[James Dolan] ① 17/4/19

Categorified:

K: certain category of sets

V: category of comm Rings

[any locally presentable category
addition is coproduct
$\widetilde{\text{Set}}$-module]

M: cartesian monoidal category of coexponents

action:

coexponentiation functor

Im(K[M]): conjecturally the category of cocomm Corings

K[M]: $\widetilde{\text{set}}$ presheaves on the category of coexponents

The category of coexponents is small.

(these are finitely generated abelian groups)

So can freely co-complete.

[Decategorified level:

Synthetic diff geom

exponential is tangent bundle

Categorified

Jacobian varieties

exponential is picard stack

moduli stacks of bundles

abelian variety as exponential in 2-cat

of line bundles over a curve]

algebra of square matrices is an internal hom (V, V)

categorified is biring

take monoid action on a vector space

$$M \to (V,V)^{\times}$$

linear span of M is a monoid ring

$$\begin{tikzcd}
K[M] \arrow[rr, "K\text{-algebra hom}"] \arrow[dr] & & (V,V) \\
& Im(K(M)) \arrow[ur] &
\end{tikzcd}$$

[illegible]

free functor

[illegible]

forgetful functor

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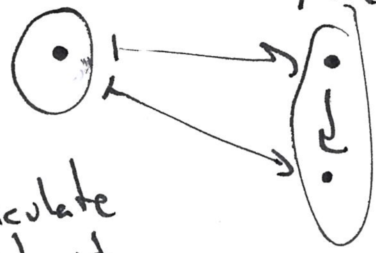
walking vertex quiver

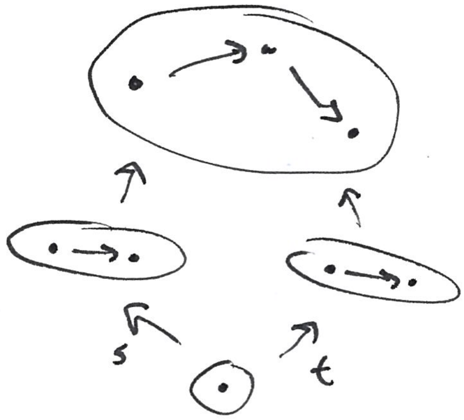
walking arrow quiver



to calculate pushout:

$$\begin{tikzcd} & V \arrow[dl, "s"'] \arrow[dr, "t"] & \\ A & & A \end{tikzcd}$$

work in the category of quivers:



~~Example Comm Ring integers~~

(Locally presentable) cocomplete category $\check{T}$

"site category" $\check{C}$ is a full, small, subcategory of $\check{T}$

inclusion $\check{C} \to \check{T}$ is full & faithful

$$\begin{tikzcd} \check{C} \arrow[d, "x \mapsto \hom(x, \cdot)"] \\ \check{Set}^{\check{C}^{op}} \end{tikzcd}$$

this is the Yoneda embedding

this is the free cocomplete category on $\check{C}$

Example

$\check{C} = \{a, b\} \to \check{Set}^{\check{C}^{op}}$

$(\check{C}, \check{Set})$

$a \mapsto \hom(a, \cdot)$

$b \mapsto \hom(b, \cdot)$

$$\begin{tikzcd} \hom(a,\cdot) \arrow[dr] & & \hom(b,\cdot) \arrow[dl] \\ & \hom(a,\cdot)+\hom(b,\cdot) & \end{tikzcd}$$

Example

$\check{C} = \{ \text{Vertex} \overset{s}{\underset{t}{\rightrightarrows}} \text{Edge} \}$

Then $\check{Set}^{\check{C}^{op}}$ is graphs

two objects & four morphisms

Example

$\check{C} = \{ \text{Vertex} \overset{s}{\underset{t}{\rightrightarrows}} \text{Arrows} \}$

yoneda

quivers: $\check{Set}^{\check{C}^{op}}$

walking vertex

walking arrow

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Exercise

take product of two representable presheaves:

this is a Square:

hom(⌞ , ⋮) × hom(⌞ , ⋮)

= is a pushout

hom(⌞ , ⋮)

↙ ↘

hom(⌞ , ⋮)   hom(⌞ , ⋮)

↘ ↙

.

generators & relators

because every presheave is a colimit of representables.

Reminder every group is a quotient of a free group. (every group is given by generators & relations).

Example

Site: $C = \{ \text{Red}, \text{Blue} \}$

Presheaf: Reds & Blues

Example

simplicial sets

Site: $C^{\text{opposite?}}$ category of nonempty finite ~~[illegible]~~ totally ordered sets

hom(⋮ , ⋮) has 10 elements

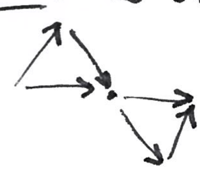
hom(⌞ , ⋮)

= 4 + 10 + ~~[illegible]~~ 20 + 35 + ...

detached 4 vertices, detached 10 edges, faces, bodies

this is the walking 3-simplex.

Exercise construct this presheaf



as an amalgamated sum of representables

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$\tilde{T}$ locally presentable category
$\Rightarrow$ cocomplete

$\tilde{C}$ small, full subcategory of $\tilde{T}$

$$
\begin{tikzcd}
\tilde{C} \arrow[r, tail, "\text{full \& faithful}"] \arrow[d, "\text{Yoneda}"'] & \tilde{T} \\
\tilde{Set}^{\tilde{C}^{op}} \arrow[ur, dashed, "\exists!"'] &
\end{tikzcd}
$$

universal arrow (Kan extension)

← free thing (has a universal property) (left adjoint functor)

Homework presheaves = free colimits.

Lemma Presheaves = free cocompletion 19/4/19