

James Nolan

① 15/7/19

$$L \otimes R \cong 1$$

$$\sigma \downarrow \quad \downarrow \in \text{End}(1) \text{ a scalar}$$

$$R \otimes L \cong 1$$

| vector space            | module                                    |
|-------------------------|---|
| field                   | comm ring                                 |
| spectrum<br>= one point | spec                                      |
| line object             | line bundle<br>over spectrum<br>of a ring |

4) a ~~line~~ D-dimensional object (Simon's idea: take the trace?)

An object  $V$  and

inverse  $L$  :  $L \otimes \text{Ext}^D V \cong 1$

Enrich over Vect  
so we get Exterior powers.  
Could also ~~potentially~~ have

neg:  $1 \rightarrow 1$

plus:  $1 \rightarrow 1 + 1$   $\begin{matrix} \text{\$ neg} \\ + \text{id}_1 \\ = \text{zero} \end{matrix}$

zero: ~~1~~  $1 \rightarrow 0$

and apply belief method  
to effectively relativise  
over abelian groups

(see also ③ 12/7/19  
for similar examples)

Belief method works in

Locally presentable categories.  
These have good adjoint functor  
theorems:

cocontinuous = left adjoint

Locally presentable categories  
= "smallly sketched"

"colimit theory"

Within doctrine of  $\text{loc pres}^{\text{sym mon}}$   
List of examples of theories

1) Euler object  $E + 1 + E \rightarrow E$

2) An adjunction  $L \rightarrow R$   
counit  $L \otimes R \rightarrow 1$   
unit  $1 \rightarrow L \otimes R$

3) An adjunction with  
counit & unit being isos.  
= An invertible object.

like a line object in algebraic geometry  
or a 1-dim vector space.

( $n^2$ ) & has solution

$n = -1$ , is this an Euler object?

Line object vs. anti-line object

~~Super~~ Super algebras

This rep category  
is the walking  
2-dim vector space  
object  $\sim$  oriented  
area form.

The generic model of the theory  
is  $V$ , it lives  
in the walking  
model of the theory  
 $\text{Rep}(G)$ .

Project: use belief  
method to recover  
 $\text{rep}(SL(2))$ .

~~Schur~~ - functors:  
these are linear  
structure types.  
use Young diagrams.

~~theory of~~  
5) over  $\mathbb{C}$ -Vect

~~theory of~~  $\text{Ext}^2 V \cong 1$  "a 2-dim  
object with a  
volume form"

relates to  $SL(2)$ .

(the secret of mathematics)  
the category of believers is the  
~~category~~ category of reps of  $SL(2)$

the believer  $V$  is the  
natural repr.  $V^* = V$   
How this tensor cat into  
another tensor cat it picks  
out a two dim vector  
bundle  $\sim$  volume form.

Eg. how this category  
into cat of vector  
bundles over a (fixed)  
complex manifold to pick  
out a 2-dim vector bundle  
w area form.

"post-

This is the "Tannakian"  
philosophy.

James Dolan

③ 19/7/19

lots of points  
non-trivial symmetry  
Algebraic  
variety

one point  
much symmetry  
Algebraic  
groups

Stacks

7) change the doctrine:

Grounded to pose

the geometric doctrine.

Interpretations are ~~still~~

left adjoints that

preserve topos structure.

eg. Category of actions of  
a group.

eg. boolean logic.

the associated vector  
bundle construct:

input: A principle  $G$ -bundle  
over a "space"  $X$

$\&$  A repr of  $G$

output: the associated vector bundle  
which is a vector  
bundle over  $X$ .

This is describing how  
the syntactic category of  
the theory of a  $G$ -torsor  
(the walking  $G$ -torsor)  
when you turn this into  
category of vector bundles  
over  $X$  you get a  
 $G$ -torsor.

5) an object  $V$

$$\& L \otimes \text{Ext}^2 V \cong 1$$

$$\& V \otimes V^*$$

$\sigma \downarrow$  is constrained  
 $V^* \otimes V$  to kill off  
the super part

gives  $\text{Rep}(GL(2))$ . "bosonic"