James Dolan

1 11/6/15

Now we wish to apply the belief method within the Doctrine of "colimits & symmetric tensor products".
This is the algebraic geometry doctrine.

A symmetric monoidal category with colimits,
this is a categorified commutative ring
We are doing categorified affine algebraic geometry.
(decategorify the believers to get a Grobner basis)

Example sketches

1) theory of an object X

$\boxed{?}$ free sym monoidal

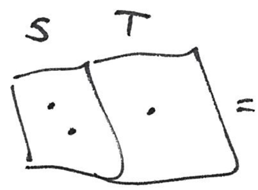
the generic model is the free symmetric monoidal cocomplete category on one object

(free sym. monoidal cat is ...

See 3 27/4/14

2) theory of a pair of objects X, Y

free monoid on X, Y
as a category it's discreet
free sym monoid on X, Y
pairs of bijections between pairs of sets



$= X \otimes X \otimes Y$

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(2) 11/6/15

Example sketches

1) Sketch: $\circlearrowleft \bullet$ $\quad \xmapsto{\text{free monoidal cat}} \quad$ $\mathbb{N}$ $\quad \xmapsto{\text{symmetric}} \quad$ Sets & bijections

Groupoid (Feynman diagrams)

2) Sketch: $\circlearrowleft \bullet\, X \quad \circlearrowleft \bullet\, Y$ $\quad \xmapsto{\text{monoidal}} \quad$ words in X, Y $\quad \xmapsto{\text{sym}} \quad$ Set pairs & bijection pairs

3) Sketch: $\circlearrowleft \bullet\, X$ s.t. $\emptyset \twoheadrightarrow 1^{\otimes}$ ~~is epi~~; initial object is $\emptyset$ $\quad \xmapsto{\text{monoidal}} \quad$ use belief method: structure types that believe $\emptyset \twoheadrightarrow 1^{\otimes}$

$$\begin{tikzcd} \emptyset \arrow[r] \arrow[dr] & 1^{\otimes} \arrow[d] \\ & B \end{tikzcd}$$

$\emptyset$ is the impossible structure type

1 is the vacuous (trivial) structure type

$$\mathrm{Hom}(\emptyset, B) \leftarrowtail \mathrm{Hom}(1^{\otimes}, B)$$

using internal Homs:

$$1 \leftarrowtail B$$

"a subobject of the terminal object is a truth value"
- topos theory

So B is just a property!

ie. a commutative quantale.

the syntactic category is the category of property types.

James Dolan

③ 11/6/19

3) (cont'd)

poset P, functor

$$P^{op} \xrightarrow{F} \left\{ \begin{matrix} \top \\ \bot \end{matrix} \right\}$$

Sub Example
$$\left\{ \begin{tikzcd}[column sep=small] & A & \\ B \arrow[ur] & & C \arrow[ul] \end{tikzcd} \right\}^{op} \to \left\{ \begin{matrix} \top \\ \bot \end{matrix} \right\}$$

i.e. downward closed subsets: the presheaf poset

This is the free cocomplete poset on P!

& implement this!

Sub example free com. monoid on X, Y

$M = \{X^2, X^2Y, \ldots\}$

Now take subsets: PM.

This is the free com quantale on X, Y.

1	x	x^2	x^3	.
y	xy	x^2y	.	.
y^2	xy^2	.	.	.
.	.	.	.	.

Eg. $\{X, XY, Y^2\} \in PM$ a "property"

or $X + XY + Y^2$.

or polynomials over {true, false}

Now add relators

$\mathbb{B} = \{0,1\}$ James Dolan ④ 11/6/19

$$M = \mathbb{B}[x, y]$$

$$Q = \mathbb{B}[x, y] \,/\, x^2 + y^2 = 1$$

using internal hom we find the believers:

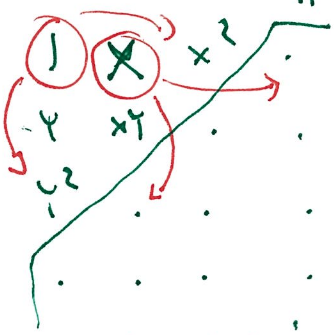
$$\mathrm{Hom}(x^2+y^2, B) = \mathrm{Hom}(1, B) = B$$

tensor-hom adjunction

$$\frac{K \to \mathrm{Hom}(x^2+y^2, B)}{K \otimes (x^2+y^2) \to B}$$

So $K \to B$ iff $K \otimes (x^2+y^2) \to B$

try B_{11}



not a believer.

$B = \sum_{q \in Q} q$ is a believer

$B = \sum x^{2n} y^{2m}$ is a believer