(James Dolan) ③ 24/8/19

Two kinds of geometry

Kleinian geometry

vs.

affine algebraic geometry

In $A(\mathbb{Z}/p)$

$X \otimes X = pX$

① 25/8/19

Given n-box young diagram D, can associate a rep of S_n (also $GL_q(n)$.) in many ways.

Functor

$\tilde{n}\text{Set} \to \tilde{\text{Set}}$

$S \mapsto$ framed D quotient of S

" rows of D partition S

e.g. D: 

$\tilde{4}\text{set} \to \tilde{\text{Set}}$

(unframed): D quotient $\{a,b,c,d\} \mapsto$ 3 partitions $\{(ab, cd), (ac, bd), (ad, bc)\}$ — opposite edges of tetrahedron

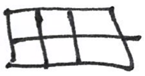
framed: D-quotient " $\mapsto$ 3 partitions mapped to D 6 of these.

total order on the parts

give perm reps of 4!

oriented D-quotients — not sure exactly how to define these so far...

e.g.

$D=$  $\tilde{6}\text{Set} \to \tilde{\text{Set}}$

...... unframed partitions 10

framed D-quotients $\frac{6!}{3!\,3!} = 20$

oriented D-quotients 20

[James Dolan] (2) 25/8/19

an orientation on a set is a parity class of total orders

for $n \geq 2$ there are two of these.

for $n = 0, 1$ we ignore these...

eg. $D =$ 

$$\widetilde{4Set} \to \widetilde{Set}$$

(unframed) D-quotients $\{a, b, c, d\} \mapsto$ ab, ac, ad, bc, bd, cd — 6

framed D-quotients " $\mapsto 12$

oriented D-quotients " $\mapsto 6$

eg. $D =$ 

$$\widetilde{6Set} \to \widetilde{Set}$$

framed D-quotients " $\mapsto 15 \times 6 = 30 \times 3 = 90$

(unframed) D-quotients " $\mapsto \frac{90}{6} = \frac{45}{3} = 15$

oriented D-quotients " $\mapsto 30$

James Dolan

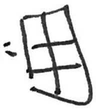
③ 25/8/19

$\{1\ 2\ 3\ 4\} \mapsto \{(1\ 2), (1\ 3), (1\ 4), (2\ 3), (2\ 4), (3\ 4)\}$

Consider the signed permutation rep given by the induced by 1-dim char given by orientation ± 1.

$Fix(g) = \{+(1\ 2)(3)(4), -(1)(2)(3\ 4)\}$

Char $= 1 - 1 = 0$

eg. $D =$ 

$S = \{1, 2, 3, 4, 5, 6\}$

with unframed D-quotient:

$K = \{(1,6), (2,4), (3,5)\}$

Conjugacy class of permutation given by $g =$ 

Conjecture: irrep for D is contained within the signed perm for D, iff  does not appear. Call these diagrams "bad".

eg $n = 6$

6
5, 1
4, 2
4, 1, 1
3 3
3 2 1
3 1 1 1
2 2 2 ← bad
2 2 1 1 ← bad
2 1 1 1 1
1 1 1 1 1 1

remember: char of perm = |Fixed points|

g act on K:

$\{(2,6)\ (1,4)\ (3,5)\}$ not fixed

$Fix(g) = \{(1\ 2)(3\ 4)(5\ 6), (1\ 2)(3\ 5)(4\ 6), (1\ 2)(3\ 6)(4\ 5)\}$

signed Char $= +3 - 0 = 3$

eg $D =$  with $g =$ 

eg $n = 7$ bad diagrams

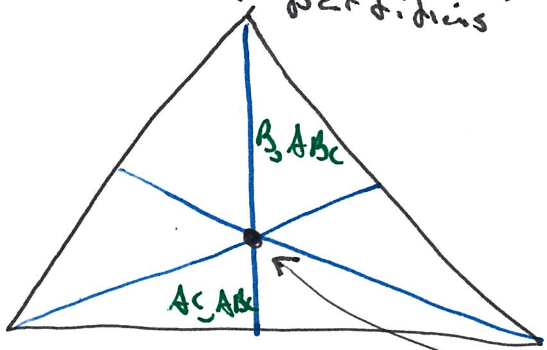
3 2 2
2 2 2 1
2 2 1 1 1

asymptotic as $n \to \infty$ $\frac{|good|}{|bad|} \to 0$.

James Nolan

(4) 25/8/19

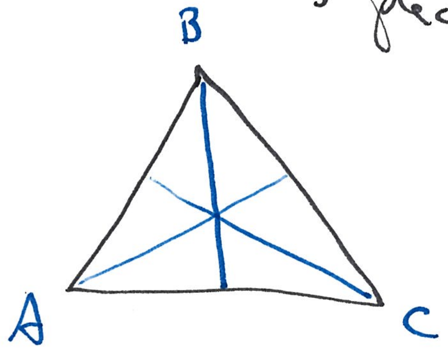
unframed partitions correspond to linear hull of the corresponding framed partitions

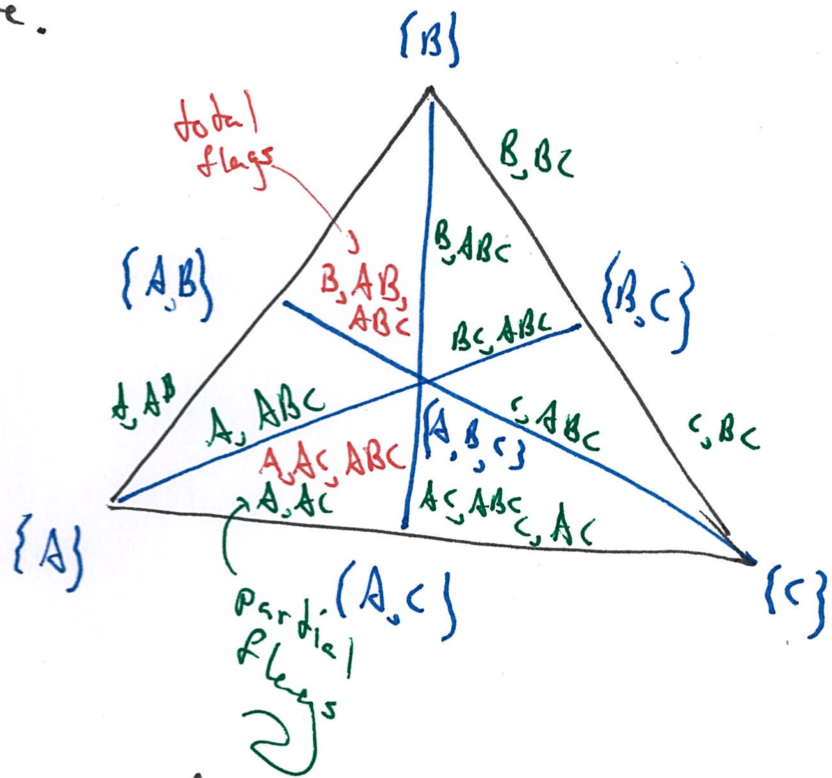


2 dim vector space with origin
(could also do this in 3 dim)
ie. taking linear hull is forgetting the frame.

question: how to fix the bad diagrams?

A-series kaleidoscope
barycentric subdivide simplices





decorate each piece with a flag

write as monotonic list of subsets or
list of differences aka framed partition.

James Nolan

5 25/8/19