James Dolan ① 27/7/18

Struct Types:
categorified polynomial construction that takes an object x and constructs a new object F(x) using tensor & colimit

Young diagrams

Repr of S_n

Structure types over the base category complex vector spaces

Base Doctrine of colimits
Belief method is presheaves

change of Doctrine
See ③ 27/4/19

Doctrine: small colimits with symmetric tensor

Syntactic category

$$\begin{tikzcd}
\widetilde{\mathrm{Set}} \arrow[r] \arrow[d, hook] & \widetilde{\mathrm{StructTypes}} \arrow[d] \\
\widetilde{\mathrm{Complex\ Vec}} \arrow[r, hook] & \widetilde{\mathrm{Complex\ StructTypes}}
\end{tikzcd}$$

Schur functors

Pushout of theories

initial theory:

$$\begin{tikzcd}
T_{\text{nothing}} \arrow[r] \arrow[d] & T_{\text{object } x} \arrow[d] \\
T_{\text{complex structure on } 1_\otimes} \arrow[r, hook, "\text{introduce young diagrams here}"] & P
\end{tikzcd}$$

Notice: no finiteness condition here

Degree n Structure types are $\widetilde{\mathrm{Set}}$ valued reps of S_n

James Dolan | 2 | 27/7/19

Complex structure types

is a $\mathbb{Z}$Vec space

$\mathbb{C}$ enriched abelian category with one irreducible object for each young diagram

Schur's lemma

$$\mathrm{Hom}(A,B) = \mathbb{C}\begin{cases} 1 & A=B \\ 0 & A\neq B \end{cases}$$

for simple A, B

matrix notation for morphisms

Young diagrams

row of columns ▷

column of rows ▽

number of boxes n

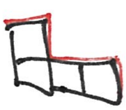
categorified irreducible with degree monomial

$n=0$ young diagram is $\mathbb{1}_\otimes$

categorified homogeneous degree n polys are $\mathbb{C}$-reps of $n!$

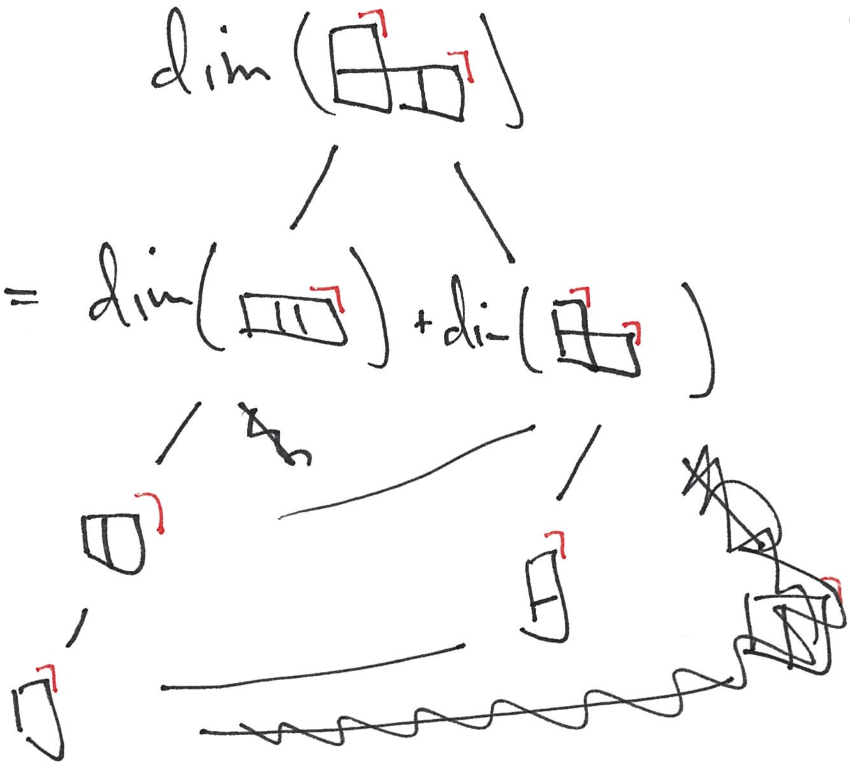
	$\mathbb{1}_\otimes$		triv 	alt 			
dim	Φ	1	1	1	1	2	1

$1^2 + 2^2 + 1^2 = 6 = 3!$

4!					
dim	1	3	2	3	1

James Dolan ③ 27/7/19

remove the bumps to get a recursive formula:



A young diagram also gives an atomic permutation repr (in two ways)

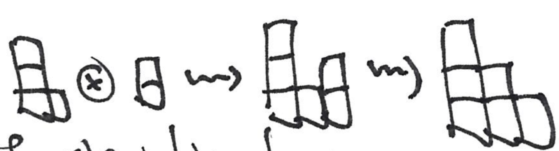
eg. the rows partition n:

 : $1+1+2$

(use categorified Gram Schmidt to find the $\mathbb{C}$-rep)

eg. the rows partition n:

 : $3+1$

tensor: 

the usual × of structure types.

James Dolan (4) 27/7/19

(SL(2) is an affine algebraic group, a comm Hopf algebra, so Tannaka Krein gives modules with G— tensor product.)

action of the free commutative monoid on $\ominus$

modules of a symmetric algebra

Adding further beliefs:

to get reps of SL(2)

add isomorphism:

$\ominus \xrightarrow{\sim} 1$

the exterior square

T

object X with $\ominus(X) \xrightarrow{\sim} 1$

<u>stage 1</u> add arrow

$\ominus \longrightarrow 1$

<u>stage 2</u> make it invertible

Believers:

$\mathrm{Hom}(\ominus, B) \xleftarrow{\sim} \mathrm{Hom}(1, B)$
$\qquad\qquad\qquad\qquad\quad \shortparallel$
$\qquad\qquad\qquad\qquad\quad B$

the monoid is and self-commutative

James Dolan

5 27/7/19

1) theory of nothing

$\mathrm{SynCat} = \mathrm{Set}^{n}$

2) morphism

$m : 5 \to 1$

believers:

$\hom(1, B) \to \hom(5, B)$

$\parallel$

$B \to B^5$

what are the algebras of this monad?

currying

$$\frac{\dfrac{B \to B^5}{B \cdot 5 \to B}}{5 \to B^B}$$

to get a com monad need all 5 operations to commute w. each other

actions of the free comm monoid on 5