Computer Science & DA

Probability and Statistics

Discrete Random Variable

Lecture No. 01

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Recap of previous lecture

Topic Bays'& theorem

Topics to be Covered

Topic

Statistics (Basic Definition) mean, median, mode, variance , S.D, Covariance etc.

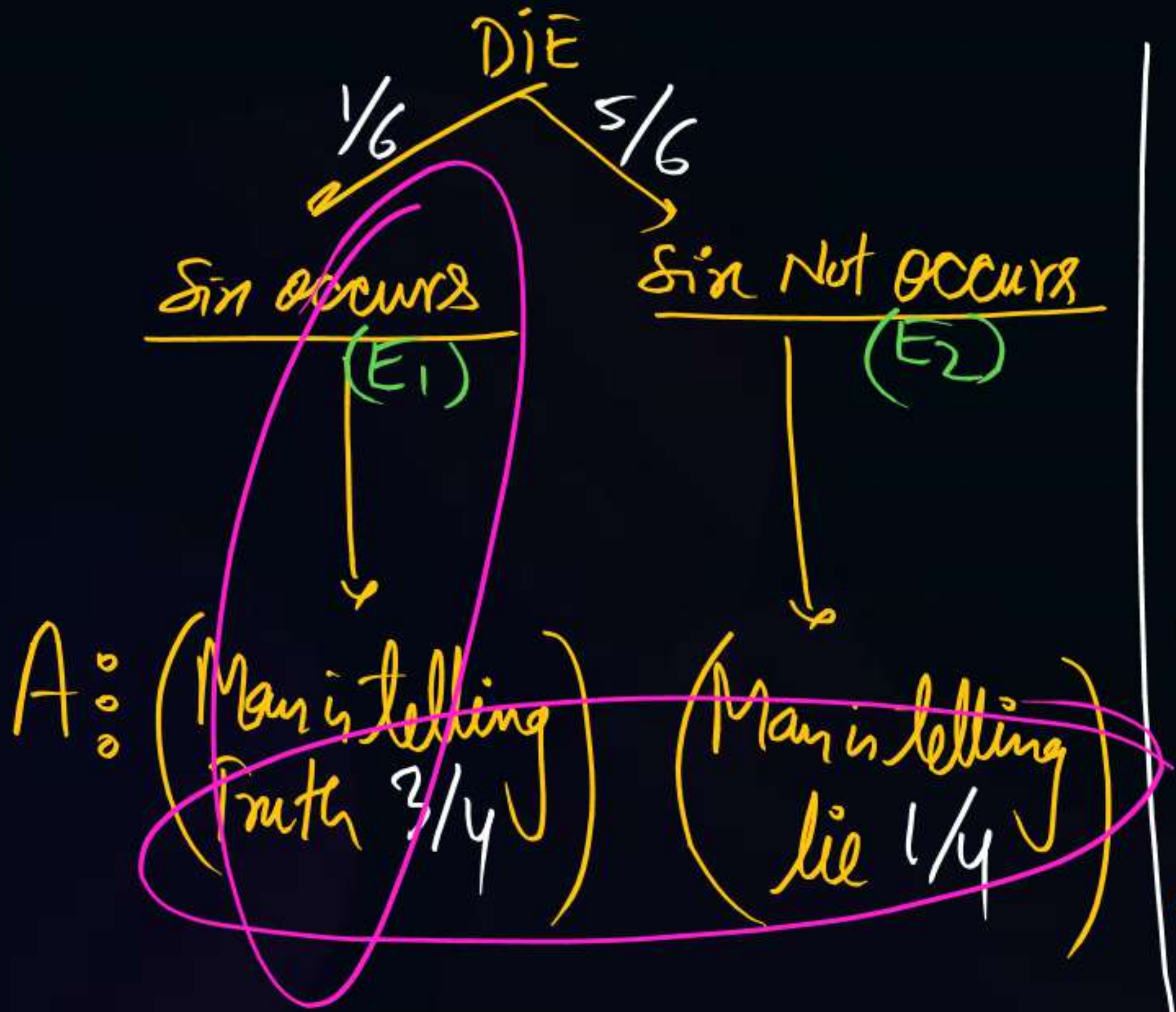
Q. A person is known to speak truth 3 out of 4 times. (He throw a die) and reports that it is six, then find the prob that it is acutally six?

R. Exp

Condition

$A = \{\text{Man Reports that it is six}\}$

$P(E_1) = \frac{1}{6}$ & $P(E_2) = \frac{5}{6} \Rightarrow E_1$ & E_2 are ME & Exh



$$P(A) = \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4} = \frac{8}{24}$$

$$P(\text{actually six}) = P(E_1/A) = \frac{\frac{1}{6} \times \frac{3}{4}}{8/24} = \frac{3}{8}$$

out of 24 Reports given by him, 8 are representing that "it is six"

& out of 8 Reports representing "Six" only 3 are Correct.

Qs In a town 60% M and 40% W in which 80% M and 50% W are employed.
(A person is selected at Random) then find the prob that person is an unemployed person?

A

Person

0.6 — 0.4

Man (E_1) — Woman (E_2)

A : (unemployed) 0.2 — 0.5

$$P(A) = (0.6 \times 0.2) + (0.4 \times 0.5)$$

$$= 0.12 + 0.20 = 0.32$$

CARDS (52)

BLACK (26) — RED (26)

Spade (13) — Club (13) — Heart (13) — Diamond (13)

FACECARDS: K, Q, J
(12)

HONOURCARDS K, Q, J, A
(16)

Q: From a pack of 52 Cards, while suffeling, one Card is lost then two cards are drawn at random then find the prob that both are spade ?

A

Cards (52)

$\frac{13}{52} = \frac{1}{4}$ → Spade lost (E_1)
(12 sp & 39 oth)

$A : \frac{{}^{12}C_2}{{}^{51}C_2}$

$\frac{39}{52} = \frac{3}{4}$ → Spade Not Lost (E_1)
(13 sp & 38 oth)

$\frac{{}^{13}C_2}{{}^{51}C_2}$

$$P(A) = \left(\frac{1}{4} \times \frac{{}^{12}C_2}{{}^{51}C_2}\right) + \left(\frac{3}{4} \times \frac{{}^{13}C_2}{{}^{51}C_2}\right)$$

$$= \frac{1}{4} \times \frac{12 \times 11}{51 \times 50} + \frac{3}{4} \times \frac{13 \times 12}{51 \times 50}$$

$$= \frac{11}{850} + \frac{39}{850} = \frac{50}{850} = \frac{1}{17}$$

eg (1) if there are $\underbrace{\text{12 Spade \& 39 other Cards}}_{\text{Total 51}}$ then $P(\text{drawing two spade cards}) = \frac{F}{T} = \frac{^{12}C_2}{^{51}C_2}$

Total ways of selecting 2 Cards $= {}^{51}C_2$

fav " " " 2 Spade Cards $= {}^{12}C_2$

Questions Based on Tree Diag —

Q: Two friends A & B play a Game of Dice alternately in which anyone can win if six appears 1^{st} time then find their respective chances of winning if A starts?

Sol: $P(W) = P(\text{six occurs}) = \frac{1}{6}$

$P(L) = P(\text{six Not occurs}) = \frac{5}{6}$

$S = \{\overset{=A}{W}, LW, \overset{=A}{LLW}, LLLW, \overset{=A}{LLLLW}, \ldots\ldots\}$

fav cases for A to win;

$A = \{W \text{ or } LLW \text{ or } LLLLW \text{ or } \ldots\ldots\}$

$P(A\,win) = P(W) + P(LLW) + P(LLLLW) + \ldots\ldots$

$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \ldots\ldots$

$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\ldots\right]$

$= \frac{1}{6}\left[\frac{1}{1-\left(\frac{5}{6}\right)^2}\right] = \frac{6}{11} \Rightarrow P(B) = \frac{5}{11}$

Note: (1) $S_\infty = a + ar + ar^2 + ar^3 + \dots\dots \infty = \frac{a}{1-r}$; $-1<r<1$

where $a = 1^{st}$ term & $r =$ Common Ratio

(2) $\boxed{1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots\dots \infty = (1-x)^{-2}}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots\dots$$

Put $x \approx -x$ & $n = -2$ you will get Result

eg $(1-x)^{-1} = \frac{1}{(1-x)} = 1 + x + x^2 + x^3 + \dots\dots \infty$

Gate: A coin is tossed until H appears then find the prob that Required number of tosses will be odd?

$P(H) = \frac{1}{2}$ & $P(T) = \frac{1}{2}$

$S = \{H, TH, TTH, TTTH, TTTTH, \dots\}$

Fav Cases $= \{H, TTH, TTTTH, \dots\}$

Req Prob $= P(H) + P(TTH) + P(TTTTH) + \dots\dots$

$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots\dots = 2/3$

STATISTICS

Random Variable → If we are not sure about the outcome of an experiment then such types of experiments are called R. Exp & Variable involve in it is called R. Variable.

① Mean(x) / Central Value / Average Value / Expected Value → $\bar{X} = E(X) = \frac{\sum X}{N}$

$(\bar{X})$ $\quad \mu_X \quad$ $E(X)$

(Mature Method)

M-I $\quad \bar{X} = \frac{\sum X}{N}$ (Childhood M.) $\quad$ M-II $\quad E(X) = \frac{\sum p_i x_i}{\sum p_i} = \sum p_i x_i$

② Variance(x) → "it is the Av. of sq. of deviations from central value"

$$\boxed{Var(X) = \frac{\sum (X-\bar{X})^2}{N}} = E(X-\bar{X})^2 = \text{-----------} = \boxed{E(X^2) - E^2(X)}$$

Explanation

$$\because \bar{X} = \frac{\sum X}{N} = E(X)$$

$$\Rightarrow Var(X) = \frac{\sum (X-\bar{X})^2}{N} = E(X-\bar{X})^2$$

② S.D(σ) = $+\sqrt{Var(X)}$
(RMSD)

③ Covariance :- it measures the simultaneous Variation of two Random Variables X & Y

$$Cov(X,Y) = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{N}$$

$$= E\{(X-\bar{X})(Y-\bar{Y})\}$$

$$= \ldots\ldots\ldots\ldots$$

$$\boxed{Cov(X,Y) = E(XY) - E(X).E(Y)}$$

Note :- if X & Y are two Ind. R.V then

$$Cov(X,Y) = 0$$

eg Cov(Ht, Age) = 0 after the age of 20 yrs.

eg Cov(Beauty, Bravery) = 0

Some Good Points –

1. $Var(X)$ or $SD(\sigma) \geq 0$ (T)

2. Var or SD $\propto \frac{1}{\text{Consistency}}$ (T)

3. $Cov(X, X) = Var(X)$ (T)

Proof: $Cov(X, X) = \frac{\sum (X-\bar{X})(X-\bar{X})}{N} = \frac{\sum (X-\bar{X})^2}{N} = Var(X)$

4. $Var(X) = \frac{\sum (X-\bar{X})^2}{N} = \cdots = E(X^2) - E^2(X)$

Some More Standard Results –

if X & Y are R.V and a, b, c are Const.

1. $E(aX \pm bY \pm C) = aE(X) \pm bE(Y) \pm E(C)$

$= aE(X) \pm bE(Y) \pm C$

2. $Var(aX + b) = a^2 Var(X) + Var(b)$

$= a^2 Var(X) + 0$

3. $Var(aX \pm bY)$

$= a^2 Var(X) + b^2 Var(Y) \pm 2ab\, Cov(X, Y)$

2M

Q. if X & Y are two Ind R.V then which is false?

(a) $E(XY) = E(X).E(Y)$ (T)

(b) $Cov(X, Y) = 0$ (T)

(c) $E(X^2Y^2) = E^2(X)E^2(Y)$ (F)

(d) $Var(X-Y) = Var(X) + Var(Y)$ (T)

$Var(X-Y) = 1^2 Var(X) + (-1)^2 Var(Y) + 2(1)(-1)\cancel{Cov(X,Y)}^{0}$

$= Var(X) + Var(Y)$

Q. If the difference b/w Expected Value of the sq. of Random Variable and sq of the Expected Value is represented by R then

(a) $R=0$ (b) $R>0$ (c) $R<0$ (d) $R \geq 0$

Let X is Random Variable then ATQ,

$E(X^2) - (E(X))^2 = R$

$= Var(X)$

Q: if X & Y are two Zero Mean, Ind Random Variables having Variances $\frac{1}{4}$ & $\frac{1}{9}$ resp then find Mean & Variance of $(2X-3Y) = ?$

Sol: $E(X) = E(Y) = 0$

$Var(X) = \frac{1}{4}$, $Var(Y) = \frac{1}{9}$

$Cov(X, Y) = 0$

Let $\boxed{Z = 2x - 3y}$ —— (1)

(i) $E(Z) = E(2X - 3Y)$

$= 2E(X) - 3E(Y) = 2(0) - 3(0) = 0$

(ii) $Var(Z) = Var(2x - 3y)$

$= 4\,Var(X) + 9\,Var(Y) + 2(2)(-3)\,Cov(X, Y)$

$= 4\left(\frac{1}{4}\right) + 9\left(\frac{1}{9}\right) + 0$

$= 2$ & $SD(\sigma) = +\sqrt{2}$

2014
2M

Apti

Which of the following Batsman is more consistent ?

Batsman	Av.	SD
K	65	6.71
L	53	4.79
M	79	4.91
L	42	5.12

$\because$ Consistency $\propto \frac{1}{SD}$

So Ans = L

eg Consider 4 children having wt 9 kg, 13 kg, 16 kg & 22 kg

① Av wt of child $= \frac{\Sigma X}{N} = \frac{9+13+16+22}{4} = 15\text{ kg}$

② Av of deviations from central Value $= ? = \frac{\Sigma (X-\bar{X})}{N} = \frac{(9-15)+(13-15)+(16-15)+(22-15)}{4} = 0\text{ kg}$

③ $\underbrace{\text{Av of squares of deviations from Central Value}}_{Var(X)}$ $= ? = \frac{\Sigma (X-\bar{X})^2}{N} = \frac{(-6)^2+(-2)^2+(1)^2+(7)^2}{4} = 22.5\text{ kg}^2$

So S.D (σ) $= +\sqrt{Var(X)} = 4.75$ kg — Best

Var = Mean sq. deviation & SD = Root M.S.D

④ Av of Modulus of Deviations from Central Value $= ? = \frac{\Sigma |X-\bar{X}|}{N} = \frac{6+2+1+7}{4} = 4\text{ kg}$

Not Best.

$\underbrace{10.25 \le \text{Av wt} \le 19.75}_{\text{Best}}$, $\underbrace{11 \le \text{Av wt} \le 19}_{\text{Not Best.}}$

R.V

Discrete R.V (X)

(Counting Related Variables)

(eg No of students, No. of vehicles No of deaths etc)

↓

Discrete Prob Dist

eg (BINOMIAL / POISSON)

Continuous R.V (x)

(If x has ∞ possibilities then it is called C.R.V)

eg Height, wt, Age

↓

Cont. Prob Dist

eg (Exp / uniform / Normal)

↓

Prob Mass func^n (p.m.f)

$\sum p_i = 1$ & $p_i \geq 0$

↓

① Mean(X):

$E(X) = \sum p_i x_i$

② Var(X)

$= E(X^2) - E^2(X)$

③ S.D(σ) $= \sqrt{\text{Var}X}$

↓

Prob Density func^n (p.d.f)

$\int_{-\infty}^{\infty} f(x)\,dx = 1$, $f(x) \geq 0$

↓

① Mean (x)

$E(x) = \int_{-\infty}^{\infty} x . f(x)\,dx$

② $\text{Var}(x) = E(x^2) - E^2(x)$

③ $SD(\sigma) = +\sqrt{\text{Var}(x)}$

THANK - YOU