James Dolan ① 4/6/15

Categorified Gröbner basis theory

presentations in co-algebra:
generators & relations
gives co-elements = maps out of

"a model is a point"

out: so it's easy to map out
this a model of the theory.

in: harder to map into a presentation: maps into = elements
this is a construction of the theory

"a construction is a polynomial"

Example

$$k[a,b,c] \Big/ \langle p(a,b,c) \rangle$$

Gröbner basis give a section of this projection.

Categorify confluent rewriting systems.

seasons
& time of day
spring vs autumn
longer/shorter
dawn/dusk

Ergodic hypothesis

Categorified linear algebra

cocomplete categories
free cocomplete = presheaves
duality between limits & colimits

example: 2-vector spaces?
needs relativisation
i.e. enrichment
(Relativise algebraic geometry over different fields, rings, etc.)

Categorify algebraic geometry
(from an algebra we relativise vector spaces to get Vec over an algebra)

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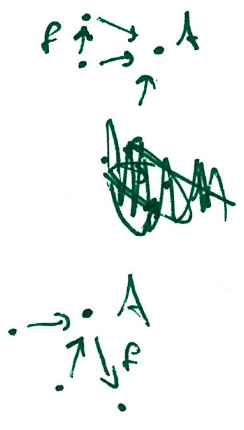
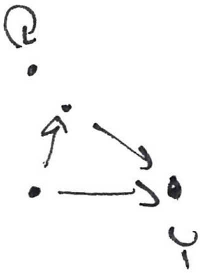
$hom(f, A)$





Amazing Fact (like Gröbner basis)

construction of the theory

=

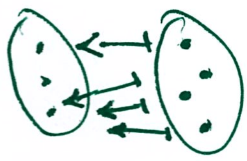
co-classical models

Example (Free)

site category

$$C = \{X \xrightarrow{e} Y\}$$

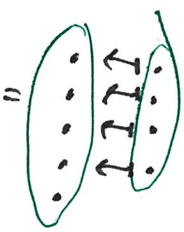
$$PSh(C) = hom(C^{op}, \check{Set}) = [C^{op}, \check{Set}]$$

$\hat{F}(C) =$  $\in PSh(C)$

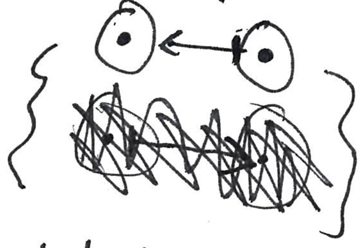
$\hat{F}(X)$ $\hat{F}(Y)$

here we construct F as a colimit of representables

$$\begin{tikzcd} & 3\check{Y} \arrow[dr, "include"] & \\ 3\check{X} \arrow[ur] \arrow[rr] \arrow[d, "fold"'] & & 4\check{Y} + \check{X} \arrow[d] \\ \check{X} \arrow[rr] & & F \end{tikzcd}$$

$4\check{Y} + \check{X} =$ 

$\check{X} = hom_C(-, X) = \{$ 

$\check{Y} = hom_C(-, Y) = \{$  $\}$

We have a presheaf morphism $hom(X, Y)$ $hom(Y, Y)$

$$e : \check{X} \to \check{Y}$$

ie. a natural transform

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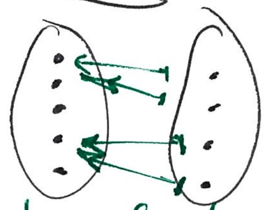
So when is

$$\hom(\check{Y}, \check{F}) \xrightarrow{\hom(e, \check{F})} \hom(\check{X}, \check{F})$$

an injection?

Example (cont'd)

F = 

Left adjoint "repairs" non-injective functions:
(See also "forcing" in logic)
by collapsing the fibres or taking the image of the map.

left adjoint functors categorify closure operators on posets.

$$\begin{array}{ccccc} 2\check{X} & \to & 4\check{Y} + 3\check{X} & \leftarrow & 2\check{X} \\ \downarrow & & \downarrow & & \downarrow \\ \check{X} & \to & \check{F} & \leftarrow & \check{X} \end{array}$$

Example (non-free)

now add relation to make e surjective

the amazing trick:
Take subcategory instead of quotient category.
Inclusion functor has left adjoint.
The presheaves that "believe" they are an epimorphism:
$\check{B} \in PSh(C)$ are such that

$$\hom_{PSh}(\check{X}, \check{B}) \xleftarrow{\hom(e, \check{B})} \hom_{PSh}(\check{Y}, \check{B})$$

ie. a co-classical model of the theory