Computer Science & DA

Probability and Statistics

Continuous Random variable

Lecture No. 01

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Recap of previous lecture

Topic Binomial Distribution & Poisson Distribution

Topics to be Covered

Topic Basics of Continuous R Variable.

Remaining theory of BINOMIAL & POISSON Dist →

BINOMIAL

① we are not sure about location of success) But sure about n & p

$X \sim B\{n, p\}$ & $\boxed{P(X = r\ \text{success}) = {}^nC_r\, p^r q^{n-r}}$

② $X = \{\text{Number of success}\}$

③ $\text{Mean}(X) = E(X) = \sum p_i x_i = \ldots\ldots = np$

④ $\text{Var}(X) = E(X^2) - E^2(X) = \ldots\ldots = npq$

⑤ $SD(\sigma) = +\sqrt{npq}$ eg In B.Dist $\boxed{\text{Mean} > \text{Variance}}$

⑥ Complete B.Dist is, $\sum p_i = \boxed{\sum_{r=0}^{n} {}^nC_r\, p^r q^{n-r} = (q+p)^n} = 1^n = 1$

POISSON

① Not sure about 'n' but sure for it's Average(λ)

$X \sim P\{\lambda\}$ & $\boxed{P(X = r\ \text{success}) = \dfrac{e^{-\lambda}\lambda^r}{r!}}$

② $X = \{\text{Number of success}\}$

③ $\text{Mean}(X) = E(X) = \sum p_i x_i = \ldots\ldots = \lambda$

④ $\text{Var}(X) = E(X^2) - E^2(X) = \ldots\ldots = \lambda$

⑤ $SD(\sigma) = +\sqrt{\lambda}$ eg In P.Dist, $\boxed{\text{Mean} = \text{Variance}}$

⑥ W.K. that $(x+a)^n = \sum_{r=0}^{n} {}^nC_r\, x^{n-r} \cdot a^r$ (Using Class 12^{th})

Similarly Complete B.Dist is $\sum p_i = \boxed{\sum_{r=0}^{n} {}^nC_r\, p^r q^{n-r} = (q+p)^n} = 1^n = 1$

⑦ if Q. is Based on B.Dist $(n \,\&\, p)$ $\Longrightarrow$ Using P.Dist (λ) (But Take Care it is Valid only when $n \to$ very large & $p \to$ very small)

$\not\Longleftarrow$

⑧ In P.Dist; $\lambda \to$ Average of X per unit data.

Q. If Mean & Variance of B.Dist are 4 & 12 resp then find Complete B.D

$\because$ Mean = 4 & Var = 12 so it is senseless Question

Q. If .. .. 12 & 4 resp .. ..

Sol: ATQ,

$\underbrace{np = 12, \; npq = 4}$, $q+p=1$

$q = \frac{1}{3}$, $p = \frac{2}{3}$, $n = 18$

So Complete B.Dist is; $= (q+p)^n$

$= \left(\frac{1}{3} + \frac{2}{3}\right)^{18}$

(HW) out of 1000 families with 5 children each how many would you expect having
(a) exactly 2 Boys, (b) either 2 or 3 Boys
(c) All Girls.

Sol: $N = 1000$ families ; for single family :

$X = \{\text{Number of } \underbrace{\text{Boys}}_{}\}$ → success

$n = 5,\ p = P(B) = \frac{1}{2},\ q = \frac{1}{2}$

$$\boxed{P(X = r \text{ success}) = {}^nC_r\, p^r q^{n-r}}$$

(i) $P(X = 2B) = {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 = \frac{10}{32} = \frac{10/32}{1}$

i.e. out of 32 families, No of families having exactly 2B = 10

" " 1000 " " " " " "

$= \frac{10}{32} \times 1000 = 312.5 = 312$

(ii) $P(X = 2B \text{ or } 3B) = P(X = 2B) + P(X = 3B)$

$= {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{20}{32}$

So Req. No of families having 2 or 3 Boys $= \frac{20}{32} \times 1000 = 625$

(iii) $P(\text{all } G) = P(\text{No Boys}) = P(X = 0B) = {}^5C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

So Req. No. of families having all G $= \frac{1}{32} \times 1000 = 31$

Q: Consider a Company that produces Bolts, with prob of defective Bolt is 0.01. Company sells Bolts in the packets of 10 and offers a replacement Guarantee that "At most one of the 10 Bolts may be defective" then what proportion of packets sold must the Company Replace? (%)

Sol: Let Company sells N = 100 packets.

For single packet:

$X = \{\text{Number of Def. Bolts}\} \rightarrow$ success

$n = 10,\ p = P(\text{Def. Bolt}) = 0.01 = \frac{1}{100}$

$q = P(\text{Non Def Bolt}) = 0.99 = \frac{99}{100}$

$P(X = r \text{ success}) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[{}^{10}C_0\, p^0 q^{10} + {}^{10}C_1\, p^1 q^9\right] = 0.004 = \frac{4}{1000}$

it is the prob that this single packet will be replaced.

So % of packets to be replaced $= 0.004 \times 100 = 0.4\%$

& Proportion of " " " $= \frac{0.004}{1} = 0.004$ Ans.

Q:- wireless sets are manufactured with 25 soldier joints, out of 1 joint in 500 is defective. then find the number of w-sets to be free from defective joints in a consignment of 10000 sets.

Sol:- Let $N = 10000$ sets, for single w-set:

$X = \{\text{Number of Def. joints}\} \rightarrow$ success

$n = 25$ joints, $p = P(\text{Def joint}) = \frac{1}{500}$

M-I using B-Dist:-

$P(X = r \text{ success}) = {}^{n}C_r \, p^r q^{n-r}$

$P(X = 0 \text{ Def joint}) = {}^{25}C_0 \left(\frac{1}{500}\right)^0 \left(\frac{499}{500}\right)^{25}$

$= 0.95118 = \frac{0.95118}{1} = \frac{9511.8}{10000}$

So No. of W-sets to be free from D-joints $= 9512$

M-II Using POISSON DIST:-

Total joints in single w-set $= 25 \ (= n)$

Av Number of def joints in single w-set $(\lambda) = np = \frac{25}{500} = \frac{1}{20}$

$P(X = r \text{ success}) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$

$P(X = 0 \text{ Def joint}) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = e^{-\frac{1}{20}} = 0.9512$

So Req Ans $= 9512$ sets.

Analysis: → ① Number of w-sets to be free from def joints = 9512 w-sets.

② Total joints in 10000 sets = $25 \times 10000 = 250000$ joints

③ Total def joints " " " $= \frac{250000}{500} = 500$ def. joints

④ these 500 def. joints are Randomly distributed in $\frac{10000 - 9512}{}$ w-sets.
$= 488$

ⓐ 9152 ⓒ 10000

ⓑ 488 ⓓ None

Geometric Dist: $\rightarrow$ when we want to know

how many trials are required to get 1^{st} success then we will use this distribution.

$X = \{$Number of trials required to get 1^{st} success$\}$

& let $p = P(\text{success})$ & $q = P(\text{failure})$

& $q + p = 1$

then P. Dist is

X :	1	2	3	4	5	6 ......
P(x) :	p	qp	q^2p	q^3p	q^4p	q^5p ......

So $E(X) = \sum p_i x_i = p(1) + qp(2) + q^2p(3) + q^3p(4) + \ldots\ldots$

$E(X) = p\left[1 + 2q + 3q^2 + 4q^3 + \ldots\ldots\right]$

$= p\left[(1-q)^{-2}\right]$

$\left[1 + 2x + 3x^2 + 4x^3 + \ldots\ldots = (1-x)^{-2}\right]$

$= \dfrac{p}{(1-q)^2} = \dfrac{p}{p^2} = \dfrac{1}{p}$

Note: ① $E(X) = \dfrac{1}{p}$, ② $Var(X) = \dfrac{q}{p^2}$

③ $\boxed{P(X = r^{th}\text{ trial}) = q^{r-1} \cdot p}$

④ $X \sim G\{p\}$

Q. A Company produces on an Average 3 out of 60 defective Bulbs. what is the prob that 1^{st} defective Bulb will be found when 6^{th} one is tested?

Note: if we take Def. Bulb $\simeq$ success then we are sure about location of success & hence Can't apply 'B' as well as Poisson

Sol: $X = \{$Number of trials required to get 1^{st} Def$\}$

$$p = P(\text{Def Bulb}) = \frac{3}{60} = \frac{1}{20}, \quad q = \frac{19}{20}$$

$$P(X = 6^{th}) = q^5 p = \left(\frac{19}{20}\right)^5 \left(\frac{1}{20}\right)$$

Q. Suppose you are playing a Game of Dart and the probability of success is 0.4. what is the Prob. that you will hit the Bull's Eye on third try?

if we take Bull's Eye $\simeq$ Success then again location of Success is sure & Can't apply Binomial as well poisson.

Sol: $X = \{$No. of trials to get 1^{st} success$\}$

$$p = P(\text{Bull's Eye}) = 0.4 \ \& \ q = 0.6$$

$$P(X = 3) = q^2 p = (0.6)^2 (0.4)$$

Continuous Random Variable (x)

eg (Height, weight, Age, time)

Let x is C.R.V & $f(x)$ is it's p.d.f then we have following Results;

① $f(x) \simeq$ Prob at 'x'

② $P(-\infty < x < \infty) = 1$

or $\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$ (Assumption)

i.e Total area under $f(x) = 1$.

(*) $f(x) \geq 0$ & $\int_{-\infty}^{\infty} f(x)\,dx = 1$

③ $\boxed{P(a < x < b) = \int_a^b f(x)\,dx}$

= Area under $f(x)$ b/w a & b

④ Let $f(x)$ is p.d.f for x and $g(x)$ be another func^n of 'x' then

$$\boxed{E\{g(x)\} = \int_{-\infty}^{\infty} g(x).f(x)\,dx}$$

$E(x) = \int_{-\infty}^{\infty} x.f(x)\,dx$ = Mean(x)

$E(x^2) = \int_{-\infty}^{\infty} x^2.f(x)\,dx$ = 2nd Moment

$E(x^3) = \int_{-\infty}^{\infty} x^3 f(x)\,dx$ = 3rd Moment

....

⑤ $\boxed{Var(x) = E(x^2) - E^2(x)}$

⑥ SD $(\sigma) = +\sqrt{Var(x)}$

Q: if $f(x)=e^{-2|x|}$ is density func^n ($=$ p.d.f) of x then evaluate $P(|x|\le 1)=?$

Sol: $P(|x|\le 1)=P(-1\le x\le 1)=\int_{-1}^{1} f(x)\,dx$

$\because f(-x)=e^{-2|-x|}=e^{-2|x|}=f(x)$ is Even func^n

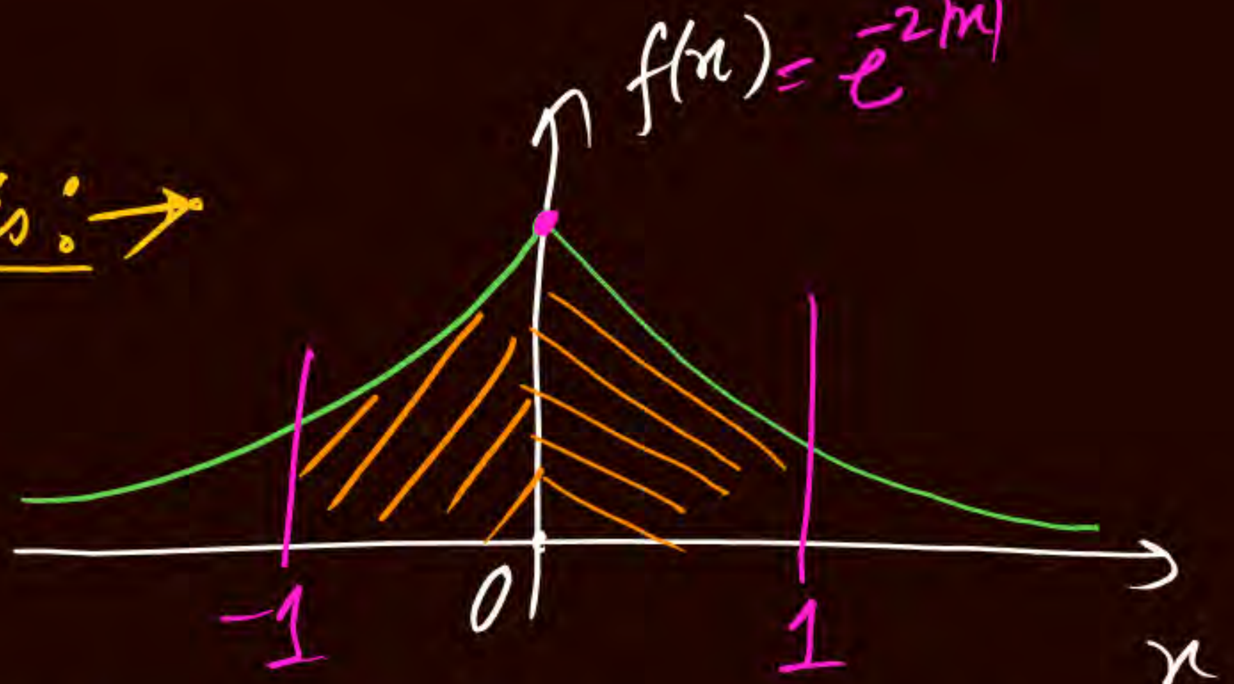
$$=2\int_0^1 f(x)\,dx=2\int_0^1 e^{-2|x|}\,dx$$

$$=2\int_0^1 e^{-2(+x)}\,dx=2\left(\frac{e^{-2x}}{-2}\right)_0^1$$

$$=-\left(e^{-2}-e^{0}\right)=1-\frac{1}{e^2}\approx 0.864$$

Analysis :→

(1)



$P(-1\le x\le 1)=0.864\simeq 86.4\%$

ie 86.4% area under $f(x)$ lies in b/n -1 & 1

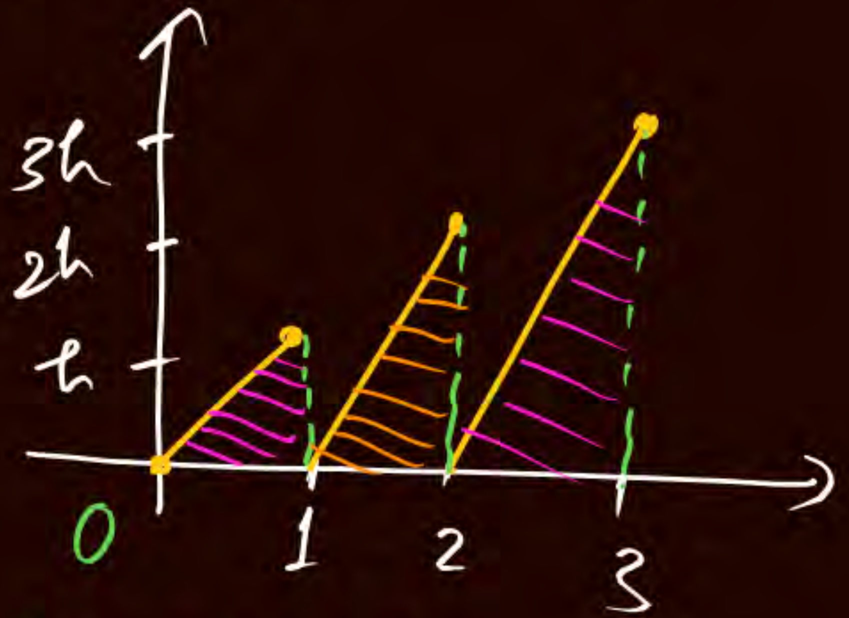
(2) Cross check :→

$$\int_{-\infty}^{\infty} f(x)\,dx=2\int_0^{\infty} e^{-2|x|}\,dx=2\int_0^{\infty} e^{-2x}\,dx=\dots\dots=1$$

(3) $|x|\le a \Rightarrow -a\le x\le a$

$|x|\ge a \Rightarrow x\le -a$ or $x\ge a$

Q: if $f(x)$ shown in the diagram is $\underbrace{\text{valid p.d.f}}$ for x then find $h = ?$



Sol: $\because f(x)$ is p.d.f so,

$\boxed{\text{Total area under } f(x) = 1}$

$$\frac{1}{2}\left[1\times h + 1\times 2h + 1\times 3h\right] = 1$$

$$h = \frac{1}{3}$$

Q: if $f(x) = Ke^{-\alpha|x|}$; $\alpha \in R^{+}$ is p.d.f for x then find $K = ?$

Sol: $\because f(x)$ is p.d.f so

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

$$\int_{-\infty}^{\infty} Ke^{-\alpha|x|}\cdot dx = 1$$

$$2K\int_{0}^{\infty} e^{-\alpha|x|}\cdot dx = 1$$

$$2K\int_{0}^{\infty} e^{-\alpha(+x)}dx = 1$$

$$2K\left[\frac{e^{-\alpha x}}{-\alpha}\right]_{0}^{\infty} = 1$$

$$-\frac{2K}{\alpha}\left[e^{-\infty} - e^{0}\right] = 1$$

$$-\frac{2K}{\alpha}(0-1) = 1$$

$$K = \frac{\alpha}{2} \quad \text{Ans}$$

2012 GATE If $f(x) = \begin{cases} 1+x, & -1 \le x \le 0 \\ 1-x, & 0 \le x \le 1 \end{cases}$ is p.d.f for C.R.V 'x' then find Mean, 2^{nd} Moment, Var & S.D

Sol: Here $f(x)$ is an Even func^n (H.W)

(i) Mean(x) = $\boxed{E(x) = \int_{-\infty}^{\infty} x f(x)\,dx}$

$= \int_{-\infty}^{-1} 0 + \int_{-1}^{1} x.f(x)dx + \int_{1}^{\infty} 0$

$= \int_{-1}^{1} \underbrace{x.f(x)}_{\text{odd func}^n}dx = 0$

(ii) 2^{nd} Moment $= \boxed{E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx} = \int_{-1}^{1} \underbrace{x^2 f(x)}_{\text{Even func}^n} dx$

$= 2\int_0^1 x^2 f(x)dx = 2\int_0^1 x^2(1-x)\,dx = 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{6}$

(iii) $\boxed{Var(x) = E(x^2) - E^2(x)}$

$= \frac{1}{6} - (0)^2 = \frac{1}{6}$ Ans

(iv) S.D(σ) $= +\frac{1}{\sqrt{6}}$ Ans

THANK - YOU