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Bialgebra:

vector space
with
mult

co-mult

(e.g. Hopf algebra)

① 14/4/19

categorified
plethory

Baez & Trimble

Biring

=

affine algebraic
comm Ring

=

pseudo-coexponent

=

comm Ring object
in category of
affine ~~schemes~~ schemes
(= comm Rings)

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Algebra
Comm Ring

co-exponent

Geometry
affine schemes

exponent (infinitesimal objects
(exponentiable object)

$A \otimes _$
right adjoint

co-base \downarrow co-exponent
 $\text{hom}(_, A)$
left adjoint

endo left adjoint functor
categorical action of ~~the~~
a coexponents on
Comm. rings.

$A \times _$ $_ \times A$
left adjoint functor right adjoint functor
 $\text{hom}(A, _)$

Cartesian internal hom
"mapping space"
Endo-~~left~~ right adjoint functor
morally these are
categorified linear operators

Strong monoidal functor
from a cartesian category
to category of endofunctors
"action"
categorified:

Coproduct is tensor
tensor product of coexponents
taken to

$\text{hom}(X, E_1 \otimes E_2)$
co-base \downarrow co-exponent

~~$\text{hom}(\text{hom}(X, E_1), E_2)$~~
 $\cong \text{hom}(\text{hom}(X, E_1), E_2)$

Cartesian product
of exponent
objects E_1, E_2

$X^{E_1 \times E_2} = (X^{E_1})^{E_2}$
 $\text{hom}(E_1 \times E_2, X)$
 $\cong \text{hom}(E_2, \text{hom}(E_1, X))$

Algebra

coadjoint

dual numbers

coexponential operator

left adjoint

ends functor "action"

"theory of model

coarse in coexp"

take presentation

↑
as formalized
or internalized
comm Ring

to get presentation of
coordinate ring of
algebraic variety
for tangent bundle

right adjoint:

tensoring with
dual numbers

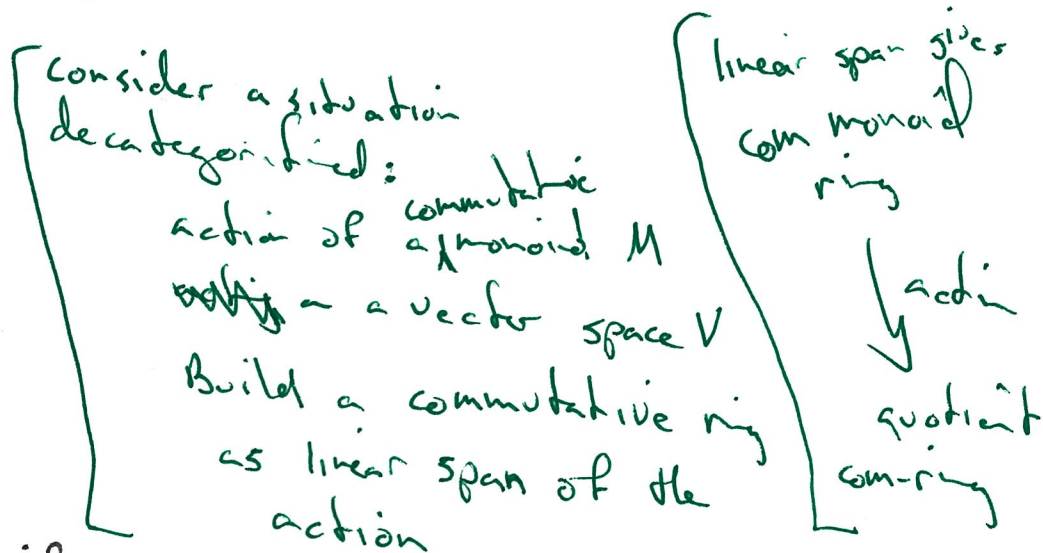
Geometry

walking tangent vector

take tangent bundle

AlgebraGeometry

Categorified monoid action

Categorify

$M: \text{co}^{\text{op}}$ Cartesian monoidal category of
coexponents

V : comonoid category of
com-rings

linear span: colimits of
endo functors built
from M

Synonyms

1) left adjoint endo functors
on the category of com-rings

2) Affine algebraic comm Ring

3) aff com-ring scheme

4) Baez & Trimble "Biring" "plethorics"

5) ~~comonoid~~

Pseudo-coexponent

?? pseudo-pseudo?

Three levels of pseudo

1) Coexponents

2) "pseudo" linear span categorified
cocomm coalgebras ?? Conjecture

3) "pseudo-pseudo": affine algebraic
com-ring

Commutative guys:
coexponents

1) Coexponents



categorified Comm Ring:

has additive group
free abelian
group finite rank

~~\$ \mathbb{Z}^n \$~~

adjoint objects in
symmetric monoidal category

see also: finite dimensional
vector spaces

Coexponents are
Comm algebra with
finite dim underlying
vector space.

symmetric monoidal nicely
co complete category
(nicely = locally presentable)

coexponents have

underlying vector space

category is a

presheaf category

these have adjoints

example category of
quasi coherent sheaves

over an
affine algebraic
variety

2) pseudo take colimits of these

Dualizing finitist algebras
can give infinite dim
coalgebras

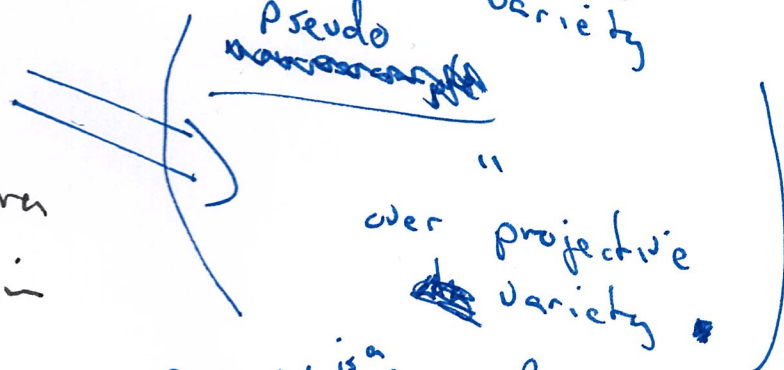
3) pseudo-pseudo

e.g.
jacobian
of a
curve
abelian
variety

internal hom
mapping space



Moduli stack of vector bundle



a module is a
functor, monoid
into Vec .

over projective variety (glued together
affine variety
colimits)

give pseudo coexponents