# ps1

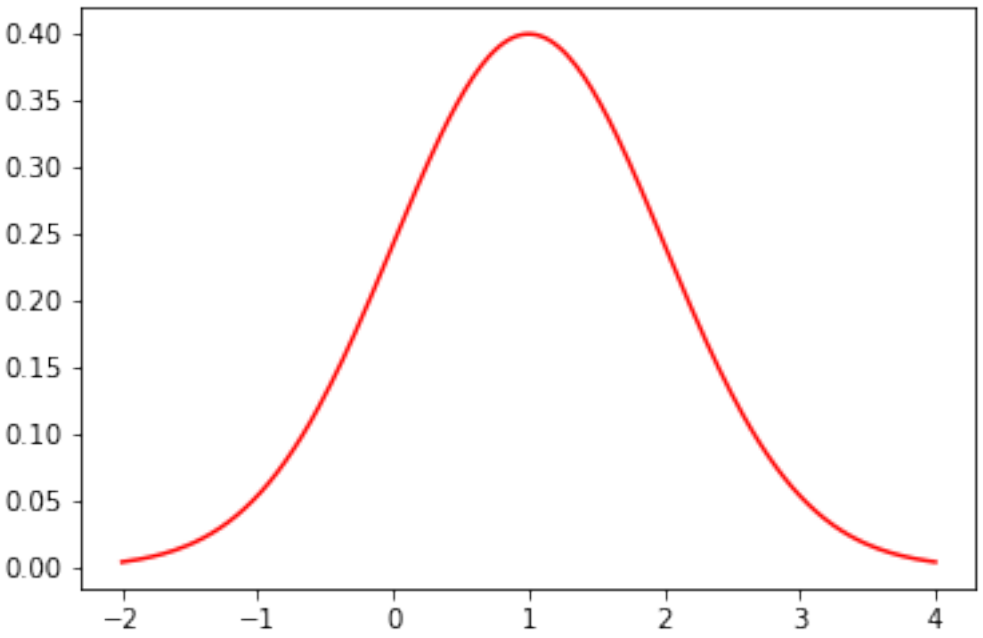
November 11, 2018

In [75]:
```
from scipy.stats import norm
import numpy as np
import matplotlib.pyplot as plt
from math import *
```

### 0.0.1 Task 1.A

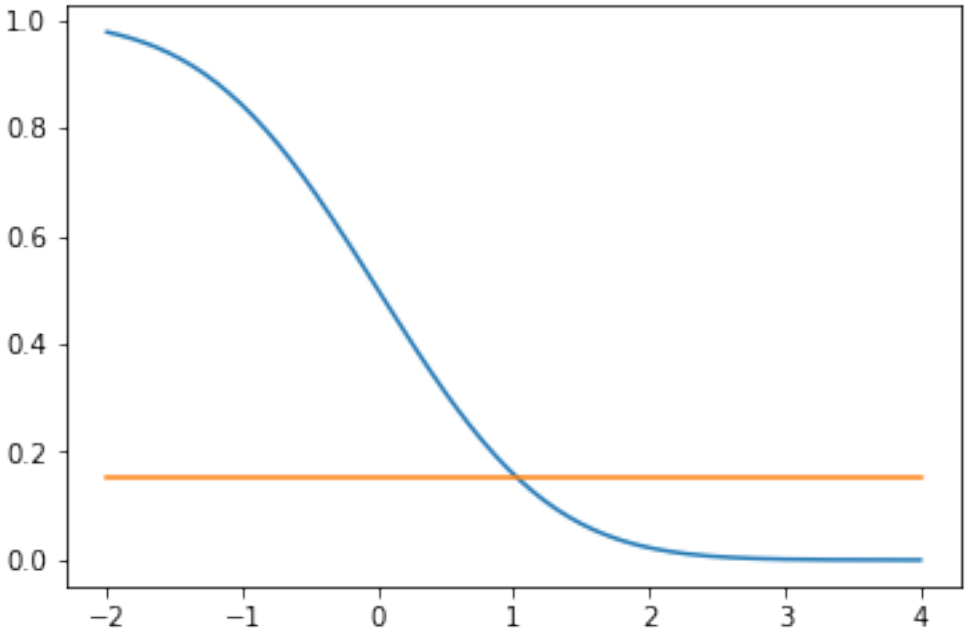
In [76]:
```
fig, ax = plt.subplots(1, 1)
mx = 1; sx = 1
x = np.linspace(-2,4,200)
p =  norm.pdf(x, mx, sx)
plt.plot(x, p, 'r-', label='norm pdf')
```

Out[76]:
```
[<matplotlib.lines.Line2D at 0x7fcfa668ac88>]
```



### 0.0.2 Task 1.B

```
In [77]: fig, ax = plt.subplots(1, 1)
         i1 = list(x).index(1.0150753768844218)
         F = 1-norm.cdf(x)
         ax.plot(x, F )
         ax.plot(x, F[i1]*np.ones_like(x) )
         plt.show()
         print("F(x>1)="+str(F[i1]))
```




```
F(x>1)=0.15503494900490788
```

### 0.0.3 Task 1.C

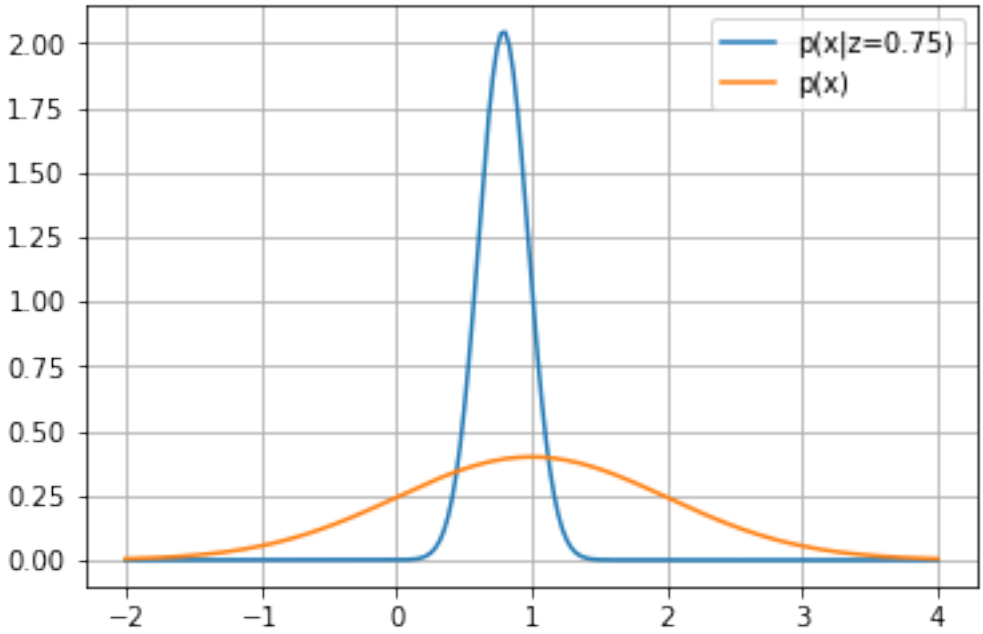
p(x | z=0.75) = p(0.75 | x) * p(x) / p(0.75)

```
In [78]: def pdf(x, mx, sigma):
             p = exp(-(x-mx)**2/2)/sqrt(2*pi*sigma**2)
             return p
```

```
In [79]: Z = 0.75
         sigma = sqrt(0.2)
         N = norm.pdf(0.75, x, sigma**2)
         p_x_given_z = N * norm.pdf(x,mx,sigma) / norm.pdf(0.75,mx,sigma)
         plt.plot(x, p_x_given_z)
```

```
        plt.plot(x, p)
        plt.grid()
        plt.legend(['p(x|z=0.75)', 'p(x)'])
```

```
Out[79]: <matplotlib.legend.Legend at 0x7fcf9fea0d30>
```




### 0.0.4 Task 1.D

```
In [80]: E = round( x[ list(p_x_given_z).index(max(p_x_given_z)) ],3)
         print("Estimate the expected value of the posterior distance to the wall"+"\n"+"E(x|0.7
```

```
Estimate the expected value of the posterior distance to the wall
E(x|0.75)=0.804
```

### 0.0.5 Task 1.E

Product rule: p(x,z) = p(x | z) * p(z)

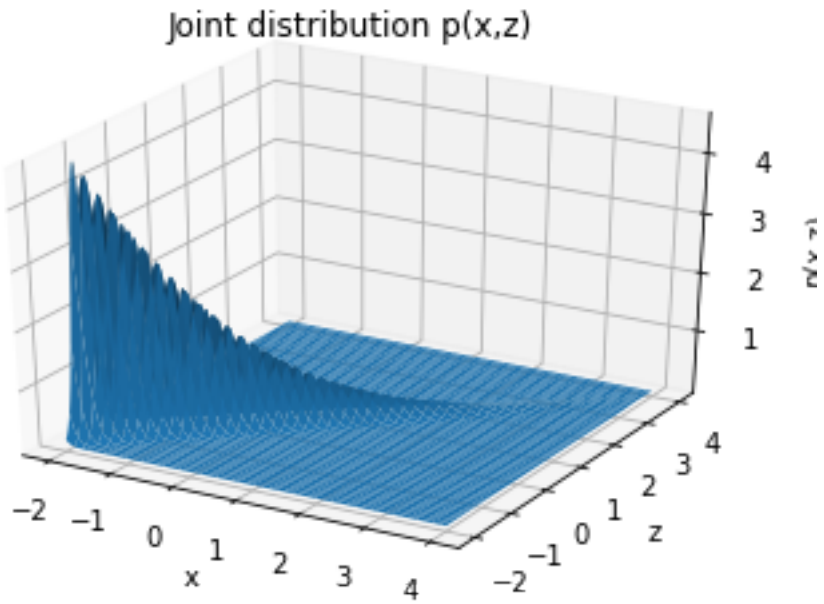
```
In [81]: def condprob(x,z=0.75,mx=1,sigma=sqrt(0.2)):
             N = norm.pdf(z, x, sigma**2)
             p_x_given_z = N * norm.pdf(x,mx,sigma) / norm.pdf(z,mx,sigma)
             return p_x_given_z
```

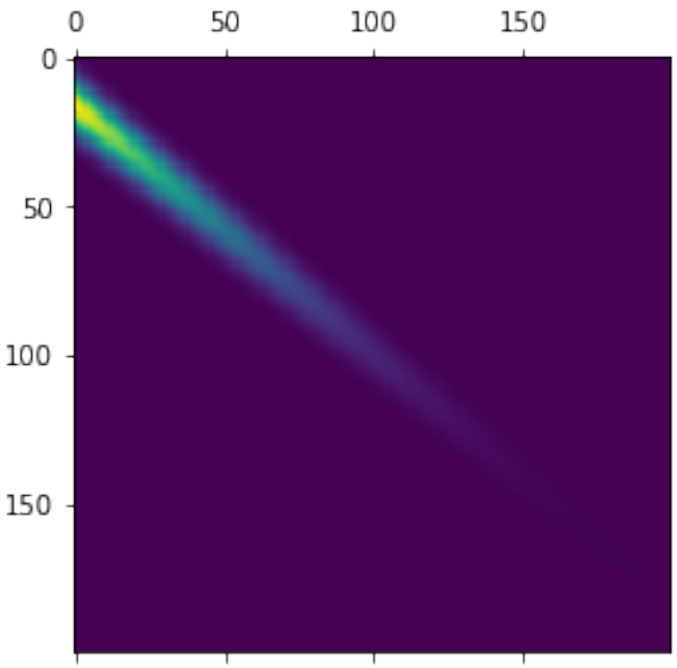
```
In [82]: z = x
         p_joint = np.zeros((len(x), len(z)))
```

```
        for X in range(len(x)):
            for Z in range(len(z)):
                p_joint[X,Z] = condprob(x[X],z[Z]) * norm.pdf(z[Z])
```

In [83]:
```
X = np.zeros((len(x),len(z)))
Z = np.zeros((len(x), len(z)))
for i in range(len(x)):
    X[i,:] = x
    Z[i,:] = z
```

In [92]:
```
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.gca(projection='3d')
ax.set_xlabel('x')
ax.set_ylabel('z')
ax.set_zlabel('p(x,z)')

X, Z = np.meshgrid(x, z)

# Plot the surface.
surf = ax.plot_surface(X, Z, p_joint)
plt.title('Joint distribution p(x,z)')
plt.matshow(p_joint)
plt.show()
```



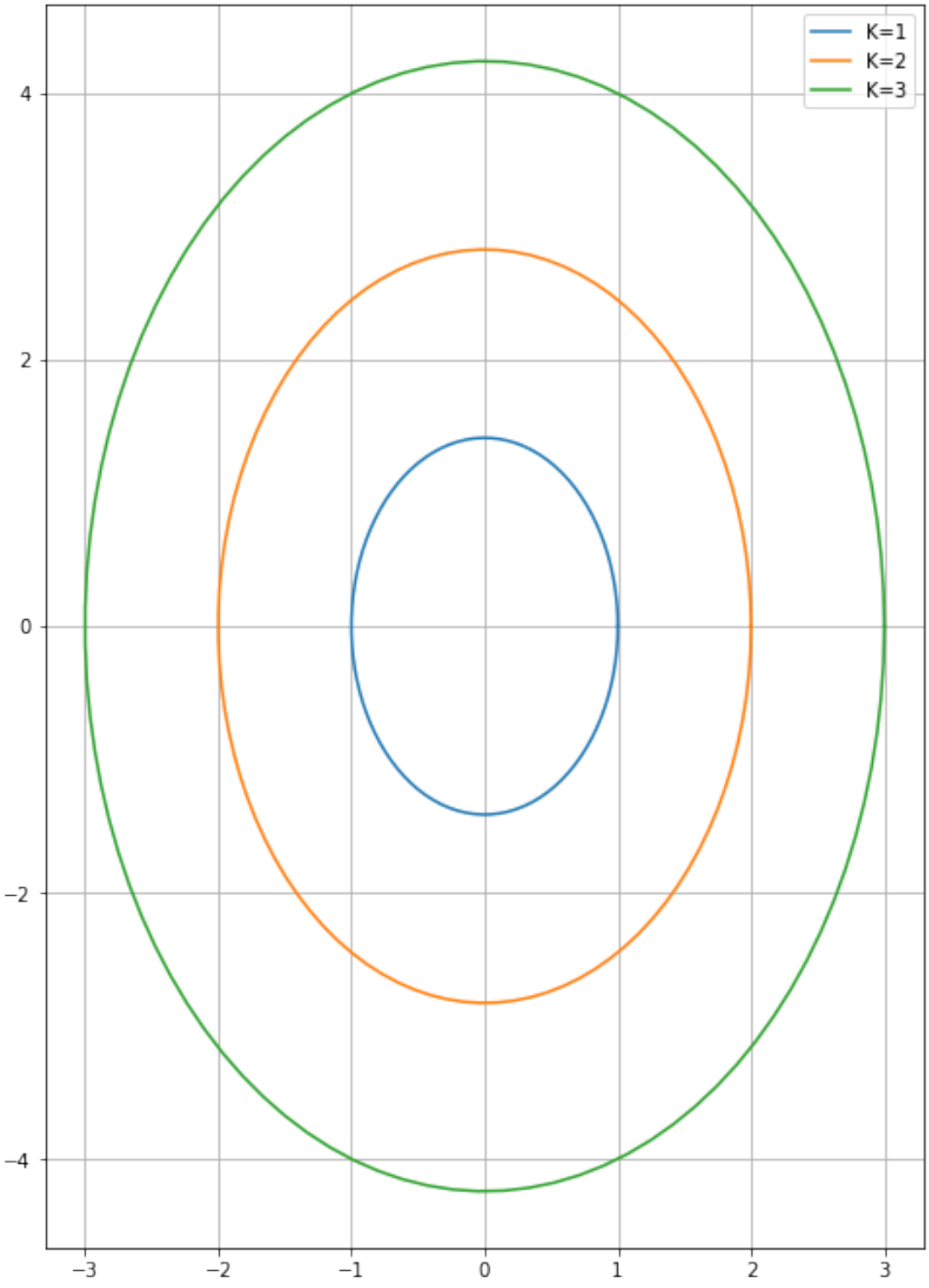



### 0.0.6 Task 2.A

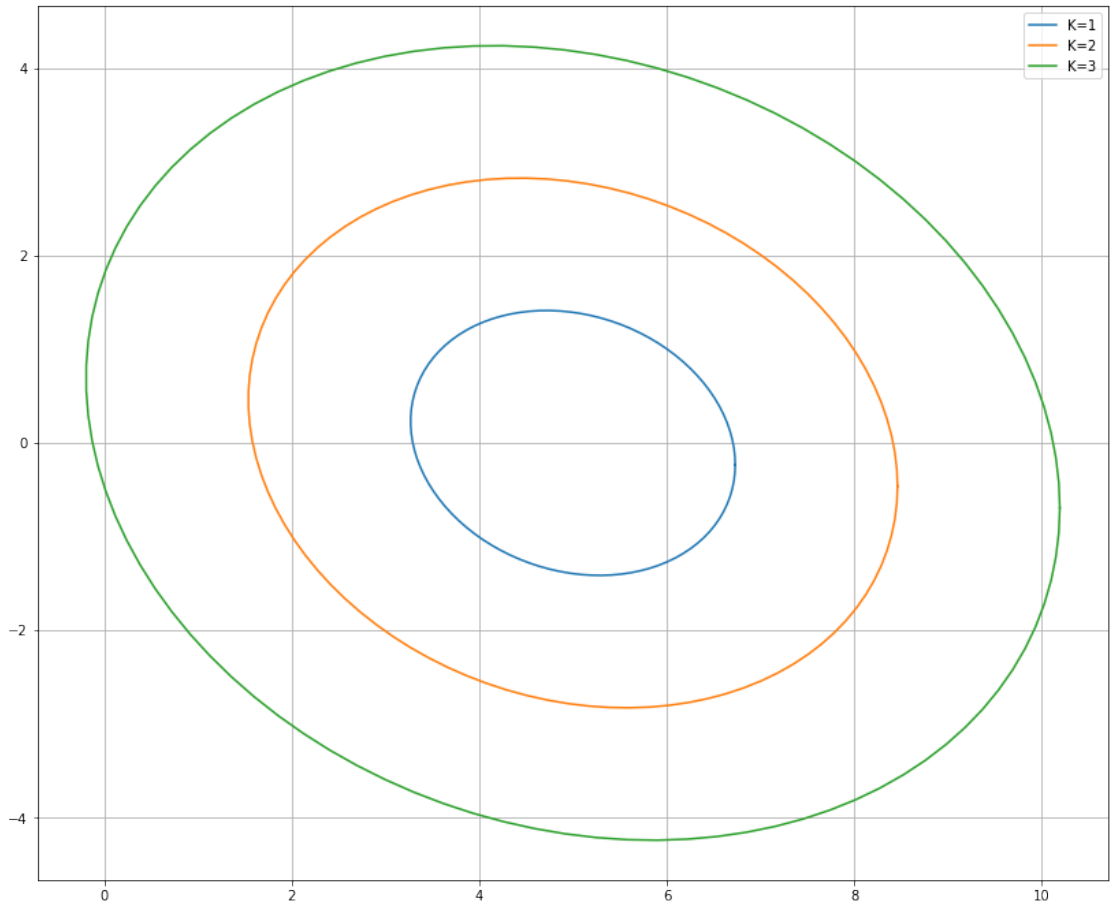
```
In [12]: from scipy.linalg import cholesky

         def plot2dcov(mean, sigma2D, k=1, n_samples=100):
             sigma2D = np.array(sigma2D)
             L = cholesky(sigma2D).T
             t = np.linspace(0,2*pi,n_samples)
             xe = np.array([]); ye = np.array([])
             for T in t:
                 [Xe, Ye] = np.dot(L, k*np.array([cos(T),sin(T)]))
                 xe=np.append(xe,Xe); ye=np.append(ye,Ye)
             xe += mean[0]; ye += mean[1]

             return xe, ye

In [13]: M1 = [0,0]; E1 = np.array([[1,0], [0,2]])
         M2 = [5,0]; E2 = np.array([[3,-0.4], [-0.4,2]])
         M3 = [2,2]; E3 = np.array([[9.1,6], [6,4]])

In [14]: def plotellipse(M,E, K=[1]):
             fig = plt.figure()
             fig.set_size_inches(40, 40)
             ax = fig.add_subplot(311, aspect='equal')
             for i in range(len(K)):
```

```
            xe, ye = plot2dcov(mean=M, sigma2D=E, k=K[i])
            plt.plot(xe,ye)
        plt.grid()
        plt.legend(['K=1','K=2', 'K=3'])
        plt.show()
```

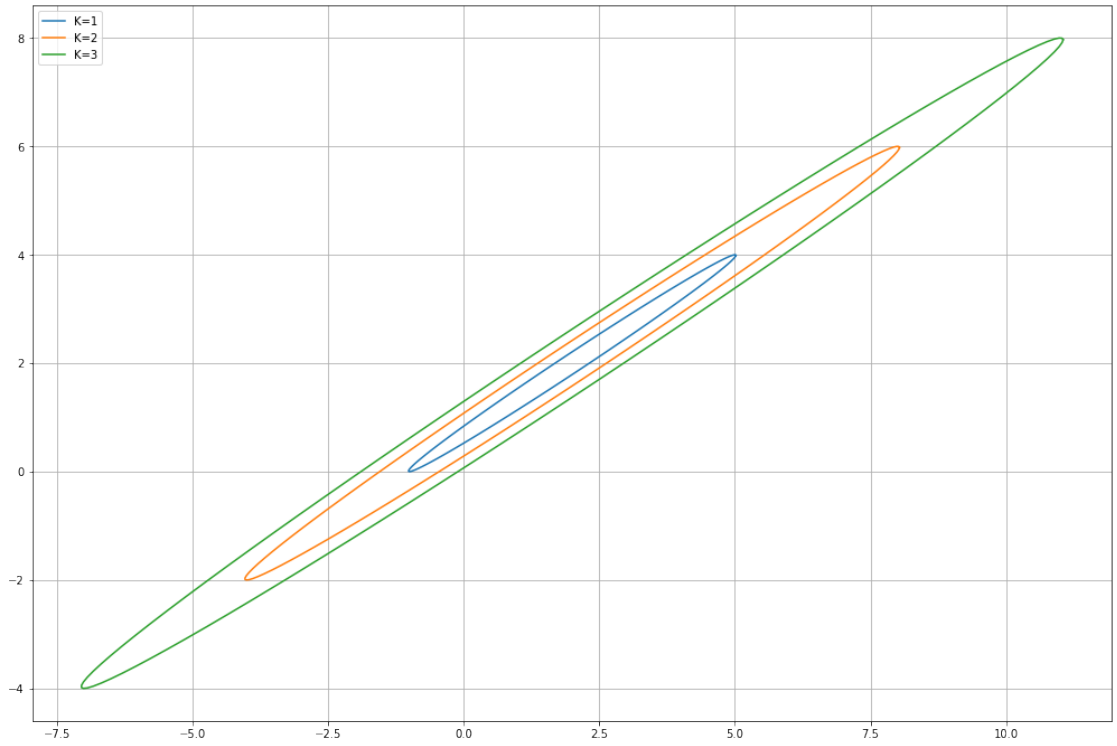
In [15]:
```
plotellipse(M1,E1, K=[1,2,3])
```




In [16]: plotellipse(M2,E2, K=[1,2,3])




```
In [17]: plotellipse(M3,E3, K=[1,2,3])
```




### 0.0.7 Task 2.B

Sample mean:

$$\bar{x} = \frac{1}{N}\sum_{i=0}^{N}\bar{x}_i$$

```
In [18]: def sample_mean(samples):
             sample_mean = np.zeros((len(samples),1))
             for i in range(len(samples)):
                 sample_mean[i] = np.mean(samples[i])
             return sample_mean
```

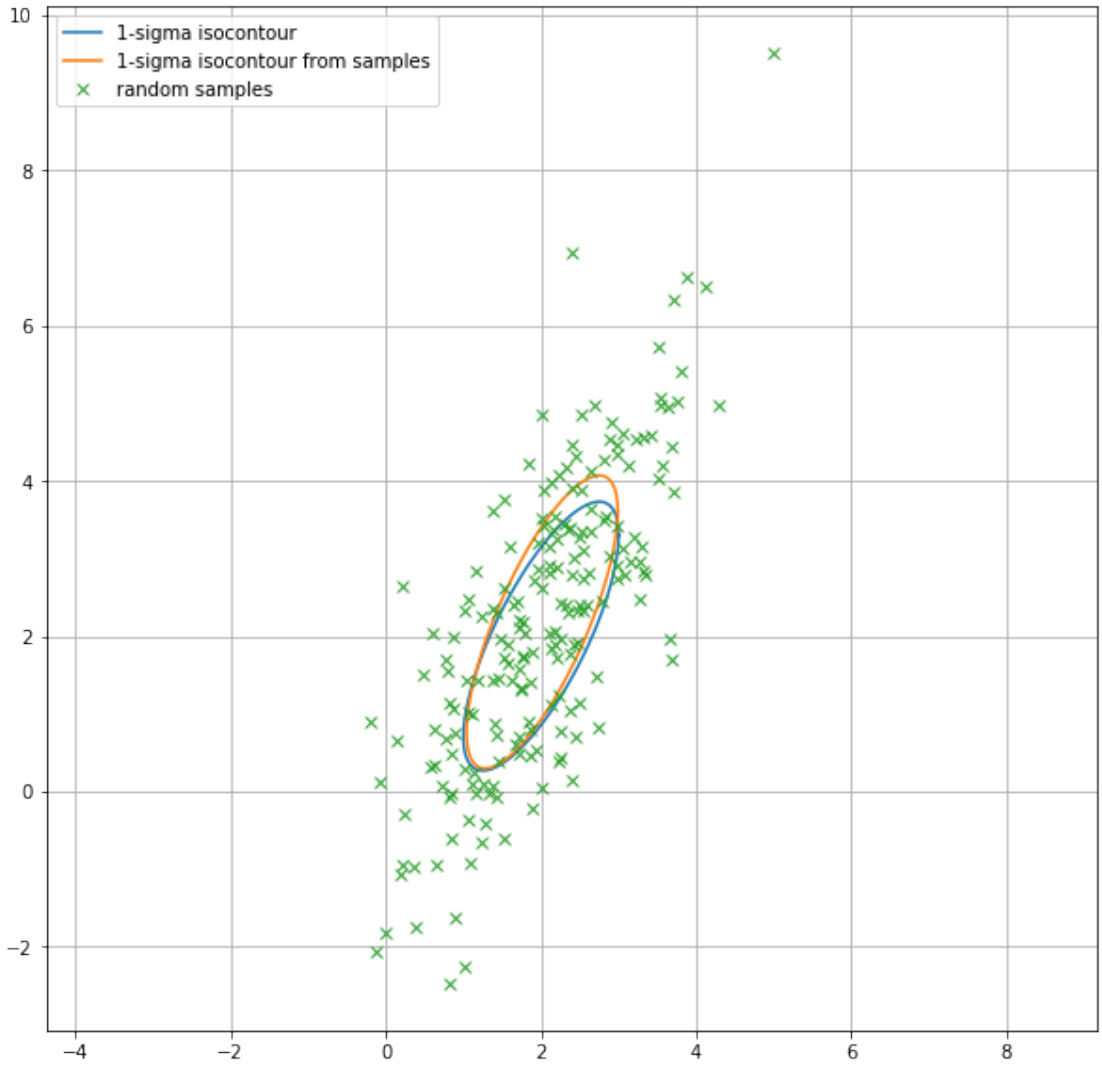
Sample covariance matrix:

$$Q = \frac{1}{N-1}\sum_{i=1}^{N}(\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})^T$$

```
In [19]: def sample_cov(samples):
             Q = np.cov(samples)
             return Q
```

### 0.0.8 Task 2.C

```
In [20]: mean = np.array([2,2]).T
         cov = np.array([[1, 1.3], [1.3, 3]])

         fig = plt.figure()
         fig.set_size_inches(10, 10)
         xe, ye = plot2dcov(mean=mean, sigma2D=cov, k=[1])
         plt.plot(xe,ye)
         n_samples = 200
         x, y = np.random.multivariate_normal(mean, cov, n_samples).T
         M = sample_mean([x,y])
         Q = sample_cov([x,y])
         xs, ys = plot2dcov(mean=M, sigma2D=Q, k=[1])
         plt.plot(xs,ys)
         plt.plot(x, y, 'x')
         plt.axis('equal')
         plt.grid()
         plt.legend(["1-sigma isocontour", "1-sigma isocontour from samples", "random samples"])
         plt.show()
         print("mean = "+str(np.round(M,2)))
         print("covariance matrix:\nQ = "+str(np.round(Q,2)))
```




```
mean = [[2.01]
 [2.18]]
covariance matrix:
Q = [[0.95 1.39]
 [1.39 3.56]]
```

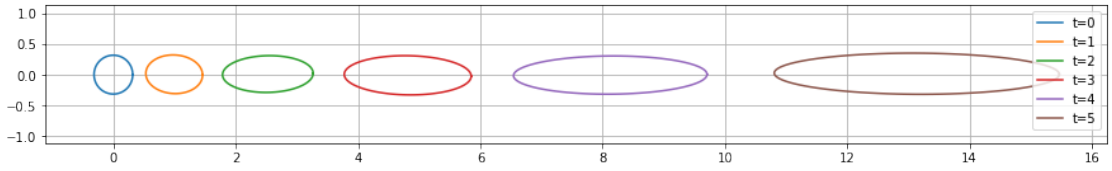
| $N_{samples}$ | 100 | | 200 | | 300 | | 400 | | 500 | | 1000 | | 10000 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean | 1.93 | | 1.93 | | 2.0 | | 2.05 | | 2.0 | | 2.03 | | 2.02 | |
| | 1.73 | | 1.87 | | 2.06 | | 2.27 | | 2.01 | | 2.1 | | 2.05 | |
| Cov | 1.16 | 1.44 | 1.13 | 1.42 | 0.93 | 1.16 | 0.88 | 1.07 | 1.06 | 1.39 | 0.94 | 1.19 | 1.07 | 1.35 |
| | 1.44 | 2.98 | 1.42 | 3.09 | 1.16 | 3.02 | 1.07 | 2.38 | 1.39 | 3.19 | 1.19 | 2.86 | 1.35 | 2.96 |

### 0.0.9 Task 3.A

$$\begin{bmatrix} x \\ y \end{bmatrix}_t = \begin{bmatrix} 1+\delta t & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_{t-1} + \begin{bmatrix} 2\delta t \\ 0 \end{bmatrix}_t$$

```
In [21]: s0 = np.array([0,0]).T
         n_samples = 500
         mean0 = s0; cov0 = np.array([[0.1, 0], [0, 0.1]])
         dt = 0.5
         A = np.array([[1+dt,0],[0,1]]); b = np.array([2*dt, 0]).T
         xs, ys = plot2dcov(mean=mean0, sigma2D=cov0, k=[1])
         fig = plt.figure()
         fig.set_size_inches(15, 2)
         plt.plot(xs, ys)
         legend_list=["t=0"]
         for i in range(5):
             s = np.dot( A, s0) + b
             s0 = s
             mean = np.dot(A, mean0) + b
             mean0 = mean
             cov = np.dot( np.dot(A, cov0), A.T )
             cov0 = cov

             x, y = np.random.multivariate_normal(mean, cov, n_samples).T
             M = sample_mean([x,y])
             Q = sample_cov([x,y])
             xs, ys = plot2dcov(mean=M, sigma2D=Q, k=[1])
             legend_list.append("t="+str(i+1))
             plt.plot(xs, ys)
             plt.legend(legend_list)
             plt.axis('equal')
             plt.grid()
             print(mean[0])
```

```
1.0
2.5
4.75
8.125
13.1875
```

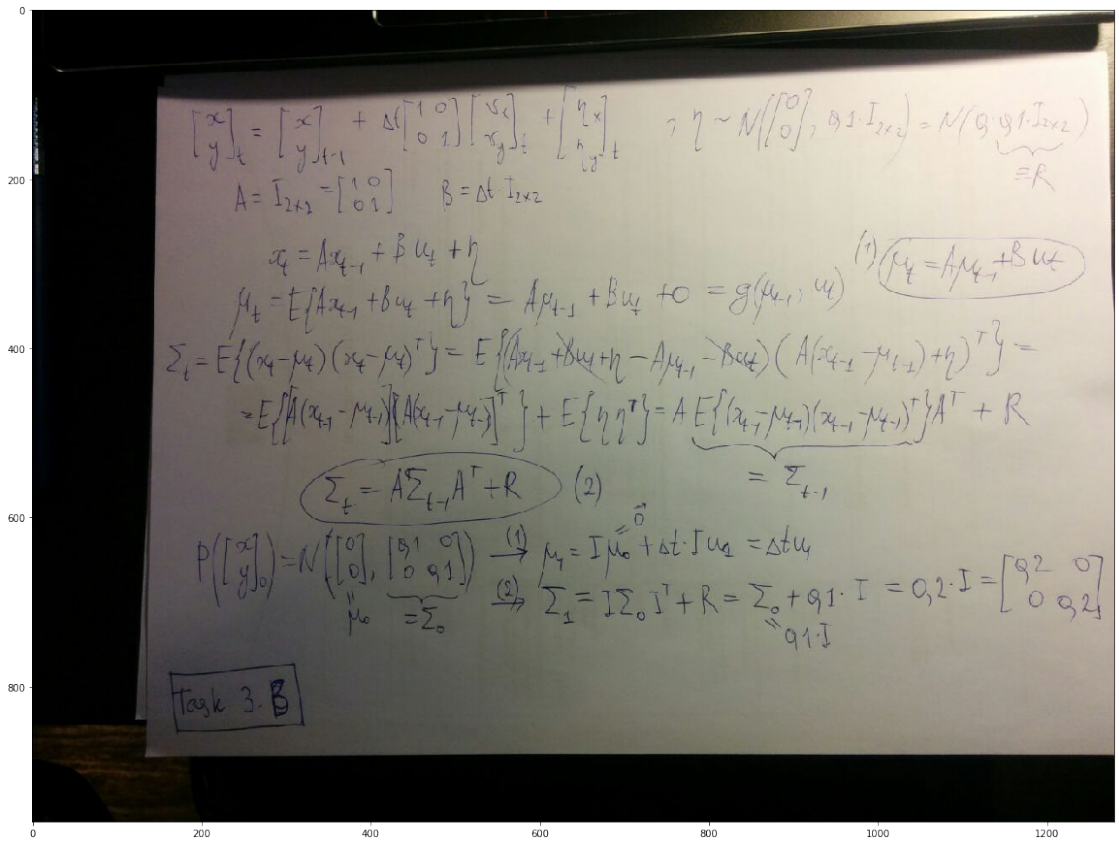


### 0.0.10 Task 3.B

```
In [22]: import matplotlib.image as mpimg
```

```
In [23]: def show_image(name):
             img=mpimg.imread(name)
             fig = plt.figure(figsize=(20,20))
             imgplot = plt.imshow(img)
             plt.show()
```

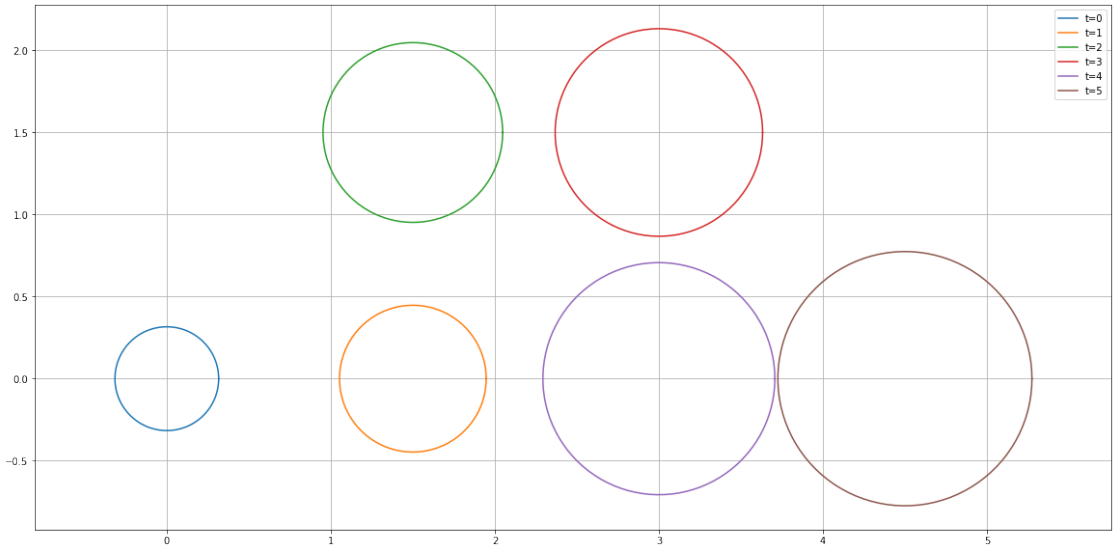
```
In [24]: show_image('task3B.jpg')
```




### 0.0.11 Task 3.C

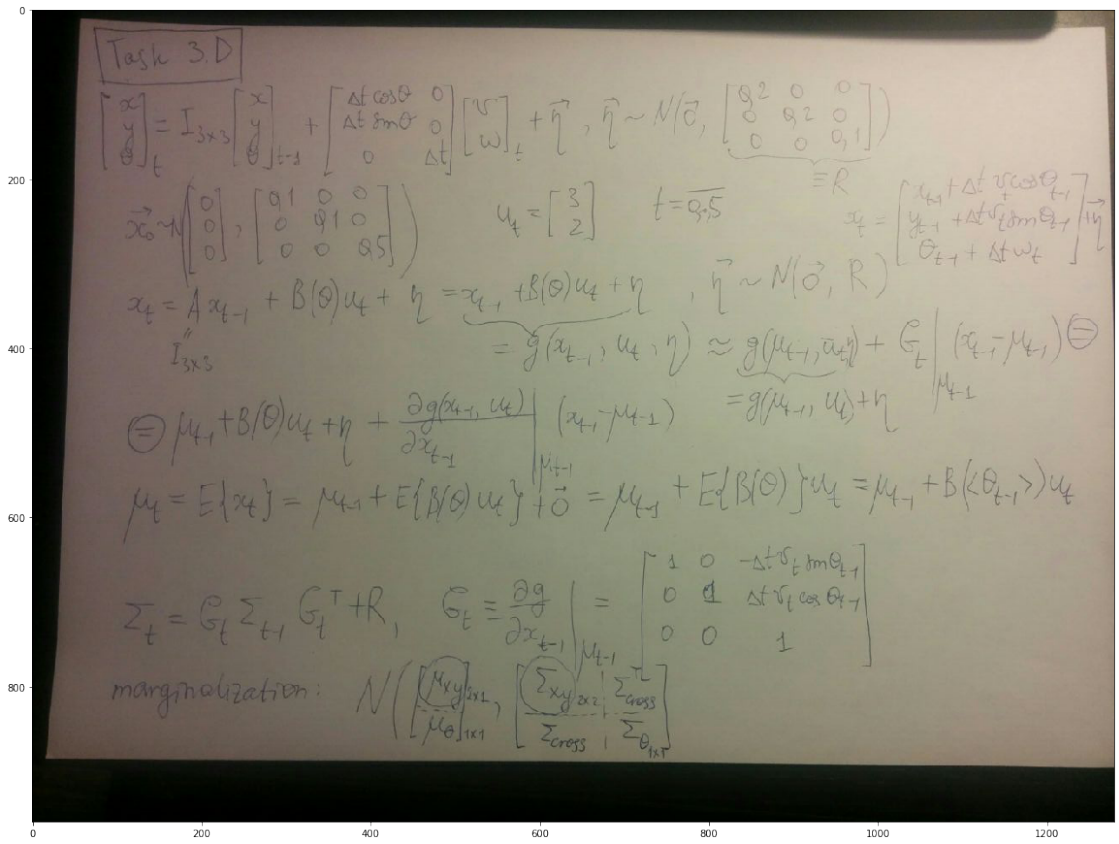
```
In [25]: u = np.array([np.array([[3],[0]]),
                       np.array([[0],[3]]),
                       np.array([[3],[0]]),
                       np.array([[0],[-3]]),
                       np.array([[3],[0]])])
         A = np.eye(2)
         B = dt * np.eye(2)
```

```
R = 0.1 * np.eye(2)

mean0 = np.zeros((2,1))
cov0 = 0.1 * np.eye(2)

plt.figure(figsize=(20,10))
legend_list=[]
xs, ys = plot2dcov(mean0, cov0, k=[1])
legend_list.append("t="+str(0))
plt.plot(xs, ys)
plt.legend(legend_list)
plt.axis('equal')
plt.grid()
for i in range(len(u)):
    mean_u = u[i]
    mean = np.dot( A, mean0 ) + np.dot( B, mean_u )
    mean0 = mean

    cov = cov0 + R
    cov0 = cov

    xs, ys = plot2dcov(mean, cov, k=[1])
    legend_list.append("t="+str(i+1))
    plt.plot(xs, ys)
    plt.legend(legend_list)
    plt.axis('equal')
```



### 0.0.12 Task 3.D

In [26]: show_image('task3D.jpg')

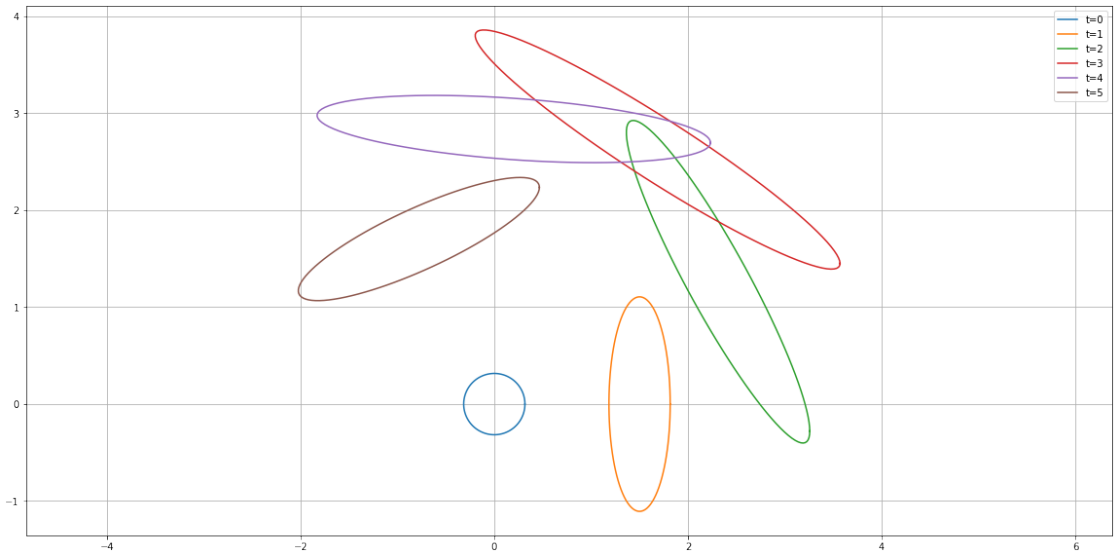



In [27]:
```
def control_matrix(dt, theta):
    return np.array([[dt*cos(theta), 0], [dt*sin(theta), 0], [0, dt]])
```

In [28]:
```
def jacobian(dt, v, theta):
    return np.array([[1, 0, -dt*v*sin(theta)],
                     [0, 1, dt*v*cos(theta)],
                     [0, 0, 1              ]])
```

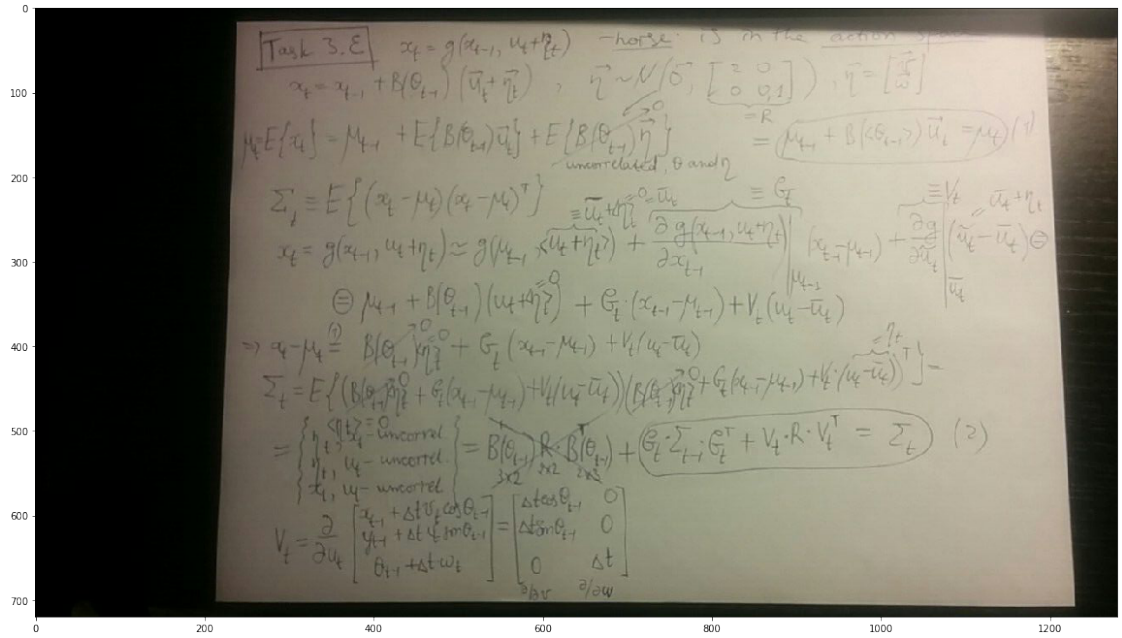
In [29]:
```
A = np.eye(3)
theta0 = 0
B = control_matrix(dt, theta0)
mean0 = np.zeros((3,1))
cov0 = np.diag([0.1, 0.1, 0.5])
R = np.diag([0.2, 0.2, 0.1])
u = np.array([[3],[2]])

legend_list = []
```

```
figure = plt.figure(figsize=(20,10))
x, y = plot2dcov(mean0[:2], cov0[:2,:2], k=[1])
legend_list.append("t="+str(0))
plt.plot(x, y)
plt.legend(legend_list)
plt.axis('equal')
for i in range(5):
    mean = mean0 + np.dot( control_matrix(dt, mean0[2]), u)
    G = jacobian(dt, u[0], mean0[2])
    cov = np.dot( np.dot( G, cov0 ), G.T )
    mean0 = mean
    cov0 = cov

    x, y = plot2dcov(mean[:2], cov[:2,:2], k=[1])
    legend_list.append("t="+str(i+1))
    plt.plot(x, y)
    plt.legend(legend_list)
    plt.axis('equal')
plt.grid()
```




### 0.0.13 Task 3.E

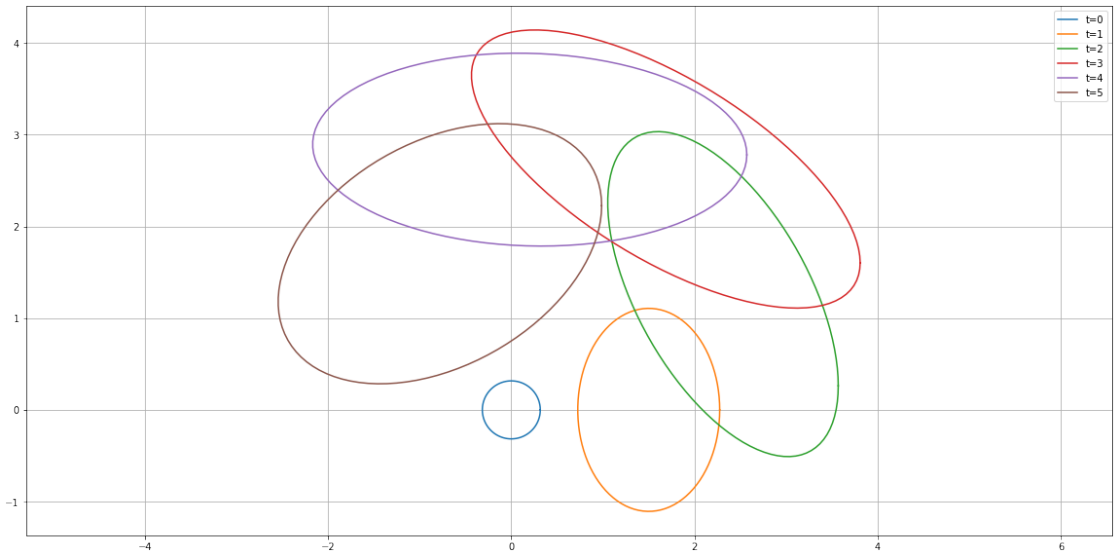
```
In [30]: show_image('task3E.jpg')
```




```
In [31]: def jacobianV(dt, theta):
             return np.array([[dt*cos(theta), 0],
                              [dt*sin(theta), 0],
                              [0,             dt]])
```

```
In [32]: A = np.eye(3)
         theta0 = 0
         B = control_matrix(dt, theta0)
         mean0 = np.zeros((3,1))
         cov0 = np.diag([0.1, 0.1, 0.5])
         R = np.diag([2, 0.1])
         u = np.array([[3],[2]])

         legend_list = []
         figure = plt.figure(figsize=(20,10))
         x, y = plot2dcov(mean0[:2], cov0[:2,:2], k=[1])
         legend_list.append("t="+str(0))
         plt.plot(x, y)
         plt.legend(legend_list)
         plt.axis('equal')
         for i in range(5):
             mean = mean0 + np.dot( control_matrix(dt, mean0[2]), u)
             G = jacobian(dt, u[0], mean0[2])
             V = jacobianV(dt, mean0[2])
             cov = np.dot( np.dot( G, cov0 ), G.T ) + np.dot( np.dot( V, R ), V.T )
             mean0 = mean
```

```
        cov0 = cov

        x, y = plot2dcov(mean[:2], cov[:2,:2], k=[1])
        legend_list.append("t="+str(i+1))
        plt.plot(x, y)
        plt.legend(legend_list)
        plt.axis('equal')
    plt.grid()
```

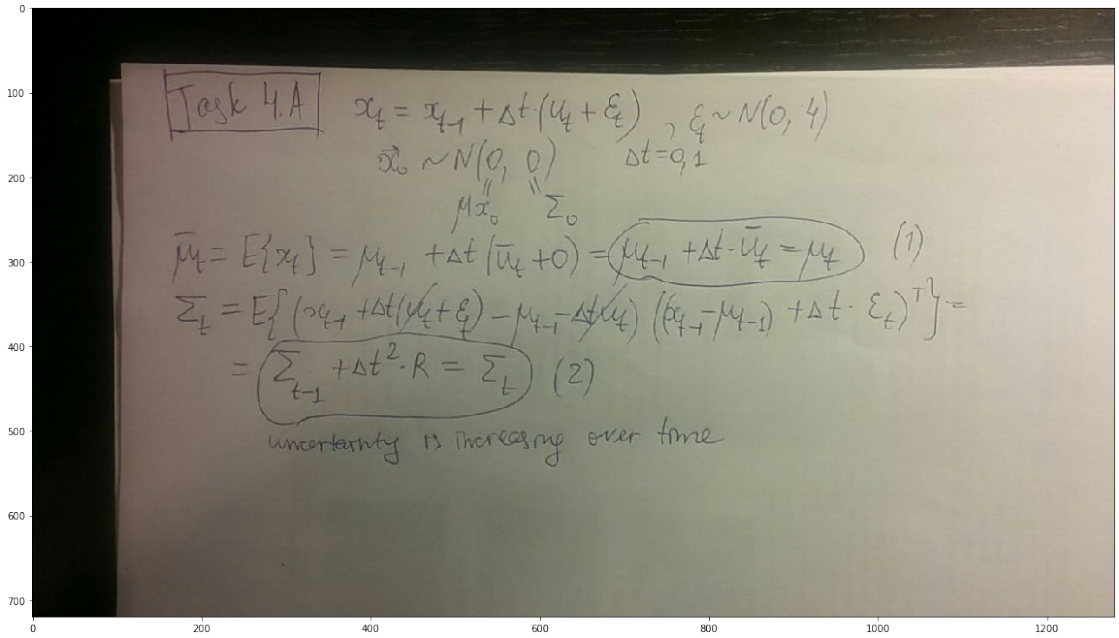



### 0.0.14 Task 4.A

In [33]:
```
data = np.load('t4')
```

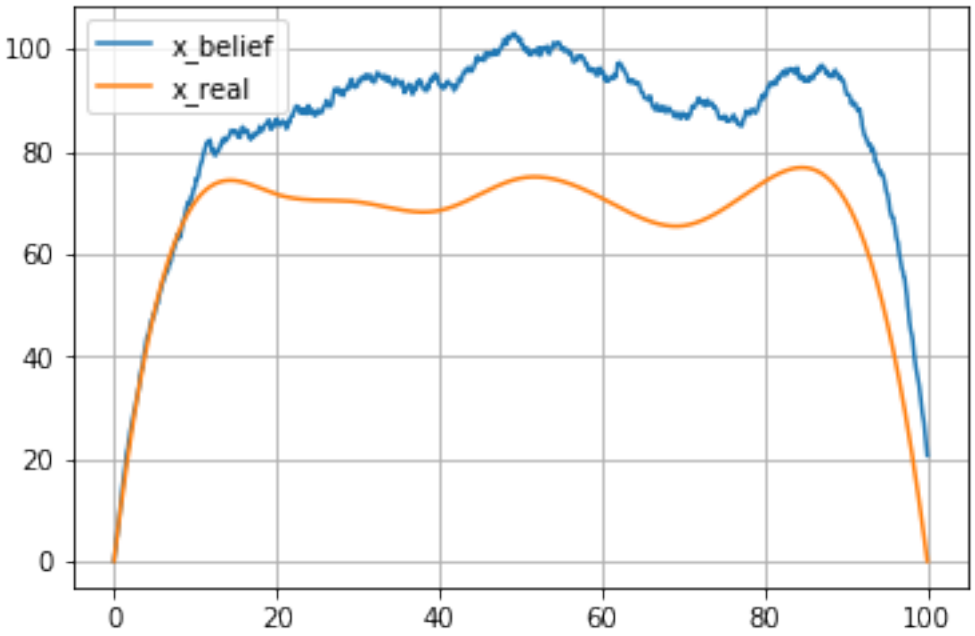
In [34]:
```
dt = float(data['dt'])
N = int(data['N'])
P_0 = float(data['P_0'])
Q = np.float(data['Q'])
R = np.float(data['R_action'])
t = np.array(data['t'])
u = np.array(data['u'])
x_0 = float(data['x_0'])
x_real = np.array(data['x_real'])
z = np.array(data['z'])
```

In [35]:
```
show_image('task4A.jpg')
```




```
In [36]: me = 0; cove =4
         x = [x_0]
         x_prev = x[0]
         for i in range(N-1):
             e = np.random.normal(me, cove)
             x.append(x_prev + dt*(u[i] + e))
             x_prev = x[-1]

         x = np.array(x)
```

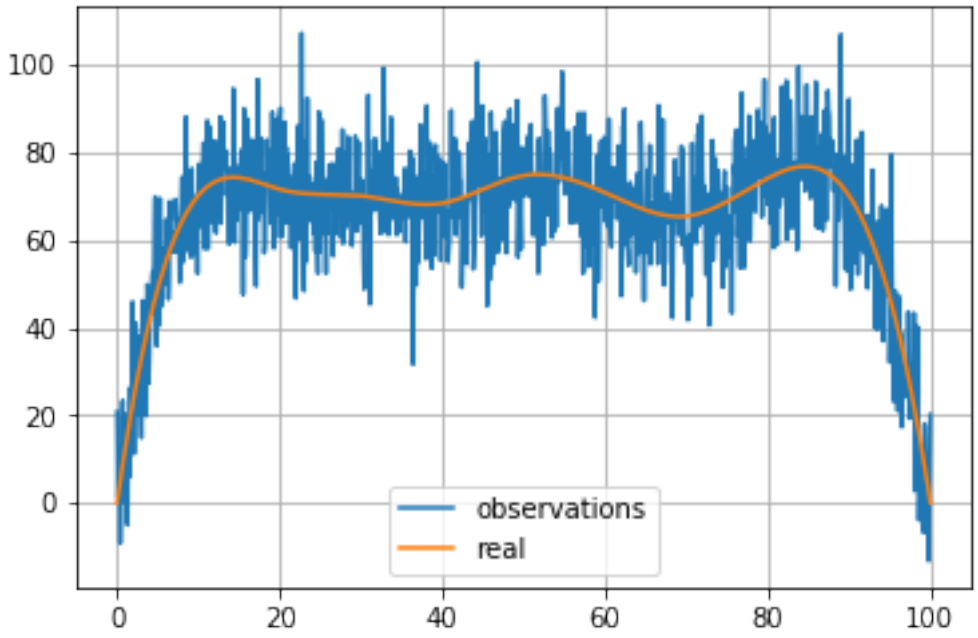
```
In [37]: plt.plot(t,x)
         plt.plot(t,x_real)
         plt.grid()
         plt.legend(['x_belief', 'x_real'])
         plt.show()
```




There is a shift between the 2 graphs: real state and belief based obly on actions information without measurements.

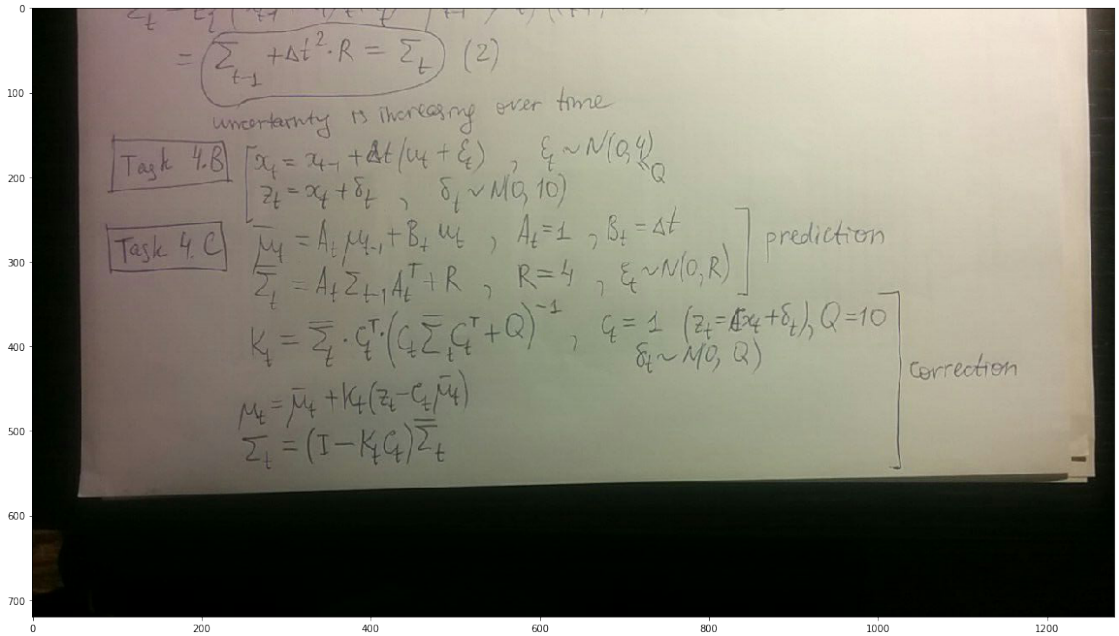
#### 0.0.15 Task 4.B

```
In [38]: md = 0; covd = Q
         z = np.zeros_like(x)
         for i in range(N):
             d = np.random.normal(md, covd)
             z[i] = x_real[i] + d

         plt.plot(t,z)
         plt.plot(t,x_real)
         plt.grid()
         plt.legend(['observations','real'])
         plt.show()
```




### 0.0.16 Task 4.C

In [39]: show_image('task4C.jpg')

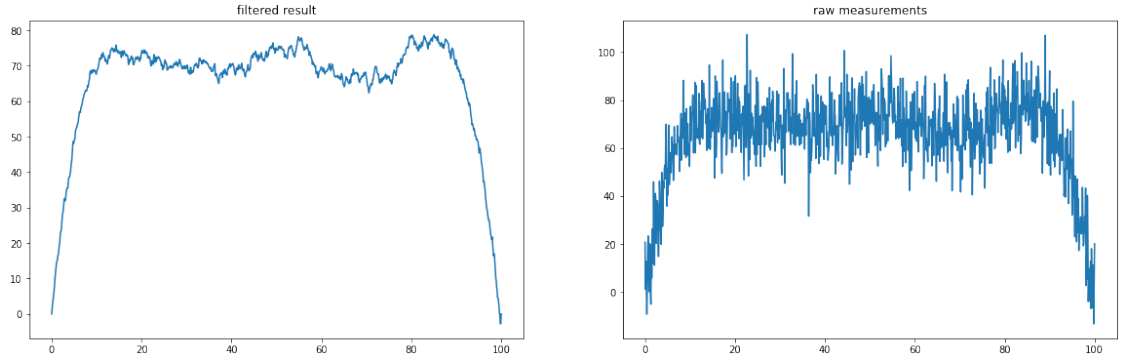


```
In [40]: A = np.array(1)
         B = np.array(dt)
         C = np.array(1)
```

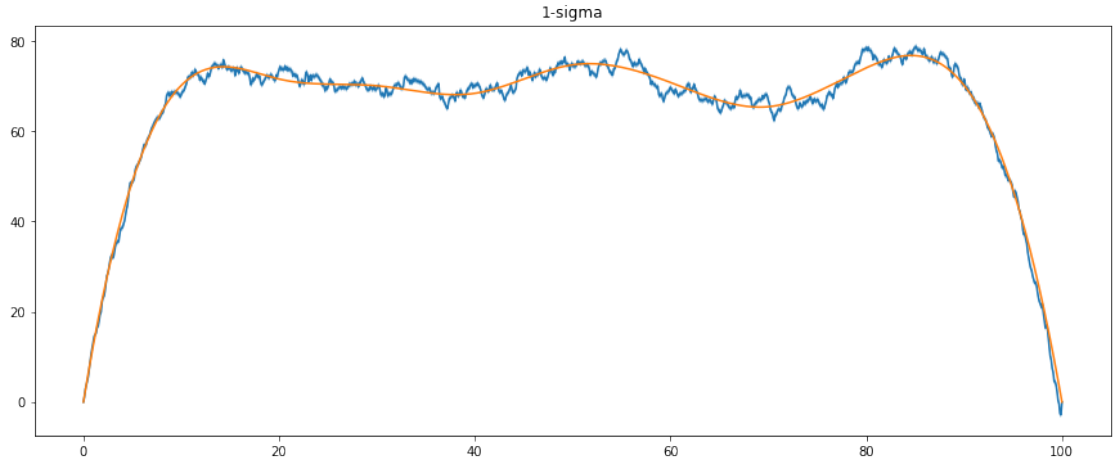
### 0.0.17 Task 4.D

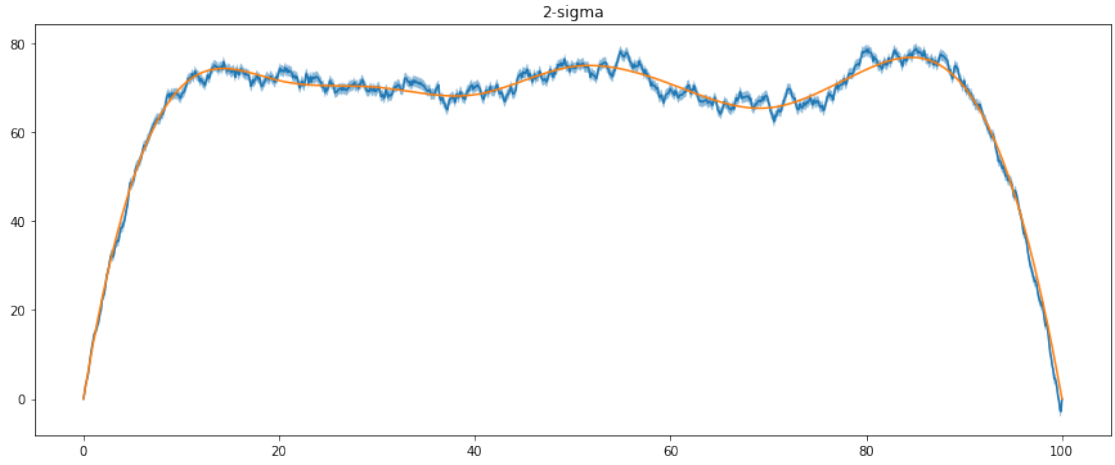
```
In [48]: def kalman(A,B,C,R,Q, u,z, mu0,cov0):
             N = len(z)
             mu_pred = np.zeros_like(z)
             cov_pred = np.zeros_like(z)
             mu = np.zeros_like(z)
             cov = np.zeros_like(z)
             mu[0] = mu0; cov[0] = cov0
             for i in range(1,N-1):
                 # prediction:
                 mu_pred[i] = np.dot(A, mu[i-1]) + np.dot(B, u[i])
                 # cov_pred[i] = np.dot( np.dot(A,cov[i-1]), A.T ) + R
                 cov_pred[i] = cov[i-1] + np.dot( np.dot(B, R), B.T )

                 # correction:
                 if len([mu[i]])<2:
                     K = np.dot( np.dot(cov_pred[i], C.T), 1.0/(np.dot( np.dot(C,cov_pred[i]), C
                 else:
                     K = np.dot( np.dot(cov_pred[i], C.T), np.linalg.inv(np.dot( np.dot(C,cov_pr
                 mu[i] = mu_pred[i] + np.dot( K, (z[i]-np.dot(C,mu_pred[i])) )
                 cov[i] = np.dot( (np.eye(len([mu[i]]))-np.dot(K,C)), cov_pred[i] )

             return mu,cov, mu_pred,cov_pred
```

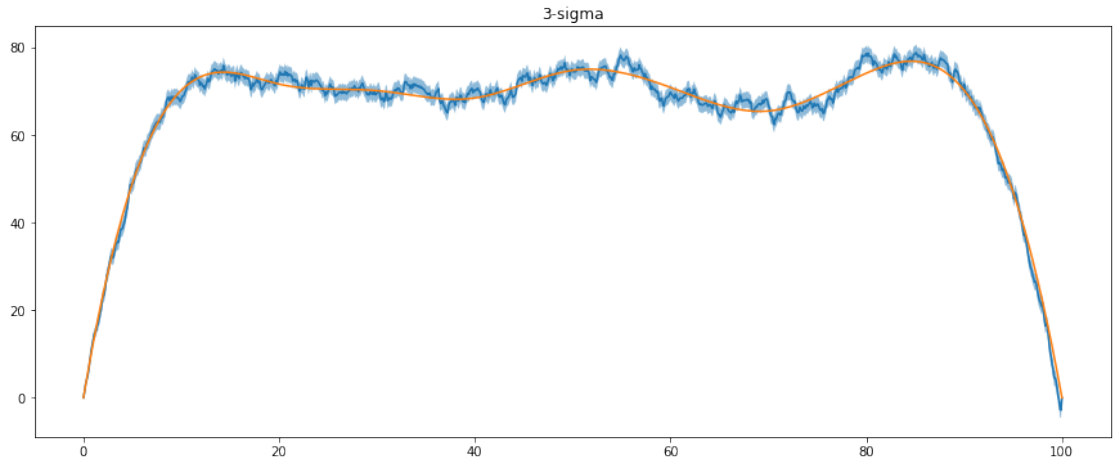
```
In [55]: x_cor, cov_cor, x_pred, cov_pred=kalman(A,B,C,R,Q, u,z, mu0=0,cov0=0)
         fig = plt.figure(figsize=(20,6))
         fig.add_subplot(121)
         plt.plot(t,x_cor)
         plt.title('filtered result')
         fig.add_subplot(122)
         plt.plot(t,z)
         plt.title('raw measurements')
         plt.show()
```



In [56]:
```
from ciplot import *
fig = plt.figure(figsize=(15,20))
fig.add_subplot(311)
ciplot(t, mu=x_cor, minus_sigma=x_cor-cov_cor, plus_sigma=x_cor+cov_cor, x_real=x_real,
plt.title('1-sigma')
fig.add_subplot(312)
ciplot(t, mu=x_cor, minus_sigma=x_cor-2*cov_cor, plus_sigma=x_cor+2*cov_cor, x_real=x_r
plt.title('2-sigma')
fig.add_subplot(313)
ciplot(t, mu=x_cor, minus_sigma=x_cor-3*cov_cor, plus_sigma=x_cor+3*cov_cor, x_real=x_r
plt.title('3-sigma')
plt.show()
```



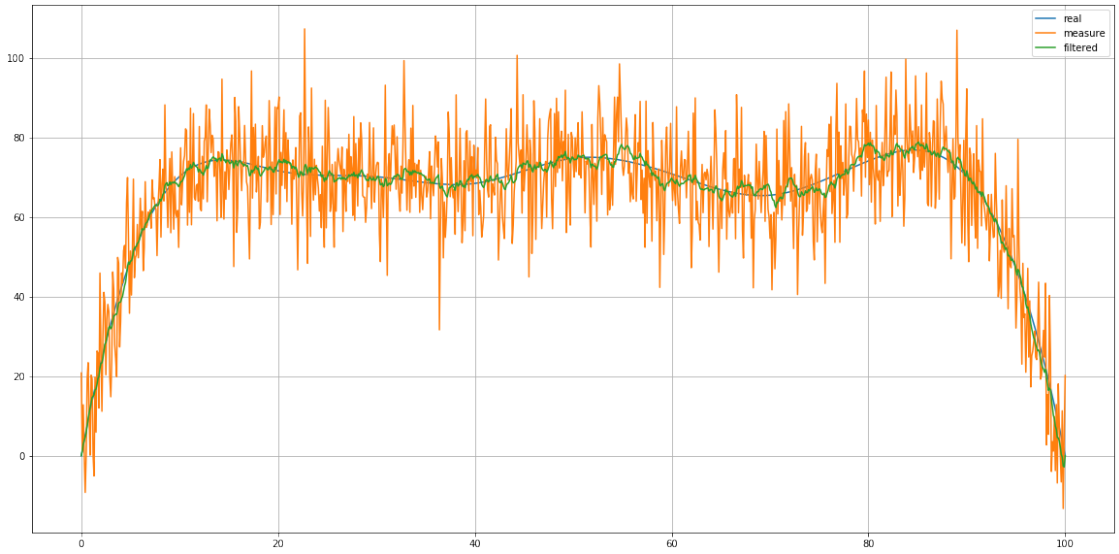




3-sigma interval contains real data completely, while more narrow intervals (1- and 2-sigma) include approximately 68% and 96% of real data.

```
In [57]: fog = plt.figure(figsize=(20,10))
         plt.plot(t,x_real)
```

```
plt.plot(t,z)
plt.plot(t,x_cor)
plt.legend(['real','measure','filtered'])
plt.grid()
plt.show()
```




In comparison to part 4A there is no shift now between filtered data and real state. That is because measurements are now included in the correction step of the Kalman filer.