Error Correction in a Fibonacci Anyon Code

Simon Burton,[1] Courtney G. Brell,[1,2] and Steven T. Flammia[1]

[1] *Centre for Engineered Quantum Systems, School of Physics, The University of Sydney, Sydney, Australia*

[2] *Institut für Theoretische Physik, Leibniz Universität Hannover, Appelstraße 2, 30167 Hannover, Germany*

Introduction. Topologically ordered systems in two dimensions are of great interest for quantum information processing and storage. These systems can be realized as ground states of local Hamiltonians and have natural robustness to local Hamiltonian perturbations [3, 4, 13]. Furthermore, these systems typically have anyonic excitations [10, 17] that may be used to realize fault-tolerant quantum computation by braiding of these particles [11, 14]. Different topologically ordered systems have different anyonic excitations, but only anyon models with sufficient complexity can be used to perform universal quantum computation in this way.

The Fibonacci anyon model is one of the simplest abstract anyon models to describe, having only one non-trivial particle type. Nonetheless, it allows for universal quantum computation by braiding [8] and so is of great theoretical interest. Furthermore, the Fibonacci anyons are experimentally motivated as the expected excitations of the $\nu = \frac{12}{5}$ fractional quantum Hall states [16], and can also be realized in several spin models [9, 12, 15].

Any scheme using anyon braiding to perform quantum computation will periodically require some form of active error correction to remove spurious thermal excitations. Left unchecked, the computation will quickly be spoiled at finite temperature. Such error-correction schemes have been well-developed for the simple class of abelian anyon models [2, 6, 7], where only trivial computation can be performed by braiding. Recently, the first error-correction schemes for non-abelian anyon models were proposed and demonstrated [5, 18]. These schemes were developed for particular anyon models that cannot be used for universal quantum computation by braiding of anyons alone. This lack of computational power was in fact a motivating feature of the study of these models, as the anyon dynamics can be efficiently simulated on a classical computer. By contrast, the fact that the Fibonacci anyons can realize universal quantum computation makes the direct simulation of these anyons on a classical computer decidedly inefficient.

In this work we use monte-carlo simulations of a phenomenological model. This ignores any microscopic structure such as an underlying lattice of spins. The two dimensional manifold of our model has the global geometry of a torus which supports a two-fold degenerate vacuum state. This space is the codespace. The noise model consists of iid pair-creation events for some varying amount of time. Error correction proceeds by first measuring a syndrome: this is the total charge in each tile of some tiling of the surface. The (classical) error correction algorithm performs repeated hierarchical clustering (fusing) of anyons, until there are no more charges remaining. This fails, destroying the encoded state, exactly when a topologically non-trivial operation occurs. Note that the error correction algorithm runs in polynomial time, but the simulation of the anyon system naively takes exponential time and memory. Here we find that a divide and conquer approach, as well as heuristics to minimise braid operations, allow simulation of systems to a reasonably large size (128x128 tiles), and we observe an error correction threshold of around 12.5%.

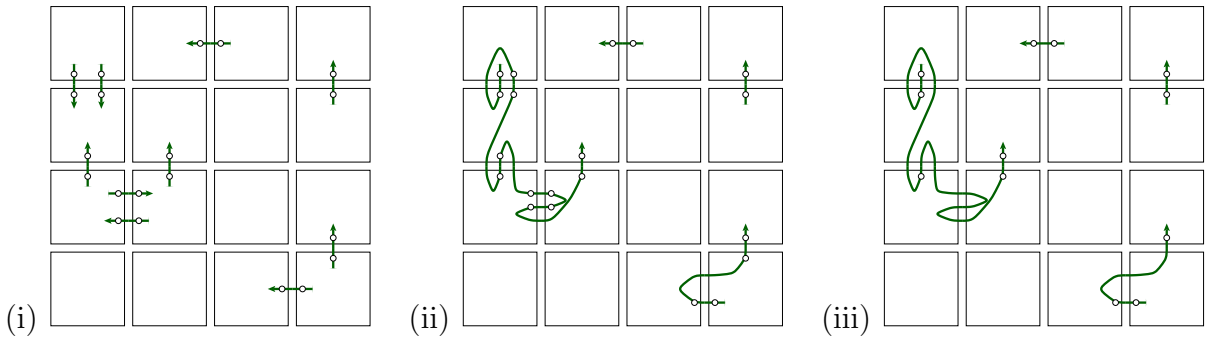
Simulation. Unlike Ising anyon systems [5], Fibonacci anyon dynamics are known to be sufficient for quantum computing [8]. In particular, the dimension of the Hilbert space associated with n anyons grows like d^n where d is the quantum dimension of the anyons, in this case $d \simeq 1.6$. Furthermore, the action of braiding Fibonacci anyons generates a dense set of unitaries on this space (for $n > 2$). Therefore, at first sight it would appear that we should not expect to be able to simulate something as complicated as a decoding threshold. However, we benefit from the following good fortune:

(1) below (bond) percolation threshold, we expect the system to decompose into separate clusters of average size $O(log(n))$ and variance $O(1)$, [1]. Taking advantage of this, the simulation proceeds by maintaining a set of disjoint sub-systems. In other words: it is a (tensor) product state, so we compute with each factor separately.

(2) even though the act of error correction (*decoding*) joins these clusters together, every fusion also decreases the number of anyons present. Moreover, the goal of the error correction algorithm is to do this as much as possible.

We cover the surface of the torus with LxL disjoint squares. These *tiles* are the regions over which charge measurements occur, in order to produce a syndrome for the decoder.

In order to keep track of the observables of the system we maintain a set of disjoint *curves*. These curves can also be thought of as a top-down view of the fusion tree. In general, this winds around the two dimensional tiles in a haphazard way determined by the progress of the simulation. For comparison, previous work used a single fixed Z-shaped curve (row-major ordering) [5].



Each round of simulation proceeds in four steps as follows:

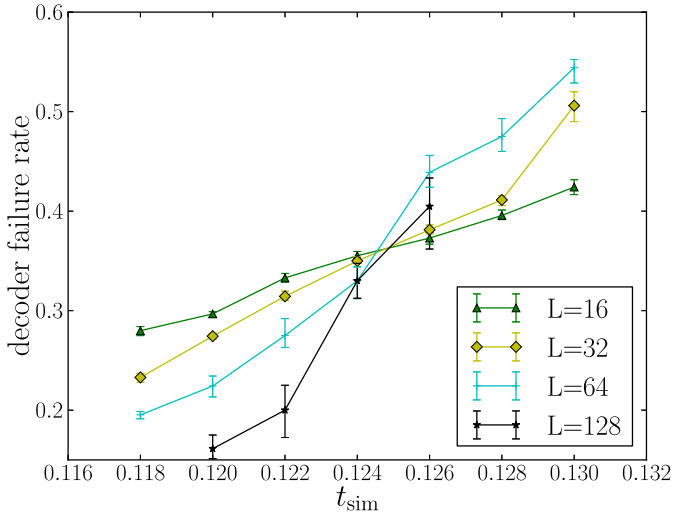
(i) The noise process is modeled by a sequence of pair-creation events distributed as a Poisson process across each edge.

(ii) We first join curves that participate in the same tile in order to,

(iii) measure the charge on each tile. Any resulting anyon is left somewhere in the tile.

(iv) The decoder examines this syndrome and clusters nearby anyons. Each cluster is then measured (fused), and the decoder continues to cluster and fuse on larger scales until there are no more anyons, or a non-trivial operation has occurred. The non-trivial operations are those that involve a path that wraps around the torus. All of these alter the encoded state, and so in this case we abandon the simulation round and declare failure to error correct. This decoding algorithm is based on the clustering "RG" algorithm [2].

Numerical Results We plot the performance of the decoder as a function of error rate for varying lattice sizes. The error rate is parameterized by the Poisson process duration t_{sim}. There is clear evidence of a decoding threshold below which decoding succeeds with asymptotic certainty as the system size increases. This threshold is at $t_{\text{sim}} \simeq 0.125 \pm 0.003$.



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