James Dolan ① 27/4/19

Borger expert on plethorys

Presheaves on $\tilde{C}$ adjoining as formal colimits.

Yoneda embedding

$\tilde{C} \to \text{Presheaves}(\tilde{C})$

Representable ⊆ presheaves "free on one generator".

Species

Presheaves on cat of finite sets & bijections.

Categorify

Example: Combinatorial species.

Example

Free symmetric monoidal category on	free abelian group on a set.
	free CommRing on one generator.
	free abelian group on monic monomials

Categorified

2-categories
2-functors

Forgetful functors (right adjoint)

CommRing → CommMonoid

↓ ↓

Ab Group → Sets

Free functors (left adjoint)

Sets → Ab Group (free)

↓ (free) ↓ (Symmetric algebra (or ring))

CommMonoid → CommRing

Small Categories → Cocomplete categories (or "locally presentable categories")

(tricky to get right adjoint)

↓ (free) ↓ (Categorified Symmetric algebra (or ring))

Small Symmetric Monoidal Categories → Sym Mon Cocomplete

"finite multisets of objects"

"presheaves"

Day Convolution ⊛

Monoid ring

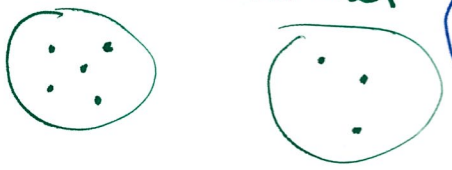
Looks like free abelian group

Convolution

extend by linearity

a morphism in Free Sym Mon($\vec{C}$) is multiset → multiset

extend by cocontinuity



Eg. Free Sym Mon on $\mathbb{F}$ is category of finite sets and bijections

the vertical arrows are exponential power series

$$a \downarrow$$

$$1 + x + \frac{x \circledast x}{2} + \frac{x^{\circledast 3}}{3!} + \dots$$

This square is distributivity

See also: Beck distributivity law

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Example