

$$P(Y=1|X) = \frac{P(Y=1) P(X|Y=1)}{P(Y=1) P(X|Y=1) + P(Y=0) P(X|Y=0)}$$

$$P(Y=1|X) = \frac{1}{1 + \frac{P(Y=0) P(X|Y=0)}{P(Y=1) P(X|Y=1)}}$$

$$= \frac{1}{1 + \exp\left(\ln\left(\frac{P(Y=0) P(X|Y=0)}{P(Y=1) P(X|Y=1)}\right)\right)}$$

As, Y has binomial distribution with $P(Y=1) = P$

$$\Rightarrow \ln \frac{P(Y=0)}{P(Y=1)} = \ln \frac{(1-P)}{P}$$

Now, if all X_i have binomial distribution
and taking $P(X_i=1|Y=1) = \theta_{i1}$

$$\Rightarrow P(X_i=0|Y=1) = (1 - \theta_{i1})$$

Also, assume $P(X_i=1|Y=0) = \theta_{i0}$

$$\Rightarrow P(X_i=0|Y=0) = (1 - \theta_{i0})$$

Assuming Naive Bayes assumption of conditional independence

$$P(X|Y=0) = \prod_i P(X_i|Y=0) = \prod_i \theta_{i0}^{x_i} (1-\theta_{i0})^{(1-x_i)}$$

$$P(X|Y=1) = \prod_i P(X_i|Y=1) = \prod_i \theta_{i1}^{x_i} (1-\theta_{i1})^{(1-x_i)}$$

$$\Rightarrow \ln\left(\frac{P(X|Y=0)}{P(X|Y=1)}\right) = \ln\left[\frac{\prod_i \theta_{i0}^{x_i} (1-\theta_{i0})^{(1-x_i)}}{\prod_i \theta_{i1}^{x_i} (1-\theta_{i1})^{(1-x_i)}}\right]$$

$$= \sum_i \left[x_i \ln \theta_{i0} + (1-x_i) \ln(1-\theta_{i0}) - x_i \ln \theta_{i1} - (1-x_i) \ln(1-\theta_{i1}) \right]$$

$$= \sum_i \left[\ln\left(\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right) \right] x_i + \sum_i \ln\left(\frac{(1-\theta_{i0})}{(1-\theta_{i1})}\right)$$

$$\Rightarrow P(Y=1|X) = \frac{1}{1 + \exp\left(\ln\left(\frac{1-p}{p}\right) + \sum_i \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right) + \sum_i \ln\left(\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right) x_i\right)}$$

$$w_0 = \ln\left(\frac{1-p}{p}\right) + \sum_i \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)$$

$$w_i = \ln\left(\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right)$$