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6. calculating beliefs & believers

List:

1. defn of a locally presentable category
2. role of young diagrams in belief method for n -dim objects
- 2'. McKay correspondence & belief in a 2-dim object with some extra structure
3. Associated vector bundle functors in post-Tannakian philosophy
4. algorithm to enumerate simple medial magmas & re Dorov generalized fields.
5. algorithm to enumerate symmetric promonoidal POSETs

Theory or sketch
Syntactic category

(2) ctd. with morphisms f
preserving the operad

$$f: b_1 \rightarrow b_2$$

$$\text{hom}(c, b_1) \rightarrow \text{hom}(c, b_2)$$

$$\text{hom}(sf) \downarrow \text{hom}(d, f)$$

$$L(c, b_2) \rightarrow L(d, b_2)$$

we are using external
hom.

functors

$$\tilde{A} \xrightleftharpoons[\text{right}]{\text{left}} \tilde{B}$$

adjoint functor theorem.

Categorical linear algebra
loc. pres. category a.k.a.
smallly sketched colimit theories

Defn recursive defn

base: terminal category:
is the theory of nothing.

recursive (given A, B
use cartesian product $A \times B$
of theories
Given family $\{A_i\}_i$ indexed by S
then $\coprod_{i \in S} A_i$ is a theory.
(this is a biproduct
ie product & coproduct.)

recursive given A we

(1) can add a new object
to the sketch:

$$\tilde{B} = \tilde{A} \times \text{Set}$$

the new
object is
(0, 1)

$$\text{hom}(c, d) \rightarrow \text{hom}(d, d)$$

(2) Given \tilde{A} , $d \in \text{Ob}(\tilde{A})$

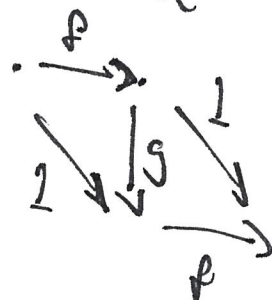
add a new morphism $f: c \rightarrow d$
 $\tilde{B} = \tilde{A} \cup \{b \in \text{Ob}(\tilde{A})\}$

$$f: \text{hom}(c, b) \rightarrow \text{hom}(d, b)$$

UFc has a belief:

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$\text{hom}(c, \text{UFc}) \rightarrow \text{hom}(d, \text{UFc})$ (3) Given \tilde{A} with
 then: unit \mapsto answer.
 add $f: d \rightarrow c$
~~add~~ invert:



$B = \{ b \in \text{Ob}(\tilde{A}) \text{ with}$
 $\text{hom}(f, b): \text{hom}(c, b) \rightarrow \text{hom}(d, b)$
 invertible in $\tilde{\text{Set}} \}$

Other questions
 how to do equalizers?
Answer invert comparison morphism

A model is a
 left adjoint functor
 "joint functor
 preserving all small colimits.

So we are requiring
 a property of our
 believers not any new
structure.

Can combine steps
~~repeat~~ (1), (2), (3)
 a set amount of times.

In (2)
 $\tilde{A} \xrightarrow{\text{Free}} \tilde{B}$
 $\tilde{B} \xleftarrow{\text{Under}} \tilde{A}$

by adjointness:
 $\text{Free}(d) \rightarrow \text{Free}(c)$

 $d \rightarrow \text{Under}(\text{Free}(c))$

Sketch	Syntactic category	Examples
nothing	1	1) cat of structure types
objects M, O	\sim $\text{Set} \times \text{Set}$	2) cat of small cats
$M \xleftarrow{d} O \xrightarrow{c} M$	$(O_B, M_B) \dots$	3) any "algebraic" category
$O \xrightarrow{d} M$	free quiver on a pair of sets	4) non-example: totally dist boolean algebra in finitary \wedge, \vee with dist.
$\begin{array}{ccc} c \downarrow & & \downarrow r \\ M & \rightarrow & P \end{array}$ composition M	Small Cat	$\text{Set}^{\text{op}} \xrightarrow{\text{pow}} \text{totally dist boolean alg.}$
etc.		
<div style="border: 1px solid red; padding: 5px; display: inline-block;"> The theory of a cocategory object. </div>		
see paper by Burroni: monad on the category of quivers whose algebras are classical constructions		
Sketch the theory of co-groups	Syntactic category of groups	(2) (ctd!) Secondary operations sketch theory of two objects M, O pair of endomorphisms of M : $M \xleftarrow{d} M \xrightarrow{c} M$

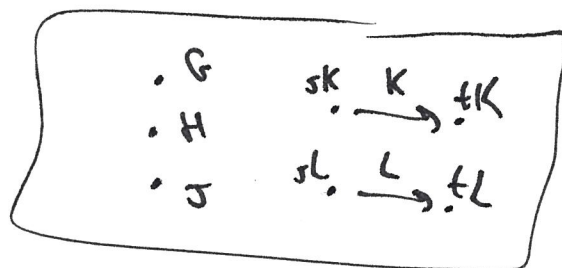
the small categories

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Novice: $O_B = \{G, H, J\}$ $O_M = \{K, L\}$

Adept: free quiver:



Under ~~the~~

$$O_M = \{G, H, J, sK, sL, tK, tL\}, M = \{K, L\}$$

So the monad ~~the~~ Under-Free is:

$$T : (O_B, M_B) \mapsto (O_B + M_B \cdot 2, M_B)$$

$$= (O_B + sM_B + tM_B, M_B)$$

$$T = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$T^2 = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$$

$$\downarrow$$

$$(O_B + 2sM_B + 2tM_B, M_B)$$

$$\rightsquigarrow (O_B + sM + tM, M)$$

A quiver is an algebra for this monad.
 When the algebras for the monad
 are equivalent to the syntactic
 category this is special: we call
 this a monadic stage (of the sketch).

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Given a monad $T: C \rightarrow C$
construct the
algebras for the monad

left adjoint t.b.e.

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Example

theory of a cocategory
object

syntactic category
is category of
small categories.

breaking the construction
into stages:

primary operations:

domain, codomain, identity

secondary operations:

composition

