MODULE - 1

FOURIER SERIES

1. Find the Fourier series expansion of the function $f(x) = |x|$ in (-π, π), hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$
2. Obtain the Fourier series for the function x^2 in $-\pi \le x \le \pi$ and hence deduce that (i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \dots \dots \dots \dots .. = \frac{\pi^2}{12}$ (ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots \dots \dots \dots .. = \frac{\pi^2}{6}$

   (iii)$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \dots \dots \dots \dots .. = \frac{\pi^2}{8}$

3. Find the Fourier series of $f(x) = x - x^2, -\pi \le x \le \pi$. Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} \dots \dots \dots \dots .. = \frac{\pi^2}{12}$

4. Obtain the Fourier expansion of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and hence $\sum \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$
5. Obtain the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$ and deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \dots \dots \dots \dots .. = \frac{\pi^2}{8}$
6. Obtain the Fourier expansion of $f(x) = \begin{cases} -k, & in\ (-\pi, 0) \\ k, & in\ (0, \pi) \end{cases}$ Hence deduce $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} \dots \dots$

7. Obtain the Fourier series expansion of $f(x) = \begin{cases} x, & if \quad 0 \le x \le \pi \\ 2\pi - x, & if \quad \pi \le x \le 2\pi \end{cases}$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \dots \dots \dots \dots .. = \frac{\pi^2}{8}$

8. Find the Fourier series expansion of the function $f(x) = x(2\pi - x)$ over the interval (0, 2π) and deduce that $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ *and* $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} \dots \dots \dots \dots .. = \frac{\pi^2}{8}$
9. Obtain the Fourier series expansion of $f(x) = \begin{cases} x, & if \quad 0 \le x \le \pi \\ x - 2\pi, & if \quad \pi \le x \le 2\pi \end{cases}$ and deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} \dots \dots$
10. Obtain the Fourier series for the function $f(x) = \begin{cases} \pi x & : 0 \le x \le 1 \\ \pi(2 - x) & : 1 \le x \le 2 \end{cases}$ and deduce that $\sum \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

11. Find the Fourier series of $f(x) = \begin{cases} 2 - x & 0 \le x \le 4 \\ x - 6 & 4 \le x \le 8 \end{cases}$ .Hence deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} \dots \dots$

12. Find the Fourier series of $f(x) = \begin{cases} 1 + \frac{4x}{3} & -\frac{3}{2} < x < 0 \\ 1 - \frac{4x}{3} & 0 \le x < \frac{3}{2} \end{cases}$ deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \dots \dots \dots \dots .. = \frac{\pi^2}{8}$

Half Range Fourier Series

13. Obtain the half range cosine series for the function $f(x) = (x-1)^2$ in the interval $0 \le x \le 1$ and hence Show That $\pi^2 = 8\left\{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \dots \dots \dots\right\}$

14. Find the Half Range Cosine Series $f(x) = \begin{cases} \frac{1}{4} - x & in\ 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & in\ \frac{1}{2} < x < 1 \end{cases}$

15. Find the Half Range Cosine Series of $f(x) = x(\pi - x)$ in $0 < x < \pi$ or $f(x) = x(l - x) in 0 \le x \le l$

16. Expand $f(x) = 2x - 1$ on a Cosine half range Fourier series in $0 < x < 1$

17. Find the Half Range Fourier sine Series of $f(x) = \begin{cases} x & if \quad 0 < x < \frac{\pi}{2} \\ \pi - x & if\ \frac{\pi}{2} < x < \pi \end{cases}$

Harmonic Analysis:-

18. Obtain the constant term and coefficients of first cosine and sine terms in the expansion of y from the following table

X	0	60°	120°	180°	240°	300°	360°
Y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

19. Obtain a_0, a_1, b_1 in the Fourier expansion of y, using harmonic analysis for the data given.

X	0	1	2	3	4	5
Y	9	18	24	28	26	20

20. Compute the constant term and the first two harmonics in the Fourier series of f(x) given by the following table

X	0	1	2	3	4	5
f(x)	4	8	15	7	6	2

21. Obtain the constant term and coefficients of sin θ and sine 2θ in the Fourier expansion of y from the following table

θ°	0	60	120	180	240	300	360
Y	0	9.2	14.4	17.8	17.3	11.7	0

22. The following table gives the variations of periodic current over a period

t(sec) :	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A(amp):	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the 1^{st}harmonic

MODULE - 2

FOURIER TRANSFORMS

Infinite Fourier Transform

1. Find the Fourier Transform of $f(x) = \begin{cases} 1 - x^2 for|x| \leq 1 \\ 0 for|x| > 1 \end{cases}$
2. Find the Fourier Transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{cos\, xt}{1+t^2} dt$
3. Find the Fourier Transform of $f(x) = \begin{cases} 1 for|x| \leq a \\ 0 for|x| > a \end{cases}$ and hence evaluate $\int_0^\infty \frac{sin\, x}{x} dx.$
4. Find the Fourier Transform of $f(x) = \begin{cases} 1 for|x| \leq 1 \\ 0 for|x| > 1 \end{cases}$ and hence evaluate $\int_0^\infty \frac{sin\, x}{x} dx.$

Fourier Sine and Cosine Transforms:-

5. Find the Fourier Sine Transform of $\frac{e^{-ax}}{x}$
6. Find the Fourier Cosine Transform of $f(x) = \begin{cases} 4x & for\ \ 0 < x < 1 \\ 4 - x & for\ \ 0 < x < 1 \\ 0 & for\ \ x > 1 \end{cases}$
7. Find the Fourier Sine Transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x\, sin\, mx}{1+x^2} dx$,m > 0
8. Find the Fourier Sine Transform of $f(x) = \begin{cases} x & 0 < x \leq 1 \\ 2 - x & 1 \leq x < 2 \\ 0 & x > 2 \end{cases}$
9. Find the Cosine and Sine Transform of $f(x) = e^{-ax}$ where a > 0
10. Find the Fourier Sine Transform of $f(x) = \frac{s}{s^2+1}$
11. Find the inverse Fourier Sine Transform of $F_x(\alpha) = \frac{1}{\alpha} e^{-a\alpha} a > 0$

Z TRANSFORM:-

12. Find the Z-transform of $(i)\ sinh\, n\, \theta (ii)\ cosh\, n\, \theta (iii) n^2$
13. Find the z-transform of $2^n + sin\left(\frac{n\pi}{4}\right) + 1$
14. Find $z_T(n^p) and z_T \left[cosh\left(\frac{n\pi}{2} + \theta\right)\right]$
15. Find the z-transform of $(i)\ sin(3n + 5)$

Inverse z-Transforms:-

16. **Find the inverse z-transform of** $\frac{2z^2+3z}{(z+2)(z-4)}$
17. **Find** $z_T^{-1}\left[\frac{z^3-20z}{(z-2)^3(z-4)}\right]$
18. **Find the inverse z-transform of** $\frac{8z^2}{(2z-1)(4z-1)}$
19. **Find the inverse z-transform of** $\frac{3z^2+2z}{(5z-1)(5z+2)}$
20. **Find the inverse z-transform of** $\frac{z}{(z-1)(z-2)}$

MODULE - 3

STATISTICAL METHOD

Correlation and Regression:-

1. Obtain the lines of regression and hence find the coefficient of correlation for the data

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

2. Find the coefficient of correlation for the following data :

x	55	56	58	59	60	60	62
y	35	38	39	38	44	43	45

3. Find the coefficient of correlation , line of regression of x on y and line of regression of y on x; given

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

4. Find the coefficient of correlation , line of regression of x on y and line of regression of y on x;

x	1	2	3	4	5
y	2	5	3	8	7

5. Find the coefficient of correlation for the following data

x	10	14	18	22	26	30
y	18	12	24	6	30	36

Curve fitting :-

6. Fit a straight line$y = ax + b$ for the data

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

7. Fit a straight line$y = ax + b$ for the data

X	50	70	100	120
Y	12	15	21	25

8. A simply supported beam carries a concentrated load **P** at its midpoint corresponding to various valuesof **P**the maximum deflection **Y** is measured and is given in the following table.

P	100	120	140	160	180	200
Y	0.45	0.55	0.60	0.70	0.80	0.85

Find the law of the form $Y = a + bP$and hence estimate **Y** at **P** is 150

Fitting of a Second Degree Parabola $y = ax^2 + bx + c$

9. Fit a parabola of second degree given

X	0	1	2	3	4	5	6
Y	14	18	23	29	36	40	46

10. Fit a Parabola $y = a + bx + cx^2$ for the data

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	2.3

11. Find the best values of a,b,c if the equation $y = a + bx + cx^2$ is to fit most closely to the following observation

X	1	2	3	4	5
Y	10	12	13	16	19

Fitting of a curve of the form $y = ae^{bx}$

12. Fit a curve of the form $y = ae^{bx}$ for the

X	0	2	4
Y	8.12	10	31.82

13. Fit a curve of the form $y = ae^{bx}$ for the data

X	1	5	7	9	12
Y	10	15	12	15	21

14. Fit a curve of the form $y = ae^{bx}$ for the data

X	77	100	185	239	285
Y	2.4	3.4	7.0	11.1	19.6

15. For the following data fit an exponential to the curve of the form $y = ae^{bx}$ by the method of least squares

x	5	6	7	8	9	10
y	133	55	23	7	2	2

Numerical Methods:-

Regula-Falsi Method

16. Find the real root of the equation $xe^x - \cos x = 0$correct to 4 decimal places

17. Find the root of the equation $2x - \log_{10} x = 7$which lies between $3.5 and 4$

18. ind the root of the equation $x^6 - x^4 - x^3 - 1 = 0$in (1,2) correct to 4 decimal places. carry out three iterations

19. Using the method of false position , find a real root of the equation $x \log_{10} x - 1.2 = 0$correct to 4 decimal places

Newton- Raphson Method

20. Find the root of the equation $x\, sin\, x + cos\, x = 0$ nearer to π, carry out three iterations upto 4 decimal places

21. Find the root of $x + log_{10} x = 3.375$ near 2.9 correct to 3 decimal places

22. Find the root of the equation $3x = cos\, x + 1$.Take $x_0 = 0.6$Perform two iterations

MODULE - 4

FINITE DIFFERENCES

Newton-Forward and Backward Interpolation:-

1. In the given table the values of $y$are consecutive terms of series of which 23.6 is the 6^{th}term find the first and tenth term of the series

X	3	4	5	6	7	8	9
Y	4.8	8.4	14.5	23.6	36.2	52.8	73.9

2. A survey conducted in a slum locality reveals the following information as classified below, Estimate the probable number of persons in the income group 20 to 25.

Income per day in Rupees 'X'	Under10	10-20	20-30	30-40	40-50
Number of persons 'Y'	20	45	115	210	115

3. Use appropriate interpolating formula to compute y(82) and y(98) for the data

X	80	85	90	95	100
Y	5026	5674	6362	7088	7854

4. From the following table estimate the number of students who have obtained the marks between 40 and 45

Marks	30- 40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

5. Find the value of f(38) and f(85) using suitable interpolation formulae

X	40	50	60	70	80	90
y=f(x)	184	204	226	250	276	304

6. The area of a circle (A) corresponding to diameter (D) is given below

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Newton's Divided Difference:-

7. Construct an interpolating polynomial for the data given below

X	2	4	5	6	8	10
f(x)	10	96	196	350	868	1746

8. Determine f(x) as a polynomial in x for the data,

X	0	1	4	8	10
f(x)	-5	-14	-125	-21	355

9. Find f(9)

X	5	7	11	13	17
f(x)	150	392	1452	2366	5202

10. Using Newton's divided difference interpolation formula, find the interpolating polynomial

X	0	1	2	3	4	5
y=f(x)	3	2	7	24	59	118

Lagrange's Interpolation and Inverse Interpolation

11. If y(1)=3, y(3)=9, y(4)=30, y(6)=132 find interpolating polynomial by Lagrange's formula

12. Using Lagrange's formulas find the interpolating polynomial that approximate the function described by the following table. Hence find f(3)

X	0	1	2	5
f(x)	2	3	12	147

13. Apply Lagrange's formula inversely to find a root of the equation f(x)=0 given that f(30)=-30, f(34)=-13, f(38)=3, f(42)=18

Numerical Integration:-

14. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Weddle's rule taking 7-ordinates and hence find $log_e 2$

15. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking 6-equal strips and by using Simpson's$\left(\frac{3}{8}\right) th$rule divide the interval into 3 equal parts hence find $log_e 2, log_e \sqrt{2}$

16. Evaluate $\int_4^{5.2} log_e x\, dx$ by Weddle's rule taking 7-ordinates

17. Evaluate $\int_0^{1.2} e^x\, dx$ by Weddle's rule taking six equal sub intervals

18. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ using (i) Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule (ii) Simpson's $\left(\frac{3}{8}\right) th$rule (iii)Weddle's rule

x	0	1	2	3	4	5	6
f(x)=$\frac{1}{1+x^2}$	1	0.5	0.2	0.4	0.0588	0.0385	0.027

19. Evaluate $\int_0^5 \frac{dx}{4x+5}$ by Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule by taking 10-equal parts and hence find $log_e 5$

MODULE - 5

VECTOR INTEGRATION

Line Integral:-

1. Find the work done by a force $\vec{F} = (2y - x^2)\hat{\imath} + 6yz\hat{\jmath} - 8xz^2\hat{k}$from the point (0,0,0) to the point (1,1,1) along the straight line joining these points
2. If $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + zx\hat{k}$, evaluate $\int_c \vec{F}.d\vec{r}$where C is the curve represented by
   $x = t, y = t^2, z = t^3, -1 \le t \le 1$

Green's Theorem:-

3. Find the area between the parabola $y^2 = 4x\ and\ x^2 = 4y$ with the help of Green's theorem in a plane
4. Verify Green's theorem in a plane $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$where C is the boundary of the region enclosed by $y = \sqrt{x} and y = x^2$
5. Verify Green's theorem in a plane $\oint_c (x^2 + y^2)dx + 3x^2y dy$where C is the circle $x^2 + y^2 = 4$ traced in the positive sense.

Stoke's Theorem:-

6. Evaluate $\int_c xydx + xy^2dy$ by Stoke's theorem where C is the square in the x-y plane with vertices (1,0) , (-1,0), (0,1) and (0,-1)
7. Use Stoke's theorem to evaluate $\int_s curl\vec{F}.d\vec{s}$where $\vec{F} = y\hat{\imath} + (x - 2xz)\hat{\jmath} - xy\hat{k}$ and S is the surface of the sphere $x^2y^2 + z^2 = a^2$above the xy-plane
8. Evaluate using Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ taken around the rectangle bounded by $x = 0, x = a, y = 0, y = b$

Gauss Divergence Theorem:-

9. Evaluate $\iint_s \vec{F}.\hat{n}ds$given $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$ by using Gauss divergence theorem
10. Using the divergence theorem, evaluate $\iint_s \vec{F}.\hat{n}ds$where $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$

11. Using the divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$ where $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ and S is the surface bounded by $x^2 + y^2 = 4, z = 0, z = 3$

Calculus of Variations:-

12. Derive Euler's equation for a variational problem in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

13. Find the extremal of the functional $\int_{x_1}^{x_2}[y' + x^2(y')^2]dx$

14. Define a geodesic on a surface. P.T the geodesics on a plane are straight lines

15. Find the geodesics on a surface given that the arc length on the surface is

$$S = \int_{x_1}^{x_2} \sqrt{x\left(1 + y'^2\right)}\, dx$$

16. Find the curve passing through the points $(x_1 y_1)$and$(x_2 y_2)$ which when rotated about the x-axis gives a minimum surface area OR A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is Catenary. (Answer is same for both questions).