

(James Nolan)

①

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Algebra

Geometry

Commutative Hopf algebra H Affine group scheme G

element of H polynomial function on G

Example: k a field

then $\text{FinVect}_k \cong \text{FinVect}_k^*$

Finite Bi Comm Hopf algebra H

then H^* is also Bi Comm Hopf algebra

(reverse arrows & swap
mul - comul
unit - counit
in definition
to get the same
definition)

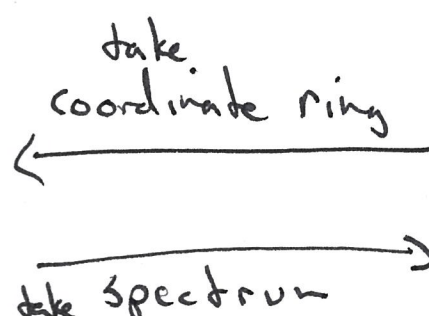
Fourier dual &
"probability" functions

Algebra

Geometry

Com-Ring

Scheme



Algebra

(pseudo-) coexponentials
theory of the
models of cobase
in the (pseudo)
coexponent

Coexponent
affine algebraic
commutative algebra

~~commutative~~
commutative
Hopf algebras

contravariant
functor \rightarrow

H

comm algebra

com bi-algebra

Hopf algebra
antipode
counit

The definition of a
Bicommutative Hopf algebras
have Hopf duality which is
a kind of Fourier duality

between finite
abelian groups

Bicommutative = commutative
& co-commutative

Geometry

exponential objects

category of
affine schemes
= opposite of category
of commutative algebras

~~commutative~~
~~category of~~
~~commutative~~
~~algebras~~

group objects in category
of affine schemes

Group object G "affine
group
scheme"

affine scheme

affine monoid

affine group
inverse
unit of group $G \rightarrow G^{-1}$

G
 \downarrow
 $G \times G$

Definition of a pseudo-coalgebra

Algebra

Geometry

affine
pseudo coalgebra
algebraic comm ring
 S

pseudo coalgebra
affine comm ring scheme
 R

Bicov Hopf algebra

Bicommut bi algebra
(Hopf algebra without antipode)
multiplication & comultiplication

underlying abelian group scheme

underlying comonoid scheme

underlying affine scheme

has:

multiplication unit

coaddition & cozero
comultiplication

comultiplication & coone

The Pseudo-coalgebra B^E

of cocommutative B -comm ring

with pseudo-coalgebra E : affine algebraic comm ring

affine scheme

affine comm ring scheme

(Glitch: not "relativised"
base ring is \mathbb{Z} .)

B^E = spectrum of theory of B in E

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B is a system of equations
model is a solution of equations.

Example tangent bundle of circle

co-base: $B = K[x, y]$
 $x^2 + y^2 = 1$

$= K[\cos, \sin]$
 $\cos^2 + \sin^2 = 1$

E : affine conic scheme

Com ring of polynomial functions on $K[x]_{x^2} = \{(a, b) \mid \text{with } (a, b) \cdot (c, d) = (ac, ad + bc)\}$
• 2-dim real vector space
 $I = (1, 0)$
 $T = (0, 1)$

R is poly fones: $K[a, b]$

(1) co-zero: $K[a, b] \rightarrow K$
 $a \mapsto 0$
 $b \mapsto 0$

(2) co-add: $R \rightarrow R \otimes R$
 $a \mapsto a_1 + a_2$
 $b \mapsto b_1 + b_2$

(4) co-one: $R \rightarrow K$
 $a \mapsto 1$
 $b \mapsto 0$

(3) co-neg: $R \rightarrow R$
 $a \mapsto -a$
 $b \mapsto -b$

$R \otimes R$ is coproduct:
 $K[a_1, b_1, a_2, b_2]$

(5) comul: $R \rightarrow R \otimes R$
 $a \mapsto a_1, a_2$
 $b \mapsto a_1 b_2 + a_2 b_1$