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Algebra		Geometry
Commutative Hopf algebra	....	Affine group scheme
H	....	G
element of H	....	polynomial function on G

Example: k a field

then $\text{FinVect}_k \cong \text{FinVect}_k^{op}$

Finite dim Bicomm Hopf algebra H

then H^{op} is also Bicomm Hopf algebra

(reverse arrows & swap
mul - comul
unit - counit
in definition
to get the same
definition)

Fourier dual &
"probability" functions

Algebra		Geometry
	take coordinate ring ⟵	
CommRing		Scheme
	⟶ take spectrum	

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Algebra	Geometry
(pseudo-) coexponentials theory of the models of cobase in the (pseudo) coexponent	exponential objects
Coexponent affine algebraic commutative algebra	category of affine schemes = opposite of category of commutative algebra
[redacted] commutative Hopf algebras $\xrightarrow[\text{functor}]{\text{contravariant}}$	~~Commutative Hopf category~~ category of group objects in category of affine schemes
H	Group object G = "affine group scheme"
comm algebra	affine sheme
com bi-algebra	affine monoid
Hopf algebra antipode counit	affine group inverse $G \to G^{-1}$ unit of group
	$G \to G \times G$

The definition of a ~~#~~ Bicommutative Hopf algebras have Hopf duality which is a kind Fourier duality between finite abelian groups

Bicommutative = commutative & co-commutative

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Definition of a pseudo-coexponent

Algebra

affine pseudo coexponent

affine algebraic commring

S

Bicomm Hopf algebra

Bicommutative bialgebra
(Hopf algebra without antipode)
multiplication & comultiplication

has:

multiplication & unit

coaddition & co-zero
conegative

comultiplication & co-one

Geometry

pseudo exponent

affine comring scheme

R

underlying abelian group scheme

underlying com monoid scheme

underlying affine scheme

The pseudo-coexponent B^E
of cobase B: commring — affine scheme
with pseudo-coexponent E: affine algebraic commring — affine comring scheme

(Glitch: not "relativised"
base ring is $\mathbb{Z}$.)

B^E = spectrum of theory of B in E

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B is a system of equations
model is a solution of equation.

Example tangent bundle of circle

cobase: $B = K[x,y] \big/ {x^2+y^2=1}$

$= K[\cos, \sin] \big/ {\cos^2 + \sin^2 = 1}$

E: affine com ring scheme

Dual numbers are not just a com ring but also an affine algebraic com ring

Com ring of polynomial functions on $K[x]/x^2 = \{(a,b) \mid$ with $(a,b)\cdot(c,d) = (ac, ad+bc)\}$

• 2-dim real vector space $a_1 b_2 \quad a_2$

$I = (1,0)$

$T = (0,1)$

R is poly funcs: $K[a,b]$

$R \otimes R$ is coproduct: $K[a_1, b_1, a_2, b_2]$

(1) cozero: $K[a,b] \to K$

$a \mapsto 0$

$b \mapsto 0$

(2) co-addition: $R \to R \otimes R$

$a \mapsto a_1 + a_2$

$b \mapsto b_1 + b_2$

(4) co-one: $R \to K$

$a \mapsto 1$

$b \mapsto 0$

(3) co-neg: $R \to R$

$a \mapsto -a$

$b \mapsto -b$

(5) comul: $R \to R \otimes R$

$a \mapsto a_1 a_2$

$b \mapsto a_1 b_2 + a_2 b_1$