# Engineering Mathematics - 4

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# Module – 5

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1. Explain the terms
   i. NULL Hypothesis
   ii. Type 1 and Type 2 errors. (5-Marks)
2. The nine items of a sample have the values 45,47,50,52,48,47,49,53,51. Does the mean of these differ significantly from the assumed mean of 47.5? (5-Marks)
3. Given the Matrix A= $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$ then show that A is a regular stochastic matrix. (6-Marks)
4. A die was thrown 9000 times and of these 3220 yielded a 3 or 4, can the die be regarded as Unbiased? (5-Marks)
5. Explain :
   i. Transient State
   ii. Absorbing State
   iii. Recurrent State (5-Marks)
6. A student's Study habits are as follows. If he studied one night , he is 70% sure not to study next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night. In the long run, how often does he study? (6-Marks)
7. In 324 throws of a six faced 'die' an odd number turned up 181 times. Is it reasonable to think that the 'die' is an Unbiased one?
8. Two horses A and B were tested according to the time(in seconds) to run a particular race with the following results:

   | | | | | | | | |
   |---|---|---|---|---|---|---|---|
   | Horse A: | 28 | 30 | 32 | 33 | 33 | 29 | 34 |
   | Horse B: | 29 | 30 | 30 | 24 | 27 | 29 | |

   Test whether you can discriminate between the two horses.($t_{0.05}$=2.2 and $t_{0.02}$=2.72 for 11 d.f)
9. Find the unique fixed probability vector for the regular stochastic Matrix, A= $\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$
10. Define the terms:
11. Prove that the Markov chain whose t.p.m P= $\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is irreducible. Find the corresponding stationary probability vector.

12. Three boys A,B,C are throwing ball to each other, A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws
    i. A has the ball
    ii. B has the ball
    iii. C has the ball

(iv) $\rho(X,Y)$ (06 Marks)

## Module-V

9. (a) Explain the terms:(i)Null hypothesis (ii)Confidence intervals (iii)Type I and Type II errors **(05marks)**

(b) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5,3,8,-1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 d.f is 2.201). **(05 marks)**

(c) Show that the Markov chain whose transition probability matrix $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Also, find the corresponding stationary probability vector. **(06 marks)**

**OR**

10. (a) In an elementary school examination the mean grade of 32 boys was 72 with a standard deviation of 8 while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis that the performance of girls is better than boys. **(05 marks)**

(b) Four coins are tossed 100 times and the following results were obtained.

| Number of Heads | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Frequency | 5 | 29 | 36 | 25 | 5 |

Fit a binomial distribution for the data and test the goodness of fit ( $\chi^2_{0.05} = 9.49$ for 4 d.f.). **(05marks)**

## Module-V

9. (a) Define the terms:(i)Null hypothesis (ii)Confidence intervals (iii)Type-I and Type-II errors **(05marks)**

(b) Ten individuals are chosen at random from a population and their heights in inches are found to be 63,63,66,67,68,69,70,70,71,71.Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05} = 2.262$ for 9 d.f.). **(05 marks)**

(c) Show that probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$ is regular stochastic matrix and find the associated unique fixed probability vector. **(06 marks)**

**OR**

9. (a) A manufacture claimed that at least 95%of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. **(05 marks)**

(b) Explain (i) transient state (ii) absorbing state (iii) recurrent state of a Markov chain. **(05marks)**

(c) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball , find the probabilities that after three throws (i) A has the ball (ii) B has the ball and (iii) C has the ball. **(06 marks)**

# Module – 2

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(b) With usual notation, prove that $J_{1/2}(x)=\sqrt{(2/\pi x)}\sin x$.

(c) c) If $x^3+2x^2-x+1=aP_0(x)+bP_1(x)+cP_2(x)+dP_3(x)$, find the values of $a,b,c,d$

(b) Express $f(x)=x^3+2x^2-x-3$ in terms of Legendre polynomials.

(c) Solve the Bessel's differential equation viz., $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-n^2)y=0$ to write th

solution in terms of $J_n(x)$.

b. Express $f(x)=3x^3-x^2+5x-2$ in terms of Legendre polynomials. **(05 M**

c. Obtain the series solution of Bessel's differential equation $x^2y''+xy'+(x^2+n^2)y=0$

Prove that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.

Derive Rodrigue's formula $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right]$.