[James Dolan]

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Bialgebra:

with Vector space

mult

co-mult

(e.g. Hopf algebra)

categorified plethory

Baez & Trimble

Biring

=

affine algebraic CommRing

=

pseudo-coexponent

=

CommRing object in category of affine ~~[illegible]~~ schemes (= $\mathrm{CommRing}^{op}$)

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Algebra	Geometry
Comm Ring	affine schemes

co-exponent

exponent (infinitesimal objects)
(exponentiable object)

$A\times \sqcup$ left adjoint functor

$\sqcup^{A}$ right adjoint functor

$\hom(A, \sqcup)$

Cartesian internal hom
"mapping space"

$A \otimes \sqcup$
right adjoint

$\hom(\sqcup, A)$ (cobase: $\sqcup$; coexponent: A)
left adjoint

endo left adjoint functor
categorified tautological action of a coexponents on comm. rings.

Endo- ~~left~~ right adjoint functor
morally these are categorified linear operators

Strong monoidal functor
from a cartesian category
to category of endo functors
"action"
categorified:

Coproduct is tensor
tensor product of coexponents taken to

$\hom(X, E_1 \otimes E_2)$ (cobase: X; coexponent: $E_1 \otimes E_2$)

$\cong \hom(\hom(X, E_1), E_2)$

Cartesian product of exponent objects E_1, E_2

$X^{E_1 \times E_2} = (X^{E_1})^{E_2}$

$\hom(E_1 \times E_2, X)$

$\cong \hom(E_2, \hom(E_1, X))$

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Algebra	Geometry
coexponent dual numbers	walking tangent vector
coexponential operator left adjoint endo functor "action"	take tangent bundle

"theory of model
cobase in coexp"

as formalized
or internalized
CommRing

take presentation
to get presentation of
coordinate ring of
algebraic variety
for tangent bundle

right adjoint:
tensoring with
dual numbers

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Algebra

Geometry

Categorified monoid action

consider a situation
decategorified:
action of a commutative monoid M
~~on~~ on a vector space V
Build a commutative ring
as linear span of the action

linear span gives
com monoid
ring
↓ action
quotient
com-ring

Categorify
M: co-cartesian monoidal category of
coexponents
V: cocomple category of
com rings

linear span: colimits of
endo functor ~~...~~ built
from M

Synonyms
1) left adjoint endo functors
on the category of Com-Rings
2) Affine algebraic comm Ring
3) aff co-ring scheme
4) Baez trimble "Biring" "plethorys"
5) ~~...~~
Pseudo-coexponent
?? pseudo-pseudo?

three levels of pseudo
1) Coexponents
Conjecture 2) "pseudo" linear span categorified
cocomm coalgebras "?conjecture
3) "pseudo-pseudo": affine algebraic
Com Ring

Commutative guys:
coexponents

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1) Coexponents $\xrightarrow{\text{categorify}}$ categorified CommRing:

has additive group free abelian group finite rank

symmetric monoidal nicely cocomplete category (nicely = locally presentable)

$ ~~[illegible]~~

adjoint objects in symmetric monoidal category

see also: finite dimensional vector spaces

coexponents have underlying vector space category is a presheaf category

coexponents are comm algebra with finite dim underlying vector space.

these have adjoints

example category of quasi coherent sheaves over an affine algebraic variety

2) pseudo take colimits of these

pseudo ~~[illegible]~~

"

over projective variety

Dualizing findim algebra can give infinite dim coalgebras

a module is a functor from a monoid into Vec.

3) pseudo-pseudo

over projective variety (glued together affine variety colimits)

e.g. jacobian of a curve abelian variety

* give pseudo coexponents

internal hom mapping space $\longrightarrow$ Moduli stack of vector bundle e.g.