

$$\bigoplus_{x \in \mathcal{A}} \mathcal{H} \left( \overset{\hat{x}}{\text{disk}(a, b)} \quad \overset{\hat{d}}{\text{disk}(x, c)} \right) \xrightarrow{F_d^{abc}} \bigoplus_{y \in \mathcal{A}} \mathcal{H} \left( \overset{\hat{d}}{\text{disk}(a, y)} \quad \overset{\hat{y}}{\text{disk}(b, c)} \right)$$

The diagram illustrates a relationship between surfaces and their direct sums. The top surface is a disk with three punctures labeled  $a$ ,  $b$ , and  $c$ , and a boundary labeled  $\hat{d}$ . It is isomorphic (indicated by  $\cong$ ) to the direct sum of two surfaces shown in the bottom row. The left surface in the bottom row is a direct sum over  $x \in \mathcal{A}$  of two disks: one with punctures  $a$  and  $b$  labeled  $\hat{x}$ , and another with punctures  $x$  and  $c$  labeled  $\hat{d}$ . The right surface in the bottom row is a direct sum over  $y \in \mathcal{A}$  of two disks: one with punctures  $a$  and  $y$  labeled  $\hat{d}$ , and another with punctures  $b$  and  $c$  labeled  $\hat{y}$ . The mapping between the bottom surfaces is labeled  $F_d^{abc}$ .