11/4/25

Q: when is a category equiv to $Set^{C^{op}}$ for some C?

Answer: presheaf topos

For abelian categories take connected projective objects

Homing out of preserves sums

almost a left adjoint

objects whose Hom functor has a right adjoint

Gives idempotent completion of C.

Example when C is a group

site category

when $Set^{C^{op}}$ is a boolean topos then C is a groupoid (?)

Free cocompletion of G is GSet

coequalizers give transitive

finite sums give the rest

Important idea: toposes & 2-rigs

think syntactically (objs & morphs)

maps into the cat

Semantically theories is a doctrine

morphisms are models

maps out of the cat

The $\overbrace{\text{2-rig of reps}}^{\text{the walking G-torsor}}$ of G is the theory of a G-torsor

A model of the theory of $\boxed{\text{G-sets}}$ is a G-torsor

hom out of the "walking G-torsor"

2-rig functors

these are topos functors

"2-boolean algebra"

Syntactic algebraic

Semantic geometric "Spectrum"