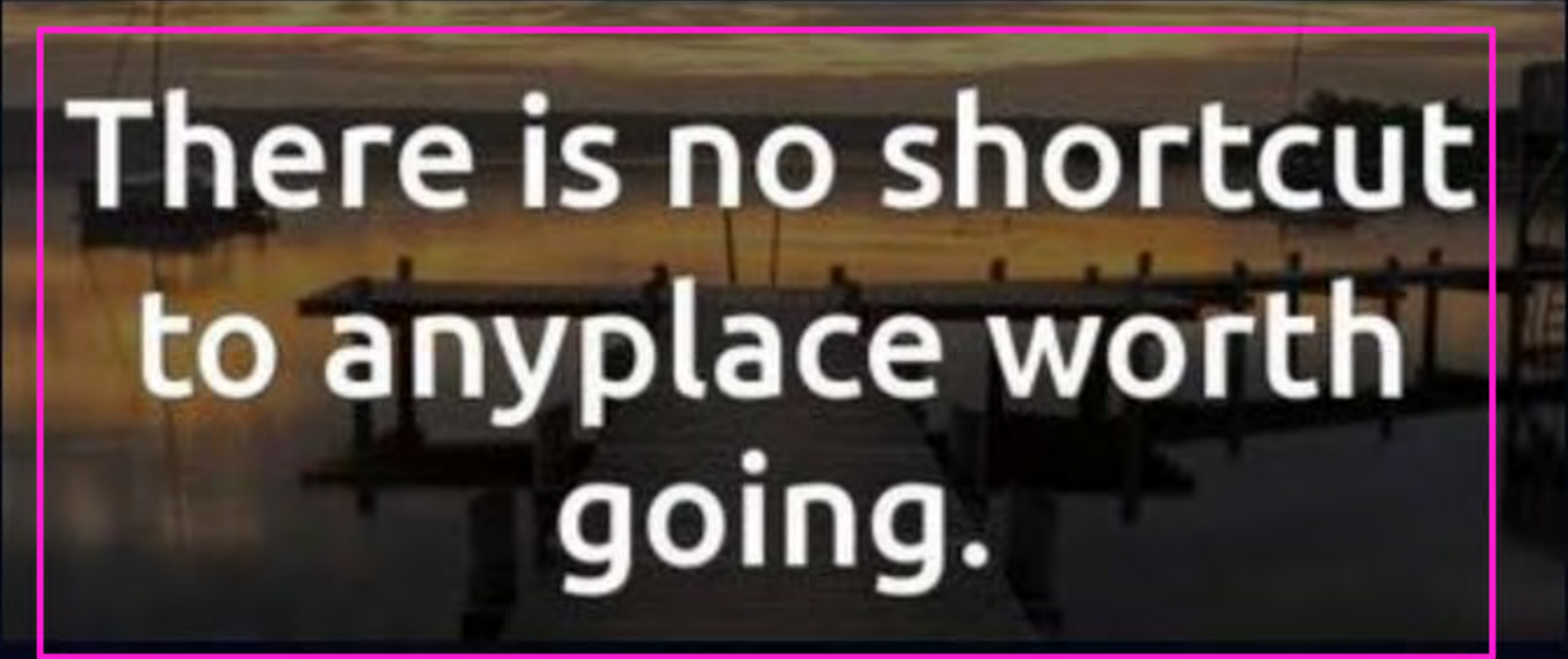
Svm

Topic

Topic

Topic

Topic

The background image shows a rowing team in a boat on a body of water, with a city skyline visible in the distance. The image is slightly blurred and has a dark overlay.

There is no shortcut  
to anyplace worth  
going.





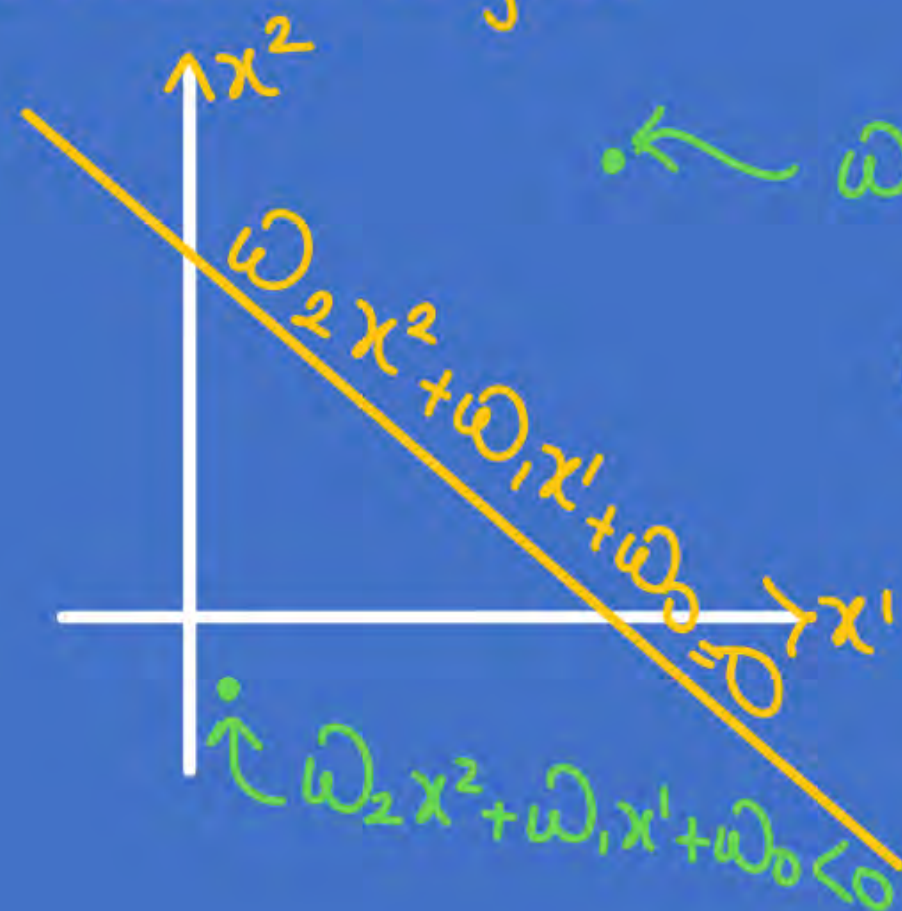
Naïve Bayes  $\Rightarrow$  done



SVMs



- The algorithm that try to give best classifier



•  $w_2 x^2 + w_1 x^1 + w_0 > 0$

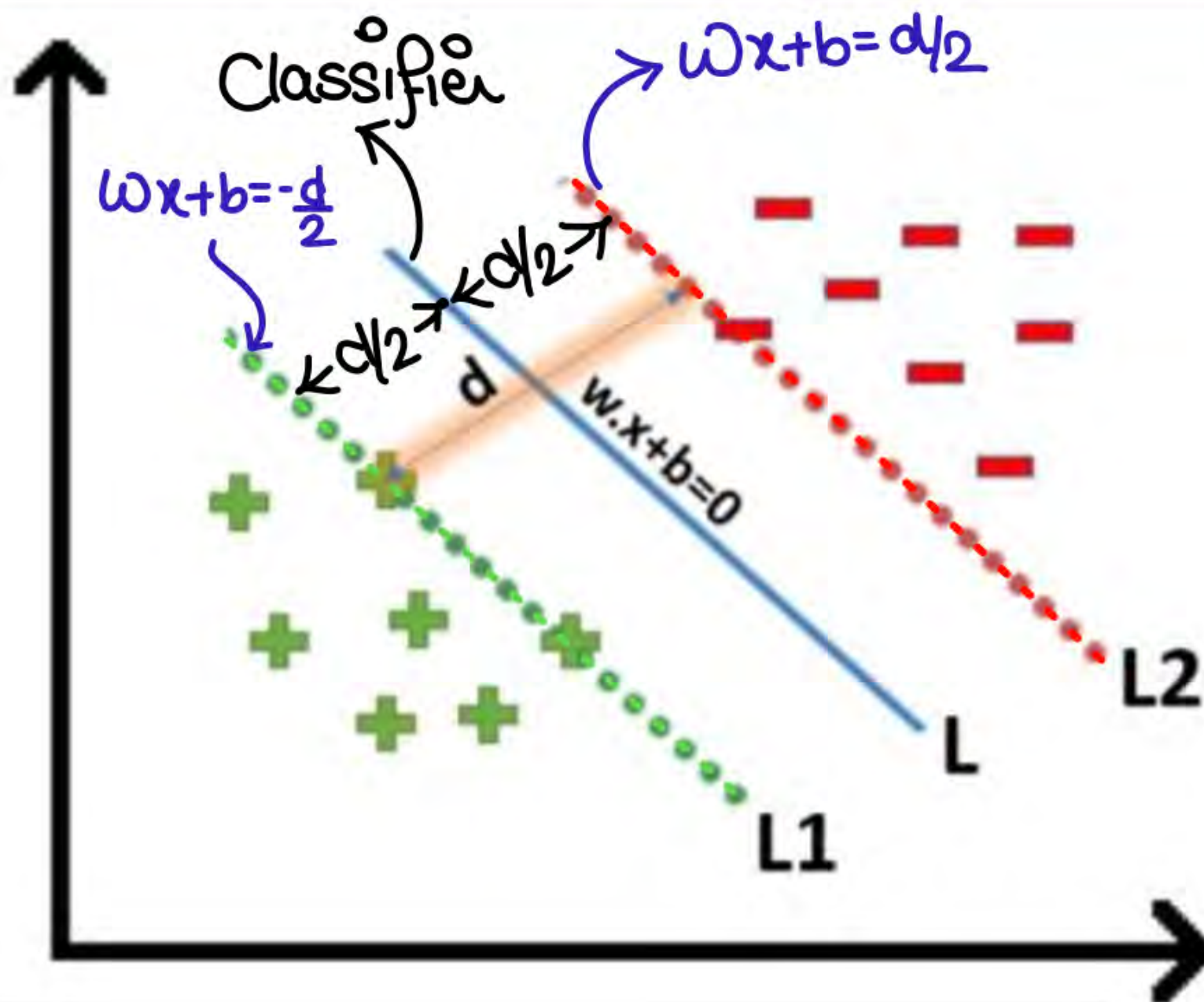
Distance of a point from a line

$$\Rightarrow \left\{ \frac{w_2 x^2 + w_1 x^1 + w_0}{\sqrt{w_2^2 + w_1^2}} \right\}$$





## Support Vector Machine



So in SVM we want the classifier which has max separation from both the classes..

$\Rightarrow$  we want that classifier must have equal distance from both classes

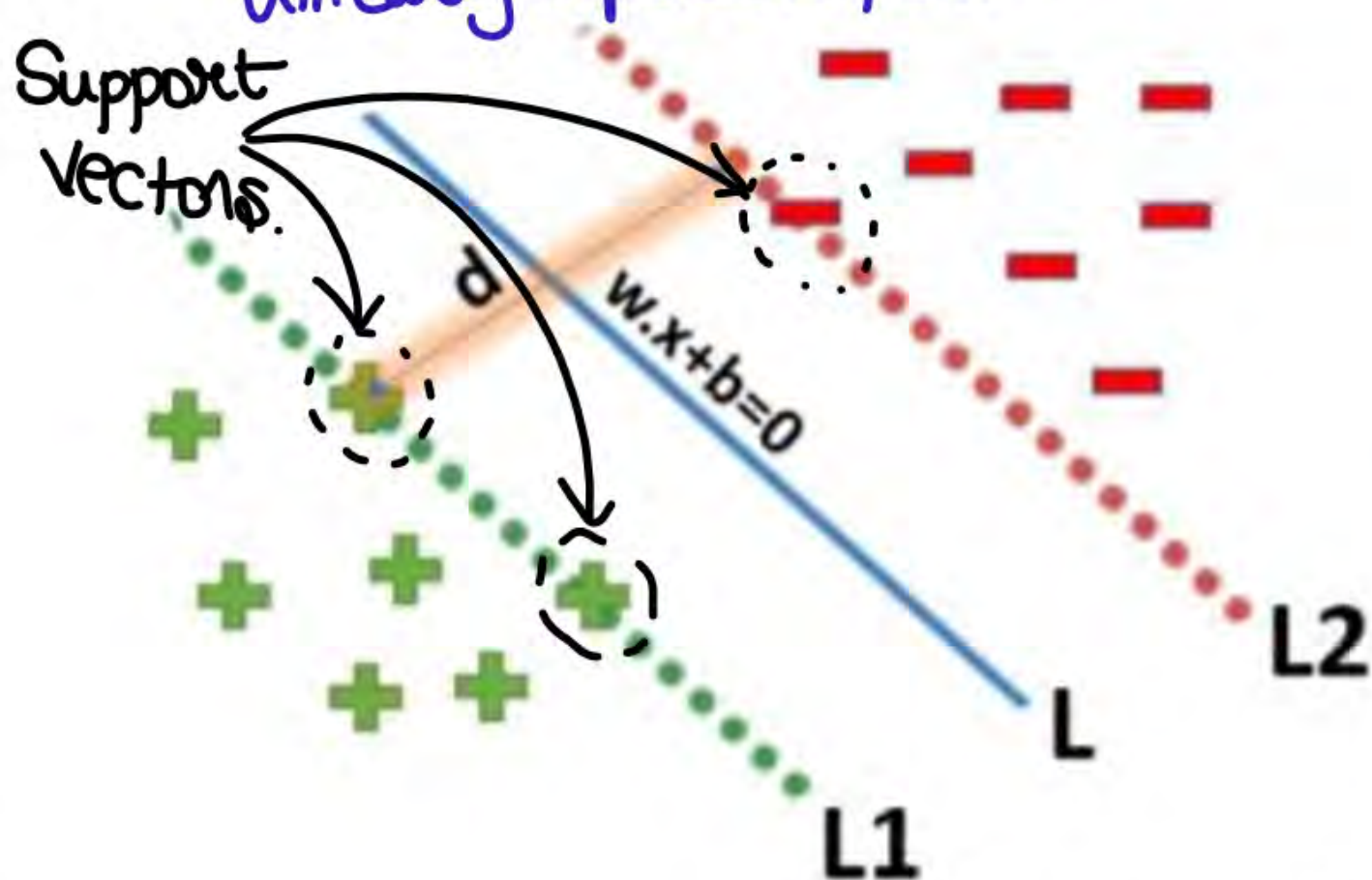




## Support Vector Machine

- we are assuming linearly separable points

Support  
vectors.



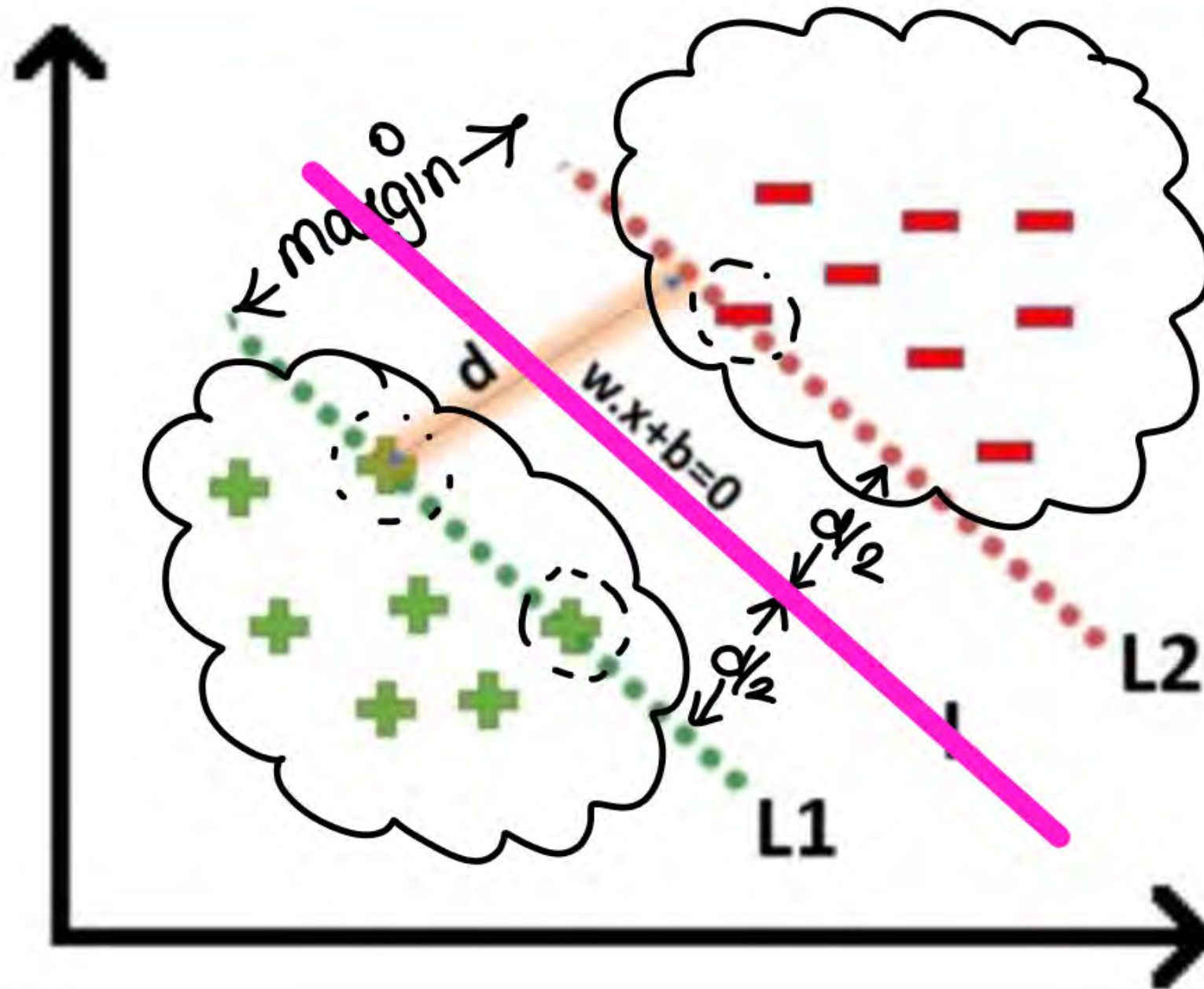
The points above the  
line have .....  
And below the line  
have .....

⇒ So here we can see that the  
Classifier eq, depend mainly  
on the Support Vectors only.



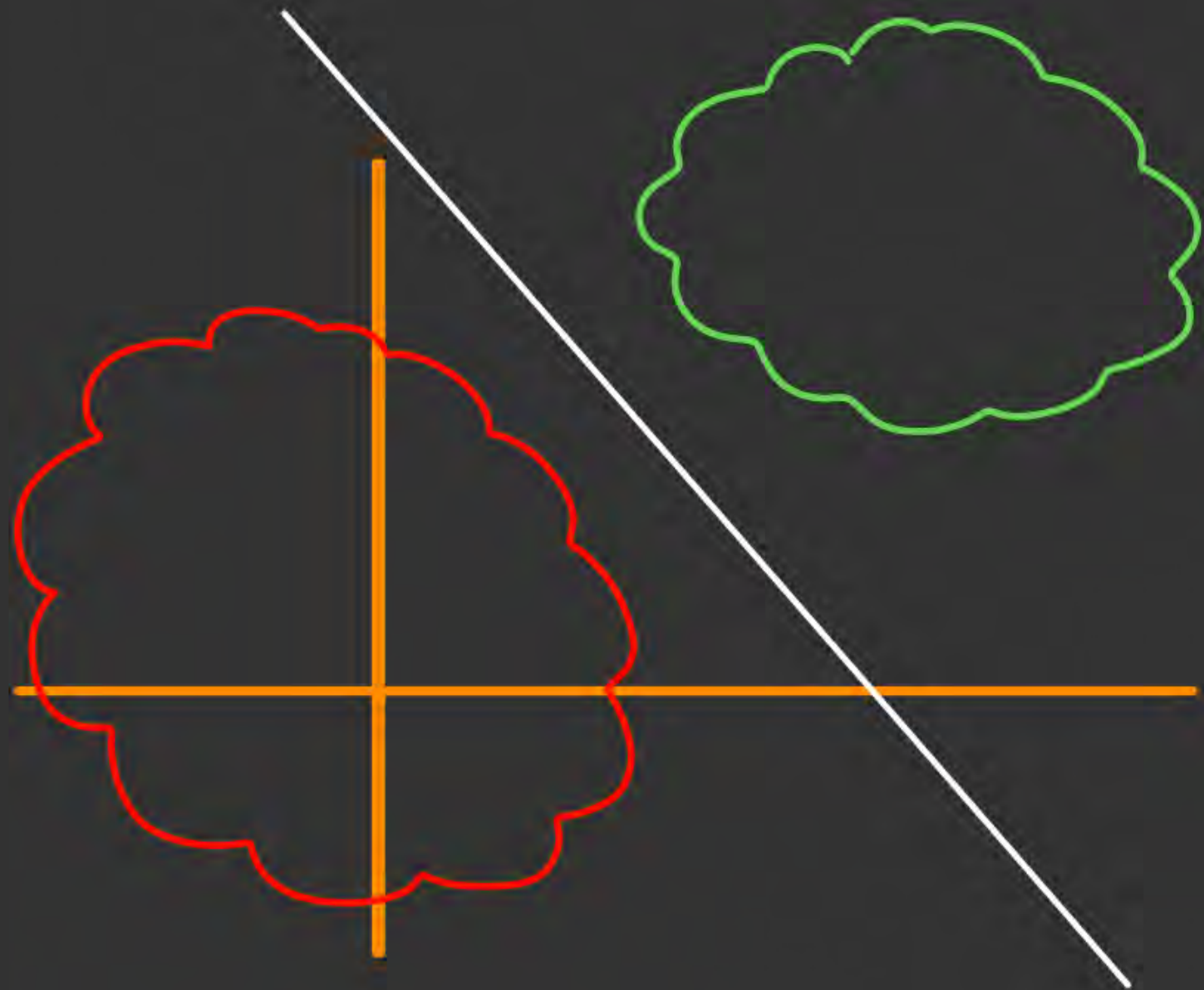


## Support Vector Machine



The equation of Line L,  
L1 and L2 will be ....

done







## Support Vector Machine

So the algorithm  $\Rightarrow$

\* The best classifier is that which separate the class 1 and Class 2 Such that the gap b/w them  $\Leftrightarrow$  margin is maximized

The equation of Line L, L1 and L2 will be ....



## Support Vector Machine

This becomes the min  
max problem...

So our task is to  
maximise the margin

...

So the classifier is  $(w \cdot x + b)$  Parameters are  $w, b$

So we find the distance of all points from classifier

So  $d/2$  is min distance.

$\therefore$





## Support Vector Machine

This becomes the min  
max problem... with a  
constraint...

So we want to  
 $\Rightarrow$  maximize  $d/2$

So our task is to  
maximise the margin  
...



## Support Vector Machine

So algorithm is  $\left[ \text{max min (distance)} \right]$

Such that  $y_i(w \cdot x_i + b) > 0$

⇒ So if class 1  $(wx_i + b) > 0$  i.e. Point shd be above classifier  
 " " " " " " " " below classifier

→ This constraint ensure that classifier does no error.

For generalising this we convert the lines L1 and L2 into...





## Support Vector Machine

This maxmin constraint problem will be used to solve for best  $w$  and  $b$ .

For generalising this we convert the lines  $L1$  and  $L2$  into...



## Support Vector Machine

lets modify the algo  $\Rightarrow$

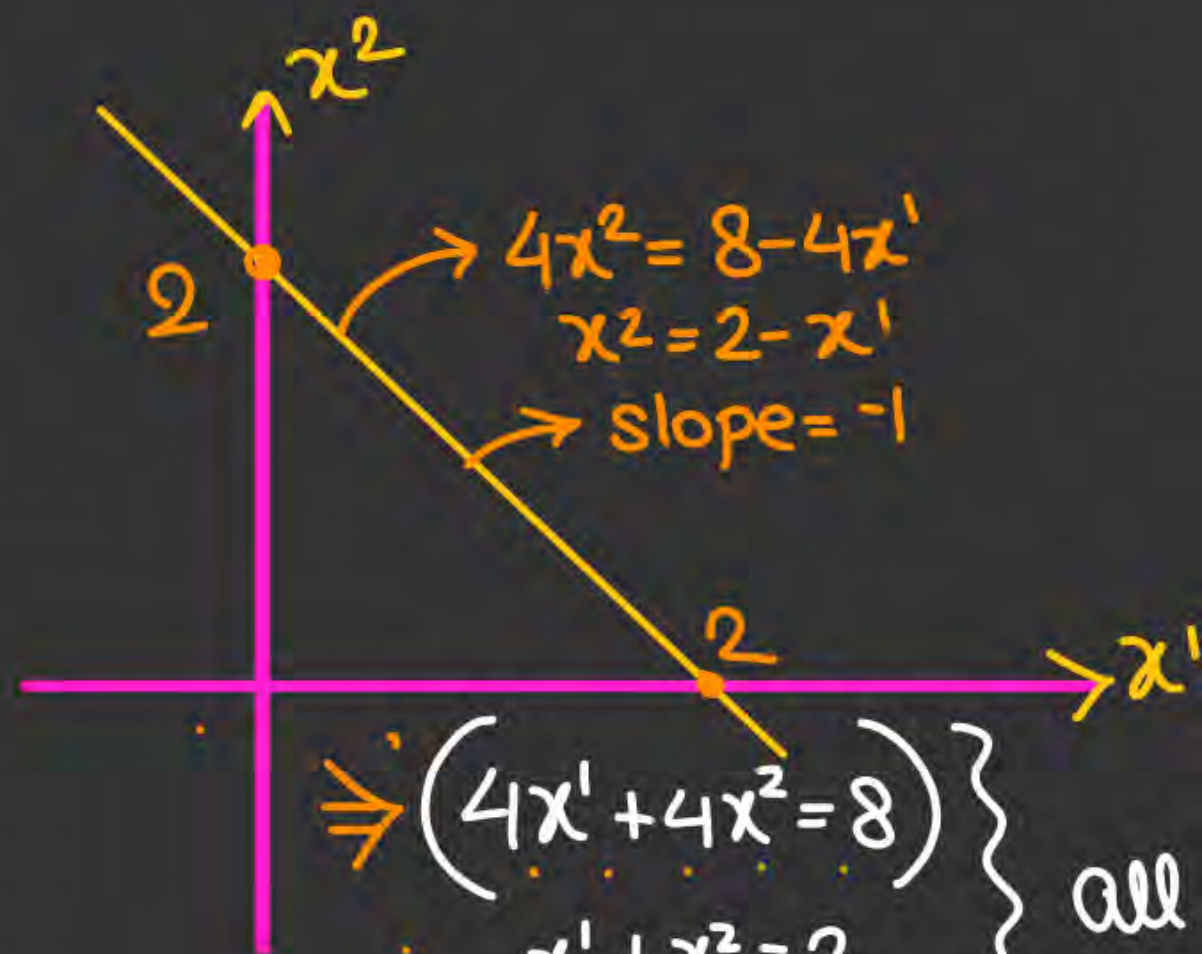
$$\underbrace{(\omega x + b)}_c = 0$$

$$\underbrace{\omega x + b}_c = d/2$$

$$\underbrace{\omega x + b}_c = -d/2$$

SVM primal equation





$$4x^2 = 8 - 4x^1$$
$$x^2 = 2 - x^1$$

$$\text{slope} = -1$$

$$\Rightarrow (4x^1 + 4x^2 = 8)$$

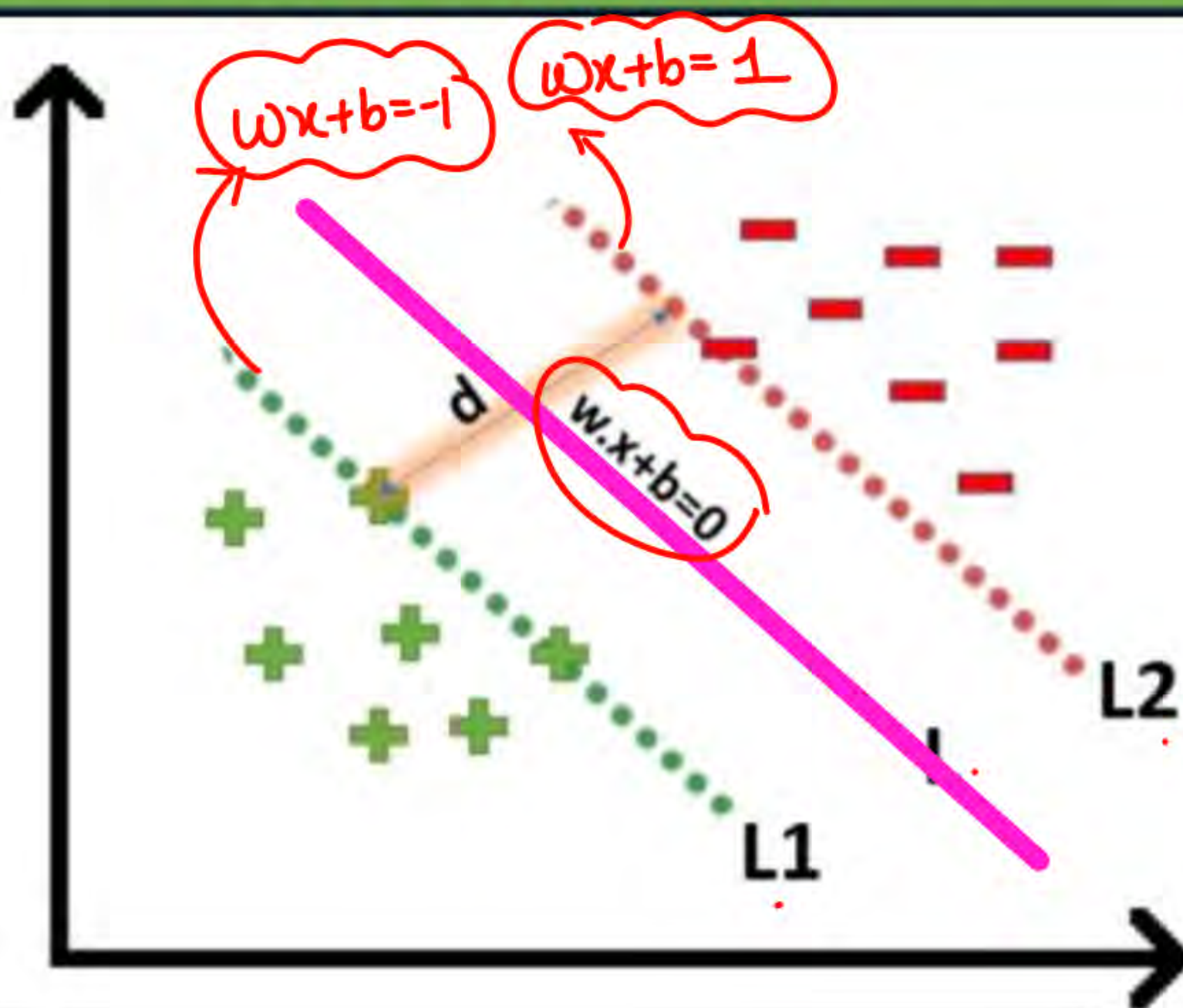
$$x^1 + x^2 = 2$$

$$\frac{1}{2}x^1 + \frac{1}{2}x^2 = 1$$

all 3 axes same eq.



## Support Vector Machine



- So we see that if we scale the straight line eq, the eq and their plot remain same
- So initially the equations were  
 $w \cdot x + b = 0, w \cdot x + b = d/2, w \cdot x + b = -d/2$   
 $\Rightarrow \frac{w \cdot x + b}{d/2} = 0, \frac{w \cdot x + b}{d/2} = 1, \frac{w \cdot x + b}{d/2} = -1$





## Support Vector Machine

So

$$\omega x + b = 0$$

$$\omega x + b = 1 \checkmark$$

$$\omega x + b = -1 \checkmark$$

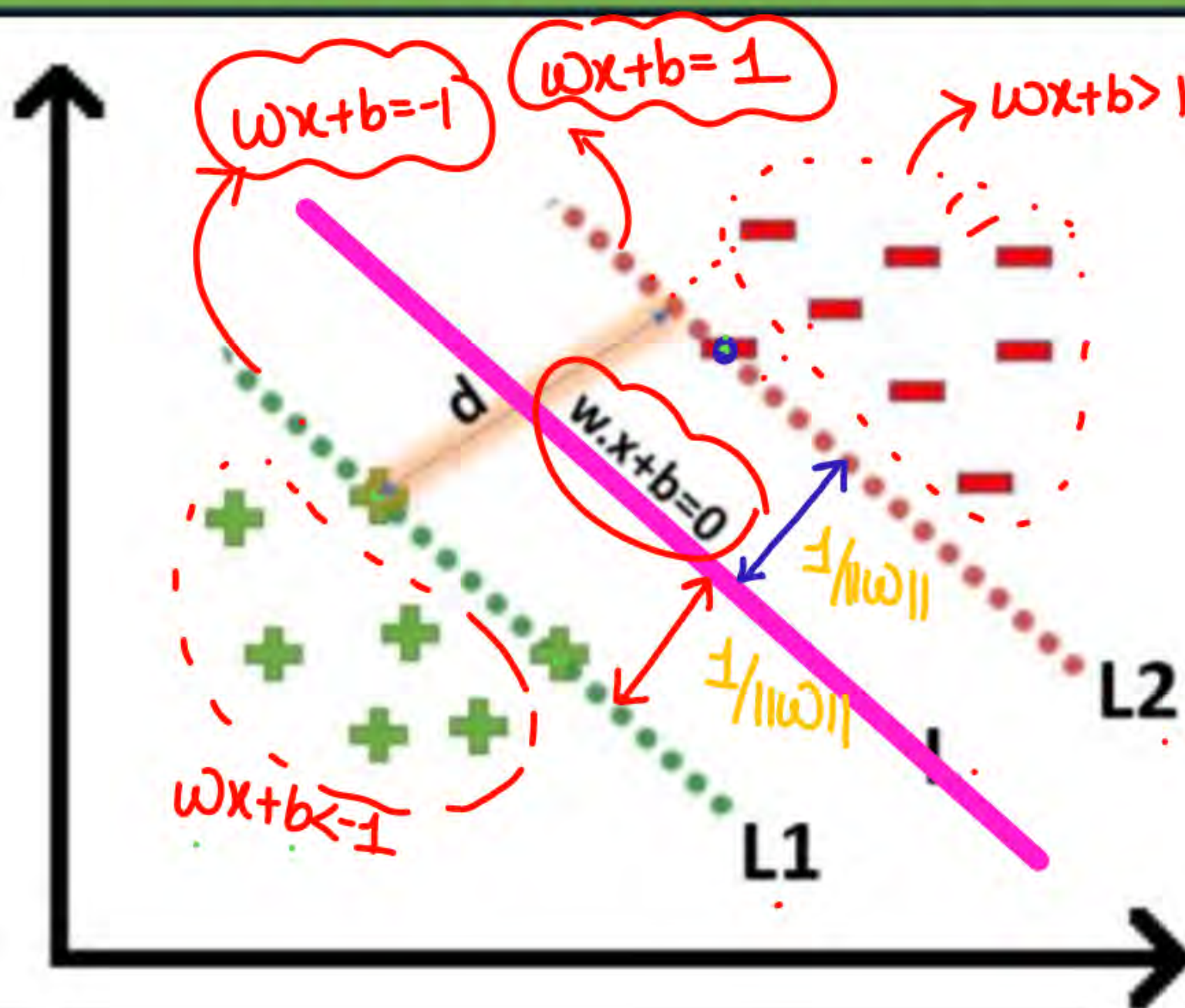
SVM primal  
equation

- So we can assume that  $\omega x + b = \pm 1$  are the lines from support vectors.



$$\|w\| = \sqrt{w_1^2 + w_2^2 + \dots}$$

## Support Vector Machine



So by removing 'd' also become much clearer

$$\Rightarrow \text{distance of SV} = \frac{|w \cdot x_i + b|}{\sqrt{w_1^2 + w_2^2 + \dots}}$$

$$\Rightarrow \text{distance of SV} = \frac{1}{\sqrt{w_1^2 + w_2^2 + \dots}}$$

$$\Rightarrow \text{total margin} = \frac{2}{\|w\|}$$





## Support Vector Machine

SVM primal  
equation

$$\max \frac{2}{\|\omega\|}$$

Such that

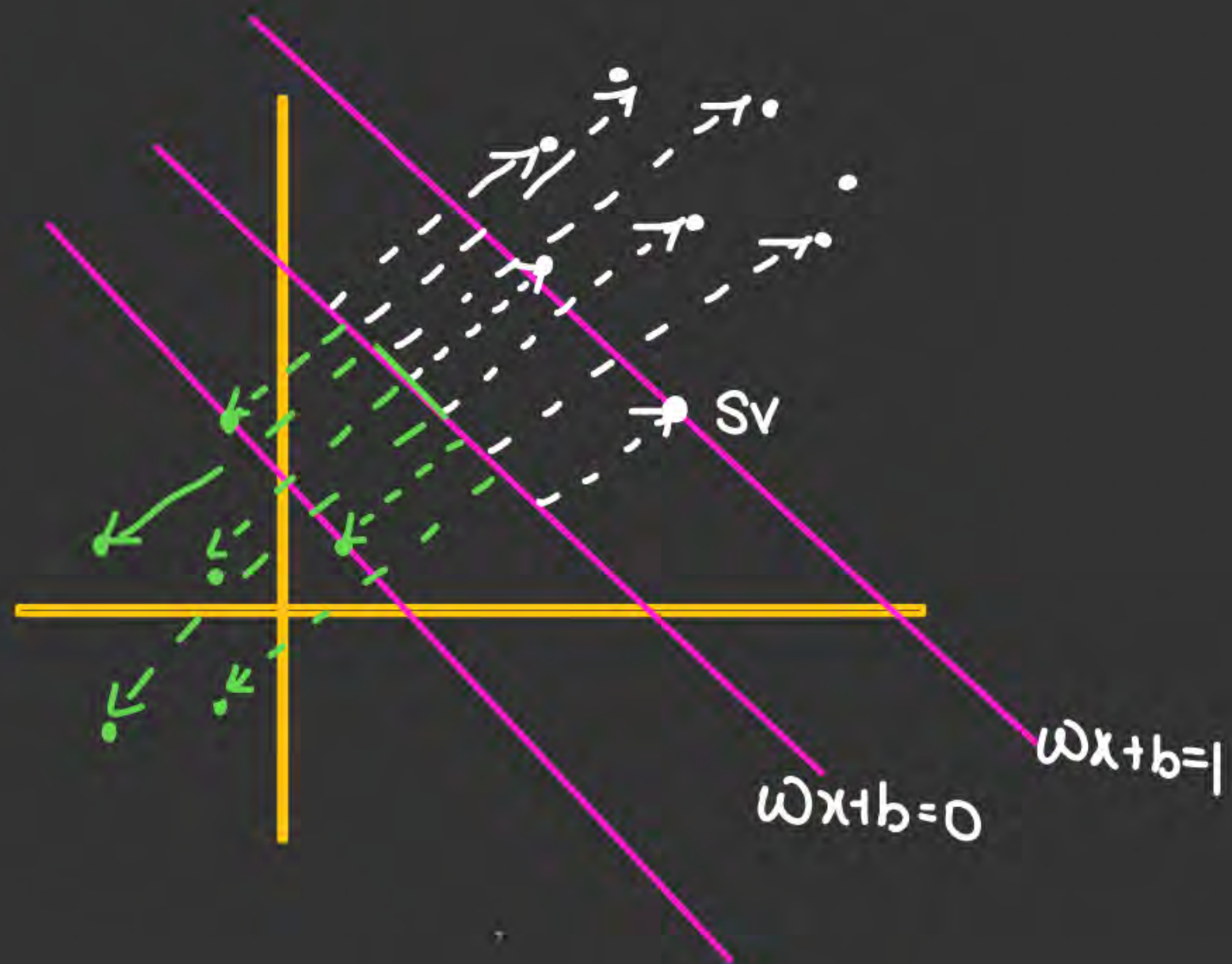
$$y_i(\omega x_i + b) \geq 1$$

→ equal to 1 for sv

→ greater than 1 for non sv.

$$\min \|\omega\|$$

$$\text{St. } y_i(\omega x_i + b) \geq 1$$







## Support Vector Machine

$$(\omega_1^2 + \omega_2^2 + \omega_3^2 - - -)$$

final algo  $\min \frac{1}{2} \|\omega\|^2$

$$\text{St. } y_i(\omega x_i + b) \geq 1$$

⇓  
This is Constraint optimization

SVM primal  
equation

## Steps in Support Vector Machine

Just a brief  $\Rightarrow$

ex  $\min f(x)$   
 $\text{st. } g(x) \leq 0$

Constraint  
 Problem

lagrangian

Lets see the  
 lagrangian...

$$\min_x \max_{\lambda} f(x) + \lambda g(x)$$

Condition  $\lambda \geq 0$

How



## Steps in Support Vector Machine

Lets see the  
lagrangian...

$$\left[ \min_x \left[ \max_{\lambda} f(x) + \lambda g(x) \right] \right]$$

$$\lambda \geq 0, g(x) \leq 0$$

$$\left( \begin{array}{l} \text{if } g(x) = 0 \\ \lambda \text{ can take} \\ \text{any value} \end{array} \right)$$

$$\left( \begin{array}{l} \text{if } g(x) < 0 \\ \lambda = 0 \end{array} \right)$$

So that term  
in bracket  
is maximized

when we solve this  
then this eq will  
convert to  $\min_x f(x)$ .

## Steps in Support Vector Machine

How lagrangian equation and our problems equate...

$$\min f(x)$$

$$g_1(x) \leq 0$$

$$g_2(x) \leq 0$$

$$g_3(x) \leq 0$$

$$\min_x \left[ \max_{\lambda_1, \lambda_2, \lambda_3} f(x) + \lambda_1 g_1(x) + \lambda_2 g_2(x) + \lambda_3 g_3(x) \right]$$

Condition  $\lambda_1, \lambda_2, \lambda_3 \geq 0$ .





**SVMs**



**The Lagrangian**

**The primal  
formulation**



## The Lagrangian

How the primal form is  
same as the original  
minimization equation





**SVMs**



## The Lagrangian

**What is the dual  
formulation...**



## The Lagrangian

**The solution of  
primal is also the  
solution of the dual**



**THANK - YOU**