

This is a letter of recommendation for James (“Jim”) Dolan for a possible position at Quantinuum, as proposed by Simon Burton.

I state without exaggeration that Jim has for decades been a legendary influence in the area of category theory, inspiring several generations of category theorists. I first learned of him in 1995, through an open letter he and John Baez sent on their groundbreaking approach to “weak n -categories”, a notoriously tricky concept I was also working on at the time, via opetopic sets and conditions of horn filler type. These ideas, published by Baez and Dolan in *Higher-Dimensional Algebra III*, continue to be studied intensively. Later I met Baez and Dolan in 1997 at a workshop on higher-dimensional categories (published as number 230 in the AMS Contemporary Mathematics series; see their paper *Categorification*). I immediately became friendly with both, and Jim particularly. I quickly got the sense that although John was officially Jim’s PhD advisor, in fact it was Jim who was John’s true teacher in category theory, and that Jim was almost certainly the idea engine throughout their long collaboration, at least with regard to the category-theoretic input.

As samples of this historic collaboration, there is firstly *Higher-dimensional algebra and topological quantum field theory* (later referred to as “HDA I”, the first of the HDA series), also groundbreaking in its programmatic sweep; for example this is where the “cobordism hypothesis” and also the “periodic table” of higher-dimensional categorical structures first appeared. Secondly I’ll mention *From finite sets to Feynman diagrams*, a short but beautiful paper in which the basic combinatorics of quantum field theory is set out in terms of “stuff, structure, properties”, a triad of concepts also due primarily to Dolan (which first appeared in the early 1990’s on the old Usenet). Today these concepts are fundamental for working category theorists. Thirdly, two lesser known but very attractive articles, *Dirichlet series and the Hasse-Weil zeta function* and *Zeta functions of \mathbb{Z} -sets*, which set forth with startling simplicity and clarity the combinatorial underpinnings of zeta functions generally, along lines similar to Joyal species and the “Montreal school” of combinatorics (as represented by Bergeron, Labelle, Leroux, et al.).

Many of Jim’s ideas have been propagated not by him but by other mathematicians who fall into conversation with him and thereby fall under their spell. For example, Martin Brandenburg’s thesis owes a large debt to Jim’s 2-categorical “doctrinal” approach to algebraic geometry (an ongoing project which may be seen someday as Dolan’s magnum opus). Another case is the Macquarie thesis of Daniel Steffen, interpreting free cartesian closed categories in terms of games with take-back strategies, directly tracing to conversations they had while Jim was briefly there in the early 2000’s, and which was again described some years later at the n -Category Cafe (blog article dated October 20, 2006, describing Jim’s guest lecture *Holodeck strategies and the lambda-calculus*). Finally, I’ll mention the case of Dolan’s response to Street’s conjectural notion of “file” as an approach to semi-strictification of weak n -categories, circa 1995: despite the fact that Street is a world leader in the area, Jim quickly realized and convinced Street and others that the approach cannot work, thereby saving us all a lot of time; a description of Jim’s thinking was written up many years

later by Bourke and Gurski, <https://arxiv.org/pdf/1412.1320>.

I have spent hundreds of hours in conversation with Dolan, who is extraordinarily generous with his time and his ideas. (An idea of what mathematical conversation with Dolan is like can be gleaned from a quick Google search of the recent recorded conversations he has been having with Baez.) What regularly amazes me is that, just working as he does with a few well-chosen examples and protracted thinking them through, he produces deep conjectures which invariably turn out to be correct. I cannot think of any counterexample.

I am going to have to stop somewhere and sum it up, otherwise I'll find myself going on about more programs he has helped inaugurate (e.g., "groupoidification" and homotopy cardinality, among others). Simply put, Jim Dolan is a genius. This is a sadly overused word, so I'll put it much more emphatically: it is my sincere opinion that he is literally one in a million. Although he is mostly regarded as a theory man, I can attest to his power to solve problems, in the spirit of Grothendieck (ask him a question in group theory, and he will insightfully answer). Provided that he is properly supported, investing in his talents would be well rewarded indeed.

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