James Dolan ① 1/8/19

Dimension:
sum over bumps:

dim( )

= dim( ) + dim( )

(Bratelli diagram?)

Categorify:

$Res^{n!}_{(n-1)!}$

and $hom(\square, -)$

Parabolic restriction

tensor is
parabolic induction

Beliefs on young diagrams

Use internal hom
when in Doctrine
of tensor products & colimits

External $Hom(a,b) = \int_a^b$

$$Hom(Y \otimes X, Z) \cong Hom(Y, hom(X,Z))$$

internal

adjoints

$R = Hom(X,-)$

$L = - \otimes X$

Check this:
Internal hom functor
preserves colimits so we
just look at irreps, ie. Young
diagrams.

Look at $hom(A,B)$
with

1) A has zero boxes

$$hom(I_\otimes, B) \cong B$$

2) $hom(\square, B)$

$$hom(\square \otimes \square, B) \cong hom(\square, hom(\square, B))$$

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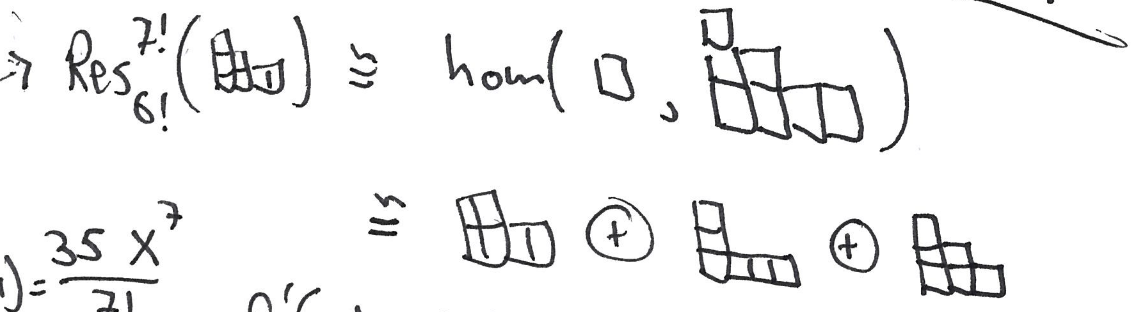
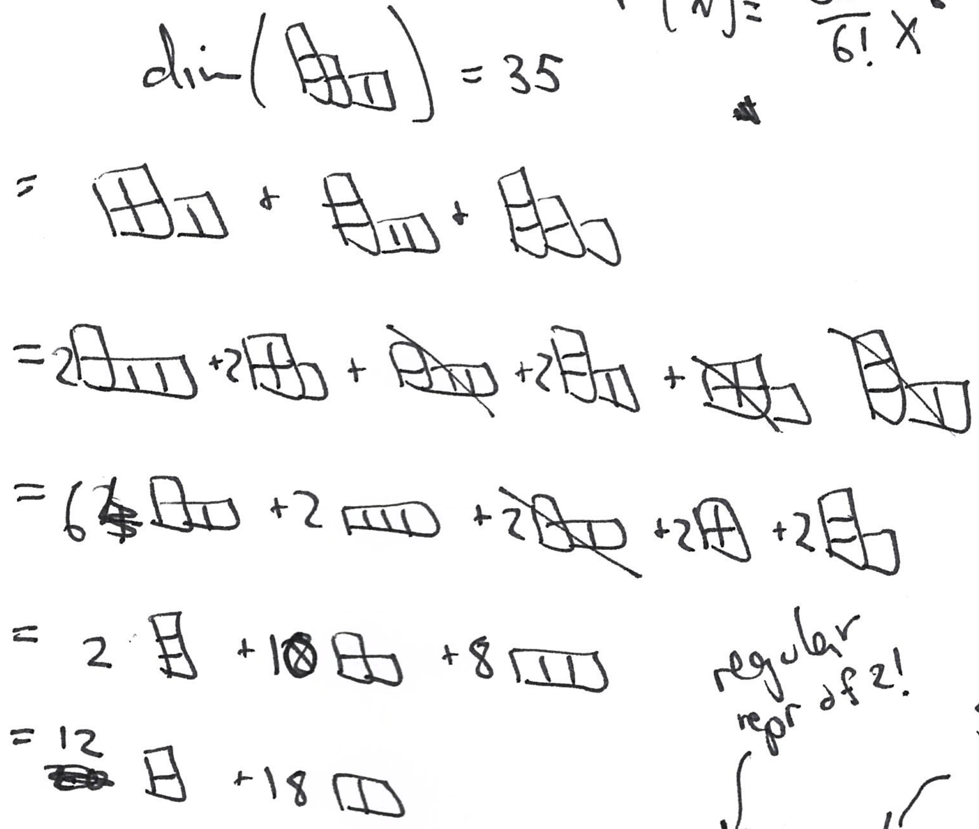
This is also the Derivative of an enriched structure type



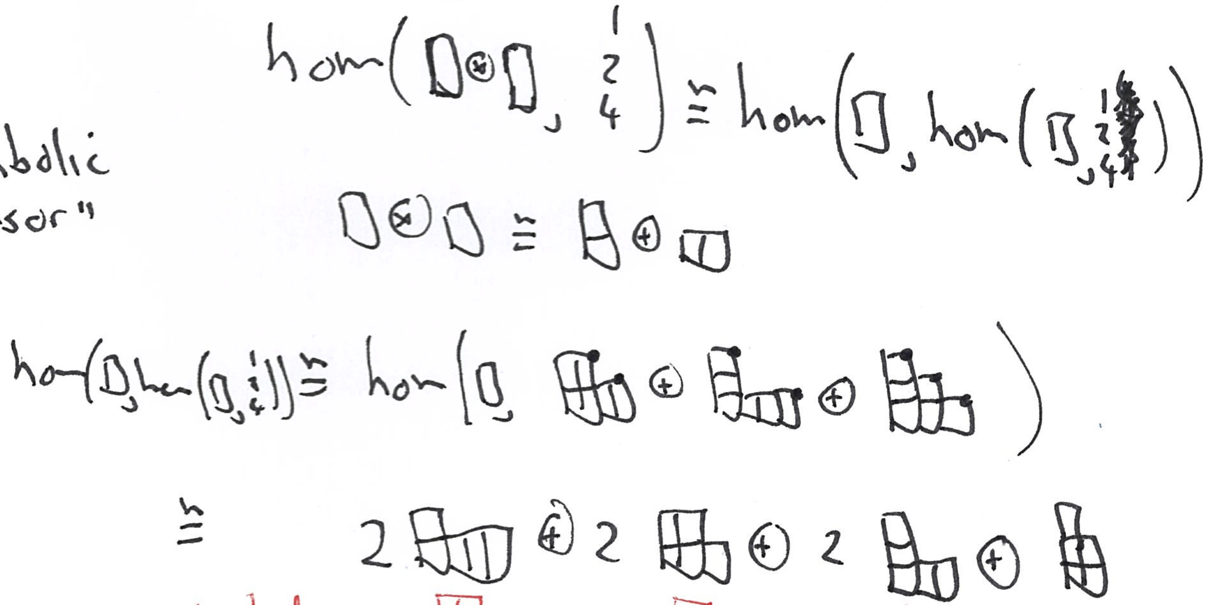
$$f(x) = \frac{35x^7}{7!}$$

$$f'(x) = \frac{35}{6!}x^6$$



regular repr of 2!

Shorthand Young diagram notation



"parabolic tensor"

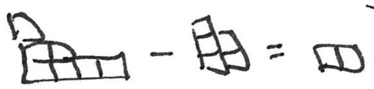
For 2 

 gets the symmetric

 gets the ~~and~~ antisymmetric

custody:

look at Y.D. remainder to determine custody



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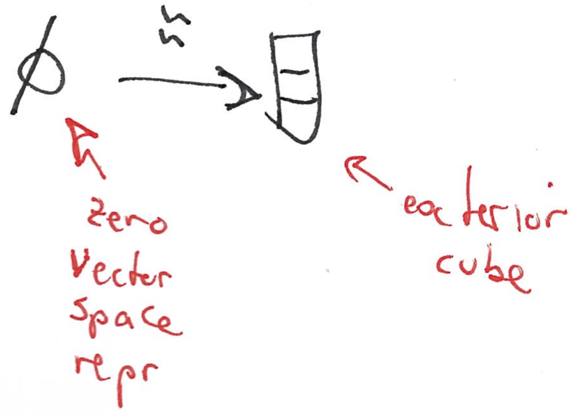
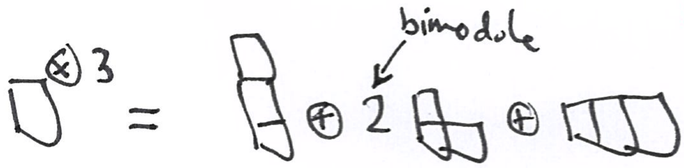
$$\hom(A_1 \oplus A_2, B)$$
$$= \hom(A_1, B) \times \hom(A_2, B)$$
$$\cong \hom(A_1, B) \oplus \hom(A_2, B)$$

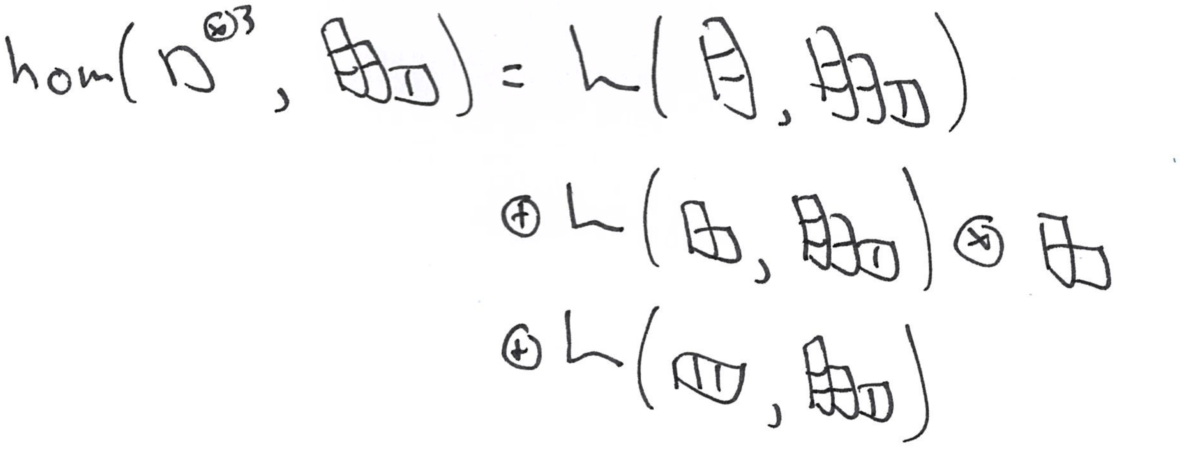
reminder: we want to believe that

For set based structure types
$\text{Singleton}^{\otimes n}$
$\cong$ total order on n elements
cayley repr of $n!$

$\square^{\otimes n}$ gives regular repr of $n!$







2Linear operator of 2Hilb spaces

repr 3! → repr 4!

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For the diagram $\yng(3,1)$ we get:

dim	$\yng(3)$ 1	$\yng(2,1)$ 2	$\yng(1,1,1)$ 1	leftover:
$\yng(4)$ 1	0	1 2	0	$\yng(2,1)$
$\yng(3,1)$ 3	1 3	2 12	1 3	$\square\otimes\square\otimes\square$
$\yng(2,2)$ 2	1 2	1 4	0	$\square\otimes\yng(2) = \yng(3)\oplus\yng(2,1)$
$\yng(2,1,1)$ 3	1 3	1 6	0	$\square\otimes\yng(1,1) = \yng(1,1,1)\oplus\yng(2,1)$
$\yng(1,1,1,1)$ 1	0	0	0	0

reverse the bump removal formula

numerology

Sum all triples: $\dim(\yng(3,1)) \times \dim(\yng(2,1)) \times 2$

$= 35$

$\hom(\yng(1,1,1,1), \yng(3,1)) = 0$ so $\yng(3,1)$ believes $\yng(1,1,1,1)$ is zero.

$\hom(\yng(1,1,1), \yng(3,1)) = \yng(2,1)$ so $\yng(3,1)$ does not believe $\yng(1,1,1)$ is zero.

There's a partial order on Y.D.

B disbelieves D is ~~[illegible]~~ zero exactly when D fits in B

So believers in $\yng(1,1,1) \cong 0$ are Y.D. with at most two rows. Just a property of believers.

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arbitrary believer will be a sum of irreducible believers.

Tensor of two believers may require "religious conversion"

Internal hom of believers is a believer.

(believing $= 0$ and super representations)

the left adjoint to the forgetful functor.

~~xxx~~

(just kill the irreps with too many rows)

GL(2) as a com Hopf alg over $\mathbb{C}$

i.e. affine algebraic group

> reprs of GL(2) are the comodules of the Hopf algebra.

algebraic variety of invertible 2x2 matrices

8 generators

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ matrix $B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$ inverse

relators:

$AB = I \quad BA = I.$

> Conjecture: the believer category ~~xxx~~ ~~xx~~ is category reprs of monoid of <u>all</u> 2x2 matrices not just invertibles