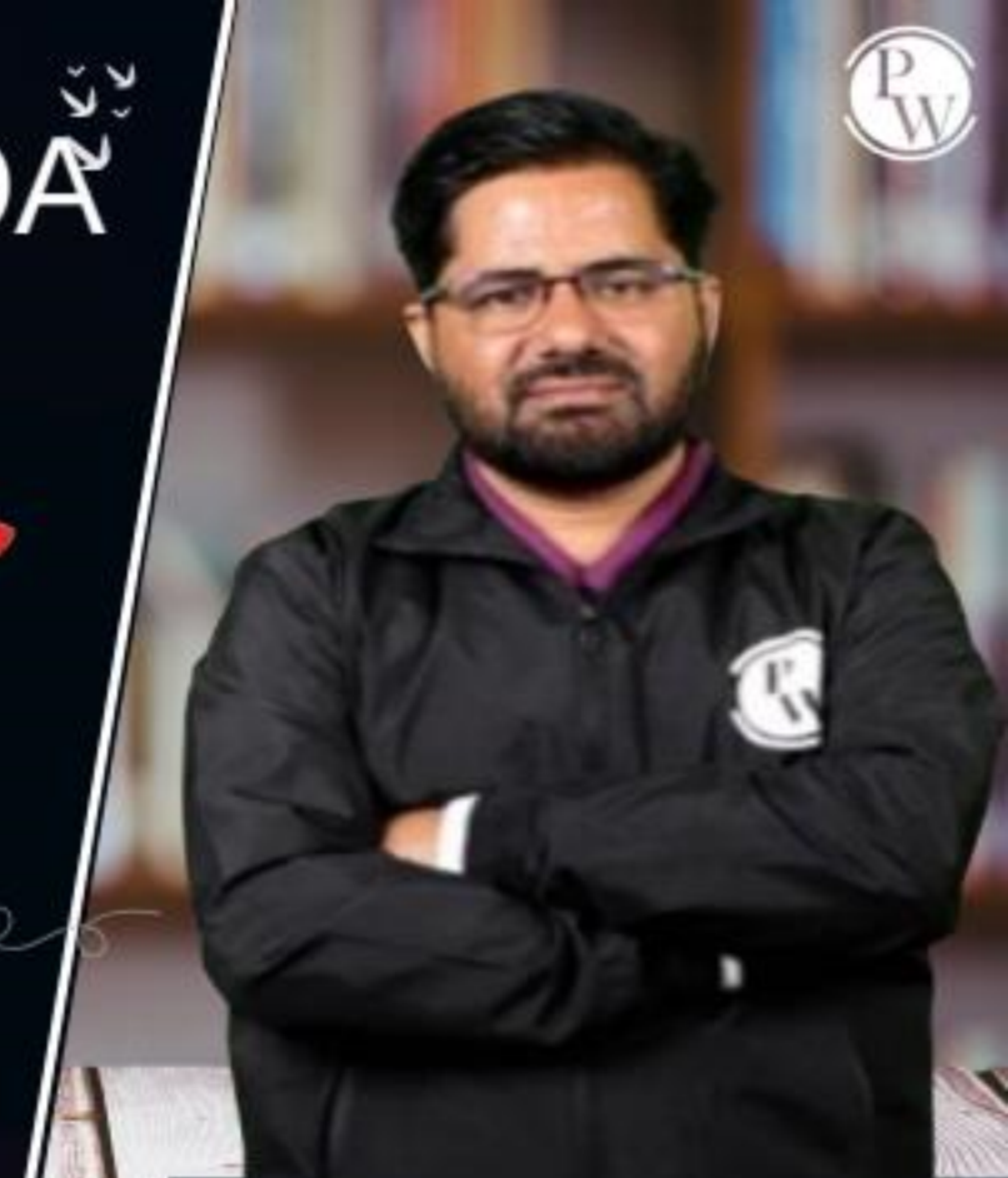
Computer Science & DA

Probability and Statistics

Discrete Random Variable

Lecture No. 02



By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic Statistics (Basic Definition) mean, median, mode, variance , S.D, Covariance etc.

① $\bar{X} = E(X) = \frac{\sum X}{N} = \sum p_i x_i$

② $Var(X) = E(X-\bar{X})^2 = \frac{\sum (X-\bar{X})^2}{N}$

$= \dots\dots = \boxed{E(X^2) - E^2(X)}$

③ $S.D(\sigma) = +\sqrt{Var(X)}$ (RMSD)

④ $Cov(X,Y) = E(X-\bar{X})(Y-\bar{Y}) = \frac{\sum (X-\bar{X})(Y-\bar{Y})}{N}$

$= \dots\dots = \boxed{E(X \cdot Y) - E(X)E(Y)}$

⑤ $Var(X)$ & $SD \geq 0$

⑥ $Var(X)$ & $SD \propto \frac{1}{\text{consistency}}$

⑦ $E(aX+b) = aE(X) + b$

⑧ $Var(aX+b) = a^2 Var(X) + 0$

⑨ $Var(aX+bY)$

$= a^2 Var(X) + b^2 Var(Y) + 2ab\,Cov(X,Y)$

Topics to be Covered

Topic Basic of Discrete Random variable

Measures of Dispersion → Variance & SD, Covariance

Measures of Central tendency → Mean, Median, Mode.

Mode:- the data having highest Frequency is Called Mode

2, 3, 3, 4, 4, 5, 5, 5, 5, 6, 7, 7, 8, 8, 8, 8, 8, 9, 9, 10 → Mode = 8

→ N = 20 (even)

Median:- After arranging the data either in Ascending order (or in descending order) the middle Most Value is Called Median.

Case I: if N = odd then Median = $\left(\frac{N+1}{2}\right)^{th}$ observation

Case II: if N = even then $Md = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th} \text{ observation}}{2}$

$$Md = \frac{\left(\frac{20}{2}\right)^{th} + \left(\frac{20}{2}+1\right)^{th}}{2}$$

$$= \frac{10^{th} + 11^{th}}{2} = \frac{6+7}{2} = 6.5$$

Note:- $\boxed{\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}}$

(GATE) Marks obtained by 100 students in a test is shown in the following Table. Then find Mean, Md and Mode of marks obtained.

Marks obtained (X)	No. of students (N)
25	20
30	20
35	40
40	20

$\Sigma X = ?$ $\quad N = 100.$

Mode = 35 Marks

$$\text{Median} = \frac{\left(\frac{100}{2}\right)^{th} + \left(\frac{100}{2}+1\right)^{th}}{2} = \frac{50^{th} + 51^{st}}{2}$$

$$= \frac{35+35}{2} = 35 \text{ Marks}$$

$$\text{Mean}(\bar{X}) = \frac{\Sigma X}{N} = \frac{20(25)+20(30)+40(35)+20(40)}{100}$$

$$= \frac{3300}{100} = 33 \text{ Marks.}$$

Sol: $\underbrace{25, 25, 25, \ldots, 25}_{20 \text{ students}}, \underbrace{30, 30, 30, \ldots, 30}_{20 \text{ students}}, \underbrace{35, 35, 35, \ldots, 35}_{40 \text{ students}}, \underbrace{40, 40, 40, \ldots, 40}_{20 \text{ students}}.$

Note: $X = \{$ which is Required should be assumed as $X\}$

Discrete Prob Dist → the Table representing Distribution of probabilities at various Values of x is called Prob. Dist.

eg A Coin is tossed thrice, then find the Prob. Dist of No. of Heads

Sol → $X = \{\text{Number of Heads}\} = \{0, 1, 2, 3\}$

$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\} = 8$

$p_1 = P(X = 0H) = P(\text{all } T) = 1/8$ & $p_2 = P(X = 1H) = 3/8$

$p_3 = P(X = 2H) = 3/8$, $p_4 = P(X = 3H) = 1/8$

So P. Dist is

X :	0	1	2	3
$P(x)$:	$\frac{1}{8}$	$3/8$	$3/8$	$1/8$

Note: $p_2 = P(X = 1H) = \frac{{}^3C_1}{2^3} = \frac{3}{8}$

$p_3 = P(X = 2H) = \frac{{}^3C_2}{2^3} = \frac{3}{8}$

Q. A coin is tossed thrice then find Mean, Variance & SD of Number of Heads?

Sol: $X = \{\text{Number of Heads}\} = \{0, 1, 2, 3\}$

$$\boxed{\begin{array}{ccccc} X: & 0 & 1 & 2 & 3 \\ P(x): & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{array}}$$

$\because \sum p_i = 1$

so all correct.

(1) $\boxed{E(X) = \sum p_i x_i} = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$

$= \frac{1}{8}(0) + \frac{3}{8}(1) + \frac{3}{8}(2) + \frac{1}{8}(3) = \frac{3}{2}$

(2) $\boxed{E(X^2) = \sum p_i x_i^2} = p_1x_1^2 + p_2x_2^2 + p_3x_3^2 + p_4x_4^2$

$= \frac{1}{8}(0)^2 + \frac{3}{8}(1)^2 + \frac{3}{8}(2)^2 + \frac{1}{8}(3)^2 = 3$

So $Var(X) = E(X^2) - E^2(X) = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4} = 0.75$

$SD(\sigma) = +\sqrt{3/4} = \sqrt{3}/2$

$\frac{\sqrt{3}/2}{3} = \frac{1}{2\sqrt{3}} \neq 0.5$

ANALYSIS:

when a coin is tossed thrice, Av Number of Heads $= 1.5$, $Var = 0.75$, $SD = \frac{\sqrt{3}}{2}$

" " " once, Av " " " $= 0.5$, $Var = 0.25$, $SD = \frac{\sqrt{3}/2}{\sqrt{3}} = 0.5$

" " " 10000 times, Av " " " $= 5000$, $Var = 2500$, $SD = 0.5 \times \sqrt{10000} = 50$

ie in Real Situation $4950 \leq \text{Av No. of Heads} \leq 5050$

Analysis 2: $X = \{\text{No. of Heads}\} = \{0, 1, 2, 3\} \Rightarrow \bar{X} = \frac{\Sigma X}{N} = \frac{0+1+2+3}{4} = 1.5$

(wrong App)

$$Var(X) = \frac{\Sigma (X-\bar{X})^2}{N} = \frac{(-1.5)^2 + (-0.5)^2 + (0.5)^2 + (1.5)^2}{4} \neq 0.75$$

Correct approach: $X = \{\text{No. of Heads}\} = \{0, 1, 1, 1, 2, 2, 2, 3\} \Rightarrow \bar{X} = \frac{\Sigma X}{N} = 1.5$

$$Var(X) = \frac{\Sigma (X-\bar{X})^2}{N} = \frac{(-1.5)^2 + 3(-0.5)^2 + 3(0.5)^2 + (1.5)^2}{8} = \ldots = 0.75$$

Q. A Man wins 5 Rupee if all Heads or all Tails occurs and he will loose 3 Rs if either one or two Head appears when 3 Coins are tossed simultaneously. The find the Expected amount wins or looses by man on an Average per Game.

(ii) if above Game is played by 5000 persons, 20 times each then find Expected amount wins or looses by Game organizer?

Sol: $N_1 = 5000$ persons

$N_2 = 20$ Games

For single person in single Game

$X = \{\text{Amount Received}\}$

$= \{+5, -3\}$

$S = \{(HHH), (HHT)(HTH)(HTT)(THH)(THT)(TTH)(TTT)\}$

$p_1 = P(X = 5\text{ Rs}) = P(HHH \text{ or } TTT) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

$p_2 = P(X = -3\text{ Rs}) = \frac{6}{8} = \frac{3}{4}$

X:	5	-3
$P(x)$:	$\frac{1}{4}$	$\frac{3}{4}$

$E(X) = \Sigma p_i X_i = \frac{1}{4}(5) + \frac{3}{4}(-3) = -1$

i.e. He will loose 1 Rs on an average per Game.

(ii) on an Average, Single person in Single Game will loose = 1 Rs

" " " " Single " " 20 Games " " = 1×20 Rs

" " " " 5000 " " 20 Games " " = 1×20×5000

= 100000 Rs

Note: Childhood Method: $X = \{5, -3, -3, -3, -3, -3, -3, 5\} \Rightarrow \bar{X} = \frac{\Sigma X}{N} = -1$

(CS-2004)

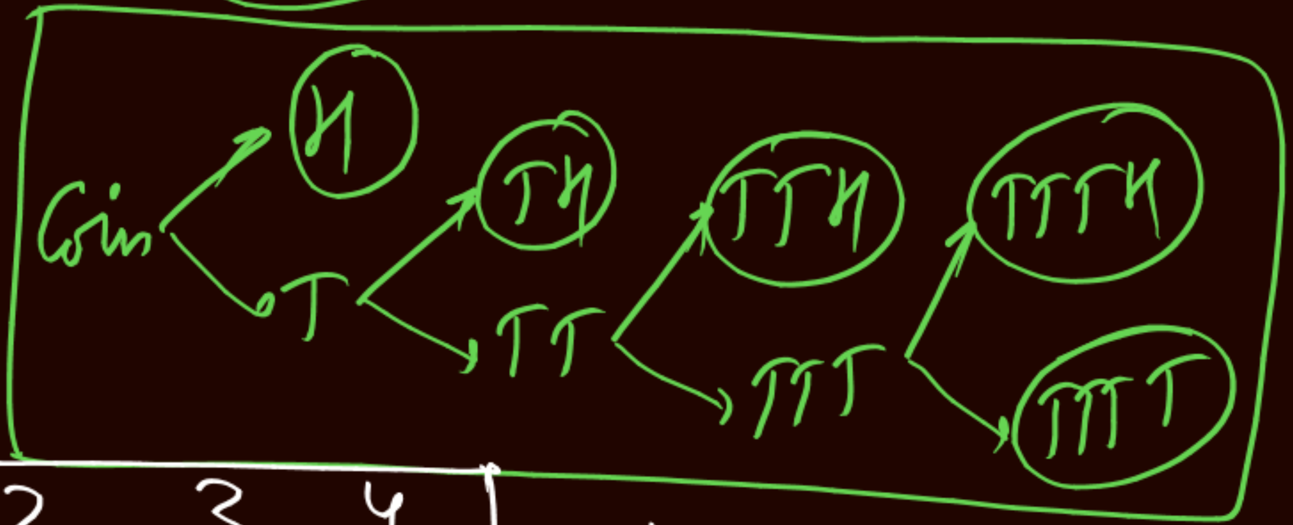
Q: 1000 students are appearing in an examination in which each student has to solve 150 MCQ's. Each Correct ans gives 1 Marks & Each incorrect ans fetches −0.25 Marks. if all the students has given all their answers Randomly then find the sum total of their Expected Marks?

$X = \{\text{Marks obtained}\}$

Ans = 9375 Marks

HW

Gate-e A Coin is tossed until Head appears or Tail appears 4 times in succession then find Average Number of tosses required ?



Sol: $X = \{\text{Number of tosses required}\} = \{1, 2, 3, 4, \}$

$S = \{H, TH, TTH, TTTH, TTTT\}$

$p_1 = P(X = 1\text{ toss}) = P(H) = \frac{1}{2}$

$p_2 = P(X = 2\text{ tosses}) = P(TH) = \frac{1}{4}$

$p_3 = P(X = 3\text{ tosses}) = P(TTH) = \frac{1}{8}$

$p_4 = P(X = 4\text{ tosses}) = P(TTTH \text{ or } TTTT)$

$= \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$

X:	1	2	3	4
$P(X)$:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

; $\sum p_i = 1$ (✓)

$E(X) = \sum p_i x_i = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$

$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{8}(4) = \frac{15}{8}$

Note: Min tosses Req = 1 & Av tosses Req = 1·89

Max " " = 4

THANK - YOU