① 9/6/19

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Sheaves vs. Presheaves

site category (non cocomplete)

free cocomplete = presheafs

Base belief doctrine: colimits

"create the Generic model of a theory by assembling the constructs which believe the theory"

= Categorified Grobner basis

example sheaves live in the geometric (or topos) doctrine:

1) small colimits
2) finite limits
3) distributivity of limits over colimits

the "beleives" are the sheaves.

(free Vec or free Ab functor)
linearize a simplicial set gives a chain complex.

Chain complex is a Presheave in the relativised case.

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a Sketch

stuff: objects X, Y

structure: morphism $e: X \twoheadrightarrow Y$

generators

properties: e is epi

relations

"invertability condition"
invert comparison morphism

Sketch is a categorified presentation

theory is a categorified algebra

	Theory	Syntactic category
1)	empty	$\{*\}$
2) "stuff"	X, Y	bi set
3)	$X \xrightarrow{e} Y$	set functions
4)	$X \overset{e}{\rightarrowtail} Y$	injections

Category of fractions:
make some morphisms invertible

Example category $\tilde{C}$
with some morphisms $\tilde{J} \subset \tilde{C}$.

zig-zags of morphisms:
zig in $\tilde{C}$
zag in $\tilde{J}$

The easy case is called "calculus of fractions":
any zig zag ⋀⋀ ··· ⋀⋀
reduces to ⋀.

The "belief method" refines this uses colimits

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$$C = \{ X \xrightarrow{e} Y \}$$

s.t.

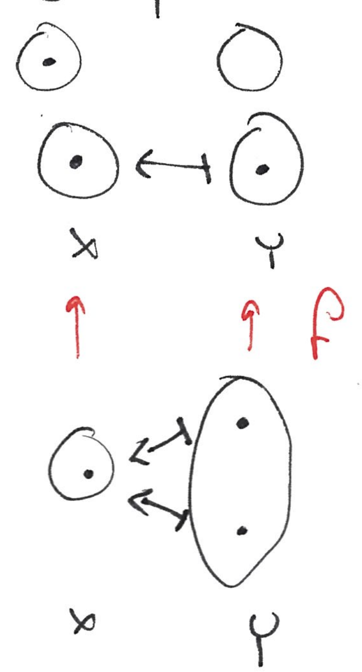
$$\begin{tikzcd}
\hat{X} \arrow[r, "\hat{e}"] \arrow[d, "\hat{e}"'] & \hat{Y} \arrow[d] \arrow[ddr, bend left, "1_{\hat{Y}}"] & \\
\hat{Y} \arrow[r] \arrow[drr, bend right, "1_{\hat{Y}}"'] & \hat{P} \arrow[dr, "\exists!\, f"] \arrow[ul, phantom, "\ulcorner"] & \\
 & & \hat{Y}
\end{tikzcd}$$

invert the comparison morphism f

$\hat{X} = \hom(-, X) =$

$\hat{Y} = \hom(-, Y) =$

$\hat{P} =$



We take presheaves that believe f is an isomorphism (i.e. invertible).

A "believer" $\hat{B}$ is such that

$$\hom(\hat{Y}, \hat{B}) \xrightarrow{f} \hom(\hat{P}, \hat{B})$$

is a bijection.

$$\begin{tikzcd}
\hat{P} \arrow[rr, "f"] \arrow[dr] & & \hat{Y} \arrow[dl] \\
 & \hat{B} &
\end{tikzcd}$$

this is a full subcategory of the $PSh(C)$

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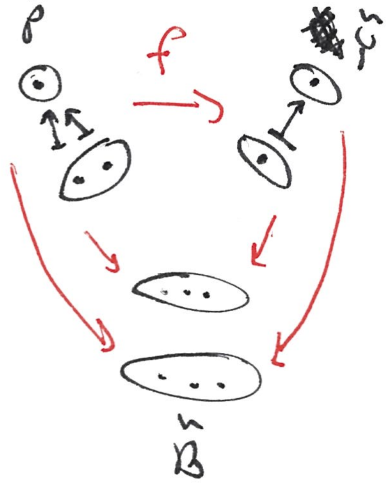
In this case
$\tilde{B}$ is a believer (i.e. being epi)
exactly when

$B = X \leftarrow Y$
is an ~~[illegible]~~ injection
i.e. a coclassical model of the theory.



Doctrine of
1) "colimits
2)
3) geometric doctrine
topos

Projects

(1) Doctrine of
commutative ~~[illegible]~~ quantales
find Grober basis analogs

(2) Algebraic-geometric
Doctrine has
colimits & tensor
products
(tensor categories)
find syntactic category
for the theory of an
adjunction: i.e. an object
V with right adjoint ~~[illegible]~~ V^*

(1)

(2) the syntactic category
will be close to the
category Rep(GL(n))