Advanced High-School Mathematics

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Preface/Acknowledgment

The present expanded set of notes initially grew out of an attempt to flesh out the International Baccalaureate (IB) mathematics "Further Mathematics" curriculum, all in preparation for my teaching this during during the AY 2007–2008 school year. Such a course is offered only under special circumstances and is typically reserved for those rare students who have finished their second year of IB mathematics HL in their junior year and need a "capstone" mathematics course in their senior year. During the above school year I had two such IB mathematics students. However, feeling that a few more students would make for a more robust learning environment, I recruited several of my 2006–2007 AP Calculus (BC) students to partake of this rare offering resulting. The result was one of the most singular experiences I've had in my nearly 40-year teaching career: the brain power represented in this class of 11 blue-chip students surely rivaled that of any assemblage of high-school students anywhere and at any time!

After having already finished the first draft of these notes I became aware that there was already a book in print which gave adequate coverage of the IB syllabus, namely the Haese and Harris text[1] which covered the four IB Mathematics HL "option topics," together with a chapter on the retired option topic on Euclidean geometry. This is a very worthy text and had I initially known of its existence, I probably wouldn't have undertaken the writing of the present notes. However, as time passed, and I became more aware of the many differences between mine and the HH text's views on high-school mathematics, I decided that there might be some value in trying to codify my own personal experiences into an advanced mathematics textbook accessible by and interesting to a relatively advanced high-school student, without being constrained by the idiosyncracies of the formal IB Further Mathematics curriculum. This allowed me to freely draw from my experiences first as a research mathematician and then as an AP/IB teacher to weave some of my all-time favorite mathematical threads into the general narrative, thereby giving me (and, I hope, the students) better emotional and

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[1]Peter Blythe, Peter Joseph, Paul Urban, David Martin, Robert Haese, and Michael Haese, MATHEMATICS FOR THE INTERNATIONAL STUDENT; MATHEMATICS HL (OPTIONS), Haese and Harris Publications, 2005, Adelaide, ISBN 1 876543 33 7

intellectual rapport with the contents. I can only hope that the readers
(if any) can find some something of value by the reading of my stream-
of-consciousness narrative.

The basic layout of my notes originally was constrained to the five
option themes of IB: geometry, discrete mathematics, abstract alge-
bra, series and ordinary differential equations, and inferential statistics.
However, I have since added a short chapter on inequalities and con-
strained extrema as they amplify and extend themes typically visited
in a standard course in Algebra II. As for the IB option themes, my
organization differs substantially from that of the HH text. Theirs is
one in which the chapters are independent of each other, having very
little articulation among the chapters. This makes their text especially
suitable for the teaching of any given option topic within the context
of IB mathematics HL. Mine, on the other hand, tries to bring out
the strong interdependencies among the chapters. For example, the
HH text places the chapter on abstract algebra (Sets, Relations, and
Groups) before discrete mathematics (Number Theory and Graph The-
ory), whereas I feel that the correct sequence is the other way around.
Much of the motivation for abstract algebra can be found in a variety
of topics from both number theory and graph theory. As a result, the
reader will find that my Abstract Algebra chapter draws heavily from
both of these topics for important examples and motivation.

As another important example, HH places Statistics well before Se-
ries and Differential Equations. This can be done, of course (they did
it!), but there's something missing in inferential statistics (even at the
elementary level) if there isn't a healthy reliance on analysis. In my or-
ganization, this chapter (the longest one!) is the very last chapter and
immediately follows the chapter on Series and Differential Equations.
This made more natural, for example, an insertion of a theoretical
subsection wherein the density of two independent continuous random
variables is derived as the convolution of the individual densities. A
second, and perhaps more relevant example involves a short treatment
on the "random harmonic series," which dovetails very well with the
already-understood discussions on convergence of infinite series. The
cute fact, of course, is that the random harmonic series converges with
probability 1.

I would like to acknowledge the software used in the preparation of these notes. First of all, the typesetting itself made use of the industry standard, LaTeX, written by Donald Knuth. Next, I made use of three different graphics resources: *Geometer's Sketchpad*, *Autograph*, and the statistical workhorse *Minitab*. Not surprisingly, in the chapter on Advanced Euclidean Geometry, the vast majority of the graphics was generated through Geometer's Sketchpad. I like Autograph as a general-purpose graphics software and have made rather liberal use of this throughout these notes, especially in the chapters on series and differential equations and inferential statistics. Minitab was used primarily in the chapter on Inferential Statistics, and the graphical outputs greatly enhanced the exposition. Finally, all of the graphics were converted to PDF format via ADOBE® ACROBAT® 8 PROFESSIONAL (version 8.0.0). I owe a great debt to those involved in the production of the above-mentioned products.

Assuming that I have already posted these notes to the internet, I would appreciate comments, corrections, and suggestions for improvements from interested colleagues and students alike. The present version still contains many rough edges, and I'm soliciting help from the wider community to help identify improvements.

Naturally, my greatest debt of gratitude is to the eleven students (shown to the right) I conscripted for the class. They are (back row): Eric Zhang (Harvey Mudd), Jong-Bin Lim (University of Illinois), Tiimothy Sun (Columbia University), David Xu (Brown University), Kevin Yeh (UC Berkeley), Jeremy Liu (University of Virginia); (front row): Jong-Min Choi (Stanford University), T.J. Young (Duke University), Nicole Wong (UC Berkeley), Emily Yeh (University of Chicago), and Jong Fang (Washington University). Besides providing one of the most stimulating teaching environments I've enjoyed over

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