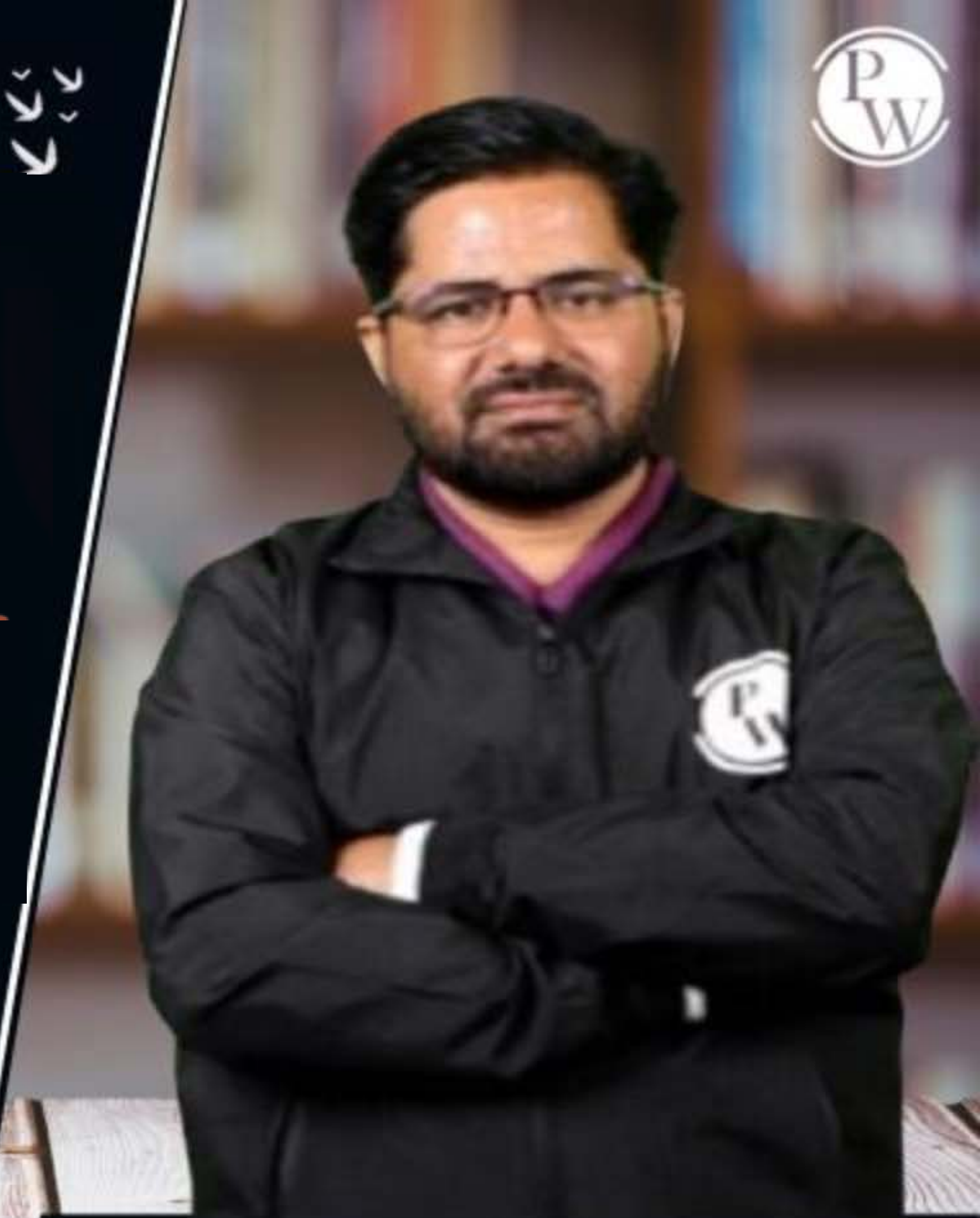
Computer Science & DA

Calculus and Optimization

Weekly Test- 01 **Discussion Notes**



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[NAT]

[1-Marks]

#Q. $\lim_{x \to \infty} \left(\frac{x + \sin x}{x} \right)$ equal to

$$= \lim_{x \to \infty} \frac{\cancel{x}\left(1 + \frac{\sin x}{x}\right)}{\cancel{x}}$$

$$= 1 + \lim_{x \to \infty} \left(\frac{\sin x}{x} \right) = 1 + 0 = 1.$$

Note $\lim_{x \to \infty} \left(\frac{\sin x}{x} \right) = \frac{\sin \infty}{\infty} = \frac{\text{Any No b/n } -1 \,\&\, 1}{\infty} = 0$

[MCQ] [1-Marks]

#Q. Find the domain of function $f(x) = \frac{\sqrt{x^2+2x}}{(x-1)(x-2)}$ $\Rightarrow x \neq 1 \,\&\, 2$

$x(x+2) \geq 0$

$\Rightarrow x \leq -2$ or $x \geq 0$

① $(x-a)(x-b) \leq 0 \Rightarrow a \leq x \leq b$

② $(x-a)(x-b) \geq 0 \Rightarrow x \leq a$ or $x \geq b$

$\boxed{a<b}$

A $\{(-\infty,-2] \cup [0,\infty)\} - \{1,2\}$

B $(-\infty,-2) \cup (0,\infty)$

C $(0,\infty)$

D $(-\infty,-2)$

$D_f = \left\{(-\infty,-2] \cup [0,\infty)\right\} - \{1,2\}$

[MCQ] [1-Marks]

#Q. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x^2-x}\right) = \lim_{x\to 0}\left(\frac{\tan x}{x}\right)\times\left(\frac{1}{x-1}\right) = 1\times\frac{1}{0-1} = -1$

A -1

B 2

C 3

D 4

[MCQ] [1-Marks]

#Q.

The value a and b so that the function:-

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} < x \le \pi \end{cases}$$

is continuous $x \in [0\ \pi]$ is: -

A $a = 0, b = 1$

B $a = \frac{\pi}{6}, b = -\frac{\pi}{12}$

C $a = 2, b = 4$

D $a = 2, b = 1$

At $x = \frac{\pi}{4}$

$LHL = RHL = f\left(\frac{\pi}{4}\right)$

$\frac{\pi}{4} + a(1) = 2 \times \frac{\pi}{4}(1) + b$

$\boxed{a - b = \frac{\pi}{4}}$ —— (1)

At $x = \frac{\pi}{2}$, $LHL = RHL$

$2\left(\frac{\pi}{2}\right) \cdot 0 + b = a(-1) - b$

$\boxed{a = -2b}$ —— (2)

[MCQ] [1-Marks]

#Q. If $\boxed{f(x + y) = f(x).f(y)}$ for all x and y and f(5) = 2, f'(0) = 3, then find f'(5)

$\Rightarrow f(5+0) = 2$

$f(5).f(0) = 2 \Rightarrow f(5).f(0) = 2\times 1$

$\Rightarrow \boxed{f(0) = 1}$

A 6

B 1

C 2

D 4

$$f'(5) = \lim_{x\to 5}\left(\frac{f(x)-f(5)}{x-5}\right) = \lim_{h\to 0}\left[\frac{f(5+h)-f(5)}{(5+h)-5}\right] = \lim_{h\to 0}\left[\frac{f(5).f(h)-f(5)}{h}\right]$$

Put $x = 5+h$

$$f'(5)=\lim_{h\to 0} f(5)\left[\frac{f(h)-1}{h}\right]=2\lim_{h\to 0}\left[\frac{f(h)-f(0)}{h-0}\right]=2\times f'(0)=2\times 3=6$$

[MCQ]

[1-Marks]

#Q. The value of ε in the MVT of $\boxed{f(b) - f(a) = (b - a) f'(\varepsilon)}$ for the function $f(x) = Ax^2 + Bx + C$ in (a, b) is

A $b + a$

B $b - a$

C $\frac{b + a}{2}$

D $\frac{b - a}{2}$

$$f(x) = Ax^2 + Bx + C \quad \begin{cases} f(a) = Aa^2 + Ba + C \\ f(b) = Ab^2 + Bb + C \end{cases}$$

$$f'(c) = 2Ac + B$$ So By LMVT

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{A(b^2 - a^2) + B(b - a)}{b - a} = A(b + a) + B$$

$$2Ac + B = A(b + a) + B \Rightarrow 2Ac = A(b + a) \Rightarrow c = \frac{a + b}{2}$$

$$\Rightarrow c = \frac{a + b}{2}$$

[MCQ] [1-Marks]

#Q. The Taylor's series for the function $x^4 + x - 2$ centered at $a = 1$

A. $5(x-1) + 6(x-1)^2 + 4(x-1)^3 + (x-1)^4$

B. $4(x-1) + 5(x-1)^2 + 6(x-1)^3 + (x-1)^4$

C. $3(x-1) + 4(x-1)^2 + 5(x-1)^3 + (x-1)^4$

D. None of these

$f(x) = x^4 + x - 2$

$f'(x) = 4x^3 + 1$

$f''(x) = 12x^2$

$f'''(x) = 24x$

$f^{iv}(x) = 24$

$\Rightarrow$

$f(1) = 0$

$f'(1) = 5$

$f''(1) = 12$

$f'''(1) = 24$

$f^{iv}(1) = 24$

$f(x) = f(1) + (x-1)f'(1) + \frac{(x-1)^2}{2!}f''(1) + \frac{(x-1)^3}{3!}f'''(1) + \frac{(x-1)^4}{4!}f^{iv}(1) + 0 + 0$

$= 0 + (x-1)5 + \frac{(x-1)^2}{2}(6) +$

[MCQ] [1-Marks]

#Q. Find the Maclaurin's series for $\ell n\,(1+x)$ and hence that for $\ell n\left(\frac{1+x}{1-x}\right)$

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$

A $2\sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}, x - \frac{x^2}{2} + \frac{x^3}{3} \cdots$

B $2\sum_{n=1}^{\infty} \frac{x^{n-1}}{n^{-1}}, x + \frac{x^2}{2} - \frac{x^3}{3}.$

C $2\sum_{n=1}^{\infty} \frac{x^{3n-1}}{3^{n-1}}, x + \frac{x^2}{2} + \frac{x^3}{3}$

D None of these

$\log\left(\frac{1+x}{1-x}\right) = \log(1+x) - \log(1-x) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} \cdots\right]$

[MCQ] [1-Marks]

#Q. The value of $\lim_{x\to\infty}\left[\sqrt{x^2+1}-x\right]$ is:

$(\infty-\infty)$

$$= \lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right)$$

$$= \lim_{x\to\infty}\left(\frac{(x^2+1)-x^2}{\sqrt{x^2+1}+x}\right) = \lim_{x\to\infty}\frac{1}{\sqrt{x^2+1}+x}$$

$$= \frac{1}{\infty+\infty}$$

$$= \frac{1}{\infty}$$

$$= 0$$

A. ∞

B. 1

C. 0

D. None of these

[MCQ] [1-Marks]

#Q. If $f(x)=\begin{cases}[x]+[-x] & x\neq 2\\ K, & x=2\end{cases}$ then f(x) is continuous at x = 2, provided K is equal to:

$LHL\ at\ (x=2) = f(2^-) = f(1.9) = [1.9]+[-1.9] = 1+(-2) = -1$

A 2

B 1

C –1

D 0

$RHL: (at\ x=2) = f(2^+) = f(2.01) = [2.01]+[-2.01] = 2+(-3) = -1$

So for Cont $K = $ App Value $= -1$

[MCQ] [1-Marks]

#Q. $f(x)=\begin{cases}\dfrac{\sqrt{1+cx}-\sqrt{1-cx}}{x} & \text{for } -2\le x<0\\ \dfrac{x+3}{x+1} & 0\le x<2\end{cases}$ is continuous on [−2,2], then c =

$RHL\ at\ (x=0)=\dfrac{0+3}{0+1}=3$

A 3

B 3/2

C $\dfrac{3}{\sqrt{2}}$

D $\dfrac{2}{\sqrt{3}}$

$LHL\ at\ (x=0)=\lim_{x\to 0}\left(\dfrac{\sqrt{1+cx}-\sqrt{1-cx}}{x}\right)\times\left(\dfrac{\sqrt{1+cx}+\sqrt{1-cx}}{\sqrt{1+cx}+\sqrt{1-cx}}\right)=\dfrac{2cx}{x\{\sqrt{1+cx}+\sqrt{1-cx}\}}$

$=\dfrac{2c}{\sqrt{1}+\sqrt{1}}=c$

$\because LHL=RHL$

$c=3$

[MCQ] [1-Marks]

#Q. If $f(x) = \begin{cases} x^p \cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then at x = 0, f (x) is:

A. Continuous if p>0 and differentiable if p>1

B. Continuous if p >1 and differentiable if p>2

C. Continuous and differentiable if p>0

D. None of these

$\lim_{x\to 0} f(x) = 0^p \times \cos(\infty)$

$= 0^p \times (\text{Any No b/w} -1 \,\&\, 1)$

$= 0^{p-1} \times 0 \times (\text{Any No b/w} -1 \,\&\, 1)$

$= 0^{p-1} \times [0] = 0^p$

$= \begin{cases} \infty \text{ (ND)}, & p < 0 \\ 0^0 \text{ (IND)}, & p = 0 \\ 0 \text{ (exist)}, & p > 0 \end{cases}$

$$f'(0) = \lim_{x\to 0}\left(\frac{f(x)-f(0)}{x-0}\right) = \lim_{x\to 0}\left(\frac{x^p\cos\frac{1}{x} - 0}{x-0}\right) = \lim_{x\to 0} x^{p-1}\cos\left(\frac{1}{x}\right)$$

this limit will exist if $p-1>0 \Rightarrow p>1$

i.e. $f(x)$ is diff for $p>1$

[MCQ]

[1-Marks]

#Q. Determine the number c which satisfy the conclusion of Rolle's theorem for $f(x) = x^2 - 2x - 8$ on $[-1, 3]$

$f(-1) = 1 - 2(-1) - 8 = -5$

& $f(3) = 9 - 2\times3 - 8 = -5$

So By R.Th ∃ c for which

$f'(c) = 0$

$2c - 2 = 0$

$c = 1$

A 1

B 2

C 3

D 4

THANK - YOU