James Dolan ① 17/7/19

These algebras form a 2-category. Morphisms are interpretations or models, of one theory in another.

The underlying category of the theory is called the syntactic category of the theory, whose objects are syntactic constructions. These syntactic constructions are the believers.

Now take theories T, U.
theory universe

Definition of belief:

Given $j : T \to U$.

A j belief on a T object:

Take T_1 a syntactic construct of theory T. (It's a skeptic of U.)
later called B

ctd...

Belief method

Categories with extra structure (small colimits) and morphisms between these categories that are left adjoint functors.

Intuition:

adjoint functor thm:

left adjoint functor = colimit preserving.

A Belief doctrine

2-category of locally presentable categories:

the 1-cells are the left adjoints

the 2-cells are the nat. trans.

Defn. A Monad on this 2-category is a belief doctrine.

Algebras of the monad are theories of the belief doctrine.

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Abstract def^n ② 17/7/14

A j-belief on T_1 is a lift of T_1 along the right adjoint of j: (called j_R).

~~Morally speaking this is ϕ_1 ... $j_R(\phi_1) = T_1$~~

This is a U_1 and an iso:

$$j_R(U_1) \xrightarrow{\cong} T_1$$

□

Concrete def^n

$$\mathrm{Hom}(-, T_1)$$

is a functor

$$T^{op} \to \mathrm{Set}$$

(will need to relativise later...)

this is a right adjoint functor.

A continuous extension of this functor along

$$j^{op} : T^{op} \to U^{op}$$

is a j-belief on T_1. □

Lemma These def^n are equivalent up to iso. □

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Use this defin^t to derive all the other rules about believers in different doctrines (e.g. symmetric tensor need comm monoid) (e.g. when to use internal hom vs. external hom)

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Sym alg on n-dim space is polys on n-vars.

Free($\overset{n}{\mathrm{Set}}^N$)

gives structure types in N variables.

<u>Example</u> T is the theory of an object x. Syntactic category of this theory is the structure types.

(<u>Example</u> monad

$\overset{n}{\mathrm{Vec}} \to \overset{n}{\mathrm{Com\,Alg}}$)

U is theory of an object x and a $x \to 1$. ← tensor unit

$j : T \to U$

The novice is the T believer

The adept is the U believer, formed from the novice together with a j-belief on it.

Take novice believer B. just a structure type.

the adept belief is

$\hom(1, B) \to \hom(X, B)$

ie. $B \to B'$

this is a creation operator.

<u>Example Belief Doctrine of Symmetric monoidal loc. pres. categories.</u>

AKA tensor categories.

How does this work as a monad?

$$\text{loc pres cad} \underset{\text{Forget}}{\overset{\text{Free}}{\rightleftarrows}} \text{sym monoidal loc pres cad}$$

Categorified free com monoid

$\exp(x)$.

<u>Example</u>

$\overset{n}{\mathrm{Set}}$ = free loc pres cat on one object

works in the Base belief doctrine

Free(Set) is the category of structure types decat: Symmetric algebra of 1-dim Vec is polys in one Variable.

how is this like a categorified differential equation?

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Syntactic category of T

Set valued functors on cat of Finset & bijections.

X structure type is the te

$X : \text{FinSetBij} \to \text{Set}$

$\{*\} \mapsto 1$

o.w. $\mapsto \emptyset$

Isolated points.

The inclusion functor is the point structure type.

$X' \cong 1$

there are no morphisms $X \to 1$.

What about Point $\to$ Point' ?

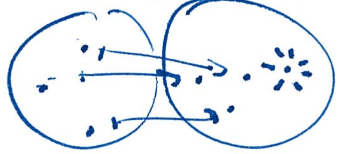
$$\text{Point} = X + \frac{2}{2!}X^2 + \frac{3}{3!}X^3 + \ldots = \sum \frac{1}{(n-1)!}X^n$$

Yes: the stabilizer of a point is $(n-1)!$. Point' = point + E

Take inclusion Point $\to$ Point'.

Alternate description of syntactic category of U: set valued functors on category of finite sets and injections.

(The creation operator "gives" injections)



Define $X : \widetilde{\text{FinSetInj}} \to \widetilde{\text{FinSet}}$ is the inclusion.

We didn't need to use colimits here: Free tensor category on an object X with a $X \to 1$ is FinSetInj^{op}. Free tensor category on object X is FinSetBij^{op}. Freely adds colimits by taking presheaves. Belief method needs the small colimits.

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Using the belief method
the adept 1 is the E structure type.
the adept X is the point structure type.

The left adjoint adept conversion is unique:

Adept = Free(Novice)

Under(Free(Novice)) not the original Novice.

The right adjoint gives canonical representatives for categorified "cosets of an ideal in a ring."

FinSet Bij

tensor is sum of sets.

FinSet Inj^{op}

tensor is sum of sets.

So $\{1,\ldots,n\} \cong X^{\otimes n}$

And $1_{\otimes} = \phi$, $X = \{1\}$.

We get $X \to 1_{\otimes}$.

Now Freely add colimits:

presheaves(FinSet Inj^{op})

$= \mathrm{Hom}(\text{FinSet Inj}, \text{Set})$.

the new X:

$X = \mathrm{Hom}(\{1\}, \_)$

the old X

structure type of Point

the new 1:

$1 = \mathrm{Hom}(\{\}, \_)$

then we have

$X \to 1$ obvious restriction