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WEEKLY TEST-01

DS AND AI

CALCULUS AND OPTIMIZATION

Q1 $\lim\limits_{x \to \infty}\left(\frac{x+\sin x}{x}\right)$ equal to

Q2

Find the domain of funtion

$f(x) = \frac{\sqrt{x^2+2x}}{(x-1)(x-2)}$

(A) $\{(-\infty, -2) \cup (0, \infty)\} - \{1, 2\}$

(B) $(-\infty, -2) \cup (0, \infty)$

(C) $(0, \infty)$

(D) $(-\infty, -2)$

Q3

Evaluate $\lim\limits_{x \to 0}\left(\frac{\tan x}{x^2-x}\right)$

(A) –1 (B) 3

(C) 2 (D) 4

Q4

The value a and b so that the function:-

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x & 0 \le x < \frac{\pi}{4} \\ 2x\cot x + b & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ a\cos 2x - b\sin x & \frac{\pi}{2} < x \le \pi \end{cases}$$

is continuous x∈[0 π] is: -

(A) a = 0, b = 1

(B) $a = \frac{\pi}{6}, b = -\frac{\pi}{12}$

(C) a = 2, b = 4

(D) a = 2, b = 1

Q5

If f(x + y) = f(x).f(y) for all x and y and f(5) = 2, f'(0) = 3, then find f'(5)

(A) 6 (B) 1

(C) 2 (D) 4

Q6

The value of ε in the MVT of f(b) – f(a) = (b – a) f'(ε) for the function f(x) = Ax^2 + Bx + C in (a, b) is

(A) b + a (B) b – a

(C) $\frac{b+a}{2}$ (D) $\frac{b-a}{2}$

Q7 The taylor's series for the function $x^4 + x - 2$ centered at a = 1

(A) $5(x-1) + 6(x-1)^2 + 4(x-1)^3 + (x-1)^4$

(B) $4(x-1) + 5(x-1)^2 + 6(x-1)^3 + (x-1)^4$

(C) $3(x-1) + 4(x-1)^2 + 5(x-1)^3 + (x-1)^4$

(D) None of these

Q8 Find the Maclaurin series for ℓn (1 + x) and hence that for $\ell n\left(\frac{1+x}{1-x}\right)$

(A) $2\sum_{n=1}^{\infty}\frac{x^{2n-1}}{2n-1}, x - \frac{x^2}{2} + \frac{x^3}{3}$

(B) $2\sum_{n=1}^{\infty}\frac{x^{n-1}}{n^{-1}}, x + \frac{x^2}{2} - \frac{x^3}{3}$

(C) $2\sum_{n=1}^{\infty}\frac{x^{3n-1}}{3^{n-1}}, x + \frac{x^2}{2} + \frac{x^3}{3}$

(D) None of these

Q9 The value of $\lim\limits_{x \to \infty}\left[\sqrt{x^2+1} - x\right]$ is :

(A) ∞ (B) 1

(C) 0 (D) None of these.

Q10 If $f(x) = \begin{cases} [x] + [-x], & x \ne 2 \\ K, & x = 2 \end{cases}$ then f(x) is continuous at x = 2, provided K is equal to :

(A) 2 (B) 1

(C) -1 (D) 0

Q11 $f(x) = \left\{ \begin{array}{ll} \frac{\sqrt{1+cx}-\sqrt{1-cx}}{x} & \text{for } -2 \le x < 0 \\ \frac{x+3}{x+1} & 0 \le x < 2 \end{array} \right\}$

is continous on $[-2,\ 2]$, then c =

(A) 3 (B) $\frac{3}{2}$

(C) $\frac{3}{\sqrt{2}}$ (D) $\frac{2}{\sqrt{3}}$

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Q12 If $f(x) = \begin{cases} x^p \cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then at x = 0, f (x) is :

(A) continuous if p > 0 and differentiable if p > 1

(B) continuous if p > 1 and differentiable if p > 2

(C) continuous and differentiable if p > 0

(D) none of these

Q13 Determine the number c which satisfy the conclusion of Rolle's Theorem for

$f(x) = x^2 - 2x - 8 \text{ on } [-1, 3]$

(A) 1 (B) 2

(C) 3 (D) 4

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Answer Key

Q1	1
Q2	(A)
Q3	(A)
Q4	(B)
Q5	(A)
Q6	(C)
Q7	(A)
Q8	(A)
Q9	(C)
Q10	(C)
Q11	(A)
Q12	(A)
Q13	(A)

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Hints & Solutions

Q1 Text Solution:

$\lim\limits_{x\to\infty}\left(\frac{x+\sin x}{x}\right)$

$\lim\limits_{x\to\infty}\left(1+\frac{\sin x}{x}\right)$

$\lim\limits_{x\to\infty}\frac{\sin x}{x}=0$

Thus the answer will be 1.

Q2 Text Solution:

Case 1 $\sqrt{x^2+2x}$ the domain must be always +ve, thus

$x^2 + 2x \geq 0$

$x(x + 2) \geq 0$

$x \in (-\infty,-2)\cup(0,\infty)$

Now $(x - 1)(x - 2) \neq 0$

$x \neq 1, x \neq 2$

Thus, $x \in \{(-\infty,-2)\cup(0,\infty)\}-\{1,2\}$

Q3 Text Solution:

$\lim\limits_{x\to 0}\frac{\tan x}{x^2-x}$

$\lim\limits_{x\to 0}\left\{\frac{\tan x}{x(x-1)}\right\}$

$\lim\limits_{x\to 0}\frac{\tan x}{x}\times\lim\limits_{x\to 0}\frac{1}{(x-1)}$

Now, $1\times\lim\limits_{x\to 0}\frac{1}{(x-1)}$

$1\times\frac{+1}{-1}=-1$

Q4 Text Solution:

$\lim\limits_{x\to\left(\frac{\pi}{4}\right)^-}=\lim\limits_{h\to 0}\left(\frac{\pi}{4}-h\right)+a\sqrt{2}\left(\sin\left(\frac{\pi}{4}-h\right)\right)$

$=\frac{\pi}{4}+a\sqrt{2}.\frac{1}{\sqrt{2}}=\frac{\pi}{4}+a$

$\lim\limits_{x\to\left(\frac{\pi}{4}\right)^+}=\lim\limits_{h\to 0}2\left(\frac{\pi}{4}+h\right)\cot\left(\frac{\pi}{4}+h\right)+b$

$=2\times\frac{\pi}{4}\times 1+b$

$=\frac{\pi}{2}+b$

$\lim\limits_{x\to\left(\frac{\pi}{2}\right)^+}=\lim\limits_{h\to 0}\left\{a\cos 2\left(\frac{\pi}{2}+h\right)\right\}$

$-b\sin\left(\frac{\pi}{2}+h\right)$

$= a \times (-1) - b \times 1$

$= -a - b$

$f\left(\frac{\pi}{2}\right)=2\times\frac{\pi}{2}\times\cos\frac{\pi}{2}+b = b$

Now, $f\left(\frac{\pi^+}{2}\right)=f\left(\frac{\pi}{2}\right)$

$= -a - b = b$

$= -a = +2b$

and $\frac{\pi}{4}+a=\frac{\pi}{2}+b$ as

$f\left(\frac{\pi}{4}\right)=f\left(\frac{\pi^+}{4}\right)$

$\frac{\pi}{4}+a=\frac{\pi}{2}+b$

$\frac{\pi}{4}-2b=\frac{\pi}{2}+b$

$3b=\frac{\pi}{4}-\frac{\pi}{2}=\frac{2\pi-4\pi}{8}$

$3b=\frac{-\pi}{12}$

$a = -2b$

$=\frac{\pi}{6}$

Thus (b) is correct option

Q5 Text Solution:

Let $x = 5, y = 0$

$f(5 + 0) = f(5).f(0) = f(5) = f(5).f(0)$

$f(0) = 1$

Now as $f(5) = 2$

$f'(a)=\lim\limits_{x\to 0}\left(\frac{f(a+x)-f(a)}{x}\right)$

$f'(5)=\lim\limits_{h\to 0}\left(\frac{f(5+h)-f(5)}{h}\right)=2.\lim\limits_{h\to 0}\left[\frac{f(h)-f(0)}{h}\right]$

$= 2 \times f'(0) = 2 \times 3 = 6.$

Q6 Text Solution:

$f(b) = Ab^2 + Bb + c$

$f(a) = Aa^2 + Ba + c$

$f'(x) = 2Ax + B$

$f'(\varepsilon) = 2A\varepsilon + B$

$2A\varepsilon + B = \frac{Ab^2+Bb+c-Aa^2-Bb-c}{(b-a)}$

$2A\varepsilon + B = \frac{A(b^2-a^2)+B(b-a)}{(b-a)}$

$2A\varepsilon+\cancel{B}=A(b+a)+\cancel{B}$

$\varepsilon=\frac{(b+a)}{2}$

Q7 Text Solution:

$f(x) = x^4 + x - 2, f(1)=0$

$f'(x) = 4x^3 + 1, f'(1)=5$

$f''(x) = 12x^2$

$f''(1)=12$

$f'''(x) = 24x, f'''(1)=24$

$f''''(x) = 24, f''''(1)=24$

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and functions derivatives after order 4 all will be 0.

Thus;

$x^4 + x - 2 = 0 + (x-1) \times 5 + \frac{(x-1)^2}{2!} \times 12$

$+ \frac{(x-1)^3}{3!} \times 24 + \frac{(x-1)^4}{4!} \times 24$

5(x – 1) + 6 (x – 1)^2 + 4 (x – 1)^3 + (x – 1)^4

Thus option (a) is correct.

Q8 Text Solution:

The Maclaurein series for ℓn (1 + x) is

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.$

and so $\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3}$

Hence, $\ln\frac{(1+x)}{1-x}$

= $\ln(1+x) - \ln(1-x)$

= $2\left(x + \frac{x^3}{3} + \frac{x^5}{5} \ldots\right)$

$2\sum_{n=1}^{\infty} \frac{x^{2n-1}}{2n-1}$

Q9 Text Solution:

Its Given -

$\lim_{x\to\infty}\left[\sqrt{x^2+1} - x\right]$

Rationalising we get -

$\lim_{x\to\infty} \frac{\left[\sqrt{x^2+1}-x\right]\left[\sqrt{x^2+1}+x\right]}{\left[\sqrt{x^2+1}+x\right]}$

$\lim_{x\to\infty} \frac{\left[x^2+1-x^2\right]}{\left[\sqrt{x^2+1}+x\right]}$

$\lim_{x\to\infty} \frac{1}{\left[\sqrt{x^2+1}+x\right]}$

$= 1/\infty$

= 0.

Q10 Text Solution:

$$f(x) = \begin{cases} [x] + [-x], & x \neq 2 \\ K, & x = 2 \end{cases}$$

Since, f(x) is continuous at x = 2

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

Now,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} ([x] + [-x]) = 1 + (-2) = -1$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} ([x] + [-x]) = 2 - 3 = -1$

$\therefore K = -1.$

Q11 Text Solution:

Given, f is continuous on $[-2, 2]$

L.H.L = $\lim_{x\to 0} \frac{\sqrt{1+cx} - \sqrt{1-cx}}{x}$

= $\lim_{x\to 0} \frac{2cx}{x\left[\sqrt{1+cx}+\sqrt{1-cx}\right]} = c$

R.H.L = $\lim_{x\to 0} \frac{x+3}{x+1} = 3$

∴ c = 3.

Q12 Text Solution:

$$f(x) = \begin{cases} x^p \cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

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$$\lim_{x->0} f\left(x\right) = 0^p \times \cos\infty = 0^p$$

$$\times \left(number\ between\ -1\ and\ 1\right)$$

$$\begin{cases} 0, p > 0 \\ 0^0, p = 0 \\ \infty, p < 0 \end{cases}$$

$$For\ continuity\ at\ x = 0, f\left(0\right)$$

$$= \lim_{x->0} f\left(x\right)$$

$$that\ is\ \lim_{x->0} f\left(x\right) = 0 => p > 0$$

$$Now\ for\ differentiability -$$

$$f'\left(0\right) = \lim_{x->0} \frac{f(x)-f(0)}{x-0} = \lim_{x->0} \frac{x^p \cos\frac{1}{x} - 0}{x-0}$$

$$= \lim_{x->0} x^{p-1} \cos\frac{1}{x} = 0^{p-1} \times \cos\infty = 0^{p-1}$$

$$\times \left(any\ number\ between\ -1\ and\ 1\right)$$

$$\begin{cases} 0, p > 1 \\ 0^0, p = 1 \\ \infty, p < 1 \end{cases}$$

$$that\ is\ f'\left(x\right)\ exists\ only\ when\ p > 1,$$

$$hence\ ans\ is\ A$$

Q13 Text Solution:

The first thing we should do is actually verify that Rolle's Theorem can be used here.

The Function is a polynomial which is continuous and differentiable everywhere and so will be continuous on $[-1,\ 3]$ and differentiable on $(-1,\ 3)$.

Next, a couple of quick function evaluations shows that $f(-1) = f(3) = -5$

Therefore, the conditions for Rolle's Theorem are met and so we can actually do the problem.

Now that we know that Rolle's Theorem can be used there really isn't much to do. All we need to do is take the derivative,

$f'(x) = 2x - 2$

and then solve f' (c) = 0

2c − 2 = 0 ⇒ c = 1