James Dolan ① 22/7/19

6. calculating beliefs & believers

List:

1. def'n of a locally presentable category

2. role of young diagrams in belief method for n-dim objects

2'. McKay correspondence & belief in a 2-di object with some extra structure

3. Associated vector bundle functors in post-tannakian philosophy

4. algorithm to enumerate simple medial magmas re Durov generalized fields.

5. algorithm to enumerate symmetric promonoidal POSETs

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Categorified linear algebra

loc. pres. category a.k.a.

small sketched colimit theories

Theory or sketch

Syntactic category

(2) ctd. with morphisms f preserving the operations

$f: b_1 \to b_2$

$$\begin{array}{ccc} \hom(c, b_1) & \to & h(d, b_1) \\ \hom(c,f) \downarrow & & \downarrow h(d,f) \\ L(c, b_2) & \to & L(d, b_2) \end{array}$$

we are using external homs.

functors

$$\check{A} \underset{\text{right}}{\overset{\text{left}}{\rightleftarrows}} \check{B}$$

adjoint functor theorem.

Defn recursive defn

base: terminal category: is the $\boxed{\text{theory of nothing}}$.

recursive (given A, B use cartesian product $A \times B$ of theories)

Given family $\{A_i\}_i$ indexed by S then $\coprod_{i \in S} \check{A}_i$ is a theory.

(this is a biproduct ie product & coproduct.)

recursive given $\check{A}$ we

(1) can $\boxed{\text{add a new object to the sketch}}$:

$\check{B} = \check{A} \times \check{Set}$

the new object is $(0, 1)$

$\hom(c, d) \to \hom(d, d)$

(2) Given $\check{A}$, $d, c \in Ob(\check{A})$

add a new morphism $f: d \to c$

$\check{B} = \{ b \in Ob(A)$

$\& f: \hom(c, b) \to \hom(d, b)$

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UFc has a belief:

$\hom(c, UFc) \to h(d, UFc)$

~~to~~ then:

unit $\mapsto$ answer.

<u>Other questions</u>

how to do equalizers?

<u>Answer</u> invert comparison morphism

A model is a left adjoint functor
"
preserving all small colimits.

(3) Given $\hat{A}$ with

$f : d \to c$

add ~~(scribble)~~ inverse:

$$\begin{tikzcd} \cdot \arrow[r, "f"] \arrow[dr, "1"'] & \cdot \arrow[d, "g"] \arrow[dr, "1"] & \\ & \cdot \arrow[r, "f"'] & \cdot \end{tikzcd}$$

$\hat{B} = \{ b \in Ob(\hat{A})$ with

$\hom(f, b) : \hom(c, b) \to h(d, b)$

invertible in $\hat{Set}$ $\}$

So we are requiring a <u>property</u> of our believers not any new <u>structure</u>.

Can ~~repeat~~ combine steps (1), (2), (3) a set amount of times.

<u>In (2)</u>

$$\tilde{A} \underset{\text{Under}}{\overset{\text{Free}}{\rightleftarrows}} \tilde{B}$$

by adjointness:

$$\frac{\text{Free}(d) \to \text{Free}(c)}{d \to \text{Under}(\text{Free}(c))}$$

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Sketch	Syntactic category
nothing	1
objects M, O	$\widehat{Set} \times \widehat{Set}$
$M \xleftarrow{d} O \xrightarrow{c} M$	(O_B, M_B) ... free quiver on a pair of sets
$\begin{array}{ccc} O & \xrightarrow{d} & M \\ {\scriptstyle c}\downarrow & \lrcorner & \downarrow \\ M & \longrightarrow & P \end{array}$ composition $P \leftarrow M$ etc.	Small Cat

$\boxed{\text{The theory of a cocategory object.}}$

See paper by Burroni: monad on the category of quivers whose algebras are the small categories.

Examples

1) cat of structure types
2) cat of small cats
3) ~~[illegible]~~ any "algebraic" category
4) non-example: totally dist boolean algebra in finitary $\wedge, \vee$ with dist.

$\widehat{Set}^{op} \xrightarrow{Pow}$ totally dist boolean alg.

It's rare for the op of a loc. pres. category to be loc. pres.

Sketch	Syntactic category
the theory of co-groups	category of groups

co classical constructions

(2) (ctd') Secondary operations

Sketch	Syntactic category
theory of two objects M, O pair of endomorphisms of M: $M \xleftarrow{d} M \xrightarrow{c} M$	$\widehat{Set}$

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<u>Novice:</u> $O_B = \{G, H, J\}$ $O_M = \{K, L\}$

<u>Adept:</u> free quiver:

$$\begin{array}{ll} \cdot\, G & sK \cdot \xrightarrow{K} \cdot\, tK \\ \cdot\, H & \\ \cdot\, J & sL \cdot \xrightarrow{L} \cdot\, tL \end{array}$$

$$\Big\downarrow \text{Under}$$

$$O = \{G, H, J, sK, sL, tK, tL\}, \quad M = \{K, L\}$$

So the monad Under $\circ$ Free is:

$$T : (O_B, M_B) \mapsto (O_B + M_B \cdot 2, M_B) = (O_B + sM_B + tM_B, M_B)$$

$$T = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$T^2 = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$$

$$\Big\downarrow T$$

$$(O_B + 2sM_B + 2tM_B, M_B) \rightsquigarrow (O_B + sM + tM, M)$$

A quiver is an algebra for this monad.

When the algebras for the monad are equivalent to the syntactic category this is special: we call this a monadic stage (of the sketch).

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Given a monad $T: C \to C$
construct the
algebras for the monad
left adjoint .... the.

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Example

theory of a cocategory object

syntactic category is category of small categories.

breaking the construction into stages:

primary operations:

domain, codomain, identity $O \overset{d}{\to} M$, $O \overset{c}{\to} M$, $M \overset{id}{\to} O$

secondary operations:

composition