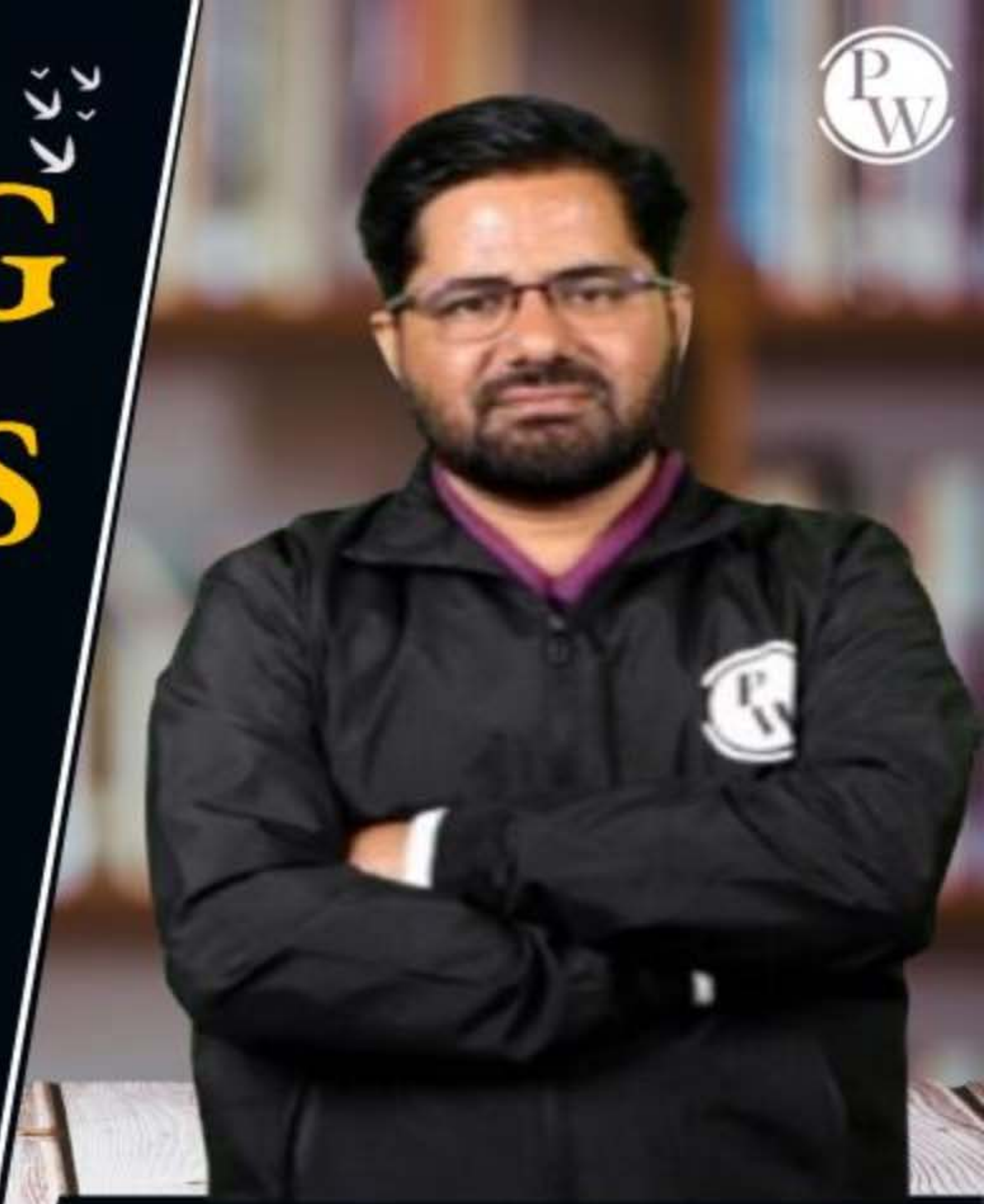
ENGINEERING MATHEMATICS

DS & AI

Calculus and Optimization

Weekly Test- 02

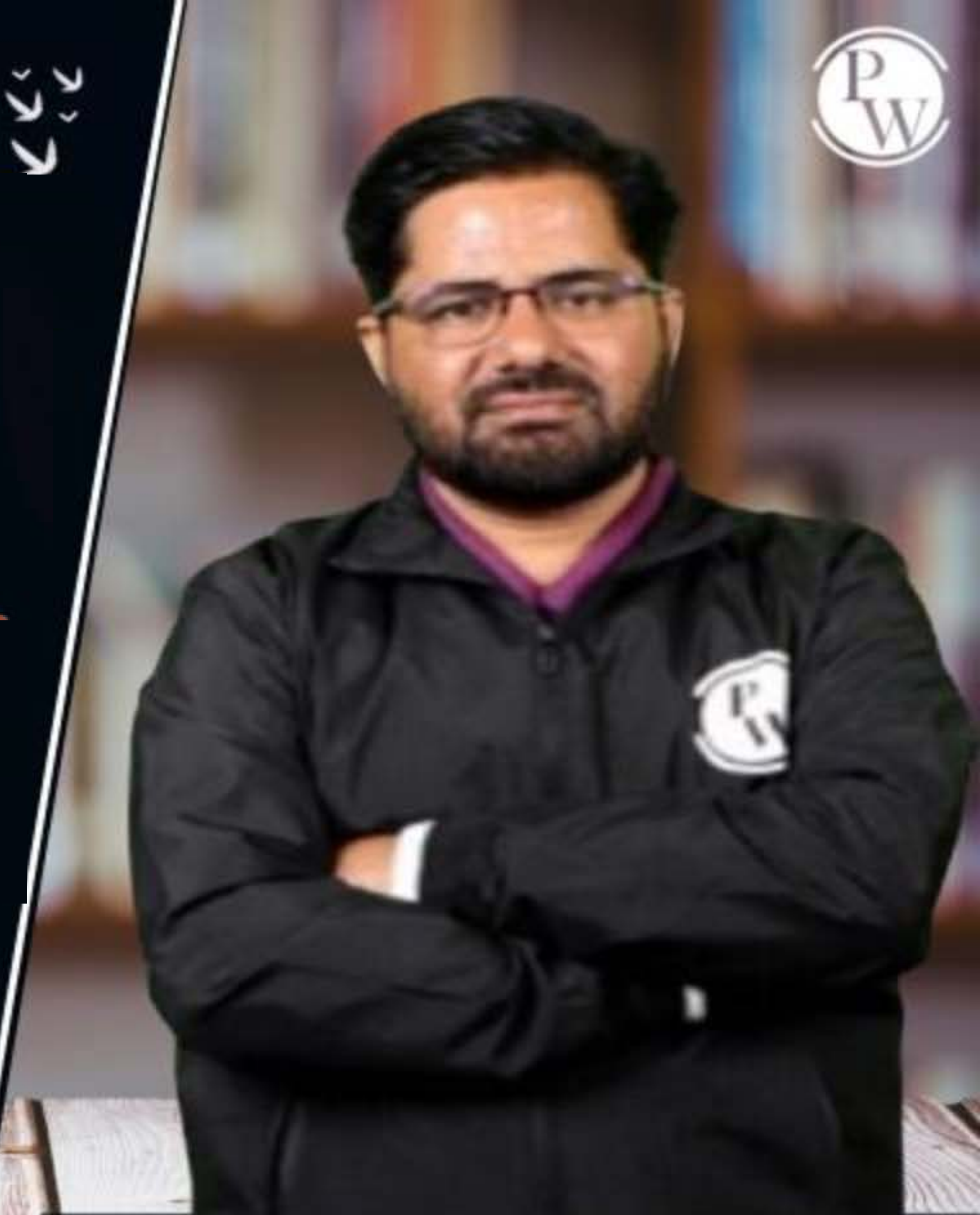


By- Dr. Puneet Sharma Sir

Computer Science & DA

Calculus and Optimization

Weekly Test- 02 Discussion Notes



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[MCQ]

#Q. The least value of the function $\boxed{f(x) = 2\cos x + x}$ in the closed interval is $\left[0, \frac{\pi}{2}\right]$

$$\boxed{f'(x) = -2\sin x + 1}$$

$$f''(x) = -2\cos x$$

T. Points are $f'(x) = 0$

$-2\sin x + 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$

A) 2

B) $\frac{\pi}{6} + \sqrt{3}$

C) $\frac{\pi}{2}$

D) None of these

+ve | -ve

0 (min) | $\pi/6$ (max) | $\pi/2$ (min)

$f(0) = 2$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} = 1.57$

[MCQ]

#Q. Find the interval in which of the following function is decreasing f(x) = 10 – 6x – 2x²

$$f(x) = 10 - 6x - 2x^2$$

$$f'(x) = -6 - 4x$$

$\Downarrow$

$$f'(x) < 0$$

$$-(6+4x) < 0$$

$$6 + 4x > 0$$

$$x > \frac{-3}{2}$$

$$\left(-\frac{3}{2}, \infty\right)$$

A $(0,1)$

B $\left(-\frac{3}{2},\infty\right)$ ✓

C $(1,\infty)$

D $\left(-\frac{3}{2},\frac{3}{2}\right)$

[MCQ]

#Q. Let $f(x)=\int_{\sin x}^{\cos x} e^{-t^2}dt$, Then $f'(\pi/4)$ equals

$$f'(x)=\frac{d}{dx}\int_{\sin x}^{\cos x} e^{-t^2}dt=\frac{d}{dx}(\cos x)e^{-\cos^2 x}-\frac{d}{dx}(\sin x)e^{-\sin^2 x}$$

A $\sqrt{\frac{1}{e}}$

B $-\sqrt{\frac{2}{e}}$

C $\sqrt{\frac{2}{e}}$

D $-\sqrt{\frac{1}{e}}$

$$=-\sin x\cdot e^{-\cos^2 x}-\cos x\, e^{-\sin^2 x}$$

$$f'\left(\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}e^{-\frac{1}{2}}-\frac{1}{\sqrt{2}}e^{-\frac{1}{2}}$$

$$=-e^{-\frac{1}{2}}\left[\frac{2}{\sqrt{2}}\right]=\frac{-\sqrt{2}}{\sqrt{e}}$$

[MCQ]

#Q. Let a be non-zero real number. Then $\lim_{x \to a} \frac{1}{x^2 - a^2} \int_a^x \sin\left(t^2\right) dt$ equals

A $\frac{1}{2a} \sin\left(a^2\right)$

B $\frac{1}{2a} \cos\left(a^2\right)$

C $-\frac{1}{2a} \sin\left(a^2\right)$

D $-\frac{1}{2a} \cos\left(a^2\right)$

$$\lim_{x\to a} \frac{\int_a^x \sin t^2\, dt}{x^2 - a^2} \simeq \frac{0}{0} = \lim_{x\to a} \frac{\frac{d}{dx}\int_a^x \sin t^2\, dt}{\frac{d}{dx}(x^2 - a^2)}$$

$$= \lim_{x\to a} \frac{\frac{d}{dx}(x)\sin(x)^2 - \frac{d}{dx}(0)\sin(0)^2}{2x} = \lim_{x\to a} \frac{\sin(x^2)}{2x}$$

$$= \frac{\sin(a^2)}{2a}$$

[MCQ]

#Q. If $I=\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$ then —— (1) $=\int_{\pi}^{-\pi}\frac{\cos^2(-t)}{1+a^{-t}}(-dt)=\int_{-\pi}^{\pi}\frac{\cos^2 t}{1+\frac{1}{a^t}}dt$

Put $x=-t$, $dx=-dt$

At $x=-\pi$, $t=\pi$

at $x=\pi$, $t=-\pi$

$=\int_{-\pi}^{\pi}\left(\frac{a^t\cos^2 t}{1+a^t}\right)dt$

$=\int_{-\pi}^{\pi}\left(\frac{a^x\cos^2 x}{1+a^x}\right)dx$ —— (2)

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

$2I=\int_{-\pi}^{\pi}\frac{(1+a^x)\cos^2 x}{1+a^x}dx=\int_{-\pi}^{\pi}\cos^2 x\,dx=2\int_{0}^{\pi}\cos^2 x\,dx$

$$2I = \int_0^{\pi} (1+\cos 2x)\,dx = \left(x + \frac{\sin 2x}{2}\right)_0^{\pi} = (\pi + 0) - (0+0)$$

$$2I = \pi$$

$$I = \frac{\pi}{2}$$

[MCQ]

#Q. Definite integration of $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is- ...(1) $= \int_2^3 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}dx$ ...(2)

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

A $\frac{1}{3}$

B $\frac{1}{4}$

C $\frac{1}{2}$ ✓

D 0

$I + I = \int_2^3 (1)dx$

$2I = (3-2)$

$I = \frac{1}{2}$

[NAT]

#Q. The minimum value $\boxed{u = xy + \frac{a^3}{x} + \frac{a^3}{y}}$ is ka^2, where k is $\underline{\quad 3 \quad}$.

$$u_x = (1)y - \frac{a^3}{x^2} + 0$$

$$u_{xx} = 0 + \frac{2a^3}{x^3}$$

$$u_y = x(1) + 0 - \frac{a^3}{y^2}$$

$$u_{yy} = 0 + \frac{2a^3}{y^3}$$

$$u_{xy} = \frac{\partial}{\partial x}(u_y)$$

$$= \frac{\partial}{\partial x}\left(x - \frac{a^3}{y^2}\right) = 1$$

T. Points are $u_x = 0$ & $u_y = 0$

$y = \frac{a^3}{x^2}$ & $x = \frac{a^3}{y^2}$

if we take $x = a = y$

So T. Point is $P(a, a)$

$r = 2$, $s = 1$, $t = 2$

$rt - s^2 = 3 > 0$ & $r > 0$

So $P(a, a)$ is the point of Minima & min Value $U_{min} = K a^2$

$$\left(xy + \frac{a^3}{x} + \frac{a^3}{y}\right)_{P(a,a)} = K a^2$$

$$3a^2 = K a^2 \Rightarrow K = 3$$

[MCQ]

#Q. Four small square of side x are cut out of a square of side 12 cm to make a tray by folding the edges. What is the value of x so that the tray has the maximum volume?

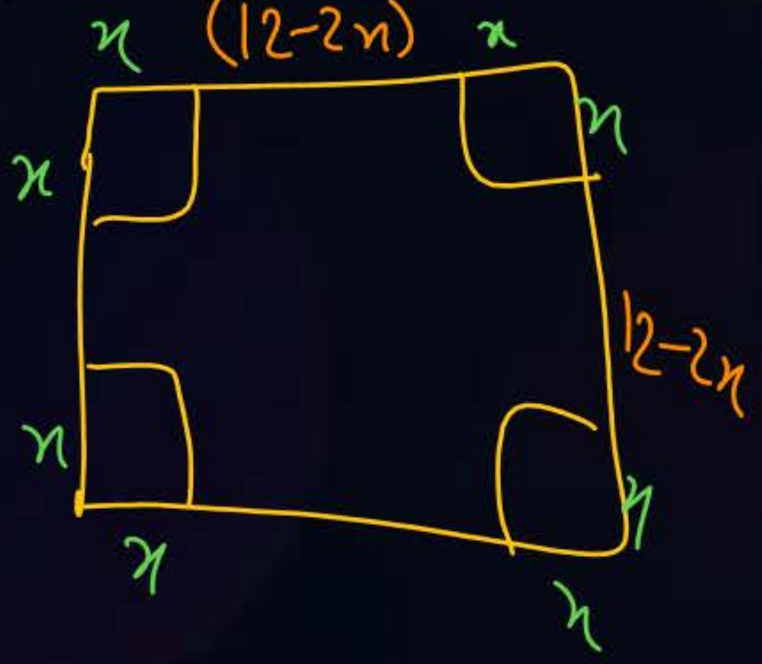
$L = 12 - 2x$
$W = 12 - 2x$
$H = x$

A 1

B 2

C 3

D 4



$\text{Volume} = (12-2x)^2 \cdot x$

$V = (144 + 4x^2 - 48x)x$

$V = 4x^3 - 48x^2 + 144x$

for Max Volume $\frac{dV}{dx} = 0$

$12x^2 - 96x + 144 = 0$

$x = 2$ & 6 ✗

[MCQ]

#Q. Find $\frac{\partial z}{\partial x}$ for the following function.

$$x^2\sin(y^3)+xe^{3z}-\cos(z^2)=3y-6z+8 \Rightarrow f(x,y,z)=c \text{ or } z=f(x,y)$$

A $\dfrac{2x\sin(y^3)+e^{3z}}{-6-3xe^{3z}-2z\sin(z^2)}$

B $\dfrac{\sin(y^3)+e^{3x}}{-6-3xe^{3z}-2z\sin(z^2)}$

C $\dfrac{e^{3x}}{-6-3xe^{3z}-2z\sin(z^2)}$

D None of them

$$(2x).\sin y^3+\frac{\partial}{\partial x}(xe^{3z})-\frac{\partial}{\partial x}\cos(z^2)=0-6\frac{\partial z}{\partial x}+0$$

$$2x\sin y^3 + 3x\,\underbrace{e^{3z}\cdot\frac{\partial z}{\partial x} + e^{3z}(1)} + \sin(z^2)\cdot 2z\cdot\frac{\partial z}{\partial x} = -6\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x}\left(3xe^{3z} + 2z\sin z^2 + 6\right) = -2x\sin y^3 - e^{3z}$$

$$\frac{\partial z}{\partial x} = \frac{-\left(2x\sin y^3 + e^{3z}\right)}{3xe^{3z} + 2z\sin z^2 + 6}$$

[MCQ]

#Q. If $u = \tan^{-1}(x+y)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

$V = \tan u = \boxed{x+y}$ & $V(\lambda x, \lambda y) = \lambda^1 (x+y)$ is V is H. funⁿ of degree $n=1$

A $\sin 2u$

B $\frac{1}{3}\sin 2u$

C $\frac{1}{2}\sin 2u = \sin u \cos u$

D None of them

E.T. for 'V'; $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = nV$

$x\frac{\partial}{\partial x}(\tan u) + y\frac{\partial}{\partial y}(\tan u) = 1 \cdot \tan u$

$x \sec^2 u \frac{\partial u}{\partial x} + y \sec^2 u \frac{\partial u}{\partial y} = \tan u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin u}{\cos u \times \sec^2 u}$

[MCQ]

#Q. If $u(x,y)=\frac{x^2+y^2}{\sqrt{x+y}}$, then value of $x\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial y}$ $= nu = \frac{3}{2}u$

$u(\lambda x, \lambda y) = \frac{\lambda^2}{\lambda}\left[\frac{x^2+y^2}{\sqrt{x+y}}\right] = \lambda^{\frac{3}{2}} u(x,y)$ so u is H. fun^n of degree $n=\frac{3}{2}$

A 3u/4

B 3u/2

C 3u/8

D 3u/9

[MCQ]

#Q. Find the 1st order partial derivatives of the following function wrt to s.

$$g(s,t,v) = t^2 \ln(s+2t) - \ln(3v)(s^3 + t^2 - 4v)$$

$$\frac{\partial g}{\partial s} = t^2 \cdot \left(\frac{1}{s+2t}\right)(1+0) - \ln(3v)\left[3s^2 + 0 - 0\right]$$

A) $\frac{\partial g}{\partial s} = \frac{t^2}{s+2t} - 3s^2 \ln(3v)$

B) $\frac{\partial g}{\partial s} = \frac{t^2}{s+2t} - 3s \ln(3v)$

C) $\frac{\partial g}{\partial s} = \frac{t^2}{s+2t} - 3s^2$

D) none of them

[MCQ]

#Q. Find the length of the curve- $y = \frac{x^5}{6} + \frac{1}{10x^3}$ between $1 \le x \le 2$?

$$\frac{dy}{dx} = \frac{5x^4}{6} - \frac{3}{10x^4}$$

A 1264/240

B 1263/240

C 1262/240

D 1261/240

$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{5x^4}{6} - \frac{3}{10x^4}\right)^2 = \left(\frac{5x^4}{6}\right)^2 + \left(\frac{3}{10x^4}\right)^2 - 2\left(\frac{5x^4}{6}\right)\left(\frac{3}{10x^4}\right) = (\quad)^2 + (\quad)^2 - \frac{1}{2}$$

$$\text{So } 1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{5x^4}{6}\right)^2 + \left(\frac{3}{10x^4}\right)^2 + 1 = \left[\frac{5x^4}{6} + \frac{3}{10x^4}\right]^2$$

Req length $s = \int_{x=1}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_1^2 \left(\frac{5}{6}x^4 + \frac{3}{10x^4}\right) dx$

$$= \left[\frac{x^5}{6} + \frac{3}{10}\left(\frac{-1}{3x^3}\right)\right]_1^2 = \left(\frac{x^5}{6} - \frac{1}{10x^3}\right)_1^2$$

$$= \left(\frac{32}{6} - \frac{1}{80}\right) - \left(\frac{1}{6} - \frac{1}{10}\right) = \frac{1261}{240}$$

(d)

THANK - YOU