A Data Fusion Algorithm for Multisensor Systems

Y. A. Vershinin
School of Engineering, Coventry University, Coventry, CV1 5FB, U. K.

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1 Introduction

Data fusion techniques are used in many tracking and surveillance systems as well as in applications where reliability is of a main concern. One solution for design of such systems is to employ a number of sensors (maybe of different types) and to fuse the information obtained from all these sensors on a central processor. Past attempts to solve this problem required an organization of a feedback from the central processor to local processor units (each of which includes a sensor and a local processor). Local estimations are then generated from the global estimation obtained from the previous step [1]. However, this causes computational bottleneck problems when data is transmitted. This problem was solved later in [2] for both cases: decentralised estimation and decentralized LQG control. The algorithms based on parallelization of the Kalman filter equations, as proposed in [3], extend the previous results allowing one to obtain the global estimation using only local estimates without transmission of information between sensors. Another method for data fusion is based on the so-called Federated Filter (square-root version of which is given in [4]). The Bayesian method based and linear sensor fusion algorithms are developed in [5] for both configurations: with a feedback from the central processor to local processing units and without such a feedback.

Information fusion can be obtained from the combination of state estimates and their error covariances using the Bayesian estimation theory [6], [7]. The two-filter method based on forward and backward solutions of Kalman filter or Bellmans's dynamic programming equations is another common method for data fusion [8]. A scattering framework [9] and decomposition of the information form of the Kalman filter [10] are also popular methods for designing the data fusion systems.

All the methods described above require the use of the central processor in order to fuse information obtained by the sensors. The main disadvantage of this approach is that in the case of central processing failure, the overall system will also fail. The method given in [11] is based on the internodal communications between local processor units without the need of any central processor. But the decentralized Kalman filter algorithms are obtained only for discrete time domain. In practice, however, continuous time implementations of a sensor fusion system are also required. A new data fusion algorithm based on the continuous time decentralized Kalman filter is proposed in this paper. In addition to the capability of combining information from different sensors, the system allows graceful degradation of the overall performance if some local units fail or interconnections are broken.

The simulation results of data fusion for three subsystems show that the performance of the overall system degrades gracefully even if the sensors of some subsystems are malfunction. Furthermore, local Kalman filters can effectively reduce subsystems and measurement noises.

2 A data fusion algorithm

The dynamics of subsystems of a complex system can be represented in the following form:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + w_i(t), \\ y_i(t) &= C_i x_i(t) + v_i(t), \qquad (i = 1,\ 2, \ldots, n) \end{aligned} \tag{1}$$

where

n – is the number of subsystems,
$x_i(t)$ - is the state of the i-th subsystem,
$u_i(t)$ - is the control signal on the i-th subsystem,
$y_i(t)$ - is the output of the i-th subsystem,
$w_i(t)$ - is the i-th subsystem noise,
$v_i(t)$ - is the measured noise of the i-th subsystem.

It is assumed that the subsystem noise $w_i(t)$ and the measured noise $v_i(t)$ are zero-mean Gaussian white noise processes with the following statistical properties:

$$E\{x_i(0)\} = E\{w_i(0)\} = E\{v_i(0)\} = 0,$$
$$E\{w_i(t)w_i^T(\tau)\} = Q_i(t)\delta(t-\tau),$$
$$E\{v_i(t)v_i^T(\tau)\} = R_i(t)\delta(t-\tau),$$
$$E\{x_i(0)w_i^T(t)\} = E\{x_i(0)v_i^T(t)\} =$$
$$E\{w_i(t)v_i^T(\tau)\} = 0$$

and $Q_i(t) \geq 0$, $R_i(t) > 0$.

State estimates are computed on each subsystem by local Kalman filters as:

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_iu_i(t) + K_i[y_i(t) - C_i\hat{x}_i(t)] \qquad (2)$$

The error covariance propagation in the information form is calculated accordingly:

$$\frac{d}{dt}(P_i^{-1}) = -P_i^{-1}A_i - A_i^TP_i^{-1} - P_i^{-1}Q_iP_i^{-1} + C_i^TR_i^{-1}C_i \qquad (3)$$

(Hereafter in the text the time notation index t is dropped for simplification of notations).

The Kalman gain matrix is calculated as:

$$K_i = P_iC_i^TR_i^{-1} \qquad (4)$$

where R_i^{-1} exist.

It is well known [6], [7] that the optimum combination of independent estimates can be accomplished in the form:

$$\hat{x}(t) = P[P_1^{-1}\hat{x}_1 + P_2^{-1}\hat{x}_2 + \ldots + P_n^{-1}\hat{x}_n] \qquad (5)$$

$$P = (P_1^{-1} + P_2^{-1} + \ldots + P_n^{-1})^{-1} \qquad (6)$$

Decentralizing algorithms (5) and (6) between the subsystems, one can obtain:

$$\hat{x}_k(t) = P_k[P_1^{-1}\hat{x}_1 + P_2^{-1}\hat{x}_2 + \ldots + P_n^{-1}\hat{x}_n] \qquad (7)$$

$$P_k = (P_1^{-1} + P_2^{-1} + \ldots + P_n^{-1})^{-1}, \quad (i = 1,2,\ldots,k,\ldots,n) \qquad (8)$$

Differentiating equations (7) and (8), the following fusion algorithm on the k-th subsystem is obtained:

$$\dot{\hat{x}}_k(t) = \dot{P}_k[\sum_{i=1}^{n} P_i^{-1}\hat{x}_i] + P_k[\sum_{i=1}^{n}\frac{d}{dt}(P_i^{-1})\hat{x}_i + \sum_{i=1}^{n} P_i^{-1}\dot{\hat{x}}_i], \qquad (9)$$

$$\frac{d}{dt}(P_k^{-1}) = \sum_{i=1}^{n} P_i^{-1}A_i - \sum_{i=1}^{n} A_iP_i^{-1} + \sum_{i=1}^{n} P_i^{-1}Q_iP_i^{-1} + \sum_{i=1}^{n} C_i^TR_i^{-1}C_i \qquad (10)$$

3 The model of the plant

The mathematical model of a conveyer driven by three DC electric motors can be obtained from the Newton's second law:

$$\begin{aligned} J_1\ddot{\theta}_1 &= \tau_1 - k_w[(r_1\theta_1 - r_2\theta_2)/l_{12} + (r_1\theta_1 - r_3\theta_3)/l_{31}] \\ J_2\ddot{\theta}_2 &= \tau_2 - k_w[(r_2\theta_2 - r_1\theta_1)/l_{12} + (r_2\theta_2 - r_3\theta_3)/l_{23}] \\ J_3\ddot{\theta}_3 &= \tau_3 - k_w[(r_3\theta_3 - r_1\theta_1)/l_{31} + (r_3\theta_3 - r_2\theta_2)/l_{23}] \end{aligned} \qquad (11)$$

where

- θ_i is angular position of a motor, (i =1,2,3)
- J_i is motor inertia,
- τ_i is motor torque,
- r_i is roll radius,
- l_{ij} is distance between rolls, (i,j =1,2,3)
- k_w is web constant.

Taking into account that $\tau_i = k_{mi}i_i$, where k_{mi} is the $i-th$ motor constant and i_i is armature current, system (11) can be written in the following form:

$$\ddot{\theta}_1 = (-k_wr_1/l_{12}J_1 - k_wr_1/l_{31}J_1)\theta_1 + (k_wr_2/l_{12}J_1)\theta_2 + (k_wr_3/l_{31}J_1)\theta_3 + (k_{m1}/J_1)i_1,$$

$$\ddot{\theta}_2 = (k_wr_1/l_{12}J_2)\theta_1 + (-k_wr_2/l_{12}J_2 - k_wr_2/_{23}J_2)\theta_2 + (k_wr_3/l_{23}J_2)\theta_3 + (k_{m2}/J_2)i_2,$$

$$\ddot{\theta}_3 = (k_wr_1/l_{31}J_3)\theta_1 + (k_wr_2/l_{23}J_3)\theta_2 + (-k_wr_3/l_{31}J_3 - k_wr_3/l_{23}J_3)\theta_3 + (k_{m3}/J_3)i_3. \qquad (12)$$

Introduce new variables as follows:

$$x_1 = \theta_1, \quad x_2 = \dot{\theta}_1, \quad x_3 = \theta_2, \quad x_4 = \dot{\theta}_2 \quad x_5 = \theta_3, \quad x_6 = \dot{\theta}_3. \qquad (13)$$

Using the representation (13) and the fact that $u_i = R_ii_i$, where u_i is the $i-th$ control input, R_i is the armature resistance, the system (12) can be written in the state-space form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a_{21} & 0 & a_{23} & 0 & a_{25} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ a_{41} & 0 & a_{43} & 0 & a_{45} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{61} & 0 & a_{63} & 0 & a_{65} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ b_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{14}$$

where

$$\begin{aligned} a_{21} &= -k_w r_1 / l_{12} J_1 - k_w r_1 / l_{31} J_1 \\ a_{23} &= k_w r_2 / l_{12} J_1 \\ a_{25} &= k_w r_3 / l_{31} J_1 \\ a_{41} &= k_w r_1 / l_{12} J_2 \\ a_{43} &= -k_w r_2 / l_{12} J_2 - k_w r_2 / l_{23} J_2 \\ a_{45} &= k_w r_3 / l_{23} J_2 \\ a_{61} &= k_w r_1 / l_{31} J_3 \\ a_{63} &= k_w r_2 / l_{23} J_3 \\ a_{65} &= -k_w r_3 / l_{31} J_3 - k_w r_3 / l_{23} J_3 \\ \\ b_1 &= k_{m1} / J_1 R_1 \\ b_2 &= k_{m2} / J_2 R_2 \\ b_3 &= k_{m3} / J_3 R_3 \end{aligned} \tag{15}$$

The observation model can be represented as

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \tag{16}$$

4 Experimental results

The simulation results of data fusion for three subsystems are shown in Figures 1 – 3.
It is assumed that all subsystems are identical. The inputs to the subsystems are sinusoidal signals with noise.

Fig.1 shows the case when all sensors are functioning.
Fig.2 shows the case when sensor 2 is malfunction.
Fig.3 shows the case when sensor 3 is malfunction.

According to the simulation results given in Figure 2, the data fusion algorithm allows the second subsystem to continue to work with minimal degradation of performance. Figure 3 shows that the third subsystem continues to work with graceful degradation of performance even though its sensor is malfunction.

5 Conclusions

The new data fusion algorithm presented in this paper allows one to combine information from different sensors in continuous time. Continuous-time decentralized Kalman filters (DKF) are used as data fusion devices on local subsystems. Such a structure gives the flexibility for reconfiguration of a control system. New subsystems can easily be added without needing any redesign of the whole system. The system does not require a central processor and therefore, in the case of failure of some local subsystems (each of which includes a local processor, sensors and actuators) the overall system will continue to work.

The simulation results show that the performance of the overall system degrades gracefully even if the sensors of some subsystems fail or interconnections are broken. Furthermore, local Kalman filters can effectively reduce subsystems and measurement noises.

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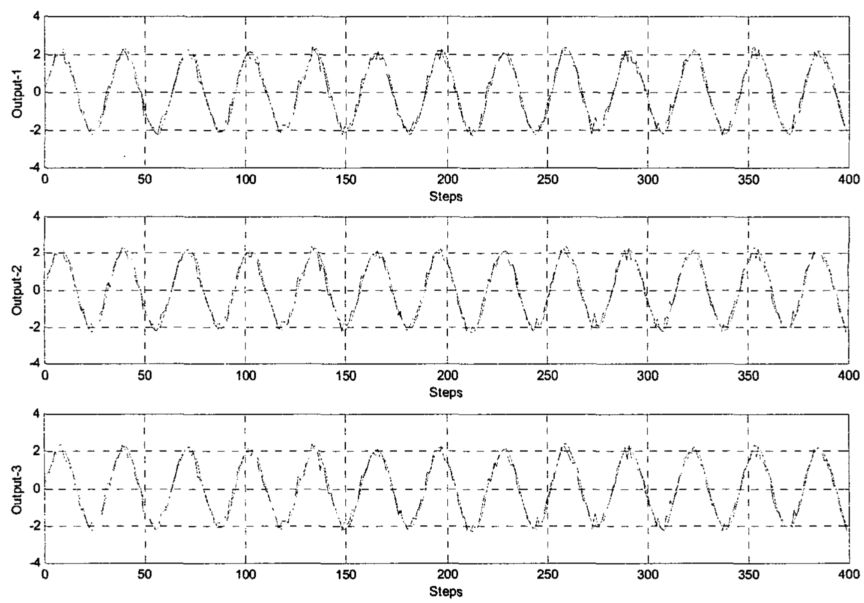

Fig. 1. ---- is the measured signal of a sensor, ——— is the output of a DKF.

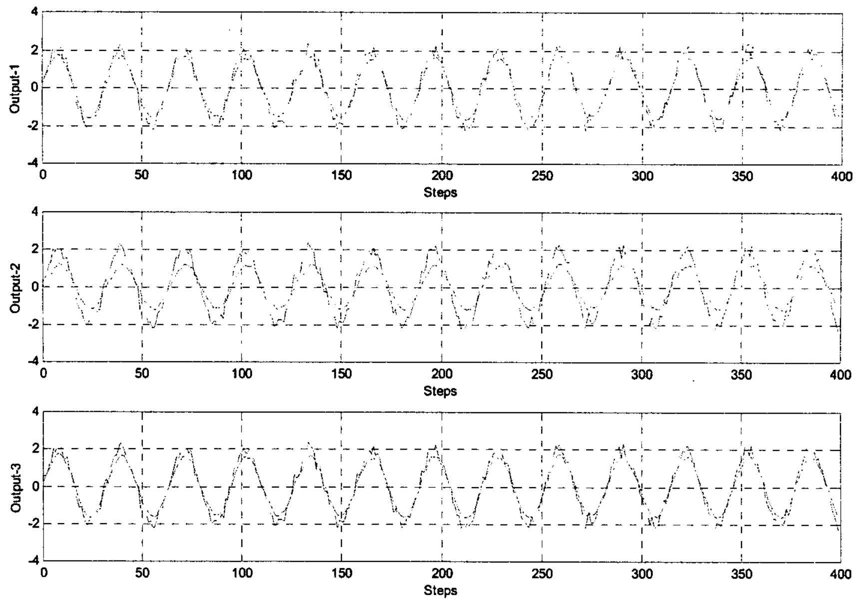

Fig. 2. - - - - is the measured signal of a sensor, ——— is the output of a DKF.

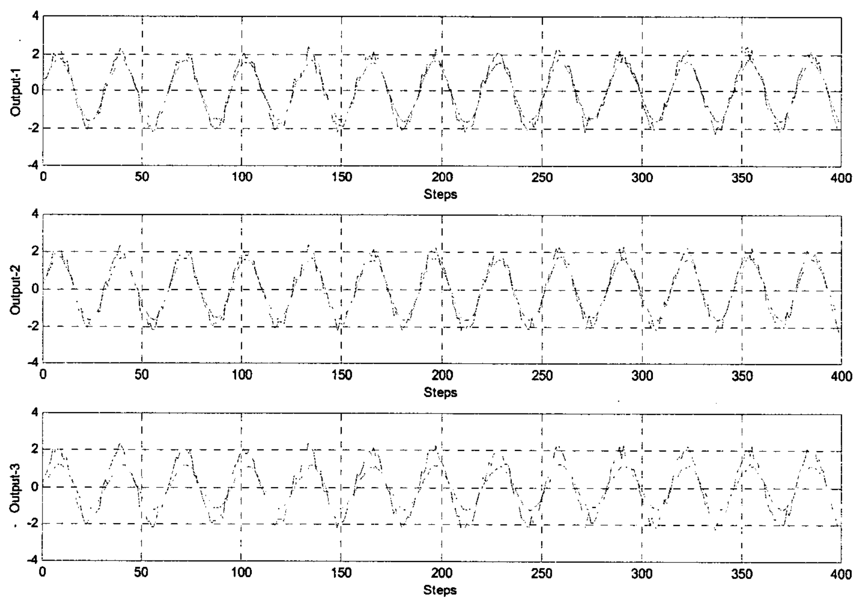

Fig. 3. - - - - is the measured signal of a sensor, ——— is the output of a DKF