CS & DA

Probability and Statistics

Continuous Random Variable

Discussion Notes

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[NAT]

#Q. If X has the probability density function

$$f(x) = \begin{cases} ke^{-3x}, & x > 0 \\ 0 & ; \text{ elsewhere} \end{cases}$$

Find k and P $(0.5 \leq X \leq 1)$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{\infty} ke^{-3x}\,dx = 1$$

$$k\left(\frac{e^{-3x}}{-3}\right)_{0}^{\infty} = 1$$

$$-\frac{k}{3}\left[e^{-\infty} - e^{0}\right] = 1$$

$$-\frac{k}{3}(0 - 1) = 1$$

$$\boxed{k = 3}$$

$$f(x) = \begin{cases} 3e^{-3x}, & x > 0 \\ 0 & , \text{ otw} \end{cases}$$

(ii) $P(0.5 \leq x \leq 1) = \int_{0.5}^{1} f(x)\,dx$

$$= \int_{0.5}^{1} 3e^{-3x}\,dx = 3\left(\frac{e^{-3x}}{-3}\right)_{0.5}^{1}$$

$$= -\left[e^{-3} - e^{-3/2}\right] = \boxed{e^{-3/2} - e^{-3}}$$

#Q. Find the distribution function of the random variable X whose probability density is given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Sol:

$$\boxed{F(a) = \int_{-\infty}^{a} f(x)\,dx}$$

When $x < 0$, $f(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} (0)\,dx = \boxed{0}$

When $0 < x < 1$, $f(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} (0) + \int_{0}^{x} (x)\,dx = \boxed{\frac{x^2}{2}}$

When $1 \leq x \leq 2$ then $f(x) = \int_{-\infty}^{x} f(x)\,dx$

$$= \int_{-\infty}^{0} (0) + \int_{0}^{1} (x)\,dx + \int_{1}^{x} (2-x)\,dx$$

$$= 0 + \left(\frac{1}{2}\right) + \left(2x - \frac{x^2}{2}\right)\Big|_{1}^{x}$$

$$= 0 + \frac{1}{2} + \left[\left(2x - \frac{x^2}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \boxed{2x - \frac{x^2}{2} - 1}$$

When $x > 2$, $F(x) = \int_{-\infty}^{x} f(x)\,dx = \cdots = \boxed{1}$

$$f(x) = \begin{cases} 0 & , \ x < 0 \\ x^2/2 & , \ 0 < x < 1 \\ 2x - \dfrac{x^2}{2} - 1 & , \ 1 \le x \le 2 \\ 1 & , \ x > 2 \end{cases}$$

#Q. If X is a random variable with density $f(x) = \dfrac{1}{2} e^{-\dfrac{|x|}{2}}$, $-\infty < x < \infty$.

Then E(|X|)_____.

= even func^n.

$$E(|x|) = \int_{-\infty}^{\infty} \underbrace{|x| \cdot f(x) dx}_{\text{Even func}^n} = 2 \int_{0}^{\infty} |x| f(x) dx$$

$$= 2 \int_{0}^{\infty} |x| \cdot \frac{1}{2} e^{-\frac{|x|}{2}} dx = \int_{0}^{\infty} x e^{-\frac{x}{2}} dx$$

Put $\dfrac{x}{2} = t$

$dx = 2 dt$

$$= \int_{0}^{\infty} 2t \cdot e^{-t} \cdot 2 dt$$

$$= 4 \int_{0}^{\infty} t \cdot e^{-t} dt$$

$$= 4 \left[t \left(\frac{e^{-t}}{-1} \right) - \int \left\{ 1 \cdot \frac{e^{-t}}{-1} \right\} dt \right]_{0}^{\infty}$$

$$= 4 \left[\left\{ 0 - (0) \right\} + \left\{ 0 - 1 \right\} \right]$$

$$= 4 \underline{\text{Ans}}$$

[MCQ]

#Q. Let X be a random variable with a continuous uniform distribution on the interval $(1, \alpha)$, where $\alpha > 1$. If $E[X] = 6\text{Var}[X]$, then $\alpha =$

A 2

B 3

C 4

D 7

for U.R.V $x \in (a, b)$ we know that

$$E(x) = \frac{a+b}{2}$$

$$\text{Var}(x) = \frac{(b-a)^2}{12}$$

in this Quest; $x \in (1, \alpha)$ then

$$E(x) = \frac{1+\alpha}{2}$$

$$\text{Var}(x) = \frac{(\alpha-1)^2}{12}$$

ATQ, $E(x) = 6\text{Var}(x)$

$$\frac{1+\alpha}{2} = 6 \cdot \frac{(\alpha-1)^2}{12}$$

$$(\alpha+1) = (\alpha-1)^2$$

$$\alpha^2 + 1 - 2\alpha - \alpha - 1 = 0$$

$$\alpha^2 - 3\alpha = 0$$

$$\alpha = 0 \text{ or } 3 \Rightarrow \boxed{\alpha = 3}$$

#Q. Suppose the probability density function of a continuous random variable
x is = $3x^2$; $0 < x < 1$.
Find 'a' and 'b' satisfying the following condition
(A) $P[x \le a] = P[x \ge a]$
(B) $P[x > b] = 0.05$

(i) $P(x \le a) = P(x \ge a)$

$P(0 < x \le a) = P(a \le x < 1)$

$$\int_0^a f(x)dx = \int_a^1 f(x)dx$$

$$\int_0^a 3x^2 dx = \int_a^1 (3x^2)dx$$

$$(x^3)_0^a = (x^3)_a^1$$

$$a^3 = 1 - a^3$$

$$a^3 = \frac{1}{2}$$

$$a = \left(\frac{1}{2}\right)^{1/3}$$

(ii) $P(x > b) = 0.05$

$$P(b < x < 1) = 0.05$$

$$\int_b^1 (3x^2)dx = \frac{1}{20}$$

$$(x^3)_b^1 = \frac{1}{20}$$

$$1 - b^3 = \frac{1}{20}$$

$$b^3 = \frac{19}{20} \Rightarrow b = \left(\frac{19}{20}\right)^{\frac{1}{3}}$$

#Q. Find whether the following function is a probability density function $= YES$?

$$f(x) = \begin{cases} \dfrac{x^2}{3}, & -1 < x < 2 \\ \\ 0, & elseweher \end{cases}$$

Also obtain $P(0 < X \le 1)$

Cross Check:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{2} f(x)\,dx$$

$$= \int_{-1}^{2} \left(\frac{x^2}{3}\right)dx = \frac{1}{9}\left(x^3\right)_{-1}^{2} = \frac{1}{9}\left(8-(-1)\right) = 1 \;\; \ddot{\smile}$$

$$P(0 < x \le 1) = \int_{0}^{1} f(x)\,dx = \int_{0}^{1} \left(\frac{x^2}{3}\right)dx$$

$$= \frac{1}{3}\left(\frac{x^3}{3}\right)_{0}^{1} = \frac{1}{9}(1-0) = \frac{1}{9}$$

#Q. The probability density function f (x) of a continuous random variable x is

defined by $f(x) = \begin{cases} \dfrac{A}{x^3}, & 5 \leq x \leq 10 \\ 0, & otherwise \end{cases}$ then the value of A is

A 100/3

B 200/3

C 50/3

D 3/200

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{5}^{10} \left(\frac{A}{x^3}\right) dx = 1$$

$$A \left(\frac{x^{-3+1}}{-3+1}\right)_{5}^{10} = 1$$

$$\frac{A}{-2}\left(\frac{1}{x^2}\right)_{5}^{10} = 1 \Rightarrow A = \frac{200}{3}$$

#Q. Let X be a random variable denoting the <u>hours of life in electric light bulb.</u>
Suppose X is distributed with density function

$$f(x) = \boxed{\frac{1}{1000} e^{-x/1000}} \text{ for } x > 0$$

Find the expected life time of such a bulb.

$$\text{p.d.f } f(t) = \begin{cases} \boxed{\mu e^{-\mu t}}, & t \geqslant 0 \\ 0, & \text{otw} \end{cases}$$

$$\Rightarrow \mu = \frac{1}{1000}$$

$$t = \{ \text{Bulb life in hrs} \}$$
$$= \text{Exp Dist}$$

$$E(t) = \int_{-\infty}^{\infty} t \cdot f(t) \, dt$$

$$= \cdots = \frac{1}{\mu} = \frac{1}{1/1000}$$

$$= 1000 \text{ hrs.}$$

[MCQ]

$$C.D.F \quad f(a) = \int_{-\infty}^{a} f(x)\,dx = \boxed{1 - e^{-\mu a}}$$

#Q.   If X has exponential distribution with mean 1/2, then $P(X < 1 \mid X < 2)$.

$$\frac{1}{\mu} = \frac{1}{2} \implies \boxed{\mu = 2}$$

A $\dfrac{(1 - e^{-2})}{(1 - e^{-4})}$

B $\dfrac{(1 - e^{-0.5})}{(1 - e^{-1})}$

C $\dfrac{(1 - e^{-1})}{(1 - e^{-2})}$

D $\dfrac{(1 - e^{-0.5})}{(1 - e^{-4})}$

$$P\left(\frac{X < 1}{X < 2}\right) = \frac{P[X < 1 \cap X < 2]}{P(X < 2)}$$

$$= \frac{P(X < 1)}{P(X < 2)} = \frac{f(1)}{f(2)}$$

$$= \frac{1 - e^{-\mu}}{1 - e^{-2\mu}} = \frac{1 - e^{-2}}{1 - e^{-4}}$$

#Q. Customers arrive randomly and independently at a service window, and the time between arrivals has an exponential distribution with a mean of 12 minutes minutes. Let X equal the number of arrivals per hour. What is P[X= 10]?

A $\dfrac{10e^{-12}}{10!}$

B $\dfrac{10^{-12}e^{-10}}{10!}$

$= \dfrac{e^{-\lambda}\lambda^{r}}{r!}$

C $\dfrac{12^{-10}e^{-10}}{10!}$

D $\dfrac{5^{10}e^{-5}}{10!}$

$= \dfrac{e^{-5} \cdot 5^{10}}{10!}$

Av. time b/w two arrivals = 12 minutes

i.e Av No. of arrivals in 12 minutes = 1

" " " " 60 mins = 5 i.e 5 per hour = λ

#Q. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \dfrac{1}{2}$. What is the probability that the repair time exceeds 2 h ?

$\mu = \dfrac{1}{2}$

$$f(t) = \begin{cases} \mu e^{-\mu t}, & t \geqslant 0 \\ 0, & \text{otw} \end{cases}, \quad t = \{ \text{Repair time} \}$$

$$P(t > 2) = \int_2^\infty f(t)\,dt = \int_2^\infty \mu e^{-\mu t} = \mu \left[\frac{e^{-\mu t}}{-\mu} \right]_2^\infty = - \left[e^{-\infty} - e^{-2\mu} \right] = \frac{1}{e^{2\mu}}$$

$$= \frac{1}{e^1} = e^{-1}$$

#Q. Students arrive at a local bar and restaurant according to an approximate Poisson process at a mean rate of 30 students per hour. What is the probability that the bouncer has to wait more than 3 minutes to card the next student ?

$$\lambda = 30 \text{ students}/hr = \frac{30}{60}/min = \frac{1}{2} \text{ per minute}$$

$$i.e_{Av.} \text{ Time b/n two students} = 2 \text{ mins} = \frac{1}{\mu} \Rightarrow \boxed{\mu = \frac{1}{2}}$$

$$t = \{\text{waiting time}\} \Rightarrow P(t > 3) = \int_3^\infty f(t)dt = \int_3^\infty \mu e^{-\mu t} dt = \mu \left[\frac{e^{-\mu t}}{-\mu}\right]_3^\infty$$

$$= -\left[e^{-\infty} - e^{-3\mu}\right] = \frac{1}{e^{3\mu}} = \frac{1}{e^{3/2}}$$

#Q. Let X be uniformly distributed over the interval [a, b], where 0 < a < b. If $E(X) = 2V(X) = 4/3$ the $P[X < 1]$ is-

A 3/4

B 1

C 1/2

D 1/4

$$E(x) = 2$$

$$\frac{a+b}{2} = 2$$

$$a + b = 4 \quad —(1)$$

$$Var(x) = \frac{4}{3}$$

$$\frac{(b-a)^2}{12} = \frac{4}{3}$$

$$(b-a)^2 = 16$$

$$b - a = +4 \quad —(2)$$

$$b - a = -4 \quad —(3)$$

Solving (1) & (2)
$$b + a = 4$$
$$b - a = 4$$
$$b = 4 \ \& \ a = 0 \checkmark$$

Solving (1) & (3)
$$b + a = 4$$
$$b - a = -4$$
$$b = 0 \ \& \ a = 4 \ ✗$$

if $a = 0, b = 4$ so p.d.f is $f(x) = \begin{cases} \frac{1}{4-0}, & 0 < x < 4 \\ 0, & o.w \end{cases}$

$$P(x < 1) = \int_0^1 f(x)dx = \int_0^1 \left(\frac{1}{4}\right)dx = \frac{1}{4} \checkmark$$

#Q. Suppose that a large conference room for a certain company can be reserved for no more than 4 hours. However the use of the conference room is such that both long and short conferences occur quite often. In fact it can be assumed that the duration X of a conference has a uniform distribution on the interval [0, 4]. What is the probability that any given conference lasts at-least 3 hours?

A 1/2

B 1/3

C 1/4

D 3/4

$$p.d.f \ is \ f(n) = \begin{cases} \dfrac{1}{4-0} & , 0 < n < 4 \\ 0 & , \ otw \end{cases} \quad , \ n = \{duration \ of \ conference\}$$

$$P(n \geqslant 3) = P(3 \leq n < 4) = \int_3^4 f(n)\,dn = \int_3^4 \left(\frac{1}{4}\right)dx = \frac{1}{4}$$

[MCQ]

#Q. X is uniformly distributed random variable that take values between 0 and 1. The value of $E(X^3)$ will be :

$$\text{p.d.f is } f(x) = \begin{cases} \dfrac{1}{1-0}, & 0 < x < 1 \\ 0, & \text{otw} \end{cases}$$

A. 0

B. 1/8

C. 1/4

D. 1/2

$$E(x^3) = \int_{-\infty}^{\infty} x^3 f(x)\, dx = \int_{0}^{1} x^3 (1)\, dx = \left(\frac{x^4}{4}\right)\Big|_{0}^{1} = \frac{1}{4}$$

#Q. Buses arrive at a specified stop at 15 min intervals starting at 7 A.M., that is, they arrive at 7:00, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 A.M. find the probability that he waits?

(a) less than 5 min for a bus and

(b) at least 12 min for a bus

$x = \{$ Arrival time of passenger at stop $\}$

$= \{$ Duration of his arrival $\}$

(a) $7:10 - 7:15$ or $7:25 - 7:30$

And, $x \in (0, 30) \Rightarrow f(x) = \begin{cases} \dfrac{1}{30-0} & 0 < x < 30 \\ 0 & \text{otw.} \end{cases}$

$P(\text{wt time} < 5 \text{ min}) = P(10 < x < 15) + P(25 < x < 30)$

$= \int_{10}^{15} \left(\dfrac{1}{30}\right) dx + \int_{25}^{30} \left(\dfrac{1}{30}\right) dx = \dfrac{5}{30} + \dfrac{5}{30} = \dfrac{1}{3}$

$$\boxed{7:00 - 7:03} \quad \text{or} \quad \boxed{7:15 - 7:18}$$

$$\text{Req Prob} = P(x \geq 12) = P(0 < x < 3) \text{ or } P(15 < x < 18)$$

$$= \int_0^3 \left(\frac{1}{30}\right) dx + \int_{15}^{18} \left(\frac{1}{30}\right) dx$$

$$= \frac{3}{30} + \frac{3}{30} = \frac{2}{10} = \boxed{\frac{1}{5}}$$

#Q. The mean height of 500 student is 151cm and the standard deviation is 15cm. Assuming that the heights are normally distributed, find how many students have heights between 120 and 155 cm? Given A(z=0 to 2.07)=0.4808 and A(z=0 to 0.27)=0.1084

$N = 500$ students

$x = \{$ Height of Student $\}$

for single st.

$\mu = 151$

$\sigma = 15$

$z_1 = \dfrac{120-151}{15} = -2.07$

$z_2 = \dfrac{155-151}{15} = 0.27$
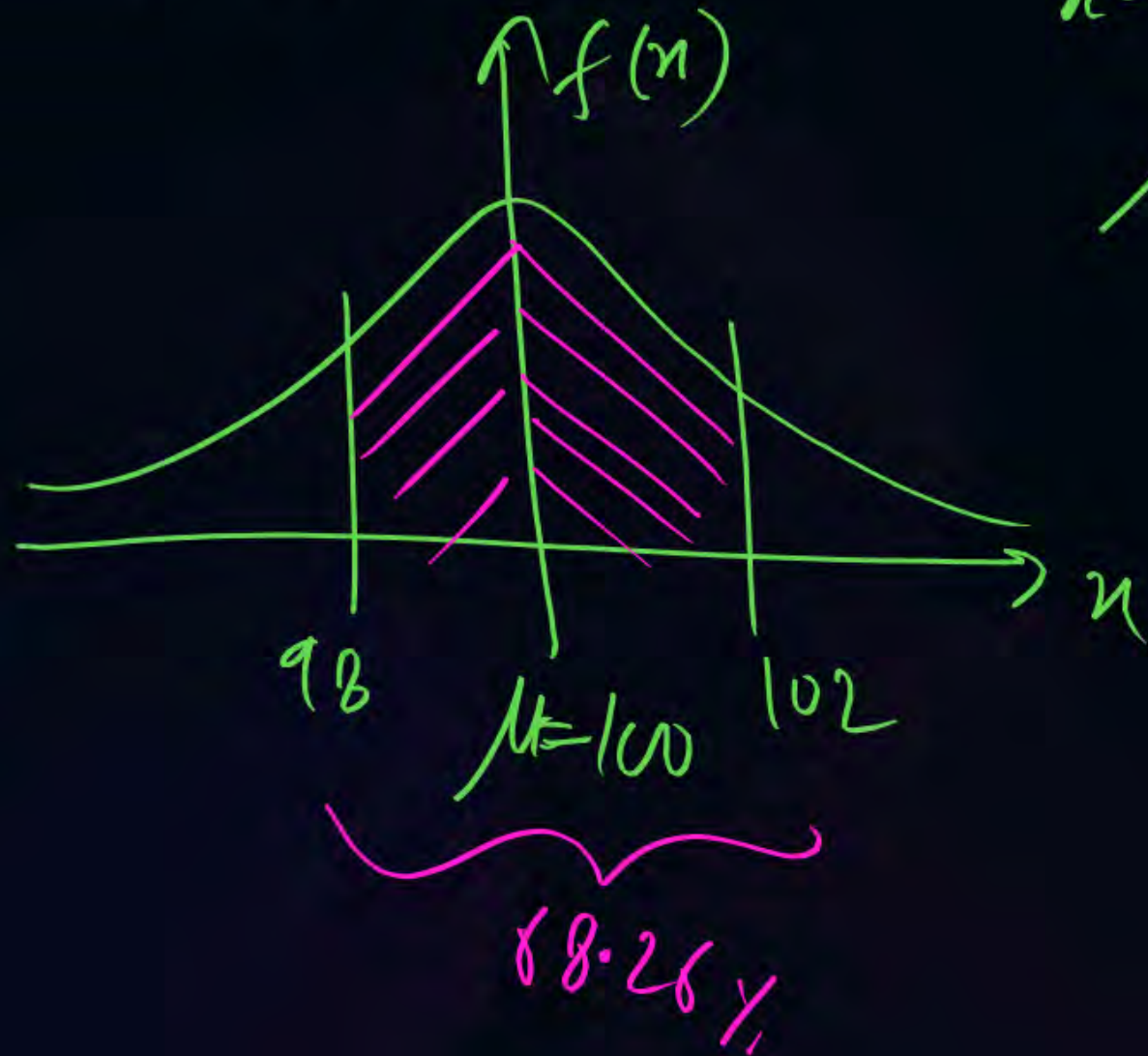
$P(120 < x < 155) = P(-2.07 < z < 0.27) =$

$= P(-2.07 < z < 0) + P(0 < z < 0.27)$

$= P(0 < z < 2.07) + P(0 < z < 0.27)$

$= 0.4808 + 0.1084 = 0.5892 = \dfrac{0.5892}{1} =$

$A_n = 0.5892 \times 500$

$= \boxed{294}$

$-2.07 \quad z=0 \quad 0.27$

$f(z)$

#Q. A manufacturer knows from experience that the resistance of resistors he produces is normal with mean $\mu = 100$ ohms and standard deviation $\sigma = 2$ ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms?

$x = \{$ Resistance of Resistor $\}$

$\mu = 100, \sigma = 2$

$\mu - \sigma = 98$

$\mu + \sigma = 102$

$P \% = 68.26 \%$

98    $\mu = 100$ 102

68.26%

#Q. If the height of 300 student are normally distributed with mean 64.5 inches and standard deviation 3.3 inches, find the height below which 99% of the students lie. Given A(z=0 to 2.327)=49%

$N = 300$, for single student: $x = \{ Height \}$

$\mu = 64.5$ inches

$\sigma = 3.3$ "

Let that height be $x = \alpha$.

$P(x < \alpha) = 0.99$

$P(z < z_1) = 0.99$

$\frac{1}{2} + P(0 < z < z_1) = 0.99$

$P(0 < z < z_1) = 0.49$

ATQ,

$z_1 = 2.327$

$z = \frac{x - \mu}{\sigma} = \frac{\alpha - 64.5}{3.3} = z_1$

$\frac{\alpha - 64.5}{3.3} = 2.327$
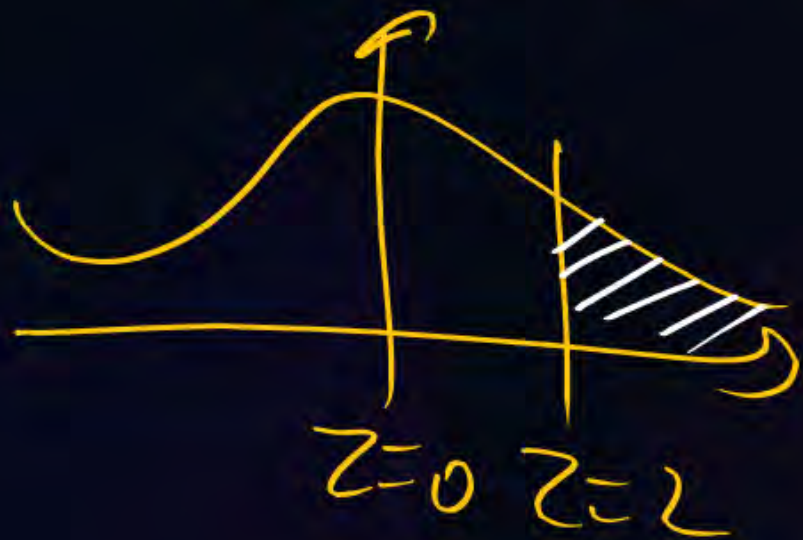
$\boxed{\alpha = 72.15 \text{ inches}}$

#Q. The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? Given $F_{(2)}$ = 0.9772

$\mu = 8, \sigma = 2, N = 5000 \text{ pair}, \text{ For single pair}: n = \{\text{Life of shoe}\}$

$$z = \frac{n-\mu}{\sigma} = \frac{12-8}{2} = 2 \quad \text{So } P(n>12) = P(z>2) = 1 - P(z \leq 2)$$

$$= 1 - F(2) = 1 - 0.9772$$

$$= 0.0228$$

So No. of shoe pair having life > 12 months $= 0.0228 \times 5000 = 114$

So No. of shoe pair to be Replaced $= 5000 - 114 = 4886$

$z = 0 \quad z = 2$

[MCQ]

#Q. The mean of a normal distribution is 50, its mode will be.

$$Me = Mo = Md = 50$$

A 25

B 40

C 50

D 100

#Q.   For the standard normal variate, the mean and variance are.

Z $\{Z \sim N\{0,1\}$

$x \sim N\{\mu, \sigma^2\}$

[A] 1, 0

[B] 0, 0

[C] 0, 1

[D] 1, 1

[MCQ]

(P W)

#Q.   The mode of a normal distribution is 80 with SD 10. Then, its median will be.

A   8

B   800

C   80

D   None of these

for N. Dist $\quad Me = Md = M_0 = 80$

$$\mu_y = 4, \quad \sigma_y^2 = 4$$

#Q. If $\log_{10} X$ is normally distributed with mean 4 and variance 4, find the probability that X lies between 1.202 and 83180000.

$y = \log_{10} X$

Given that $\log_{10} 1202 = 3.08$ and $\log_{10} 8318 = 3.92$ and $A(|z| < 1.96) = 95\%$

$$\log_{10}(1\cdot202) = \log_{10}\left(\frac{1202}{1000}\right) = \log_{10} 1202 - \log_{10} 10^3 = 3\cdot08 - 3 = 0\cdot08$$

A 1.05

$$\log_{10}(83180000) = \log_{10}(8318 \times 10^4) = \log_{10}(8318) + \log_{10} 10^4 = 3\cdot92 + 4 = 7\cdot92$$

B 0.95

C 0.78

$$P(1\cdot202 < x < 83180000)$$

$$P\left(\log_{10} 1\cdot202 < \log_{10} x < \log_{10} 83180000\right)$$

D None of these

$$P(0\cdot08 < y < 7\cdot92) = P(-1\cdot96 < z < 1\cdot96) = P(|z| < 1\cdot96) = 95\%$$

$$z = \frac{y - \mu_y}{\sigma_y} \begin{cases} z_1 = \dfrac{0\cdot08 - 4}{2} = -1\cdot96 \\[2mm] z_2 = \dfrac{7\cdot92 - 4}{2} = +1\cdot96 \end{cases}$$

[MCQ]

#Q. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal then, how many students score between 12 and 15 ? Given : Area from Z = 0 to 0.4 is 0.1554 and form Z = 0 to 0.8 is 0.2881.

A 0.443

B 44.3

C 444

D 82

$\mu = 14, \ \sigma = 2.5, \ N = 1000,$ for single student; $x = \{$ Score of student $\}$

$Z = \dfrac{x - \mu}{\sigma}$

$Z_1 = \dfrac{12 - 14}{2.5} = -0.8$

$Z_2 = \dfrac{15 - 14}{2.5} = 0.4$

$P(12 < x < 15) = P(-0.8 < Z < 0.4) = P(-0.8 < Z < 0) + P(0 < Z < 0.4)$

$= 0.2881 + 0.1554 = 0.4435$

So No. of students getting score b/w 12 & 15 $= 443.5 = 444$
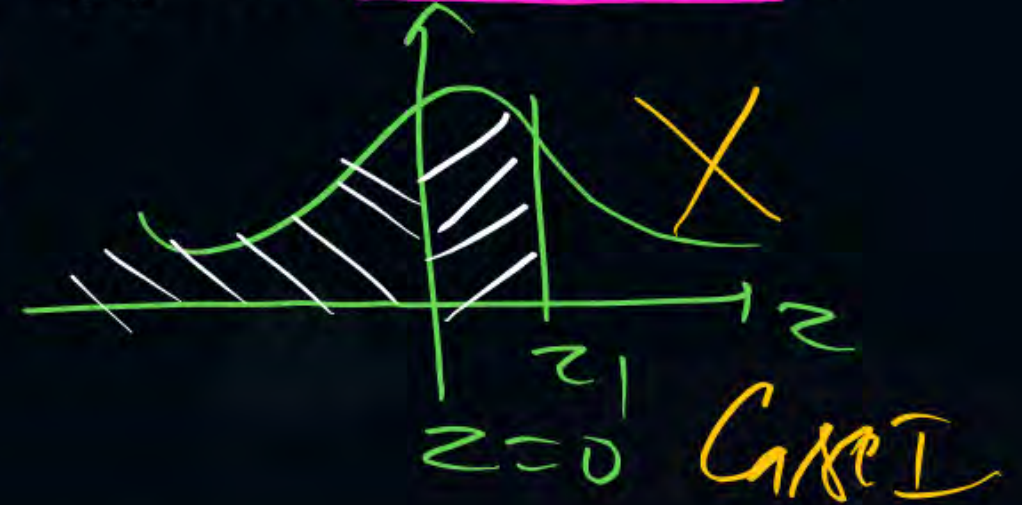
#Q. If the actual amount of instant coffee which a filling machine puts into '6-ounce' jars is a RV having a normal distribution with SD = 0.05 ounce and if only 3% of the jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars? Given that area under f(z) from 0 to 1.808 is 47%

$\sigma = 0.05$, $n = \{$ Actual amount of, Coffee in Jar $\}$
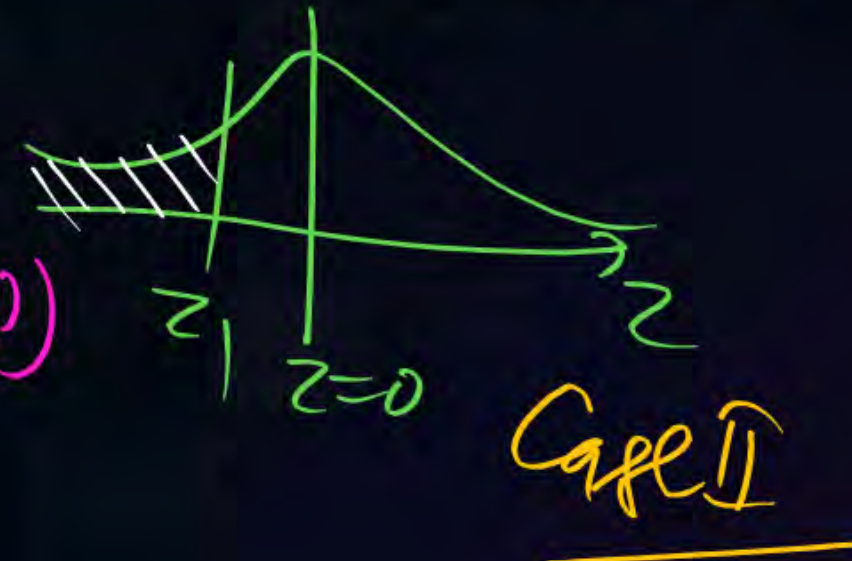
$\mu = ?$

A.Q. $P(n < 6) = 0.03$

$P(z < z_1) = 0.03$

$z = \dfrac{n-\mu}{\sigma} = \left(\dfrac{6-\mu}{0.05}\right) = z_1$

Case I: $\Bigg(0.5 + P(0 < z < z_1) = 0.03$

$\Rightarrow P(0 < z < z_1) = -0.47$ (N.P)

Case II: $0.5 - P(z_1 < z < 0) = 0.03$

$P(z_1 < z < 0) = 0.47$

z_1

$z = 0$ Case I

z_1

$z = 0$ Case II

$$P(z_1 < z < 0) = 0.47$$

$$P(0 < z < -z_1) = 0.47$$

ATQ, $P(0 < z < 1.808) = 47\%$

$$\Rightarrow -z_1 = 1.808$$

$$-\left(\frac{6-\mu}{0.05}\right) = 1.808$$

$$\mu = 6.094 \text{ ounces}$$

#Q. If the two regression lines are known, then r = ...

A A.M of the two regression coefficients

B G.M of the two regression coefficients

C H.M of the two regression coefficients

D Product of the two regression coefficients

$$r = \pm \sqrt{b_{yx} \cdot b_{xy}}$$

where $b_{yx} = r \dfrac{\sigma_y}{\sigma_x}$

$$b_{xy} = r \dfrac{\sigma_x}{\sigma_y}$$

$$r = GM \text{ of } b_{yx} \ \& \ b_{xy}$$

#Q. If the two lines of regression are perpendicular then the correlation coefficient r = _____.

$$\theta = \frac{\pi}{2}$$

$$\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$$

$$\tan\frac{\pi}{2} =$$

$$\infty =$$

$$\Rightarrow r = 0$$

#Q. If the two-regression coefficient are 0.8 and 0.2, what would be the value of co - efficient of correlation.

b_{yx} b_{xy}

$$r = +\sqrt{b_{yx} \cdot b_{xy}}$$
$$= +\sqrt{0.8 \times 0.2}$$
$$= +\sqrt{0.16} = 0.4 \text{ Ans}$$

#Q. Two random variables have the regression lines with equations $3x + 2y = 26$ and $6x + y = 31$. Find the mean values and the correlation coefficient between x and y.

$(\bar{x}, \bar{y}) = ?$

$r = ?$

(i)
$$\left.\begin{array}{l} 3x+2y=26 \\ 6x+y=31 \end{array}\right\} \Rightarrow \begin{array}{l} 3x+2y=26 \\ 12x+2y=62 \end{array}$$

$$\frac{}{-9x=-36}$$

$$\boxed{x = 4}$$

& $6(4)+y=31 \Rightarrow \boxed{y=7}$

so $(\bar{x}, \bar{y}) = (4, 7)$

R.Line y on x: $3x+2y=26$

$$y = \left(\frac{-3}{2}\right)x + 13 \quad \text{—(1)}$$

i.e. $b_{y/x} = -3/2$

R.line x on y: $6x+y=31$

$$x = -\frac{1}{6}y + \frac{31}{6} \quad \text{i.e } b_{x/y} = -\frac{1}{6}$$

$$r = -\sqrt{b_{y/x} \, b_{x/y}} = -\sqrt{\left(\frac{-3}{2}\right)\left(\frac{-1}{6}\right)} = \frac{-1}{2}$$