James Dolan (1) 19/4/19

Set valued functors & Presheaves

(Presheaves & diagrams)

diagram with commutativity

diagram scheme

abstract diagram

(concrete) diagram in a category

Presheaf or set valued functor

"Algebra"
generators & relations,
quotient of a free,
coequalizer

Finitely presented category: as a diagram scheme or algebraic theory (unary)

Example objects A, B

morphisms

$$A \xrightarrow{c} B \circlearrowright d$$

one relation:

$$c = d^2 c$$

$$\begin{tikzcd} A \arrow[r, "c"] \arrow[d, "c"'] & B \arrow[d, "d"] \\ B & B \arrow[l, "d"] \end{tikzcd}$$

hom	A	B
A	$\{1_A\}$	$\{c, dc\}$
B	$\{\}$	$\{1_B, d, d^2, \dots\}$

(columns: tgt; rows: src)

"clumps" correspond to rows

Yoneda lemma

Hom functor is free on one generator. That generator is the identity morphism.

a Natural tran

Consider a presentation for an algebra F:

$F(A) = \{e, f\}$

$F(B) = \{g\}$

$$F(A) \xrightarrow{F(c)} F(B)$$

$e \longmapsto ce,\ dce$ $F(d)$

$f \longmapsto cf,\ dcf$

$g, dg, d^2g, \dots$

each "clump" is a representable functor

now add a relation:

$$ce = d^5 g$$

then we get

$$g \to dg \to d^2g \to d^3g \to d^4g \to d^5g \rightleftarrows d^6g$$

$d^5g = ce$, $d^6g = d^5g$

here representable functor (hom functor) corresponds to a free algebra

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<u>Yoneda</u> Hom functor is free on one generator, the identity morphism:

hom	A	B
A	$\{1_A\}$	$\{c, d_c\}$
B	$\{\}$	$\{1_B, d, d^2, \ldots\}$

$\mathrm{Hom}(A, \_)$

$1_A \in \mathrm{Hom}(A, A)$

$F(A) = \{e, f\}$

$F(B) = \{g\}$

$$F(A) \xrightarrow{F(c)} F(B) \circlearrowleft F(d)$$

Then we have the following Natural transforms defined by value on the universal element

$\mathrm{Hom}(A, \_) \to F$

$1_A \mapsto e$

$\mathrm{Hom}(A, \_) \to F$

$1_A \mapsto f$

$\mathrm{Hom}(B, \_) \to F$

$1_B \mapsto g$

<u>Yoneda</u> statement about universal property
= "of a universal element"

take <u>Coequalizer</u>:

$$\mathrm{Hom}(B, \_)$$

$1_B \mapsto ce$ $1_B \mapsto dg$

e f g

$$\mathrm{Hom}(A, \_) \quad \mathrm{Hom}(A, \_) \quad \mathrm{Hom}(B, \_)$$

$$\downarrow$$

$$\mathrm{Hom}(A, \_) + \mathrm{Hom}(A, \_) + \mathrm{Hom}(B, \_)$$

$$\downarrow$$

to get F

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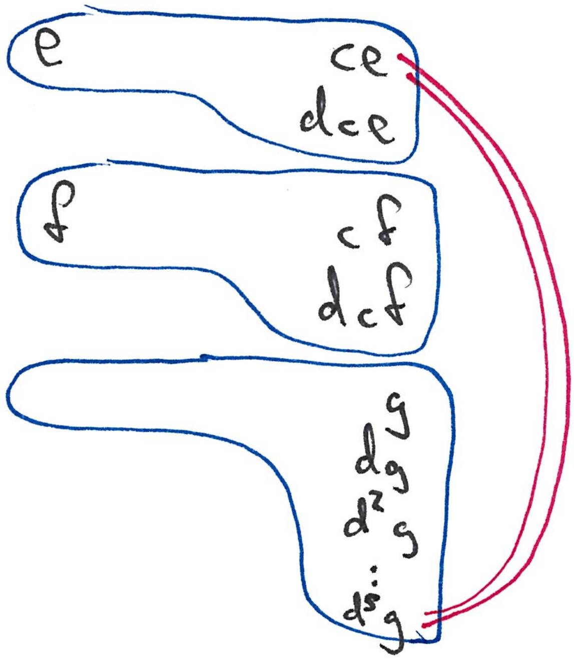
colimit of three "clumps"



Homework simplicial sets are presheaves on Δ

Δ = category of simplexes

"walking simplex"

(Graphs have just 0 & 1 dimensional simplexes)