Review (BI, BII) to PF.

$\boxed{\text{IOE 1610}}$

1) Random variables. for instance: $p(\xi)=$ 'uniform for all t'

Laser $\Omega = \{ t \mid t \in (0,T) \}$ (propag. c ~speed of light.)

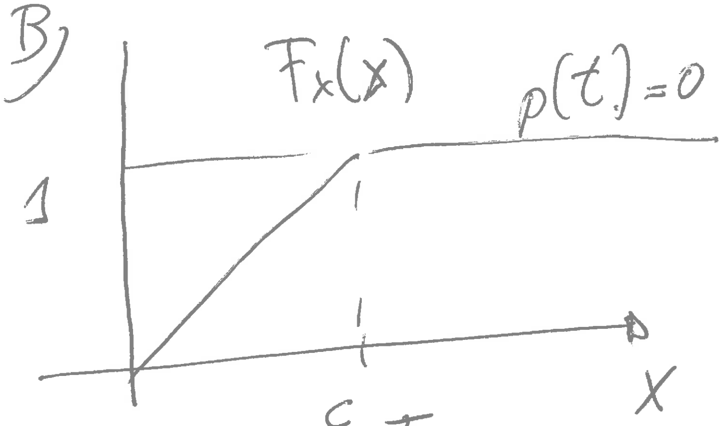
A) Define a R.V. (distance) travele!

B) Sketch $F_X(x)$ cumulative distribution function

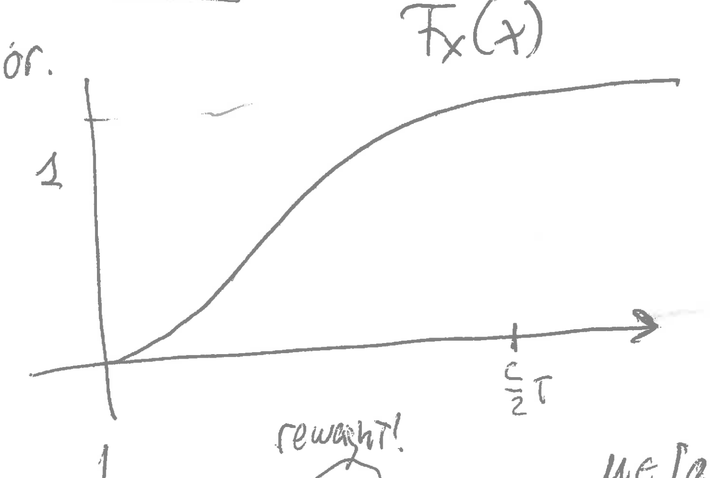
C) pdf. (probability density function)

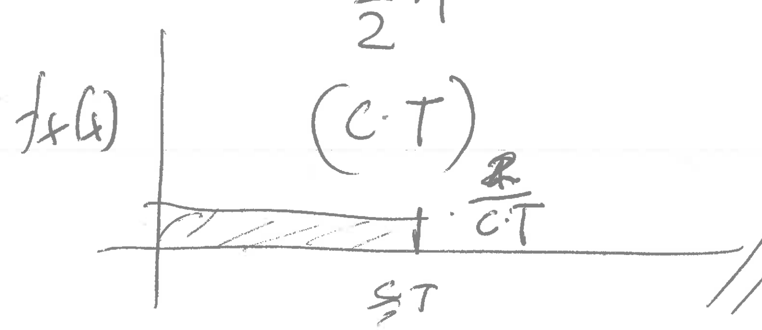
A) $X(\xi) = c \cdot t$, $\boxed{X(\xi) = \dfrac{c \cdot t}{2}}$

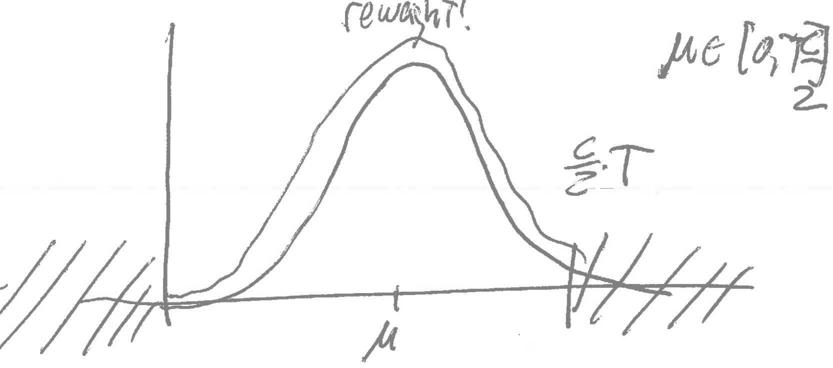
B)



or.







2.

A) $\begin{bmatrix} 5 & -3 \\ 3 & 6 \end{bmatrix}$ Not symetric.

B) $\begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix}$ $\boxed{\text{Correct.}}$ Incorrect. $\begin{bmatrix} 2 & \\ & 0 \end{bmatrix}$ $N(\mu, \sigma^2)$ semi definite positive.

c) $\begin{bmatrix} -2 & 1 \\ 1 & -6 \end{bmatrix}$ Incorect.

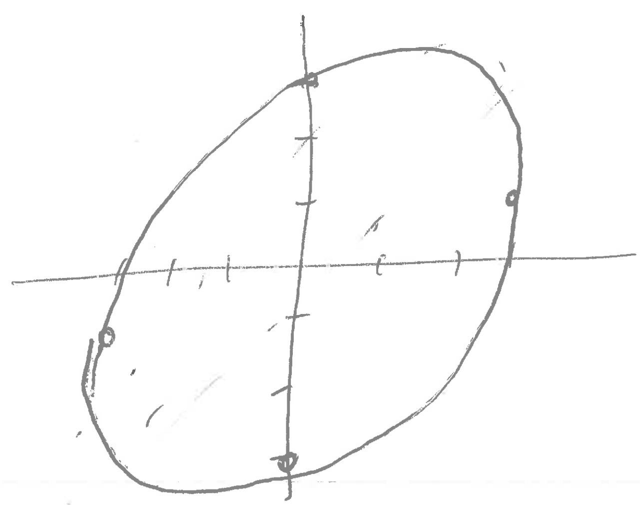
d) $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ Correct.

e) $\begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$ Incorrect. $\det \Sigma = -1.$

3.) Iso-contour. 1-sigma! $\mu = [0, 0]^T$ $L = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$

$$\begin{bmatrix} 9 & 3 \\ 3 & 10 \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

$$L \quad I \cdot L^T$$



$a^2 = 9 \quad a = 3$

$ba = ab = 3 \Rightarrow b = 1$

$c = \sqrt{10 - b^2} = \sqrt{9} = 3$

4) $p(x) = N(x; 1, 1)$

$p(z|x) \rightarrow$ Gaussian. $\mu_{z|x} = X$, $\sigma = 0.5$

$\boxed{p(x|z)}$ $z = 0.8$

$p(x, z) = p(z|x)\, p(x)$

$p(x|z) = \dfrac{p(x,z)}{p(z)} = \eta \cdot p(x,z)\Big|_{z=0.8}$

$p(z)$ not needed.

develop the joint Gaussian and evaluate $z=0.8$

$$p(x,z) = c_1 \exp\left\{-\frac{1}{2}\frac{(x-1)^2}{1}\right\} \cdot c_2 \exp\left\{-\frac{1}{2}\frac{(0.8-x)^2}{\underbrace{0.25}_{\sigma^2}}\right\}$$

$$= c \cdot \exp\left\{\underbrace{-\frac{1}{2}(x-1)^2 - \frac{1}{2}(x-0.8)^2 \cdot 4}_{\Delta}\right\}$$

$$\Delta = -\frac{1}{2}(x^2 - 2x + 1) - \frac{1}{2}\left(4x^2 - \overset{(6.4)}{4 \cdot 1.6} \cdot x + 4 \cdot 0.8^2\right)$$

$$\boxed{\Delta_{ideal} = \frac{-1}{2}\frac{x^2}{\sigma^2} + \frac{x}{\sigma^2} \cdot m - \frac{1}{2}\frac{m^2}{\sigma^2}}$$

$$\Delta = -\frac{1}{2}x^2\underbrace{(1+4)}_{1/\sigma^2} + x\underbrace{(1 + \overset{3.2}{2 \cdot 1.6})}_{m/\sigma^2} + ct.$$

$$\boxed{\sigma^2 = \frac{1}{5}}$$

$$\frac{m}{\sigma^2} = (1 + \overset{3.2}{2 \cdot 1.6}) \Rightarrow m = \sigma^2 \cdot 4.2 = \frac{4.2}{5} = \boxed{0.84}$$

(5) EKF 1 step. $X_{t-1} \sim N(1, \overset{\sigma^2}{1})$

$R=1$

$X_t = g(x_{t-1}, u_t, \varepsilon_t) = X_{t-1}^2 + u_t + \varepsilon_t \quad , \varepsilon_t \sim N(0,R)$

$z_t = X_t + \delta_t \quad , \quad \delta_t \sim N(0,Q) \quad , Q=3$

A) prop. $u_t = 1$

B) correction. $z_t = 3$

A) $g(x_{t-1}, u_t) \simeq \boxed{g(\mu_{t-1}, u_t)} + G_t \cdot \Delta x_{t-1}$, $\quad \Delta x_{t-1} = (x_{t-1} - \mu_{t-1})$

$$G_t = \left.\frac{\partial g(x_{t-1}, u_t)}{\partial x_{t-1}}\right|_{\mu_{t-1}} = \left. 2x \right|_{1} = 2.$$

5

I $\bar{\mu}_t = g(\mu_{t-1}, u_t, 0) = 1^2 + 1 = \boxed{2}$

Taylor. expan. $\Delta x = 0$ $\quad G \cdot \cancel{\Delta x} = 0$

II $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R = 2 \cdot 1 \cdot 2 + 1 = \boxed{5}$

B) $z_t = x_t + \delta_t$ $\quad H = 1$ $\quad (c = 1)$

III $K_t = \bar{\Sigma}_t \cdot H_t^T (H_t \bar{\Sigma}_t H_t^T + Q)^{-1} = \frac{5 \cdot 1}{5 + 3} = \frac{5}{8}$

$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$

IV $= 2 + \frac{5}{8}(3 - 2) = 2 + \frac{5}{8} = \boxed{2.625}$

$\Sigma_t = (I - K_t \cdot H)\bar{\Sigma}_t = (1 - \frac{5}{8} \cdot 1) \cdot 5 = \frac{3}{8} \cdot 5 = \boxed{\frac{15}{8}}$

1,875