James Dolan ① 27/9/19

a	b
c	d

two schur functors F, G:

$$\begin{array}{c} \xrightarrow{\quad R \quad} \\ \Downarrow \mu \\ \xrightarrow{\quad G \quad} \\ \Downarrow \lambda \\ \xrightarrow{\quad F \quad} \end{array}$$

R nat trans

$\mu\lambda$

$\lambda\mu$

extract idempotent projectors "Hecke eigenspaces" in the group ring of 4!

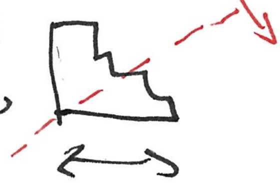
$\mathbb{C}[S_4]$

twisted average for fermionic

untwisted average for bosonic

Super vector spaces

anti symmetrize



symmetrize

flip young diagram gives supersymmetric partner

take pure odd component $X_\bullet$

 : $\dfrac{X_\bullet \otimes X_\bullet}{2!}$

 : $X_\bullet \wedge X_\bullet$

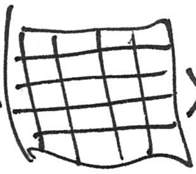
 $\}n$

$=$

if $\dim(X) = n$ then $\bigwedge_{n+1} X = 0$

For super vector spaces $\dim(X) = (n, m)$

eg. $(3,4) = \dim(X)$ then

then dim( X) $= 0$

Schur functor

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$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}$: fermionic:

$$P = \overset{abcd}{||||} + X|| + ||X + XX$$

for bosonic:

$$Q = |||| - \divideontimes| - |\divideontimes + ⋇⋇$$

up to scalar these give projectors

$Im(P) \cap Im(Q)$ gives irrep

Alternately: compose P & Q to get a projector (miracle).

Now "super" this

with $\dim(X) = (1,1)$

$$X = \langle a, b \rangle$$

even ← odd basis el

~~$\dim(X^{\otimes 4}) = (8,8)$~~

$$\dim\left(X^{\otimes 4}\right) = (8,8)$$

how does $\mathbb{C}[S_4]$ act on this super space $X^{\otimes 4}$?

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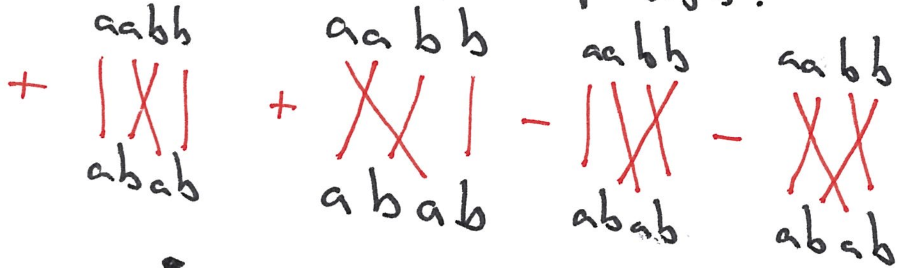
e.g. in $X^{\otimes 4}$:

$$\left.\begin{array}{c} aabb \\ \downarrow \\ abab \end{array}\right\} \text{bosonic}$$

QP has 16 terms ($\neq PQ$)

the action of $\mathbb{C}[S_4]$ picks up signs:



* swapping fermionic components picks up a minus sign.

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4 27/9/19

ex 3

co-simplicial space

"instructions for what the abstract simplexes in a simp set should be realized"

example 1

[0] [1] [2]

S^3

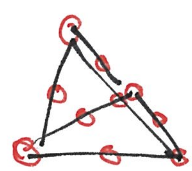
example 2

FinSet

[1] [2] [3] [4]

subdividing each line into 2 pieces

$$10 = \binom{5}{2} = \binom{5}{3}$$



[4]