

Mathematics

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(Chapter – 1) (Relations and Functions)

(Class – XII)

Exercise 1.1

Question 1:

Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3 \dots 13, 14\}$ defined as

$$R = \{(x, y): 3x - y = 0\}$$

(ii) Relation R in the set \mathbf{N} of natural numbers defined as

$$R = \{(x, y): y = x + 5 \text{ and } x < 4\}$$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$$R = \{(x, y): y \text{ is divisible by } x\}$$

(iv) Relation R in the set \mathbf{Z} of all integers defined as

$$R = \{(x, y): x - y \text{ is an integer}\}$$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y): x \text{ and } y \text{ work at the same place}\}$

(b) $R = \{(x, y): x \text{ and } y \text{ live in the same locality}\}$

(c) $R = \{(x, y): x \text{ is exactly 7 cm taller than } y\}$

(d) $R = \{(x, y): x \text{ is wife of } y\}$

(e) $R = \{(x, y): x \text{ is father of } y\}$

Answer 1:

(i) $A = \{1, 2, 3 \dots 13, 14\}$

$$R = \{(x, y): 3x - y = 0\}$$

$$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

R is not reflexive since $(1, 1), (2, 2) \dots (14, 14) \notin R$.

Also, R is not symmetric as $(1, 3) \in R$, but $(3, 1) \notin R$. [$3(3) - 1 \neq 0$]



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Also, R is not transitive as $(1, 3), (3, 9) \in R$, but $(1, 9) \notin R$. [$3(1) - 9 \neq 0$]

Hence, R is neither reflexive, nor symmetric, nor transitive.

(ii) $R = \{(x, y): y = x + 5 \text{ and } x < 4\} = \{(1, 6), (2, 7), (3, 8)\}$

It is clear that $(1, 1) \notin R$.

$\therefore R$ is not reflexive.

$(1, 6) \in R$ But, $(1, 6) \notin R$.

$\therefore R$ is not symmetric.

Now, since there is no pair in R such that (x, y) and $(y, z) \in R$, then (x, z) cannot belong to R.

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii) $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y): y \text{ is divisible by } x\}$

We know that any number (x) is divisible by itself.

So, $(x, x) \in R$

$\therefore R$ is reflexive.

Now,

$(2, 4) \in R$ [as 4 is divisible by 2]

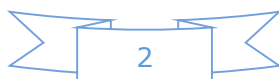
But, $(4, 2) \notin R$. [as 2 is not divisible by 4]

$\therefore R$ is not symmetric.

Let $(x, y), (y, z) \in R$. Then, y is divisible by x and z is divisible by y .

$\therefore z$ is divisible by x .

$\Rightarrow (x, z) \in R$



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$\therefore R$ is transitive.

Hence, R is reflexive and transitive but not symmetric.

(iv) $R = \{(x, y): x - y \text{ is an integer}\}$

Now, for every $x \in \mathbf{Z}$, $(x, x) \in R$ as $x - x = 0$ is an integer.

$\therefore R$ is reflexive.

Now, for every $x, y \in \mathbf{Z}$, if $(x, y) \in R$, then $x - y$ is an integer.

$\Rightarrow -(x - y)$ is also an integer.

$\Rightarrow (y - x)$ is an integer.

$\therefore (y, x) \in R$

$\therefore R$ is symmetric.

Now,

Let (x, y) and $(y, z) \in R$, where $x, y, z \in \mathbf{Z}$.

$\Rightarrow (x - y)$ and $(y - z)$ are integers.

$\Rightarrow x - z = (x - y) + (y - z)$ is an integer.

$\therefore (x, z) \in R$

$\therefore R$ is transitive.

Hence, R is reflexive, symmetric, and transitive.

(v)

(a) $R = \{(x, y): x \text{ and } y \text{ work at the same place}\}$

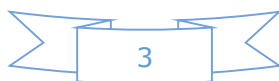
$\Rightarrow (x, x) \in R$ [as x and x work at the same place]

$\therefore R$ is reflexive.

If $(x, y) \in R$, then x and y work at the same place.

$\Rightarrow y$ and x work at the same place.

$\Rightarrow (y, x) \in R$.



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$\therefore R$ is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow x$ and y work at the same place and y and z work at the same place.

$\Rightarrow x$ and z work at the same place.

$\Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

Hence, R is reflexive, symmetric and transitive.

(b) $R = \{(x, y): x \text{ and } y \text{ live in the same locality}\}$

Clearly, $(x, x) \in R$ as x and x is the same human being.

$\therefore R$ is reflexive.

If $(x, y) \in R$, then x and y live in the same locality.

$\Rightarrow y$ and x live in the same locality.

$\Rightarrow (y, x) \in R$

$\therefore R$ is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow x$ and y live in the same locality and y and z live in the same locality.

$\Rightarrow x$ and z live in the same locality.

$\Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

Hence, R is reflexive, symmetric and transitive.

(c) $R = \{(x, y): x \text{ is exactly 7 cm taller than } y\}$

Now, $(x, x) \notin R$

Since human being x cannot be taller than himself.

$\therefore R$ is not reflexive.



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Now, let $(x, y) \in R$.

$\Rightarrow x$ is exactly 7 cm taller than y .

Then, y is not taller than x . [Since, y is 7 cm smaller than x]

$\therefore (y, x) \notin R$

Indeed if x is exactly 7 cm taller than y , then y is exactly 7 cm shorter than x .

$\therefore R$ is not symmetric.

Now,

Let $(x, y), (y, z) \in R$.

$\Rightarrow x$ is exactly 7 cm taller than y and y is exactly 7 cm taller than z .

$\Rightarrow x$ is exactly 14 cm taller than z .

$\therefore (x, z) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(d) $R = \{(x, y): x \text{ is the wife of } y\}$

Now,

$(x, x) \notin R$

Since x cannot be the wife of herself.

$\therefore R$ is not reflexive.

Now, let $(x, y) \in R$

$\Rightarrow x$ is the wife of y .

Clearly y is not the wife of x .

$\therefore (y, x) \notin R$

Indeed if x is the wife of y , then y is the husband of x .

$\therefore R$ is not transitive.

Let $(x, y), (y, z) \in R$



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$\Rightarrow x$ is the wife of y and y is the wife of z .

This case is not possible. Also, this does not imply that x is the wife of z .

$\therefore (x, z) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(e) $R = \{(x, y): x \text{ is the father of } y\}$

$(x, x) \notin R$

As x cannot be the father of himself.

$\therefore R$ is not reflexive.

Now, let $(x, y) \notin R$.

$\Rightarrow x$ is the father of y .

$\Rightarrow y$ cannot be the father of y .

Indeed, y is the son or the daughter of y .

$\therefore (y, x) \notin R$

$\therefore R$ is not symmetric.

Now, let $(x, y) \in R$ and $(y, z) \notin R$.

$\Rightarrow x$ is the father of y and y is the father of z .

$\Rightarrow x$ is not the father of z .

Indeed x is the grandfather of z .

$\therefore (x, z) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.



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Question 2:

Show that the relation R in the set \mathbf{R} of real numbers, defined as

$R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Answer 2:

$$R = \{(a, b): a \leq b^2\}$$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$, since, $\frac{1}{2} > \left(\frac{1}{2}\right)^2$

$\therefore R$ is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$ But, 4 is not less than 1^2 .

$\therefore (4, 1) \notin R$

$\therefore R$ is not symmetric.

Now,

$$(3, 2), (2, 1.5) \in R \quad [\text{as } 3 < 2^2 = 4 \text{ and } 2 < (1.5)^2 = 2.25]$$

But, $3 > (1.5)^2 = 2.25$

$\therefore (3, 1.5) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 3:

Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b):$

$b = a + 1\}$ is reflexive, symmetric or transitive.

