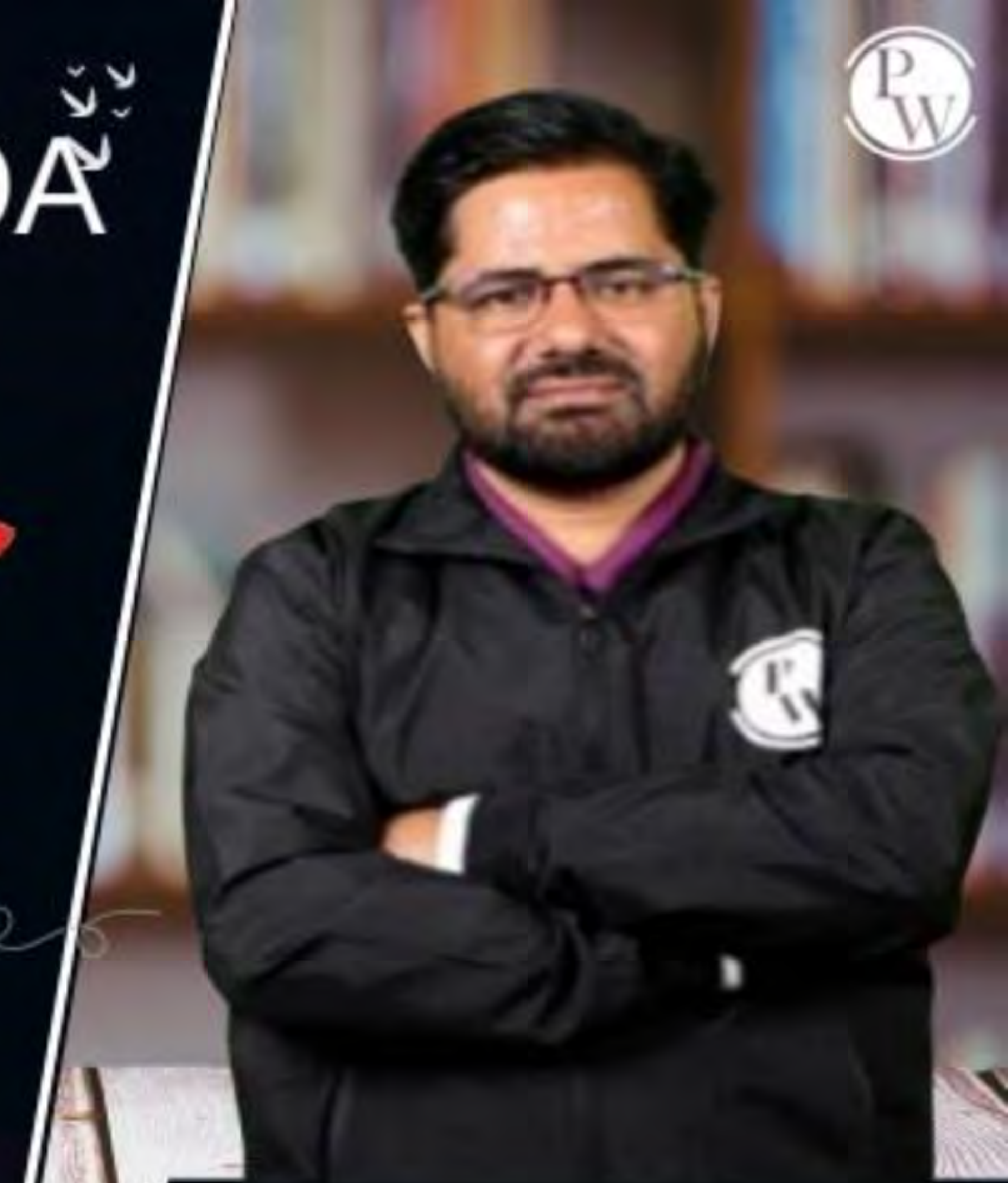
Computer Science & DA

Probability and Statistics

Discrete Random variable

Lecture No. 03



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Recap of previous lecture

Topic Basic of Discrete Random variable

Topics to be Covered

Topic Binomial Distribution & Poisson Distribution

Q:- 1000 students are appearing in an examination in which each student has to solve 150 MCQ's. Each correct ans gives 1 Marks and each incorrect ans fetches −0.25 M. If all the students have given all their answers Randomly then find the sum total of their expected (E) marks?

(all will follow Guess method.)

Sol: $N_1 = 1000$ students

$N_2 = 150$ Questions

For single student in single Q:

$X = \{\text{Marks obtained}\} = \{1, \frac{-1}{4}\}$

$p_1 = P(X = 1 \text{ Marks}) = P(\text{Correct Ans}) = \frac{1}{4}$

$p_2 = P(X = \frac{-1}{4} \text{ Marks}) = P(\text{Wrong Ans}) = \frac{3}{4}$

X:	1	$-1/4$
P(X):	$1/4$	$3/4$

Now $E(X) = \sum p_i x_i = p_1 x_1 + p_2 x_2 = \frac{1}{16}$ Marks

i.e. Av. Marks obtained by single student in single Q $= \frac{1}{16}$ Marks

So Total Av " " " 1000 " " 150 Q = ?

$= \frac{1}{16} \times 150 \times 1000 = 9375$ Marks.

Note: Max Marks that can be obtained $= 1000 \times 150 \times (1) = 150000$

Min " " " " " $= 1000 \times 150 \times (-\frac{1}{4}) = -37500$

$P(\text{Correct Ans by Guess Method}) = \dfrac{f}{T} = \dfrac{{}^1C_1}{{}^4C_1} = \dfrac{1}{4}$

$P(\text{wrong } \text{" " "}) = \dfrac{f}{T} = \dfrac{{}^3C_1}{{}^4C_1} = \dfrac{3}{4}$

Marks obtained by Smart students

$= 1000 \times 150 \times (0) = 0$ Marks.

Q. Each of the (9) words of the sentence "THE QUICK BROWN FOX JUMPS OVER THE LAZY DOG" are written on a separate pieces of paper, and these pieces are kept in a box. Now one piece is drawn at random then find Expected length of the word drawn?

(Word lengths: 3, 5, 5, 3, 5, 4, 3, 4, 3)

Sol: $X = \{\text{Length of the word drawn}\} = \{3, 4, 5\}$

$p_1 = P(X=3) = \dfrac{4}{9}$

$p_2 = P(X=4) = \dfrac{2}{9}$

$p_3 = P(X=5) = \dfrac{3}{9}$

X:	3	4	5
$P(X)$:	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

$E(X) = \sum p_i x_i = \dfrac{4}{9}(3) + \dfrac{2}{9}(4) + \dfrac{3}{9}(5)$

Av length of word drawn $= \dfrac{35}{9} = 3.88$

BINOMIAL DIST

There are four Necessary Conditions of B-Dist.

(i) Number of R-Exp (Trial) should be finite

(ii) Each R.Exp (Trial) should be Ind

(iii) Each R.Exp has only two possibilities known as Success & failure.

(iv) Prob of Success for each R.Exp should be constant.

Trick:→ whenever we are not sure about the location of success, we can apply B.Dist.

Defn: Let X is D.R.V s.t it's p.m.f is defined as;

$$\boxed{P(X=r\ \text{success}) = {}^nC_r\, p^r q^{n-r}} \quad \text{where } q+p=1$$

then X is called $\underbrace{\text{Binomial Random Variable}}_{\text{B.R.V}}$ with parameters n & p.

① $p = P(\text{success})$, $q = P(\text{failure})$

② Parameter/(Statistical Attributes) → that information, w/o which we can't apply standard Result, known as Parameter & it is denoted as $X \sim B\{n, p\}$

③ $X = \{$which is Required should be assumed as X$\}$ = Success eg 10 ships are going in an Atlantic ocean. find the prob that only 3 will come back? if History suggest that out of 11000 ships only 10000 ships came back.

Sol: $n = 10$ finite, Each ship is Ind, Each ship → Either it will come back ≃ Success
→ or " " not " " ≃ failure

$P(\text{ship will come back}) = P(\text{success}) = \frac{10000}{11000} = \frac{10}{11}$ & it is const for each ship.

$\boxed{P(X = r \text{ success}) = {}^nC_r\, p^r q^{n-r}}$ where $n = 10$, $p = \frac{10}{11}$, $q = \frac{1}{11}$ & $r = 3$ success

Let $X = \{\text{Number of ships coming back}\}$ → success.

$$P(X = 3) = {}^{10}C_3 \left(\frac{10}{11}\right)^3 \left(\frac{1}{11}\right)^{10-3}$$

Some ships are going — — .

A: Some

$C_{10}(\ )^{10}(\ )^{?}$

Q: A Coin is tossed 10 times then find the Prob that exactly 3 tosses produces head

Sol: App II Total elements in S. Space

$= \underline{2}.\underline{2}.\underline{2}.\underline{2}.\underline{2}.\underline{2}.\underline{2}.\underline{2}.\underline{2}.\underline{2} = 2^{10}$

fav. Elements $= {}^{10}C_3 = \frac{10!}{3!\,7!} = 120.$

Req Prob $= \frac{f}{T} = \frac{{}^{10}C_3}{2^{10}} = \frac{120}{1024}$

~~M-II~~ $X = \{\text{Number of Heads}\} \rightarrow$ success

$n = 10,\ p = P(H) = \frac{1}{2},\ q = P(T) = \frac{1}{2}$

$P(X = r \text{ success}) = {}^{n}C_r\, p^r q^{n-r}$

$P(X = 3H) = {}^{10}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7 = \frac{{}^{10}C_3}{2^{10}} = \frac{120}{1024}$

Q: A Coin is tossed 10 times then find the Prob that only 1st three tosses produces Head ?

we are sure about the location of H

So we Can't apply B. Dist.

Sol: Req Prob $= P(HHHTTTTTTT) = \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^7$

Q: A Box Contains 10% defective items. If 10 items are choosen at Random then find the prob that (i) there will be No defective (ii) there will be at most 2 defective ?

Sol:- $X = \{\text{Number of def. items}\} \rightarrow$ success

$n = 10, \quad p = P(\text{def. item}) = 10\% = 0.1$

$q = P(\text{Non Def item}) = 0.9$

$$\boxed{P(X = r \text{ success}) = {}^{n}C_{r}\, p^{r} q^{n-r}}$$

(1) $P(X = 0 \text{ Def}) = {}^{10}C_0 (0.1)^0 (0.9)^{10} = (0.9)^{10}$

(2) $P(X \le 2) = P(X = 0 \text{ or } 1 \text{ or } 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \left({}^{10}C_0\, p^0 q^{10}\right) + \left({}^{10}C_1\, p^1 q^{10-1}\right) + \left({}^{10}C_2\, p^2 q^{10-2}\right)$

(iii) $P(\text{getting at least one def item}) = ?$

$= 1 - P(\text{No def})$

$= 1 - (0.9)^{10}$

Q: In a Bike Race, the probability of a motorist being killed in an accident during a year is $\frac{1}{2400}$. then find the Prob that In a Race having $\boxed{200}$ motorist, there will be no fatal accident. (ii) there will be at least one F. Accident.

Sol: $X = \{\text{Number of } \underbrace{\text{f. Accidents}}_{\text{success}}\}$

$n = 200$, $p = P(\text{success}) = P(F.A) = \frac{1}{2400}$

Very large   Very small

$q = P(\text{failure}) = \frac{2399}{2400}$

$P(X = r \text{ success}) = {}^{n}C_r\, p^r q^{n-r}$

$P(X = 0\ F.A) = {}^{200}C_0\, p^0 q^{200} = \frac{0.92}{1} = \frac{92}{100}$

(ii) $P(\text{at least one F.A}) = 1 - P(\text{None}) = 0.08$

$P(X \geq 1) = 1 - P(X = 0) = 0.08$

Note: out of 100 RACE of above type

Number of RACES when No one will be killed $= 92$

M-II Using poisson Distⁿ

$n = 200, \; p = \frac{1}{2400}, \; \lambda = np = 200\left(\frac{1}{2400}\right) = \frac{1}{12}$

$$P(X=0) = \frac{e^{-\lambda}\lambda^{0}}{0!} = e^{-\frac{1}{12}} = 0.92 \quad \underline{\underline{Ans}}$$

POISSON DIST

it is a particular case of B. Dist under following Restriction:

(i) $n \to \infty$ (very large)

(ii) $p \to 0$ (very small)

(iii) $np \to$ Const. $(= \lambda)$

Trick: whenever we are not sure about (n) but we can find it's average value (λ) then we can apply P. Dist.

Def^n: Let X is D.R.V s.t it's p.m.f is defined as

$$\boxed{P(X = r \text{ success}) = \frac{e^{-\lambda}\lambda^r}{r!}}$$

then X is called Poisson Random Variable having parameter λ & it is denoted as $X \sim P\{\lambda\}$

Note:

(1) Here λ = Av per unit data & $\lambda = np$

(2) if a Quest is Based on B. Dist, it can also be solved by using P. Dist (But $n \to$ very large & $p \to$ very small)

$$\underset{(n, p)}{\text{B. Dist}} \Longrightarrow \underset{(\lambda)}{\text{P. Dist}}$$

$$\underset{(n, p)}{\text{B. Dist}} \not\Longleftarrow \underset{(\lambda)}{\text{P. Dist}}$$

(2) If on an Average 5 customers arrive at ticket window per minute the $\lambda = 5/\text{min}$

" " " 1 " " " " " in every 5 minutes then $\lambda = \frac{1}{5}/\text{min}$

" " " 3 " " " " " in a time span of 5 mins then $\lambda = \frac{3}{5}/\text{min}$

Q. A certain airport receives on an average 4 number of aircrafts per hr. then find the prob that exactly 3 aircrafts will land in a particular 2 hrs period.

Sol: X = { Number of aircrafts land in 2 hrs } → success

Av Number of aircrafts lands (λ) = 4 aircrafts / hr = 8 aircrafts / (two hrs)

$$\boxed{P(X = r \text{ success}) = \frac{e^{-\lambda}\lambda^{r}}{r!}} \Longrightarrow P(X=3) = \frac{e^{-\lambda}.\lambda^{3}}{3!} = \frac{e^{-8}.8^{3}}{3!} = 0.0286$$

$$= \frac{28}{1000}$$

Q: An observer counts on an average 240 vehicles/hr on a specific highway location then find the prob that, No vehicle will arrive in a 30 sec time interval

Sol: X = {Number of vehicles in 30 sec} → success

Av. Number of Vehicles (λ) = 240 veh/hr = 240 veh/60 mins = 4 veh/min = 2 veh/30 sec

$$\boxed{P(X = r \text{ success}) = \frac{e^{-\lambda}\lambda^{r}}{r!}} \Rightarrow P(X=0) = \frac{e^{-\lambda}\lambda^{0}}{0!} = e^{-\lambda} = e^{-2} = 0.135$$

$$= \frac{0.135}{1} = \frac{135}{1000}$$

out of 1000 intervals of thirty secs each, Number of Intervals when No vehicle will arrive = 135

Note → $\lambda = 240\,veh/hr = 2\,veh/30\,sec \neq \frac{1}{15}\,veh$

$\lambda = 240\,veh/hr = 2\,Veh\,/\,thirty\ sec$

$\lambda = 240\,veh/hr = 4\,veh/min \rightarrow 2\,veh/30\,sec$

$\rightarrow \frac{1}{15}\,veh/sec$

ie An for 30 sec is $\lambda = 2$ ✓ (It is for our Quest)

& " " 1 sec is $\lambda = \frac{1}{15}$ (if it is for future Q.)

THANK - YOU