16-782
Planning & Decision-making in Robotics

Interleaving Planning & Execution: Real-time Heuristic Search

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Planning during Execution

- Planning is a repeated process!
  - partially-known environments
  - dynamic environments
  - imperfect execution of plans
  - imprecise localization

- Need to be able to re-plan fast!

- Several methodologies to achieve this:
  - anytime heuristic search: return the best plan possible within T msecs
  - incremental heuristic search: speed up search by reusing previous efforts
  - real-time heuristic search: plan few steps towards the goal and re-plan later

this lecture

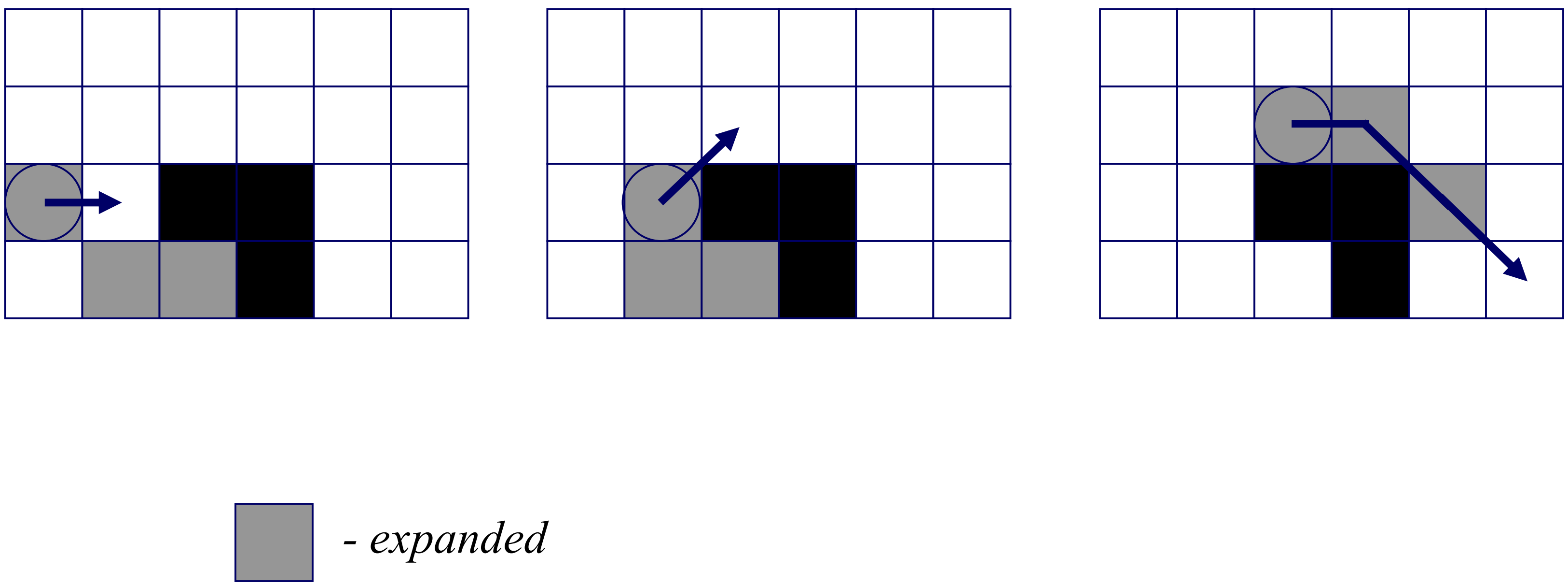
Real-time (Agent-centered) Heuristic Search

Enforce a strict limit on the amount of computations (no requirement on planning all the way to the goal)

Real-time (Agent-centered) Heuristic Search

1. Compute a partial path by expanding at most N states around the robot
2. Move once, incorporate sensor information, and goto step 1

Example in a fully-known terrain:



Real-time (Agent-centered) Heuristic Search

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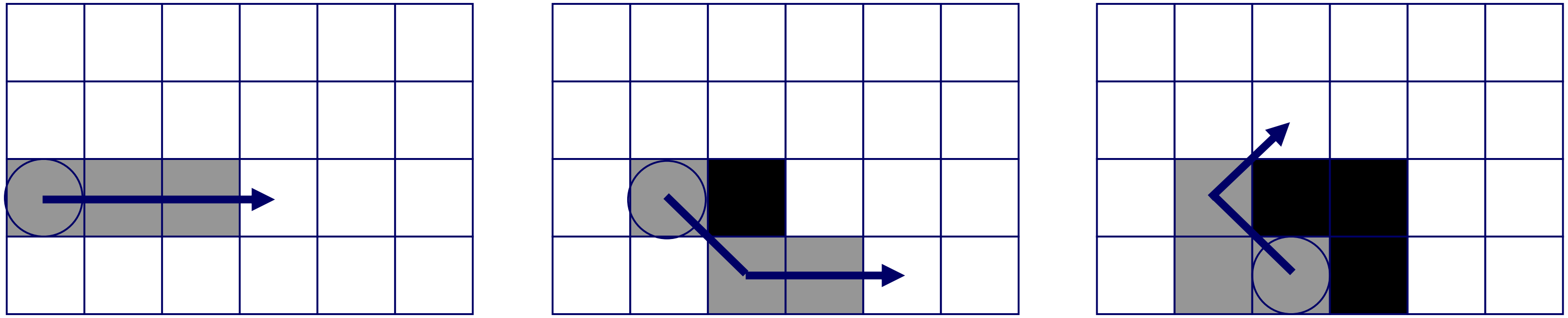
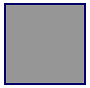
Example in an unknown terrain (planning with Freespace Assumption):

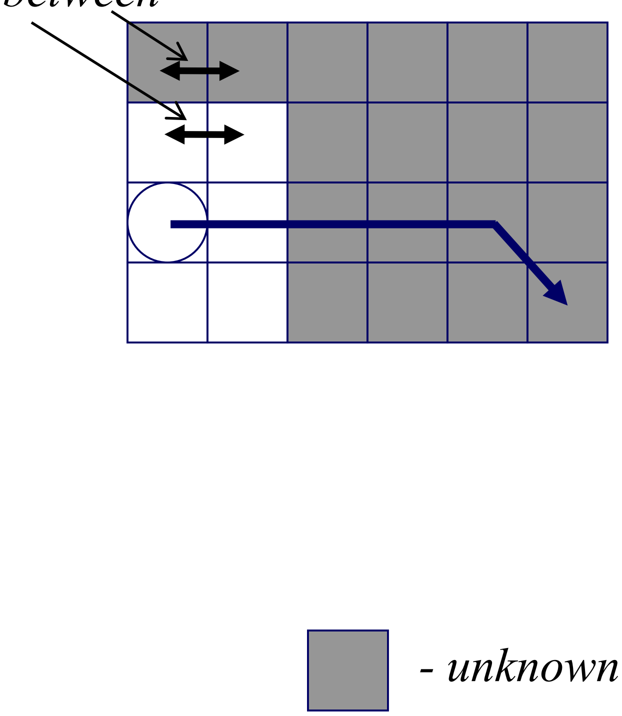



- *expanded*

Planning with Freespace Assumption [Nourbakhsh & Genesereth, '96]

- Freespace Assumption: all unknown cells are assumed to be traversable
- Planning with the Freespace Assumption: always move the robot on a shortest path to the goal assuming all unknown cells are traversable
- Replan the path whenever a new sensor information received

costs between unknown states is the same as the costs in between states known to be free



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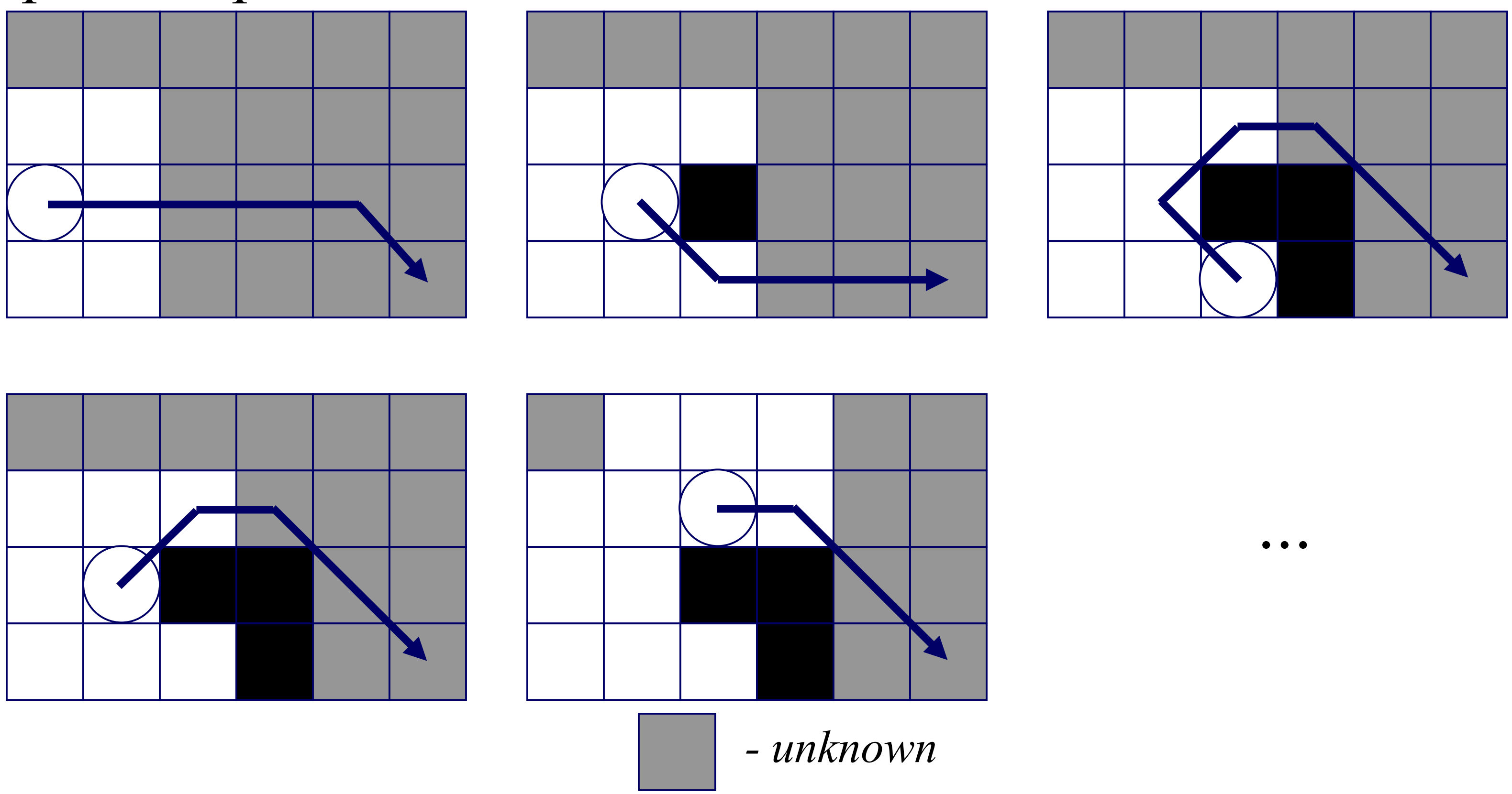

Real-time (Agent-centered) Heuristic Search

1. Compute a partial path by expanding at most N states around the robot
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Research issues:

- how to compute partial path

- how to guarantee complete behavior (guarantee to reach the goal)

- provide bounds on the number of steps before reaching the goal

Real-time (Agent-centered) Heuristic Search

1. Compute a partial path by expanding at most N states around the robot
2. Move once, incorporate sensor information, and goto step 1

Research issues:

- how to compute partial path *Any ideas?*

- how to guarantee complete behavior (guarantee to reach the goal)

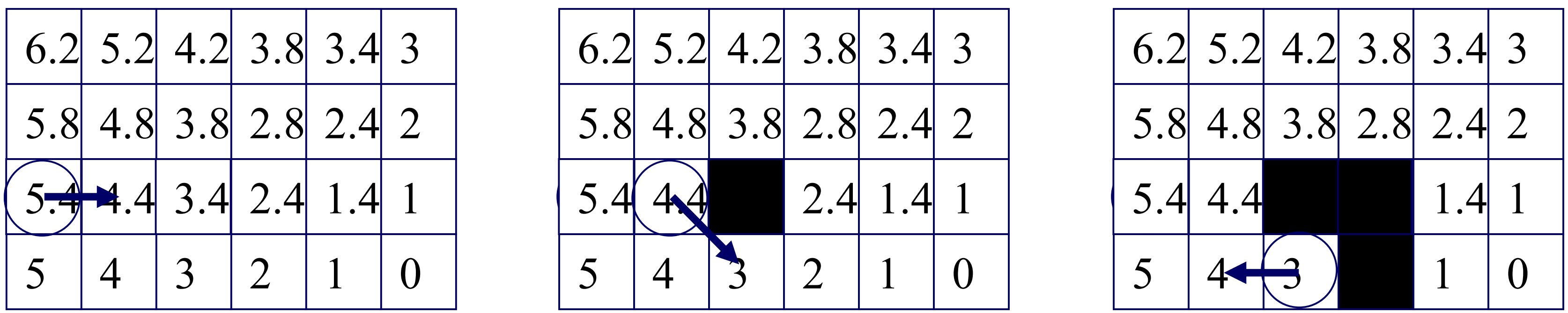
- provide bounds on the number of steps before reaching the goal

Learning Real-Time A* (LRTA*) [Korf, ‘90]

- Repeatedly move the robot to the most promising adjacent state, using heuristics

1. *always move as follows:* $s_{start} = \arg\min_{s \in succ(s_{start})} c(s_{start}, s) + h(s)$

$$h(x,y) = \max(abs(x-x_{goal}), abs(y-y_{goal})) + 0.4 * \min(abs(x-x_{goal}), abs(y-y_{goal}))$$



Any problems?

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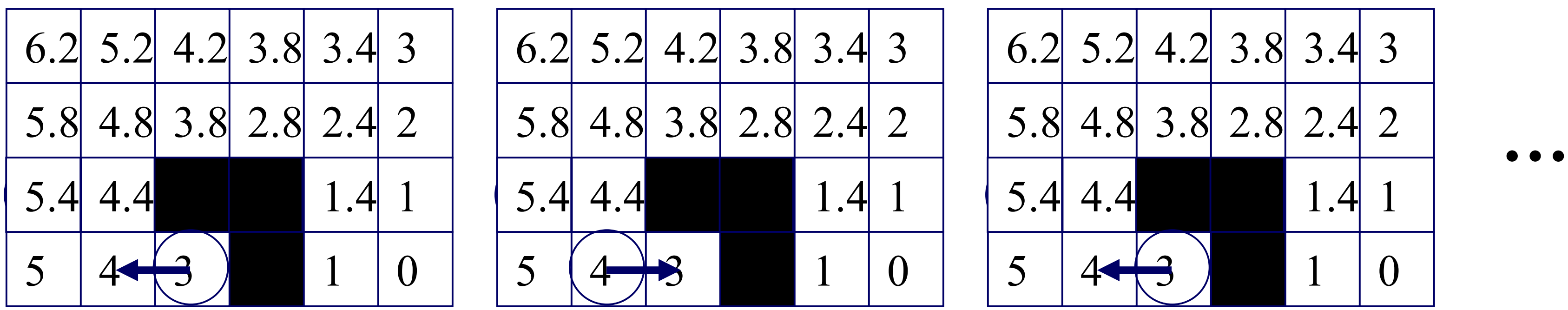


Local minima problem (myopic or incomplete behavior)

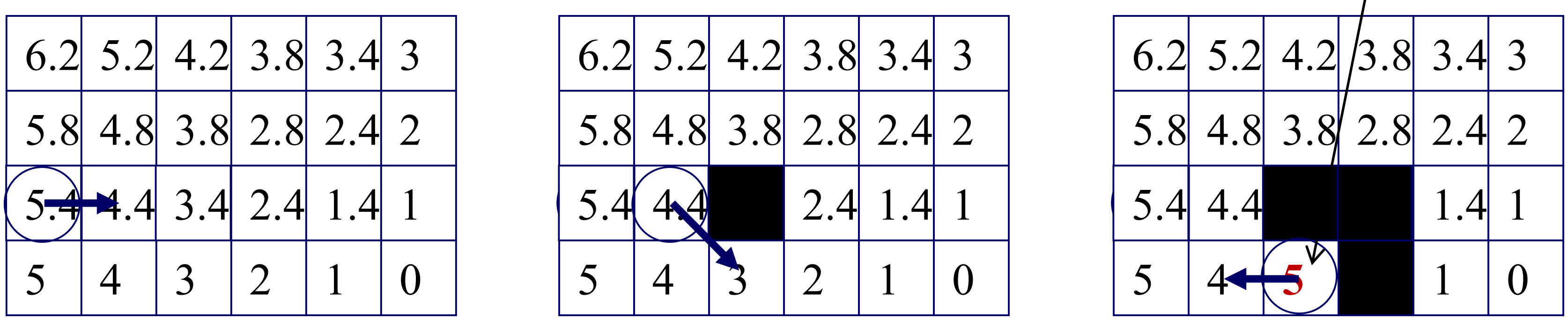
Any solutions?

Learning Real-Time A* (LRTA*) [Korf, '90]

- Repeatedly move the robot to the most promising adjacent state, using **and updating** heuristics

makes h-values more informed

1. *update* $h(s_{start}) = \min_{s \in succ(sstart)} c(s_{start}, s) + h(s)$
2. *always move as follows:* $s_{start} = \arg\min_{s \in succ(sstart)} c(s_{start}, s) + h(s)$



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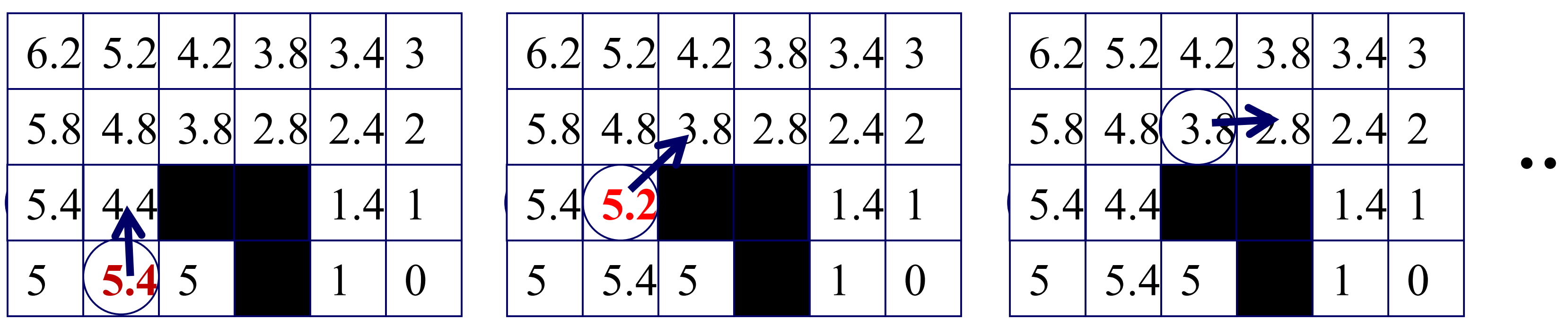

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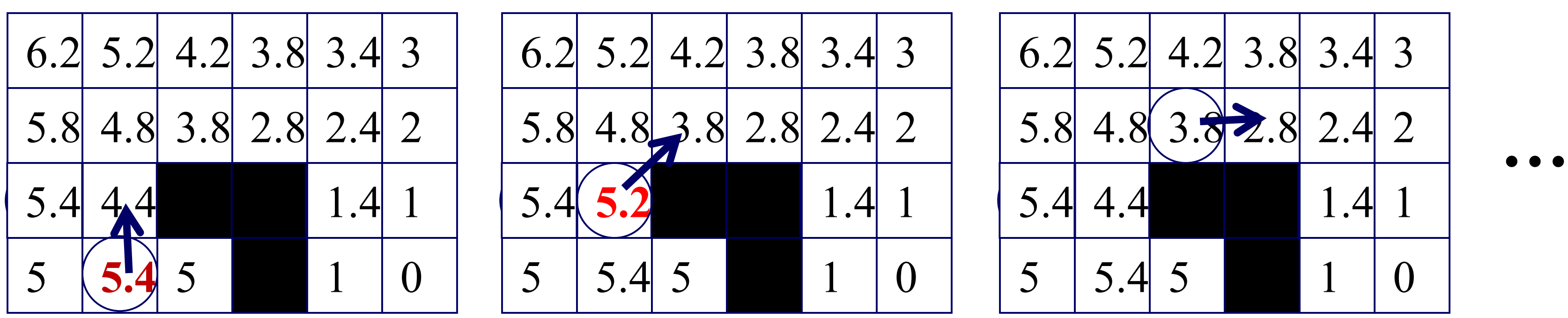

h-values remain admissible and consistent

proof?

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6.2	5.2	4.2	3.8	3.4	3
5.8	4.8	3.8	2.8	2.4	2
5.4	4.4			1.4	1
5	**5.4**	5		1	0

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5.4	**5.2**			1.4	1
5	5.4	5		1	0

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…

robot is guaranteed to reach goal in finite number of steps if:

- all costs are bounded from below with $\Delta > 0$
- graph is of finite size and there exists a finite-cost path to the goal
- all actions are reversible

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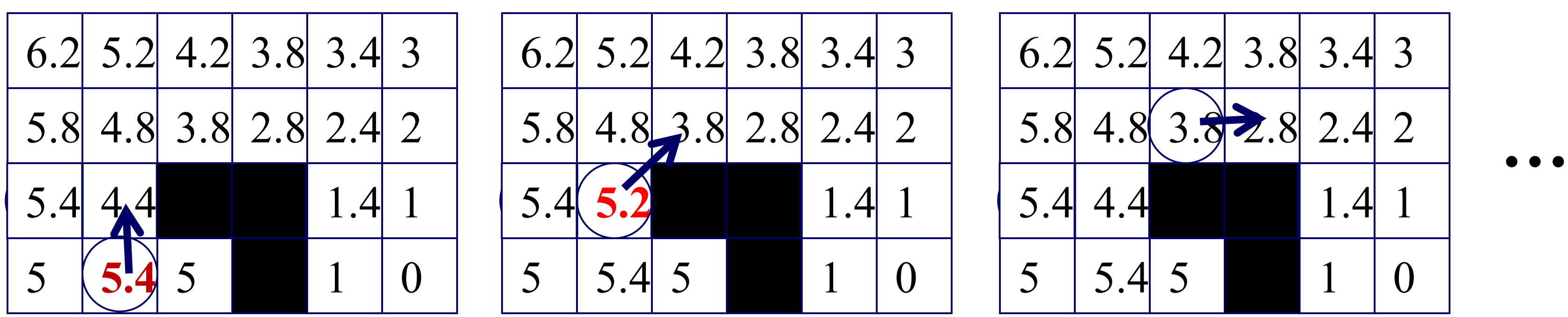

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Why conditions?

Learning Real-Time A* (LRTA*)

- LRTA* with $N \geq 1$ expands [Koenig, '04]

  1. *expand N states*
  2. *update h-values of expanded states by Dynamic Programming (DP)*
  3. *move on the path to state* $s = \arg\min_{s' \in OPEN} g(s') + h(s')$

necessary for the guarantee to reach the goal

- expanded

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state *s*:

- the state that minimizes cost to it plus heuristic estimate of the remaining distance

- the state that looks most promising in terms of the whole path from current robot state to goal

- expanded

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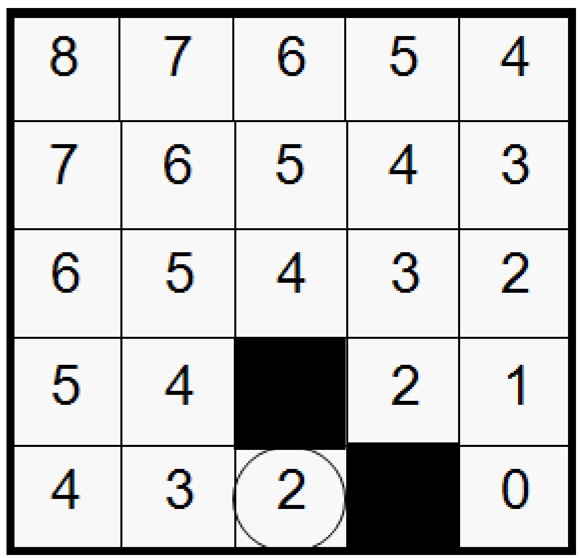

4-connected grid (robot moves in 4 directions)

example borrowed from ICAPS'06 planning summer school lecture (Koenig & Likhachev)

- expanded

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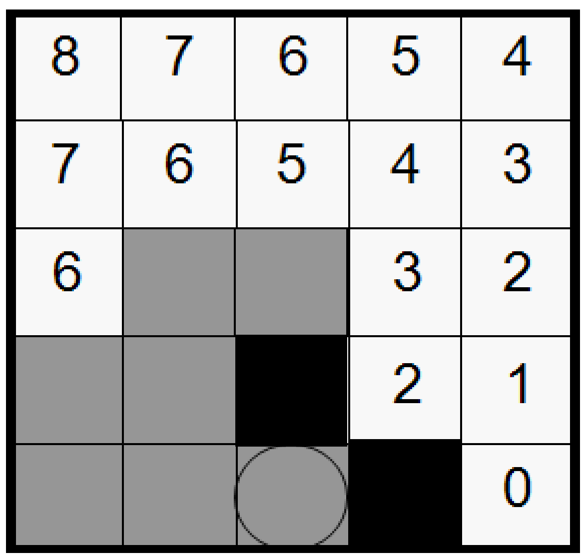

expand N=7 states

- expanded

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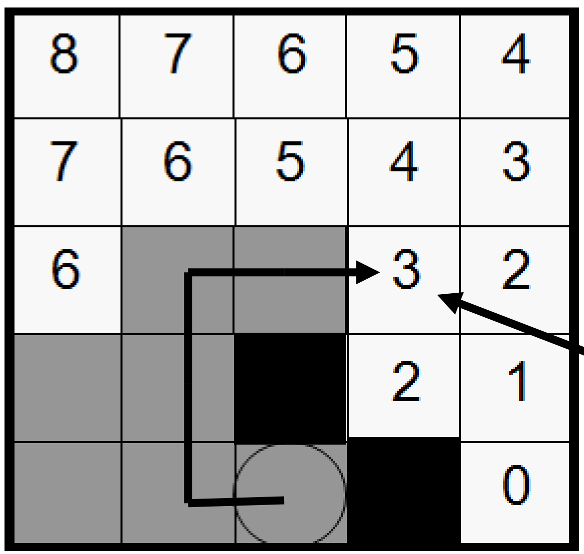

expand N=7 states

unexpanded state with smallest g + h (= 5 + 3)

- *expanded*

Learning Real-Time A* (LRTA*)

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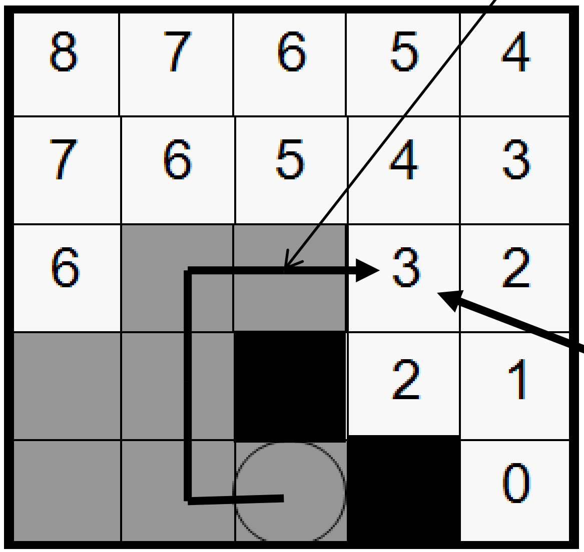
How path is found?

1. *expand N states*
2. *update h-values of expanded states by Dynamic Programming (DP)*
3. *move on the path to state $s = \text{argmin}_{s' \in OPEN}\, g(s') + h(s')$*



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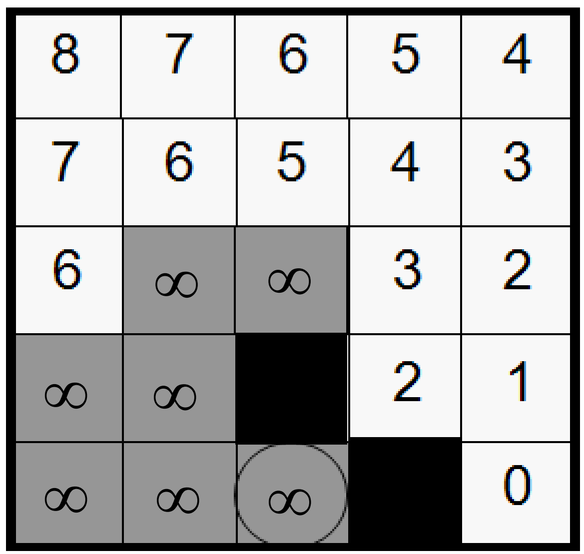
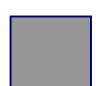


update h-values of expanded states via DP:
compute $h(s) = \min_{s' \in succ(s)} (c(s,s') + h(s'))$
until convergence

 *- expanded*

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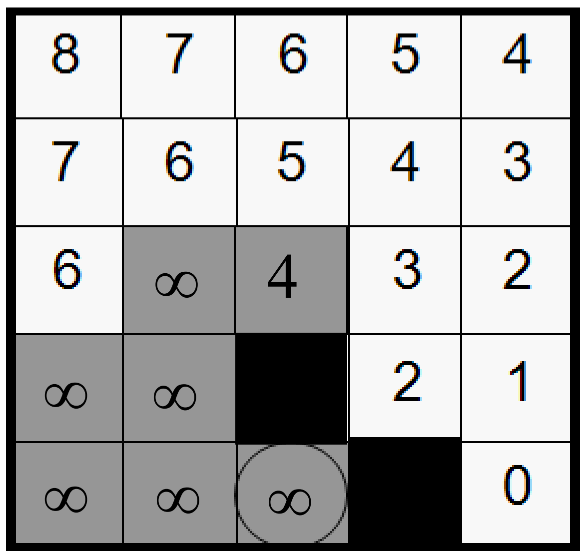

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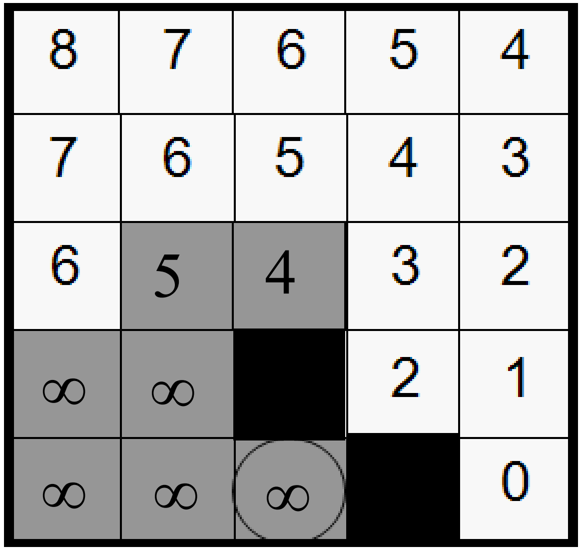

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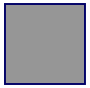 *- expanded*

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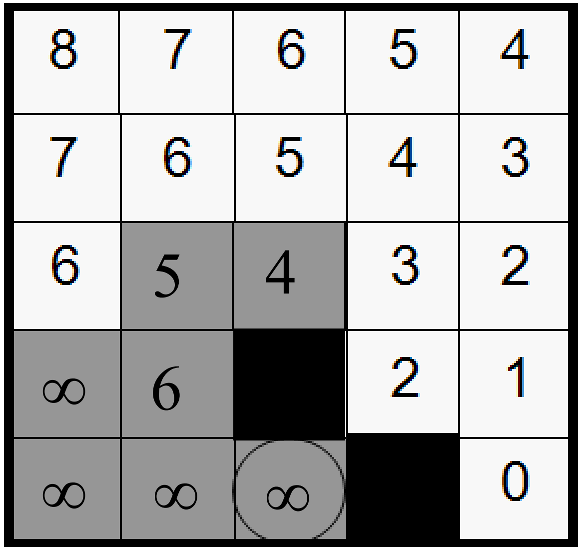

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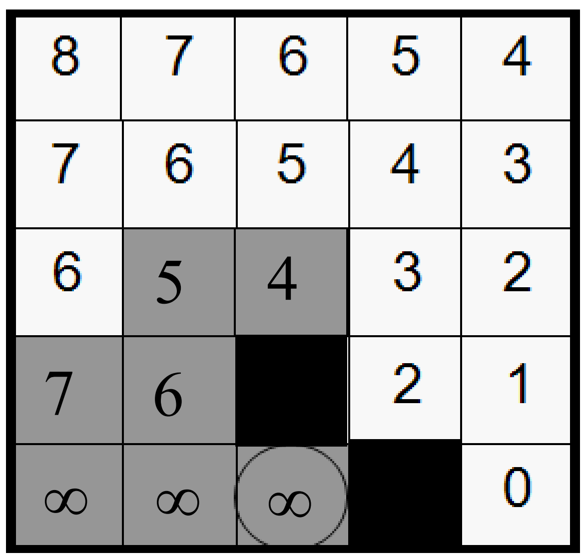

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Does it matter in what order?

- expanded

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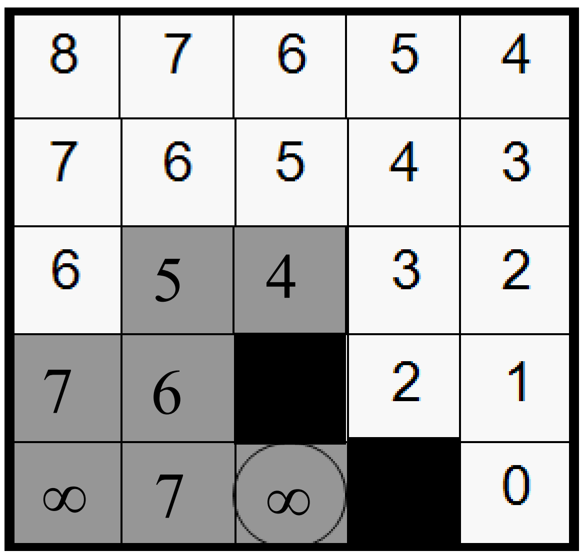

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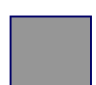 *- expanded*

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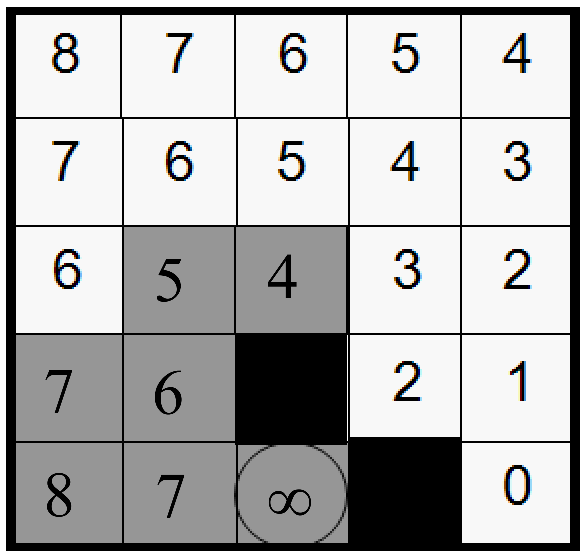

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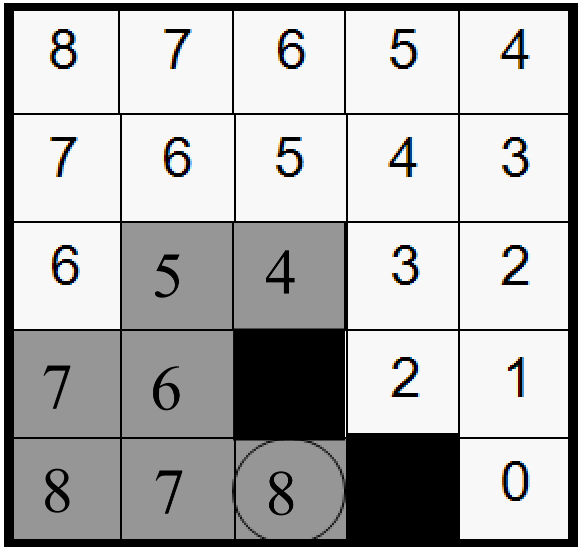

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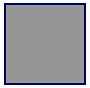

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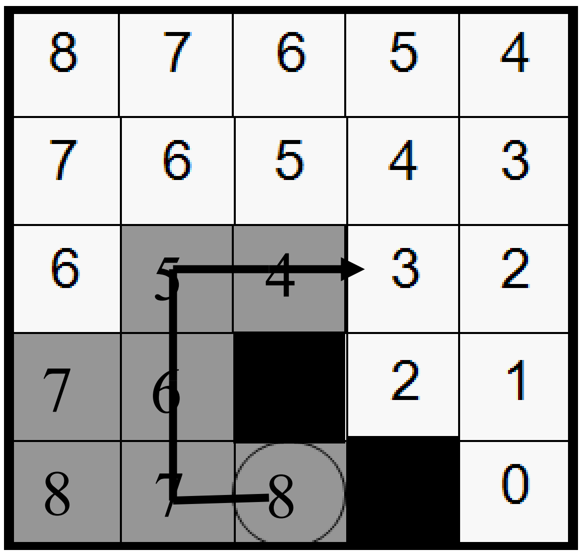


make a move along the found path and repeat steps 1-3

Drawbacks compared to A?*

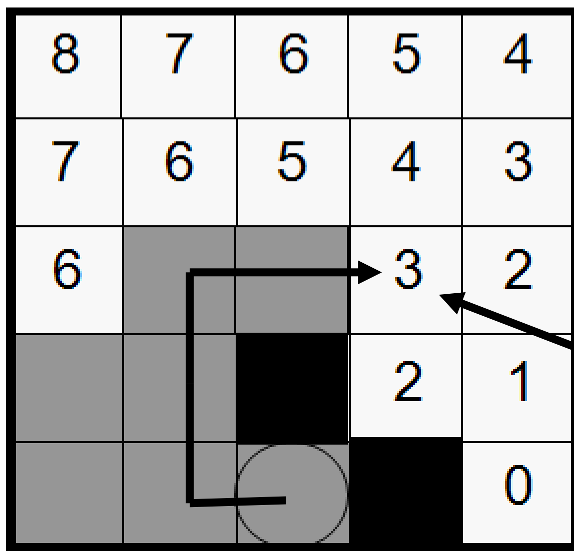
- expanded

Real-time Adaptive A* (RTAA*) [Koenig & Likhachev, '06]

- RTAA* with $N \geq 1$ expands

one linear pass, and even that can be lazy(postponed)

1. *expand N states*
2. *update h-values of expanded states u by h(u) = f(s) – g(u),*
   where $s = \text{argmin}_{s' \in OPEN}\, g(s') + h(s')$
3. *move on the path to state* $s = \text{argmin}_{s' \in OPEN}\, g(s') + h(s')$



expand N=7 states

unexpanded state *s* with smallest *g* + *h* (= 5 + 3)

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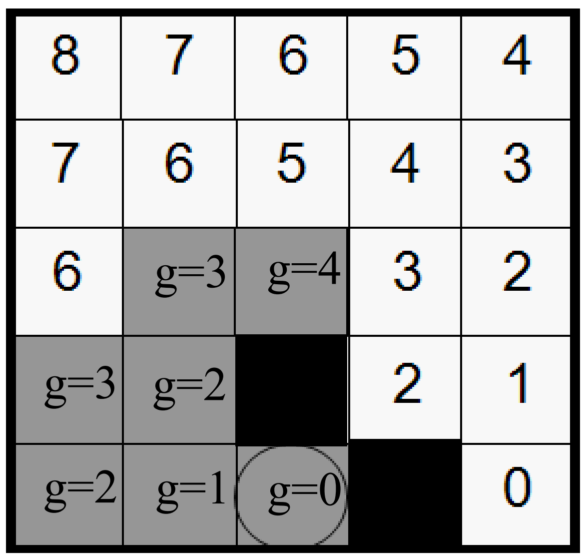

update all expanded states u:

$h(u) = f(s) - g(u)$

unexpanded state *s* with smallest

f(s) = *8*

- expanded

Real-time Adaptive A* (RTAA*)

- RTAA* with $N \geq 1$ expands

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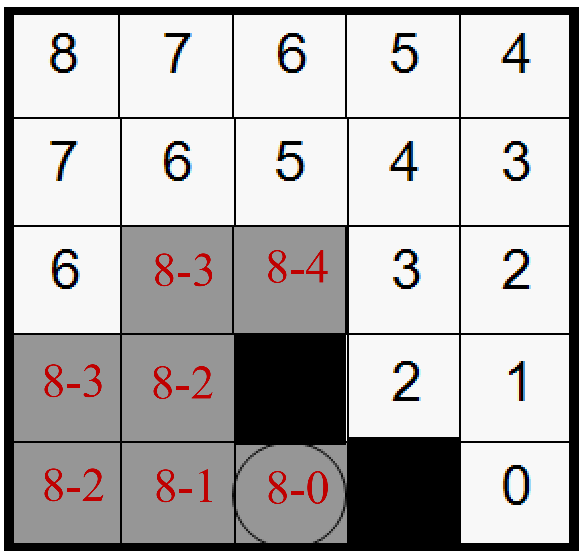

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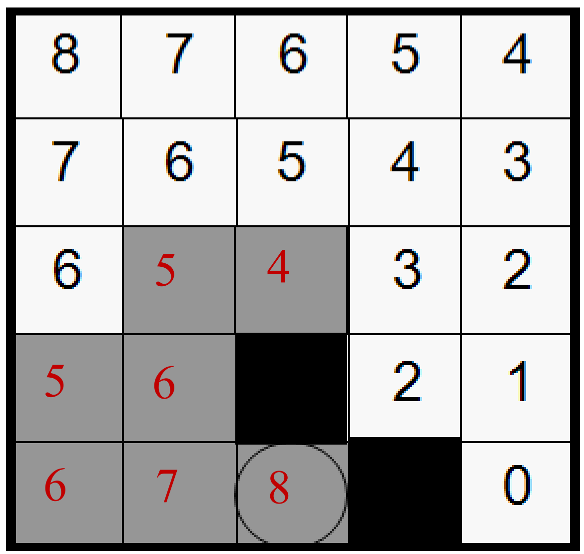

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- RTAA* with $N \geq 1$ expands

1. *expand N states*
2. *update h-values of expanded states u by* $h(u) = f(s) - g(u)$,

   where $s = \text{argmin}_{s' \in OPEN}\, g(s') + h(s')$
3. *move on the path to state* $s = \text{argmin}_{s' \in OPEN}\, g(s') + h(s')$

proof of admissibility:

h() – true cost-to-goal*

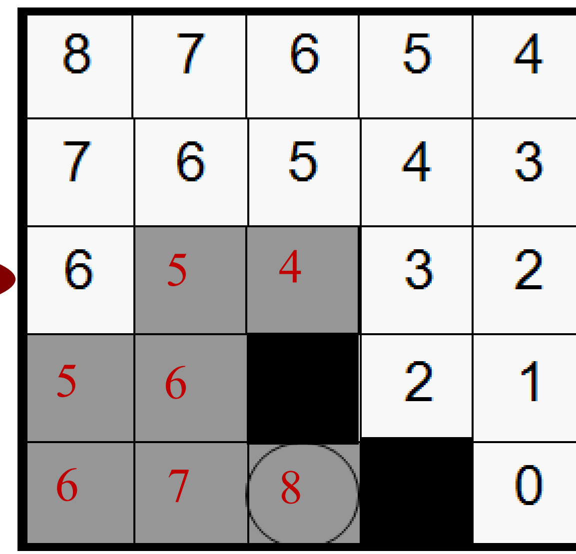
$$g(u) + h^*(u) \geq h^*(s_{start})$$

$$h^*(u) \geq h^*(s_{start}) - g(u)$$

because $f(s) \leq h^*(s_{start})$

$$h^*(u) \geq f(s) - g(u)$$

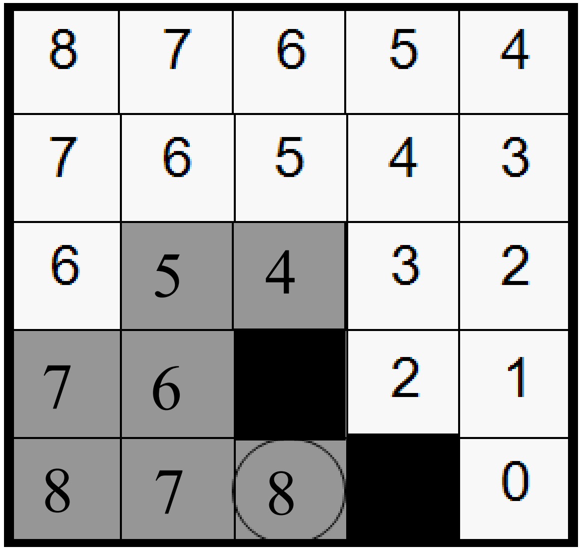
$$h^*(u) \geq h_{updated}(u)$$



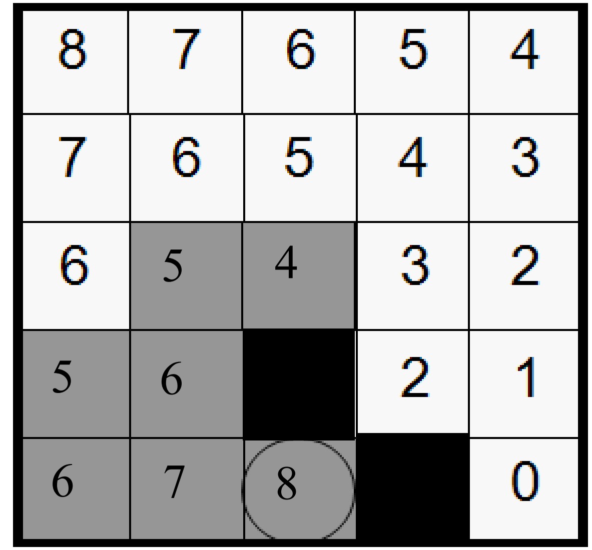
- expanded

LRTA* vs. RTAA*

LRTA*



RTAA*



- Update of *h*-values in RTAA* is much faster but not as informed
- Both guarantee adimssibility and consistency of heuristics
- For both, heuristics are monotonically increasing
- Both guarantee to reach the goal in a finite number of steps (given the conditions listed previously)

What You Should Know…

- What Freespace Assumption means
- Why we need to update heuristics in the context of Real-time Heuristic Search
- The operation of LRTA*
- Pros and cons of LRTA*
- What domains LRTA* is useful in and what domains it is not really applicable
- What RTAA* is