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free theory on an object $X$.

comb spec:
categorified polynomials

$$\mathrm{Rep}(M_2) \longrightarrow \mathrm{Rep}(GL(2))$$

For $GL(2)$:

$$\xrightarrow{\cong} \mathbb{C}$$

this rep involves $\det(\cdot)^{-1}$
which explains why
this is not a $M_2$ rep.

<u>Example</u> theory of a
two dim object.

Hom into a "universe of models"
Variable model is a
bundle of $T_1$ models parametrized
by the spectrum of $T_2$.

If $T_2$ is initial theory then
this is a constant model
(spectrum is trivial.)

"Associated vector bundle"
see: homotopy theory
or D.G.

Combinatorial species
in doctrine of

sym mon loc pres categories
categorified commutative
algebra over a base ring

enrich over
base: category of sets

base: complex Vec
leads to young diagram

extra beliefs:

$$\cong 0 \quad \text{a property}$$

internal hom into believer

$$\hom( \ , B) \cong 0$$

gives full subcategory
of young diagrams with
two rows or less,
(tensor: kill the tall diagram)

looking for cat of
reps of $GL(2)$ but found
reps of $M_2$:

$$GL(2) \leq M_2$$

affine algebraic
monoid of all
2×2 operators

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Think of vector bundles over a line, like the mobius strip: a bundle over $\mathbb{R}P^1$.

Torsor for $GL(2)$ is a princip. bundle for $GL(2)$ now get associated vector bundle from reps of $GL(2)$.

this is a tensor functor $T_1 \to T_2$.

<u>Example</u> over $\mathbb{C}$:

~~[illegible]~~

theory $T_2$ of a quantity

$q: \mathbb{C} \to \mathbb{C}$

believers:

$\hom(\mathbb{C}, B) \to \hom(\mathbb{C}, B)$

ie. $B \to B$

syntactic category is modules of polynomials in one variable "$q$"

a "classical model" of $T_2$ is just a complex number

$T_1$ is theory of 2-dim vector space (syntactic category is comodules of co-Hopf algebra for alg group $GL(2)$)

~~$\hom(T_1, T_2)$~~

a complex tensor functor

$T_1 \to T_2$

is "a bundle of $T_1$ models over the spectrum of $T_2$"

ie. two dim algebraic vector bundles over the affine line

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Models of $T_1$
have non-trivial
automorphisms
stacky point(s)

Models of $T_2$
have only points
non-stacky.

(For projective line we
use two variables
or coordinate charts: $q, q^{-1}$)

Blowup in algebraic
geometry
involves anomalous
fibres.

see also: renormalization
group,
anomalous symmetry
at critical
point.

Idea: theory of
$\boxed{\ \ \ \ }$ = 0
fermionic objects
of dim $\leq 2$.

Idea $\boxplus = 0$ (1,1) dim bosonic-fermionic.

Future: intertwiners between
tensors of young diagrams

Example $T_1$: theory of
objects dim $\leq 2$

rep$(M_2) \cong$ comodules of
com bialgebra for $M_2$

$T_2$ is as above.

look for a model $T_1 \to T_2$

e.g. free $Q$-module on a, b: $\mathbb{C}Q\{a,b\}$
($Q$ = {poly over $q$})
trivial 2-dim vector bundle.

e.g. any quotient module
of above module:

$$\frac{\mathbb{C}Q\{a,b\}}{(a+b)(q-1)(q-2)(q-3)}$$

$$\Lambda^3\, \mathbb{C}Q\{a,b\} = 0$$

generic fibres are 1-dim
but fibres over $q=1,2,3$
are full rank: 2-dim

non-locally trivial vector bundle
a.k.a.
quasi coherent sheaves over
the affine line