Module - 04 Vector Differentiation.

Vector is a quantity having both magnitude & direction.

Vector quantities like force, velocity, acceleration etc.

* The derivative of the vector $\vec{r}(t)$ is denoted by, $\frac{d\vec{r}}{dt}$,

where $\vec{r} = xi + yj + zk$.

* Velocity & acceleration:

i) velocity of the particle at time t, $v = \frac{d\vec{r}}{dt}$.

ii) Acceleration of the particle at time t, $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$.

~~Problems~~: * Unit tangent vector normal to the space curve: $\hat{T} = \frac{d\vec{r}}{dt}$

$$\hat{T} = \frac{\vec{T}}{|\vec{T}|}$$

Problems:

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1) A particle moves along the curve $\vec{r} = (1-t^3)\hat{i} + (1+t^2)\hat{j} + (2t-5)\hat{k}$.
Determine the velocity & acceleration.

Solⁿ: $\vec{r} = (1-t^3)\hat{i} + (1+t^2)\hat{j} + (2t-5)\hat{k}$ (diff r w.r.t t).

velocity, $v = \frac{d\vec{r}}{dt} = -3t^2 i + 2tj + 2k$. (diff v w.r.t t)

acceleration, $a = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = -6ti + 2j$.

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2) A particle moves along the curve $c: x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$
where t denotes time. Find velocity & acceleration at $t = 2$.

Solⁿ: w.k.T $\vec{r} = xi + yj + zk$

$$\vec{r} = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k.$$

diff w.r.t t

$$\text{Velocity, } \vec{v} = \frac{d\vec{r}}{dt} = (3t^2 - 4)i + (2t+4)j + (16t - 9t^2)k.$$

diff w.r.t t

$$\text{acceleration, } \vec{a} = \frac{d\vec{v}}{dt} = 6ti + 2j + (16 - 18t)k.$$

(2). velocity at $t=2$, $(\vec{v})_{t=2} = (3(4)-4)i + (2(2)+4)j + (16(2)-9(4))k$.

$= 8i + 8j + 4k = 4(2i+2j+k)$

Acceleration at $t=2$, $(\vec{a})_{t=2} = 6(2)i + 2j + (16-18(2))k$

$= 12i + 2j + 20k = 2(6i+j+10k)$

3) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t-5$, where t is the time, find the veloct & acceleration at $t=1$.

Sol^n: $\vec{r} = xi + yj + zk$

$\vec{r} = 2t^2 i + (t^2-4t)j + (3t-5)k$.

velocity, $\vec{v} = \frac{d\vec{r}}{dt} = 4ti + (2t-4)j + 3k$.

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = 4i + 2j$

$(\vec{v})_{t=1} = 4(1)i + (2(1)-4)j + 3k = 4i - 2j + 3k$.

$(\vec{a})_{t=1} = 4i + 2j$.

4) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$; $z = 2\sin 3t$, where t is the time. Find the velocity & acceleration.

Sol^n: $\vec{r} = xi + yj + zk = e^{-t}i + 2\cos 3t\, j + 2\sin 3t\, k$.

velocity, $\vec{v} = \frac{d\vec{r}}{dt} = -e^{-t}i + 2(-3\sin 3t)j + 2(3\cos 3t)k$

$\vec{v} = -e^{-t}i - 6\sin 3t\, j + 6\cos 3t\, k$.

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = -(-e^{-t})i - 6(3\cos 3t)j + 6(-3\sin 3t)k$.

$\vec{a} = e^{-t}i - 18\cos 3t\, j - 18\sin 3t\, k$.

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5) Find the angle b/w the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ at the points $t = \pm 1$.

Sol^n: The tangent vector, $\vec{T} = \frac{d\vec{r}}{dt} = 2ti + 2j - 3t^2k$.

$(\vec{T})_{t=-1} = 2(-1)i + 2j - 3(-1)^2 k = -2i + 2j - 3k. = \vec{A}$ (say)

$(\vec{T})_{t=1} = 2(1)i + 2j - 3(1)k = 2i + 2j - 3k = \vec{B}$ (say).

Let θ be the angle between the tangents at $t = \pm 1$.

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\cdot|\vec{B}|} = \frac{(-2)(2) + (2)(2) + (-3)(-3)}{\sqrt{4+4+9}\ \sqrt{4+4+9}} = \frac{-\cancel{4}+\cancel{4}+9}{\sqrt{17}\ \sqrt{17}}$$

$$\cos\theta = \frac{9}{(\sqrt{17})^2} = \frac{9}{\sqrt{17}} \Rightarrow \theta = \cos^{-1}(9/\sqrt{17})$$

Thus, $\theta = \cos^{-1}(9/\sqrt{17})$ is the required angle.

6) If $x = t^2+1$, $y = 4t-3$, $z = 2t^2-6t$ represents the parametric equation of a curve, find the angle between the tangents at $t=1, t=2$.

Soln: W.K.T $r = xi + yj + zk. = (t^2+1)i + (4t-3)j + (2t^2-6t)k$

The tangent vector, $\vec{T} = \frac{d\vec{r}}{dt} = 2ti + 4j + (4t-6)k$

$(\vec{T})_{t=1} = 2i + 4j - 2k = \vec{A}$ (say).

$(\vec{T})_{t=2} = 4i + 4j + 2k = \vec{B}$ (say).

Let θ be the angle between the tangents at $t=1$ & $t=2$.

$$\text{W.K.T}\quad \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(2)(4) + (4)(4) + (-2)(2)}{\sqrt{4+16+4}\ \sqrt{16+16+4}}$$

$$\cos\theta = \frac{8+16-4}{\sqrt{24}\ \sqrt{36}} = \frac{\cancel{20}^{10}}{\sqrt{24}\,(\cancel{6})_3} = \frac{\cancel{10}^{5}}{\cancel{2}\sqrt{6}\times 3} = \frac{5}{3\sqrt{6}}$$

$\Rightarrow \theta = \cos^{-1}(5/3\sqrt{6})$ is the required angle.

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7) A particle moves along the curve $c: x = 2t^2$, $y = t^2-4t$, $z = 3t-5$ where t is the time. Find the components of velocity & acceleration at $t=1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

Soln: $\vec{r} = 2t^2 i + (t^2-4t)j + (3t-5)k$.

$\vec{v} = \frac{d\vec{r}}{dt} = 4ti + (2t-4)j + 3k$.

(4.) $\vec{a} = \frac{d\vec{v}}{dt} = 4i+2j$

$(\vec{v})_{t=1} = 4i-2j+3k, = \vec{V}$ (say).

$(\vec{a})_{t=1} = 4i+2j = \vec{A}$ (say).

Direction, $\vec{D} = i-3j+2k$.

Unit vector in the given direction, $\hat{n} = \frac{i-3j+2k}{\sqrt{1+9+4}} = \frac{i-3j+2k}{\sqrt{14}}$

velocity component, $\vec{V}\cdot\hat{n} = (4i-2j+3k)\cdot\frac{(i-3j+2k)}{\sqrt{14}}$

$\vec{V}\cdot\hat{n} = \frac{(4)(1)+(-2)(-3)+(3)(2)}{\sqrt{14}} = \frac{16}{\sqrt{14}}$

Acceleration Component,

$\vec{A}\cdot\hat{n} = (4i+2j)\frac{(i-3j+2k)}{\sqrt{14}} = \frac{(4)(1)+(2)(-3)+0}{\sqrt{14}} = \frac{-2}{\sqrt{14}}$

8) A particle moves along the curve whose parametric eqn are $x=t^3+1$, $y=t^2$, $z=2t+5$ where t is the time, Find the components of velocity & acceleration at $t=1$ in the direction $i+j+3k$.

Soln: $\vec{r} = (t^3+1)i + t^2 j + (2t+5)k$.

$\vec{v} = \frac{d\vec{r}}{dt} = 3t^2 i + 2tj + 2k$, $\vec{a} = \frac{d^2\vec{r}}{dt} = 6ti + 2j$.

$(\vec{v})_{t=1} = 3i+2j+2k = V$ (say).

$(\vec{a})_{t=1} = 6i+2j = A =$ (say).

Direction, $\vec{D} = i+j+3k$, $\hat{n} = \frac{i+j+3k}{\sqrt{1+1+9}} = \frac{i+j+3k}{\sqrt{11}}$

velocity component, $\vec{V}\cdot\hat{n} = (3i+2j+2k)\frac{(i+j+3k)}{\sqrt{11}} = \frac{(3)(1)+(2)(1)+(2)(3)}{\sqrt{11}}$

$\vec{V}\cdot\hat{n} = \frac{11}{\sqrt{11}} = \sqrt{11}$.

Acceleration Component, $\vec{A}\cdot\hat{n} = (6i+2j)\frac{(i+j+3k)}{\sqrt{11}} = \frac{(6)(1)+(2)(1)+0}{\sqrt{11}} = \frac{8}{\sqrt{11}}$.

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Scalar & vector point functions:

If to every point (x, y, z) of a region R in space there corresponds,

a) a scalar $\phi(x, y, z)$ then ϕ is called a scalar point function.

b) a vector $\vec{A}(x, y, z)$ then $\vec{A}$ is called a vector point function.

eg: Scalar point function : i) $\phi = x^2+y^2+z^2$, ii) $\phi = xy^2z^3$.

Vector point function : i) $\vec{A} = x^2 i + y^2 j + z^2 k$, ii) $\vec{A} = xyz i + yz j + z k$.

Operators: 1) The vector differential operator ∇, (Del).

$$\boxed{\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k = \Sigma \frac{\partial}{\partial x} i}$$

The Laplacian operator ∇^2 is defined by,

ii) The Laplacian operator

$$\boxed{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \Sigma \frac{\partial^2}{\partial x^2}}$$

1) Gradient : $\boxed{\text{grad } \phi = \nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k}$. $\nabla\phi$ is vector quantity.

2) Divergence : $\boxed{\text{div } \vec{A} = \nabla \cdot \vec{A} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}}$ div $\vec{A}$ is a scalar quantity.

3) Curl: $\text{curl } \vec{A} = \nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ a_1 & a_2 & a_3 \end{vmatrix}$

NOTE: i) Laplacian of $\phi = \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$.

ii) Laplacian of $\vec{A} = \nabla^2 \vec{A} = \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2}$.

iii) $\nabla\phi$ is a vector normal to the surface $\phi(x, y, z) = c$,

Unit vector normal to the surface, $\hat{n} = \frac{\nabla \phi}{|\nabla \phi|}$.

* Directional Derivative: If $\phi(x, y, z)$ is a scalar function & $\vec{d}$ is a given direction then $\nabla\phi \cdot \hat{n}$ where $\hat{n} = \vec{d}/|\vec{d}|$ is called as directional derivative of ϕ along $\hat{n}$.

problems:

1) Find grad ϕ when $\phi = 3x^2y - y^3z^2$ at the point $(1,-2,-1)$.

Sol^n: W.K.T grad $\phi = \nabla\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$.

$$\text{grad}\,\phi = \frac{\partial}{\partial x}(3x^2y - y^3z^2)\, i + \frac{\partial}{\partial y}(3x^2y - y^3z^2)\, j + \frac{\partial}{\partial z}(3x^2y - y^3z^2)\, k.$$

$$\text{grad}\,\phi = 6xy\, i + (3x^2 - 3y^2z^2)\, j + (-2y^3z)\, k.$$

$$(\text{grad}\,\phi)_{(1,-2,-1)} = 6(1)(-2)\, i + (3(1)^2 - 3(4)(+1))\, j + (-2(-8)(-1))\, k$$

$$= -12i - 9j - 16k.$$

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2) Find the divergence & curl of the vector,

$$\vec{F} = (xyz + y^2z)\hat{i} + (3x^2 + y^2z)\hat{j} + (xz^2 - y^2z)\hat{k}.$$

Sol^n: divergence, $\text{div}\,\vec{F} = \nabla\cdot F = \frac{\partial}{\partial x}(xyz + y^2z) + \frac{\partial}{\partial y}(3x^2 + y^2z) + \frac{\partial}{\partial z}(xz^2 - y^2z)$.

$$\text{div}\,\vec{F} = \underline{yz} + 0 + 0 + \underline{2yz} + 2xz - y^2.$$

$$\text{div}\,\vec{F} = 3yz + 2xz - y^2$$

curl, $\text{curl}\,\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (xyz + y^2z) & (3x^2 + y^2z) & (xz^2 - y^2z) \end{vmatrix}$

$$\text{curl}\,\vec{F} = i(-2yz - y^2) - j(z^2 - xy - y^2) + k(6x - xz - 2yz)$$

3) Find the divergence & curl of the vector $\vec{V} = (xyz)i + (3x^2y)j + (xz^2 - y^2z)k$ at the point $(2,-1,1)$

Sol^n: $\text{div}\,\vec{V} = \nabla\cdot\vec{V} = \frac{\partial}{\partial x}(xyz) + \frac{\partial}{\partial y}(3x^2y) + \frac{\partial}{\partial z}(xz^2 - y^2z)$

$$= yz + 3x^2 + 2xz - y^2$$

$$(\text{div}\,\vec{V})_{(2,-1,1)} = (-1)(1) + 3(4) + 2(2)(1) - (+1) = -1 + 12 + 4 - 1$$

$$= 16 - 2 = 14.$$

$$\text{curl}\,\vec{V} = \nabla\times\vec{V} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xyz & 3x^2y & xz^2 - y^2z \end{vmatrix}$$

$$\text{curl } \vec{v} = i(-2yz-0) - j(z^2-xy) + k(6xy-xz)$$

$$(\text{curl } \vec{v})_{(2,-1,1)} = i(-2(-1)(1)) - j(1-2(-1)) + k(6(2)(-1)-(2)(1))$$

$$= 2i - 3j - 14k$$

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4.) Find unit normal vector to surface $Q = x^2yz + 4xz^2$ at $(1,-2,-1)$.

Sol^n: Let $\phi = x^2yz + 4xz^2$.

$\nabla\phi$ is a vector normal to the surface,

W.K.T $\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$

$$\nabla\phi = (2xyz + 4z^2) i + x^2 z j + (x^2y + 8xz) k.$$

$$(\nabla\phi)_{(1,-2,-1)} = (2(1)(-2)(-1) + 4(1)) i + (1)(-1) j + (1(-2) + 8(1)(-1)) k.$$

$$= 8i - j - 10k.$$

The required unit vector normal, $\hat{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{8i-j-10k}{\sqrt{64+1+100}}$

$$\hat{n} = \frac{8i-j-10k}{\sqrt{165}}$$

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5.) If $\phi = 2x^3y^2z^4$ find div (grad ϕ).

Sol^n: grad $\phi = \nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$

$$= \frac{\partial(2x^3y^2z^4)}{\partial x} i + \frac{\partial(2x^3y^2z^4)}{\partial y} j + \frac{\partial(2x^3y^2z^4)}{\partial z} k$$

$$\text{grad } \phi = 6x^2y^2z^4 i + 4x^3yz^4 j + 8x^3y^2z^3 k.$$

$$\text{div}(\text{grad } \phi) = \nabla \cdot \text{grad } \phi = \frac{\partial}{\partial x}(6x^2y^2z^4) + \frac{\partial}{\partial y}(4x^3yz^4) + \frac{\partial}{\partial z}(8x^3y^2z^3)$$

$$\text{div}(\text{grad } \phi) = 12xy^2z^4 + 4x^3z^4 + 24x^3y^2.$$

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6.) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the vector $i + 2j + 2k$.

Sol^n: Let $\phi = xy^2 + yz^3$

$$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k = y^2 i + (2xy + z^3) j + 3yz^2 k.$$

(8)

$$[\nabla\phi]_{(2,-1,1)} = (+1)i + (2(2)(-1)+1)j + 3((-1)(1))k$$
$$= i - 3j - 3k.$$

The unit vector in the direction of, $i+2j+2k$, is

$$\hat{n} = \frac{i+2j+2k}{\sqrt{1+4+4}} = \frac{i+2j+2k}{\sqrt{9}} = \frac{i+2j+2k}{3}$$

$\therefore$ The required directional derivative is,

$$\nabla\phi\cdot\hat{n} = (i-3j-3k)\cdot\left(\frac{i+2j+2k}{3}\right)$$

$$\nabla\phi\cdot\hat{n} = \frac{(1)(1)+(-3)(2)+(-3)(2)}{3} = \frac{1-6-6}{3} = \frac{-11}{3}$$

7) Find the directional derivative of $\phi = x^2+y^2+2z^2$ at $P(1,2,3)$ in the direction of line $\vec{PQ} = 4i-2j+k$.

Soln: $\phi = x^2+y^2+2z^2$.

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k = 2xi+2yj+2zk$$

$$[\nabla\phi]_{(1,2,3)} = 2i+4j+6k.$$

The unit vector in the direction of $4i-2j+k$ is,

$$\hat{n} = \frac{4i-2j+k}{\sqrt{16+4+1}} = \frac{4i-2j+k}{\sqrt{21}}$$

$\therefore$ The required directional derivative is,

$$\nabla\phi\cdot\hat{n} = (2i+4j+6k)\left(\frac{4i-2j+k}{\sqrt{21}}\right)$$

$$= \frac{(2)(4)+(4)(-2)+(6)(1)}{\sqrt{21}} = \frac{8-8+6}{\sqrt{21}} = \frac{6}{\sqrt{21}}.$$

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8) Find the angle b/w the surfaces $x^2+y^2+z^2=9$ & $z=x^2+y^2-3$ at $(2,-1,2)$.

Soln: Given: $x^2+y^2+z^2=9$ & $x^2+y^2-z=3$.

Let $\phi_1 = x^2+y^2+z^2$ & $\phi_2 = x^2+y^2-z$.

W.K.T $\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$.

$\nabla\phi_1 = 2xi + 2yj + 2zk$ & $\nabla\phi_2 = 2xi + 2yj - k$.

$[\nabla\phi_1]_{(2,-1,2)} = 4i - 2j + 4k$ $\quad [\nabla\phi_2]_{(2,-1,2)} = 4i - 2j - k$.

If θ is the angle b/w these two normals we have,

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1| \cdot |\nabla\phi_2|} = \frac{(4)(4)+(-2)(-2)+(4)(-1)}{\sqrt{16+4+16}\ \sqrt{16+4+1}} = \frac{16+\cancel{4}-\cancel{4}}{\sqrt{36}\sqrt{21}}$$

$$\cos\theta = \frac{\cancel{16}^{\,8}}{{}_3\cancel{6}\sqrt{21}} = \frac{8}{3\sqrt{21}} \Rightarrow \theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$$ is the required angle.

Solenoidal & irrotational vectors.

A vector $\vec{F}$ is said to be solenoidal if $\boxed{\operatorname{div}\vec{F} = 0}$ & irrotational if $\boxed{\operatorname{Curl}\vec{F} = 0}$.

Irrotational vector field is also called as conservative field (or) potential field.

When $\vec{F}$ is irrotational there always exists a scalar point function ϕ such that $\nabla\phi = \vec{F}$. Then ϕ is called a scalar potential of $\vec{F}$.

Problems:

1) Find a, for which $f = (x+3y)i + (y-2z)j + (x+az)k$ is solenoidal.

Soln: W.K.T solenoidal means, $\operatorname{div} \cdot f = 0$.

$$\Rightarrow \operatorname{div} f = \nabla \cdot f = 0.$$

$$\Rightarrow \frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(y-2z) + \frac{\partial}{\partial z}(x+az) = 0.$$

$$1+0+1-0+0+a = 0$$

$$2+a = 0 \Rightarrow \boxed{a=-2}.$$

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2.) If $\vec{F} = (ax+3y+4z)\hat{i} + (x-2y+3z)\hat{j} + (3x+2y-z)\hat{k}$ is solenoidal, find a.

Soln: W.K.T solenoidal means, $\operatorname{div}.\vec{F} = 0$.

$$\Rightarrow \operatorname{div}.\vec{F} = \nabla \cdot \vec{F} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(ax+3y+4z) + \frac{\partial}{\partial y}(x-2y+3z) + \frac{\partial}{\partial z}(3x+2y-z)$$

$$\Rightarrow (a+0+0)\hat{i} + (0-2+0)\hat{j} + (0+0-1)\hat{k} = 0$$

$$a-2-1=0$$

$$a-3=0 \Rightarrow \boxed{a=3}$$

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3. Show that $\vec{f} = (2xy^2+yz)\hat{i} + (2x^2y+xz+2yz^2)\hat{j} + (2y^2z+xy)\hat{k}$ is irrotational.

Sol^n: We have to show curl $\vec{f} = \vec{0}$.

$$\text{curl}\,\vec{f} = \nabla\times\vec{f} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy^2+yz & 2x^2y+xz+2yz^2 & 2y^2z+xy \end{vmatrix}$$

$$\text{curl}\,\vec{f} = i\left(\frac{\partial}{\partial y}(2y^2z+xy) - \frac{\partial}{\partial z}(2x^2y+xz+2yz^2)\right) - j\left(\frac{\partial}{\partial x}(2y^2z+xy) - \frac{\partial}{\partial z}(2xy^2+yz)\right)$$
$$+ k\left(\frac{\partial}{\partial x}(2x^2y+xz+2yz^2) - \frac{\partial}{\partial y}(2xy^2+yz)\right)$$

$$\text{curl}\,\vec{f} = i\left((4yz+x)-(x+4yz)\right) - j(y-y) + k\left((4xy+z)-(4xy+z)\right)$$

$$\text{curl}\,\vec{f} = i(0) - j(0) + k(0) = \vec{0}$$

$\therefore \vec{f}$ is irrotational.

4. verify whether $\vec{A} = (2x+yz)i + (4y+zx)j - (6z-xy)k$ is irrotational.

Sol^n: We have to show, curl $\vec{A} = \vec{0}$.

$$\text{curl}\,\vec{A} = \nabla\times\vec{A} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x+yz & 4y+zx & -6z+xy \end{vmatrix}$$

$\because -(6z-xy) = -6z+xy.$

$$\text{curl}\,\vec{A} = i\left(\frac{\partial}{\partial y}(-6z+xy) - \frac{\partial}{\partial z}(4y+zx)\right) - j\left(\frac{\partial}{\partial x}(-6z+xy) - \frac{\partial}{\partial z}(2x+yz)\right) + k\left(\frac{\partial}{\partial x}(4y+zx) - \frac{\partial}{\partial y}(2x+yz)\right)$$

$$\text{curl}\,\vec{A} = i(x-x) - j(y-y) + k(z-z) = i(0) - j(0) + k(0)$$

$$\text{curl}\,\vec{A} = \vec{0}$$

$\therefore \vec{A}$ is irrotational.

5) Find the constants a, b, c such that the vector, $\vec{F} = (x+y+az)\hat{i} + (x+cy+2z)\hat{k} + (bx+2y-z)\hat{j}$ is irrotational.

Sol^n: $\vec{F}$ is irrotational.

$\therefore$ curl $\vec{F} = 0$.

$\nabla \times \vec{F} = 0$.

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y+az & x+cy+2z & bx+2y-z \end{vmatrix} = 0.$$

$$\Rightarrow i\left(\frac{\partial}{\partial y}(bx+2y-z) - \frac{\partial}{\partial z}(x+cy+2z)\right) - j\left(\frac{\partial}{\partial x}(bx+2y-z) - \frac{\partial}{\partial z}(x+y+az)\right) + k\left(\frac{\partial}{\partial x}(x+cy+2z) - \frac{\partial}{\partial y}(x+y+az)\right) = 0.$$

$$\Rightarrow i(2-2) - j(b-a) + k(1-1) = 0.$$

$$\Rightarrow b-a = 0 \Rightarrow b = a.$$

6) Find the constants $a, b \& c$ if $\vec{F} = (x+y+az)i + (bx+2y-z)j + (x+cy+2z)k$ such that $\vec{F}$ is irrotational.

Sol^n: $\vec{F}$ is irrotational, curl$\vec{F} = \vec{0}$

$\nabla \times \vec{F} = 0$.

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y+az & bx+2y-z & x+cy+2z \end{vmatrix} = 0.$$

$$\Rightarrow i(c+1) - j(1-a) + k(b-1) = 0.$$

$\Rightarrow c+1 = 0$, $-(1-a) = 0$, $b-1 = 0$.

$\Rightarrow \boxed{c=-1}$ | $-1+a = 0 \Rightarrow \boxed{a=1}$ | $\Rightarrow \boxed{b=1}$

$\therefore$ $a=1$, $b=1$, $c=-1$ are the required values.