

James Dolan

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central idempotents:

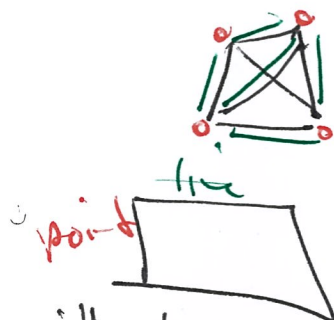
$$\begin{pmatrix} 0 & & \\ & 1 & \\ & & \ddots & \\ & & & 0 \end{pmatrix}$$

in this basis.

Q

How to understand
~~all~~ all this?

Use Mackey operators
& Klien's geometry



We will also use
twisted reps
= induced reps
equivariant vector bundles
over homogeneous spaces
(i.e. G -sets.)

Permutation rep is a
 G -set, with trivial
~~non-trivial~~ line bundle
over the G -set.

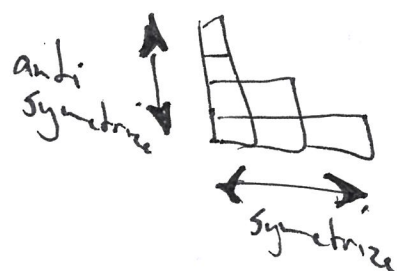
Induced reps are
non-trivial line bundle

see: Mackey

Belief method

the free symmetric
category on an object
= Yang diagrams

Textbooks say



projection operators P, Q
live in $\mathbb{C}[G]$ but not
central.
central idempotents
that decompose $\mathbb{C}[G]$
into multiplicity sectors

semi-simple:

$$\mathbb{C}[G] = \bigoplus M_i$$

matrix
algebras
of dimension

Lemma this is
a decategorified
span with
surjective legs.

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A.

Wild guess conjecture:
fix n box yang diagram n
extract G -sets:

rows, cols

take S with n -elements

$$H = \left\{ f: S \rightarrow \text{rows} \text{ s.t. } |f^{-1}(\text{row})| = |\text{row}| \right\}$$

For $f \in X$, $\text{Stab}(f)$ is
a Young subgroup

$$V = \left\{ f: S \rightarrow \text{cols} \text{ s.t. } |f^{-1}(\text{col})| = |\text{col}| \right\}$$

the yang diagram gives a
Hecke operator in $H \times V$

"Crackpot matrix"

take $S = \{1, \dots, n\}$ with two
different kinds of "coloring"

Example $n=9$

3 2 3 1 \leftarrow partition

4	1	2	0	1
4	2	0	2	0
1	0	0	1	0

partition \rightarrow

For a Yang diagram:

	3	3	2	1	
4	1	1	1	1	
4	1	1	1	1	
1	1	1	0	0	

whoops
no
young
diag
fits here

vertical will be twisted

horiz will be untwisted
(crackpot matrix multiplication)

A Super crackpot matrix

each row & column

also gets a sign

and if an ending is
not cut the signs
must agree for
that row & column.

Because: $0!$ and

$1!$ are the only
perm-groups where
twisted rep =
untwisted rep.

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(James Hecke algebras Endo algebra)
Vaughan Jones to oe

Example

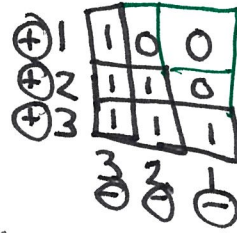
Hecke algebras:

$$H \leq G$$

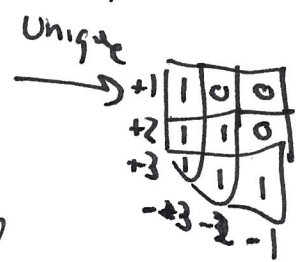
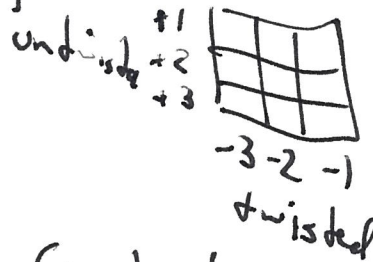
take subquotient of $\mathbb{C}[G]$

for solving $\mathbb{C}[H] \leq \mathbb{C}[G] \dots???$

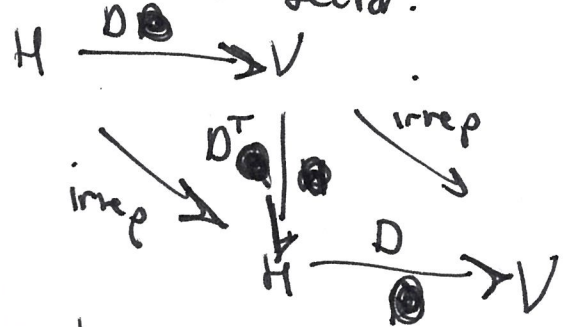
This is where the knot book definit. comes from.



also:



Construct projector:



Categorified Gram-Schmidt

More about eigenspaces of Hecke ops
think kernels & cokernels of Hecke ops.

(See Langlands: eigen spaces of Hecke operators)

As irreps $D \xleftrightarrow{\det} D^+$

As perm reps $D \leftrightarrow D^+$

"bigger" \leftrightarrow "smaller"
perfectly compatible
irrep content
except for a "digger"
irrep "that you want to extract"

Application

Conjecture: rectangular Young diagrams annihilate super vector spaces of $GL(L)$.