

South Carolina Investment Analysis

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Prepared on 10/24/2021

Executive Summary

Below, you will find an investment recommendation for South Carolina's pension fund. Following an introduction and preliminary model considerations, we discuss the theory behind our work and additional points to note. After a detailed analysis of multiple scenarios, we conclude that Scenario (c)(2) offers the most applicable insights to this panel. Our detailed recommendation as well as the calculations behind it can be found within the respective subsections, the appendix, and the attached spreadsheet. Lastly, we discuss weaknesses of this model and conclude the paper with our recommendation.

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Introduction

In the mid 1990s South Carolina is one of the only remaining states prohibiting stock-market investments with the states pension fund. All state pension funds are exclusively invested into government and corporate bonds, yielding anywhere between 5% and 8% annually (as expressed by US treasury rates at the time (multpl, 2021)).

With an expressed target return on the upper end of the scale, continuous budget deficit increases are to be expected. Having managed to pass our legislation, we are now able to invest about \$1.75 billion into the open market, and an additional \$5.25B for a net total of roughly \$7 billion in stock-market investments until May 2002.

Exhibit A – for amplification see appendix

With year-over-year inflation assumed as 2.51% currently, we expect a net increase of 5.49% of our total portfolio value by the end of 1998, missing the targeted return of 8% (Assumption (3)). If we match our 8% target without considering inflation, opportunity costs as expressed in equation (1)b. for 1997 equal to $33.36\% - 8\%$ or 25.36% of our total portfolio value, or roughly \$4.44 billion.

Theory + Metrics

Using assumptions (1) through (7), we believe we can substantially increase return whilst decreasing risk. Our scenarios and subsequent recommendations include:

- (a) an investment strategy for targeting a return of 10% per year before adjusting for inflation
- (b) minimum risk (b)
- (c) maximum Sharpe ratio, and
- (d) maximum added value.

Mathematically we utilized these formulae:

- (8) Portfolio Standard Deviation (also: Risk and/or Volatility)
$$= | \text{weight}_{\text{Asset}(n)} * \text{deviation}_{\text{Asset}(n)} - \text{weight}_{\text{Asset}(m)} * \text{deviation}_{\text{Asset}(m)} |$$
- (9) Return with and without inflation congruent to assumption (1)
- (10) Sharpe Ratio = (Portfolio Growth – Risk-Free Rate)/(Portfolio Volatility), where
 - a. Portfolio Growth = Expected Return of Asset n * Portfolio Weight of Asset n + Portfolio Percentage of Risk-Free Investment * Borrowing Rate (when Risk-Free Investment < 0) or * Risk Free Rate (when risk-free investment > 0), and
 - b. Risk-Free Rate = Average Return of 10- and 30- year bonds (assumption (6))
- (11)
Added Value over Inflation = Portfolio Growth (as expressed in (9)a.) – 0.5 * Risk Aversion Factor * Portfolio Variance, where
 - a. Portfolio Variance = (Portfolio Volatility)²
 - b. Risk Aversion Factor = 5.5

(8) through (11) all based on Essentials of Investments (Bodie, Kane, & Marcus, 2018).

Sharpe Ratio

The Sharpe Ratio expresses how much added value we can expect for an additional unit of risk. Here, we maximize the portfolios utility while considering the risk-free rate. A value of 0.5 would tell us that for any additional percentage of risk (volatility) we are comfortable with, we can expect a half percentage increase in return over the risk-free rate. Maximizing the possible return output of increased risk (scenario (c)) is more desirable over scenarios (a), (b), and (d). In scenario (a) we will achieve reasonably consistent returns with fairly consistent risk at the expense of maximum possible returns and minimum possible risk. Scenario (b) will achieve the lowest value at risk (see chapter below), at the expense of maximum return. Scenario (d) will achieve maximum return, at the expense of security through minimum possible risk. Therefore, we ought to maximize our portfolios utility by considering both return, and risk; something scenario (c) will achieve.

Value At Risk (VAR)

The Value-at-Risk metric expresses the maximum possible loss in a worst-case scenario. For this worst-case scenario, we are working with 99% confidence levels, meaning that only 1% of all cases will yield different results. Ideally, we would be able to give a definite (i.e. 100% confidence) answer, however our portfolio return is normally distributed (in a Bell Curve); therefore we cannot manage to achieve perfect confidence. 99% confidence roughly corresponds with 5 sigma and will therefore account for all reasonable assumptions.

Further Considerations

To maximize theoretically possible levels of return, risk, utility, and respectively desired outcomes in each scenario, only the 30 companies composing the Dow Jones Industrial (DJI) will be considered. Each share is analyzed individually for its expected return. This expected return is a (standardized) measure of the following: An individual stocks' comparative performance to

- the Dow Jones Industrial overall (Beta),
- its forward (i.e. expected) Price to Earnings Ratio compared to the remaining analyzed shares,
- its historical Price-Book Ratio compared to the remaining analyzed shares, and
- its Relative Strength as expressed through comparative performance over the past 12 months.

Knowing expected returns, we can then analyze the impact of our more conservative approach with a high level of risk-aversion and adjust accordingly. Additionally, we can run scenarios for different possible outcomes to further solidify our predictions' quality (see chapter x.y.).

Now, software can determine optimal investment values (expressed as percentages of our overall portfolio) for each stock individually, under any given scenario.

There are three possibilities when choosing the optimal portfolio:

1. We buy shares of company M
2. We short shares of company M (sell borrowed shares of M at price x with the expectation of lower future share value (price y) of M and liquidate the margin between x and y)
3. We do not buy nor short shares of company M

Additionally, we considered the following constraints when simulating the optimal portfolio for each given scenario:

1. No individual buy or short position may be more than our overall portfolio value
2. We can invest a maximum of 40% of the total portfolio value, meaning our risk-free investments must be always equal to or higher than 60% (to reflect the proposed gradual increase of 10%/year)
3. No individual share can be both bought and shorted at the same time

Scenario Simulations

Scenario (a)

Inputs:

1. Risk Aversion Factor: 5.5
2. Probability: 1% (or 5-sigma)
3. Risk-Free Investment must equal or surpass 60% of the entire portfolio value
4. As outlined in preceding chapters and preliminary considerations

Scenario solves for a target return of 10%.

Outputs:

	a	a+	a-	a+a-	Dollar Value
Bank of America	0.8726%	0.87%	0.00%	0.00%	\$ 152,703,472.68
Apple	0.3298%	0.33%	0.00%	0.00%	\$ 57,706,874.99
Visa	0.0000%	0.00%	0.00%	0.00%	\$ -
McDonalds	0.0000%	0.00%	0.00%	0.00%	\$ -
Caterpillar	0.6425%	0.64%	0.00%	0.00%	\$ 112,430,728.41
UnitedHealth Group	0.3752%	0.38%	0.00%	0.00%	\$ 65,666,303.88
Nike	0.0000%	0.00%	0.00%	0.00%	\$ -
Microsoft	0.3439%	0.34%	0.00%	0.00%	\$ 60,189,768.67
Wal-Mart Stores	0.0000%	0.00%	0.00%	0.00%	\$ -
3M	0.0000%	0.00%	0.00%	0.00%	\$ -
American Express	1.4796%	1.48%	0.00%	0.00%	\$ 258,928,565.24
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%	\$ -
Home Depot	0.0000%	0.00%	0.00%	0.00%	\$ -
DowDuPont	0.5547%	0.55%	0.00%	0.00%	\$ 97,063,821.18
United Technologies	0.3601%	0.36%	0.00%	0.00%	\$ 63,026,066.74
Coca-Cola	0.0000%	0.00%	0.00%	0.00%	\$ -
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%	\$ -
JPMorgan Chase	23.8336%	23.83%	0.00%	0.00%	\$ 4,170,875,336.76
Merck & Co	0.3299%	0.33%	0.00%	0.00%	\$ 57,726,555.46
Cisco Systems	0.6524%	0.65%	0.00%	0.00%	\$ 114,164,724.23
Travelers Companies	0.4519%	0.45%	0.00%	0.00%	\$ 79,079,062.47
Pfizer	0.3408%	0.34%	0.00%	0.00%	\$ 59,646,051.35
Disney	0.3440%	0.34%	0.00%	0.00%	\$ 60,197,010.89
Intel	0.5888%	0.59%	0.00%	0.00%	\$ 103,048,606.25
Goldman Sachs	2.4963%	2.50%	0.00%	0.00%	\$ 436,844,156.41
Chevron	0.4389%	0.44%	0.00%	0.00%	\$ 76,813,516.03
Verizon Communications	0.0000%	0.00%	0.00%	0.00%	\$ -
Exxon Mobil	0.3315%	0.33%	0.00%	0.00%	\$ 58,011,810.99
IBM	0.3324%	0.33%	0.00%	0.00%	\$ 58,164,856.16
GE	0.4556%	0.46%	0.00%	0.00%	\$ 79,726,519.33
Total	35.55%	35.55%	0.00%	0.00%	
Risk-Free Investment	64.446%				

Exhibit B – For amplification, see appendix.

Analysis:

Though achieving a desired return of 10% per annum, this portfolio increases risk compared to our baseline of 9.38% (ref. preliminary consideration exhibit A) by 4.00% to 13.38%. We will therefore not consider it further as outlined in Chapter Theory, Sharpe Ratio. While we add 5.08% value over the risk-free rate, this portfolio puts over 21% (roughly \$3.7B) at risk.

Scenario (b)

Inputs:

1. Risk Aversion Factor: 5.5
2. Probability: 1% (or 5-sigma)
3. Risk-Free Investment must equal or surpass 60% of the entire portfolio value
4. As outlined in preceding chapters and preliminary considerations

Scenario solves for minimum volatility.

Output:

	a	a+	a-	a+*a-	Dollar Value
Bank of America	0.6934%	0.69%	0.00%	0.00%	\$ 121,346,542.2
Apple	0.3079%	0.31%	0.00%	0.00%	\$ 53,877,156.6
Visa	-0.8384%	0.00%	0.84%	0.00%	\$ 146,726,262.1
McDonalds	-0.4422%	0.00%	0.44%	0.00%	\$ 77,382,894.2
Caterpillar	0.5232%	0.52%	0.00%	0.00%	\$ 91,551,692.8
UnitedHealth Group	0.3468%	0.35%	0.00%	0.00%	\$ 60,682,166.6
Nike	-0.5742%	0.00%	0.57%	0.00%	\$ 100,481,493.2
Microsoft	0.3353%	0.34%	0.00%	0.00%	\$ 58,670,253.0
Wal-Mart Stores	0.0168%	0.02%	0.00%	0.00%	\$ 2,937,271.0
3M	-2.3221%	0.00%	2.32%	0.00%	\$ 406,375,645.4
American Express	0.8578%	0.86%	0.00%	0.00%	\$ 150,110,429.5
Johnson & Johnson	-1.4903%	0.00%	1.49%	0.00%	\$ 260,799,366.2
Home Depot	-0.7275%	0.00%	0.73%	0.00%	\$ 127,318,700.6
DowDuPont	0.4992%	0.50%	0.00%	0.00%	\$ 87,355,446.7
United Technologies	0.3149%	0.31%	0.00%	0.00%	\$ 55,108,912.0
Coca-Cola	-0.7813%	0.00%	0.78%	0.00%	\$ 136,733,086.5
Procter & Gamble	-0.3212%	0.00%	0.32%	0.00%	\$ 56,214,300.1
JPMorgan Chase	-1.3390%	0.00%	1.34%	0.00%	\$ 234,333,405.1
Merck & Co	0.3092%	0.31%	0.00%	0.00%	\$ 54,101,638.3
Cisco Systems	0.5722%	0.57%	0.00%	0.00%	\$ 100,131,296.1
Travelers Companies	0.4258%	0.43%	0.00%	0.00%	\$ 74,513,883.7
Pfizer	0.3289%	0.33%	0.00%	0.00%	\$ 57,199,290.0
Disney	0.3184%	0.32%	0.00%	0.00%	\$ 55,712,518.2
Intel	0.4106%	0.41%	0.00%	0.00%	\$ 71,859,168.7
Goldman Sachs	0.6644%	0.66%	0.00%	0.00%	\$ 116,273,226.0
Chevron	0.3913%	0.39%	0.00%	0.00%	\$ 68,473,979.8
Verizon Communications	-0.5894%	0.00%	0.59%	0.00%	\$ 103,148,144.8
Exxon Mobil	0.3144%	0.31%	0.00%	0.00%	\$ 55,012,758.6
IBM	0.3169%	0.32%	0.00%	0.00%	\$ 55,457,254.0
GE	0.5274%	0.53%	0.00%	0.00%	\$ 92,301,056.7
Total	-0.95%	8.47%	9.43%	0.00%	
Risk-Free Investment	86.815%				

Exhibit C

Portfolio growth	5.3276%	5.3276%
a'G	0.0008	0.0007
(a+)'G	0.0046	0.0041
(a-)'G	0.0038	0.0034
Portfolio variance	0.01%	0.01%
Portfolio volatility	0.77%	
Added value	5.31%	
Sharpe ratio	-0.811609728	
Probability	1.00%	
VAR	3.54%	
Inflation 1998	2.51%	
Inflation Adjusted Return	2.8176%	

Exhibit D

For amplification see appendix.

Analysis:

While this scenario achieves 3.54% growth in the worst-case scenario (\$610M), we must emphasize the negative Sharpe Ratio of -0.812. Effectively, that means that a riskier portfolio would lead to decreased returns. Considering the efficient frontier as displayed below, we are generating lower y values (return) at increasing x values (risk) – geometrically, we are looking for the higher y-value that corresponds to this x value, representing a positive Sharpe ratio.

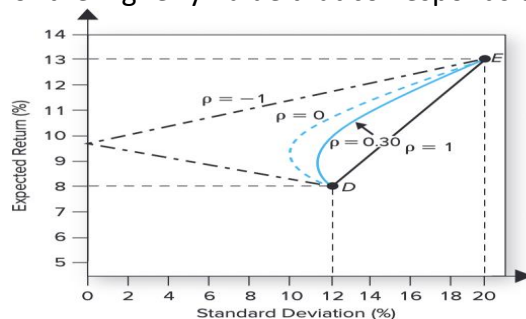


Exhibit E - for amplification see appendix (Bodie, Kane, & Marcus, 2018).

Scenario (c)

Inputs:

1. Risk Aversion Factor: 5.5
2. Probability: 1% (or 5-sigma)
3. Risk-Free Investment must equal or surpass 60% of the entire portfolio value
4. As outlined in preceding chapters and preliminary considerations

Scenario solves for maximum Sharpe Ratio.

Output:

	a	a+	a-	a+a-	Dollar V
Bank of America	7.4705%	7.47%	0.00%	0.00%	\$ 1,307.33
Apple	0.0000%	0.00%	0.00%	0.00%	\$
Visa	0.0000%	0.00%	0.00%	0.00%	\$
McDonalds	0.0000%	0.00%	0.00%	0.00%	\$
Caterpillar	0.0000%	0.00%	0.00%	0.00%	\$
UnitedHealth Group	0.0000%	0.00%	0.00%	0.00%	\$
Nike	0.0000%	0.00%	0.00%	0.00%	\$
Microsoft	0.0000%	0.00%	0.00%	0.00%	\$
Wal-Mart Stores	0.0169%	0.02%	0.00%	0.00%	\$ 2.96
3M	0.0000%	0.00%	0.00%	0.00%	\$
American Express	0.0000%	0.00%	0.00%	0.00%	\$
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%	\$
Home Depot	0.0000%	0.00%	0.00%	0.00%	\$
DowDuPont	0.0000%	0.00%	0.00%	0.00%	\$
United Technologies	0.0000%	0.00%	0.00%	0.00%	\$
Coca-Cola	0.0000%	0.00%	0.00%	0.00%	\$
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%	\$
JPMorgan Chase	1.9736%	1.97%	0.00%	0.00%	\$ 345.37
Merck & Co	1.5654%	1.57%	0.00%	0.00%	\$ 273.93
Cisco Systems	2.9321%	2.93%	0.00%	0.00%	\$ 513.11
Travelers Companies	0.7137%	0.71%	0.00%	0.00%	\$ 124.89
Pfizer	0.0000%	0.00%	0.00%	0.00%	\$
Disney	0.0000%	0.00%	0.00%	0.00%	\$
Intel	3.6513%	3.65%	0.00%	0.00%	\$ 638.97
Goldman Sachs	2.5862%	2.59%	0.00%	0.00%	\$ 452.59
Chevron	0.7209%	0.72%	0.00%	0.00%	\$ 126.16
Verizon Communications	0.0000%	0.00%	0.00%	0.00%	\$
Exxon Mobil	0.0000%	0.00%	0.00%	0.00%	\$
IBM	3.4986%	3.50%	0.00%	0.00%	\$ 612.24
GE	0.0000%	0.00%	0.00%	0.00%	\$
Total	25.13%	25.13%	0.00%	0.00%	
Risk-Free Investment	74.871%				

Exhibit F

Portfolio growth	8.1419%	8.1419%
a'G	0.0151	0.0114
(a+)'G	0.0151	0.0114
(a-)'G	0.0000	0.0000
Portfolio variance	0.36%	0.36%
Portfolio volatility	6.04%	
Added value	7.14%	
Sharpe ratio	0.363012509	
Probability	1.00%	
VAR	-5.91%	
Inflation 1998	2.51%	
Inflation Adjusted Return	5.6319%	

Exhibit G

For amplification see appendix.

Analysis:

As theoretically predicted (see chapter: Theory + Metrics), maximizing the Sharpe Ratio outputs the most desirable investment strategy. While not offering portfolio growth as substantially as scenario (d), risk is further decreased to a mere 6.04% (\$1.06B), putting only 5.91% of the portfolio at risk of being lost in our five sigma considerations. Added portfolio value of 7.14% per year over the risk-free rate offers ample growth opportunity even after including inflation. Maximizing this portfolios utility will require re-analysis when 40% of the total fund can be invested (May 2002); we will therefore take another look at an ideal portfolio when keeping 90% of the current allocation in risk-free assets.

Scenario (c) (2)

Inputs:

1. Risk Aversion Factor: 5.5
2. Probability: 1% (or 5-sigma)
3. Risk-Free Investment must equal or surpass 90% of the entire portfolio value
4. As outlined in preceding chapters and preliminary considerations

Scenario solves for maximum Sharpe Ratio.

Outputs:

	a	a+	a-	a+*p-	Dollar ¹
Bank of America	2.9655%	2.97%	0.00%	0.00%	\$ 518,96
Apple	0.0000%	0.00%	0.00%	0.00%	\$
Visa	0.0000%	0.00%	0.00%	0.00%	\$
McDonalds	0.0000%	0.00%	0.00%	0.00%	\$
Caterpillar	0.0000%	0.00%	0.00%	0.00%	\$
UnitedHealth Group	0.0000%	0.00%	0.00%	0.00%	\$
Nike	0.0000%	0.00%	0.00%	0.00%	\$
Microsoft	0.0000%	0.00%	0.00%	0.00%	\$
Wal-Mart Stores	0.0165%	0.02%	0.00%	0.00%	\$ 2,85
3M	0.0000%	0.00%	0.00%	0.00%	\$
American Express	0.0000%	0.00%	0.00%	0.00%	\$
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%	\$
Home Depot	0.0000%	0.00%	0.00%	0.00%	\$
DowDuPont	0.0000%	0.00%	0.00%	0.00%	\$
United Technologies	0.0000%	0.00%	0.00%	0.00%	\$
Coca-Cola	0.0000%	0.00%	0.00%	0.00%	\$
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%	\$
JPMorgan Chase	0.7609%	0.76%	0.00%	0.00%	\$ 133,15
Merck & Co	0.6113%	0.61%	0.00%	0.00%	\$ 106,98
Cisco Systems	1.1684%	1.17%	0.00%	0.00%	\$ 204,47
Travelers Companies	0.2882%	0.29%	0.00%	0.00%	\$ 50,44
Pfizer	0.0000%	0.00%	0.00%	0.00%	\$
Disney	0.0000%	0.00%	0.00%	0.00%	\$
Intel	1.4392%	1.44%	0.00%	0.00%	\$ 251,85
Goldman Sachs	1.0420%	1.04%	0.00%	0.00%	\$ 182,34
Chevron	0.2903%	0.29%	0.00%	0.00%	\$ 50,86
Verizon Communications	0.0000%	0.00%	0.00%	0.00%	\$
Exxon Mobil	0.0000%	0.00%	0.00%	0.00%	\$
IBM	1.4177%	1.42%	0.00%	0.00%	\$ 248,05
GE	0.0000%	0.00%	0.00%	0.00%	\$
Total	10.00%	10.00%	0.00%	0.00%	
Risk-Free Investment	90.00%				

Exhibit H

Portfolio growth	6.8208%	6.8208%
a'G	0.0060	0.0045
(a+)'G	0.0060	0.0045
(a-)'G	0.0000	0.0000
Portfolio variance	0.06%	0.06%
Portfolio volatility	2.40%	
Added value	6.66%	
Sharpe ratio	0.363009761	
Probability	1.00%	
VAR	1.24%	
Inflation 1998	2.51%	
Inflation Adjusted Return	4.3108%	

Exhibit I

For amplification see appendix.

Analysis:

Though not immediately surpassing 8% annual growth, this suggested portfolio allocation substantially decreases any volatility (by about 7%) compared to status quo. With additional allocations as outlined in the legislation, we predict to achieve consistent annual growth of 8% or higher within three years. Even in the worst possible scenario, we should expect to see 1.24% growth; in the best possible scenario the portfolio will grow by 12.82% annually. Based on this analysis, we expect a 31.15% chance of surpassing our 8% goal by May 1999; leading us to recommend higher initial investments.

Scenario (d)

Inputs:

1. Risk Aversion Factor: 5.5
2. Probability: 1% (or 5-sigma)
3. Risk-Free Investment must equal or surpass 60% of the entire portfolio value
4. As outlined in preceding chapters and preliminary considerations

Scenario solves for maximum added value.

Output:

	a	a+	a-	a+a-	Dollar
Bank of America	5.7165%	5.72%	0.00%	0.00%	\$ 1,000.3
Apple	0.3138%	0.31%	0.00%	0.00%	\$ 54.5
Visa	0.0000%	0.00%	0.00%	0.00%	\$
McDonalds	0.0000%	0.00%	0.00%	0.00%	\$
Caterpillar	1.3449%	1.34%	0.00%	0.00%	\$ 235.3
UnitedHealth Group	0.7442%	0.74%	0.00%	0.00%	\$ 130.2
Nike	0.0000%	0.00%	0.00%	0.00%	\$
Microsoft	0.0000%	0.00%	0.00%	0.00%	\$
Wal-Mart Stores	0.0174%	0.02%	0.00%	0.00%	\$ 3.0
3M	0.0000%	0.00%	0.00%	0.00%	\$
American Express	0.7568%	0.76%	0.00%	0.00%	\$ 132.4
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%	\$
Home Depot	0.0000%	0.00%	0.00%	0.00%	\$
DowDuPont	1.5482%	1.55%	0.00%	0.00%	\$ 270.5
United Technologies	0.6430%	0.64%	0.00%	0.00%	\$ 112.5
Coca-Cola	0.0000%	0.00%	0.00%	0.00%	\$
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%	\$
JPMorgan Chase	1.6022%	1.60%	0.00%	0.00%	\$ 280.3
Merck & Co	0.7748%	0.77%	0.00%	0.00%	\$ 135.5
Cisco Systems	3.0489%	3.05%	0.00%	0.00%	\$ 533.5
Travelers Companies	1.0763%	1.08%	0.00%	0.00%	\$ 188.3
Pfizer	0.6034%	0.60%	0.00%	0.00%	\$ 105.6
Disney	0.2553%	0.26%	0.00%	0.00%	\$ 44.6
Intel	1.7714%	1.77%	0.00%	0.00%	\$ 309.5
Goldman Sachs	2.9137%	2.91%	0.00%	0.00%	\$ 509.5
Chevron	1.1969%	1.20%	0.00%	0.00%	\$ 209.4
Verizon Communications	0.0000%	0.00%	0.00%	0.00%	\$
Exxon Mobil	0.6083%	0.61%	0.00%	0.00%	\$ 106.4
IBM	0.9115%	0.91%	0.00%	0.00%	\$ 159.5
GE	0.5306%	0.53%	0.00%	0.00%	\$ 92.6
Total	26.38%	26.38%	0.00%	0.00%	
Risk-Free Investment	73.622%				

Exhibit J

Portfolio growth	8.2413%	8.2413%
a'G	0.0158	0.0125
(a+)'G	0.0158	0.0125
(a-)'G	0.0000	0.0000
Portfolio variance	0.42%	0.42%
Portfolio volatility	6.48%	
Added value	7.09%	
Sharpe ratio	0.353532511	
Probability	1.00%	
VAR	-6.84%	
Inflation 1998	2.51%	
Inflation Adjusted Return	5.7313%	

Exhibit K

For amplification see appendix.

Analysis:

This scenario is more desirable than our current investment structure as it increases expected portfolio return (by 0.241%), while significantly reducing portfolio risk (by 2.90%); a feat scenarios (a) and (b) fail to achieve respectively. Though not ideally maximizing utility of any possible portfolio, we can confidently recommend implementation of it should other options not be feasible for implementation. A unit of increased risk is rewarded with 0.35 points higher return.

Note all scenarios thus far only consider our fully invested portfolio in 2002. Due to their respective flaws, we will not explore a year-over-year investment strategy; except when considering scenario (c).

Model Weaknesses and Scientific Bias

All calculated scenarios output probabilities. There are never any guarantees for any given scenario to be precisely correct. Moreover, there are clearly identifiable weaknesses in this model this chapter oughts to address.

Firstly, all six assumptions outlined in chapter 2 of this paper can be determined differently, yielding different results for all scenarios. Secondly, this model relies overwhelmingly on historical performance of any given asset. Uncertainties (for example introduced by black swan events) cannot be accounted for, and would substantially alter all predicted outcomes. Thirdly, and most importantly; this model has been iterated and reiterated hundreds of times. Nevertheless, it is prone to human error, and one false calculation would alter the conclusions significantly. Peer review to this process is recommended. Additionally, there are millions of possible starting points, altering outputs in any possible way.

Lastly, and regardless of outlined imperfections, maximizing Sharpe Ratio as a means to determine maximum portfolio utility to any given market conditions has been proven correct frequently in (real) market conditions. Offering a sound theoretical approach does not lead to guarantees neither however. Therefore, we want to emphasize that though we believe in the theories' validity and our own recommendations, there are other outcomes possible, and acceptable.

Discussion

In seeking an optimal investment strategy for the 1.75 billion US-Dollars this panel looks to allocate into the open market, we considered multiple different scenarios. Analyzing all relevant metrics, and putting emphasis on risk and reward, we can strongly recommend exchanging some of the current uncertainty with slightly lower, but much more secure growth in the short term, and higher overall growth with lower uncertainty in the long term. This is further outlined by both scenarios (c) and (c)(2), respectively offering highest utility to the people of South Carolina under different initial inputs.

There is an argument to be made under these unprecedented times of growth to choose a riskier portfolio at the exchange of security, as expressed through scenario (d); however that portfolio as well as a hypothetical maximum growth scenario rely on less diversification. Market conditions can quickly change, and with ever-possible economic downsides a portfolio offering positive returns with high confidence will prove more desirable. Ultimately, a hybrid strategy of focusing on (d) in the short-term and (c) in the long-term is possible but challenging to implement and maintain.

For further discussion and explanation do not hesitate to reach out to the authors.

Conclusion

After analyzing the current environment, and discussing assumptions, five simulations are ran to predict a portfolio maximizing investment utility for the people of South Carolina. Though theory clearly indicates to seek out the any asset allocation that maximizes Sharpe ratio (risk-to-reward measure), further discussion reveals multiple possibilities about determining the best short-, medium-, and long-term strategies. Academically, there is no question, that using scenarios (c)(2) and (c) will offer the best possible theoretical outcome.

In writing this case many simulation weaknesses and questions were raised, it is strongly recommended to conduct more in-depth audits of possible investment strategies with additional data availability over longer periods of time.

Appendix

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Exhibits

Exhibit A – for amplification see appendix

	Mean Return	St Dev		CORREL									Treasury Bi	Treasury Bc	Corporate Bonds	
A	6.57%	2.59%			A	B	C						1965	3.93%	0.71%	-0.46%
B	8.29%	12.00%		A	1	0.0714465	0.054722						1966	4.76%	4.69%	0.20%
C	8.43%	11.63%		B	0.071447	1	0.960203						1967	4.21%	-9.18%	-4.95%
				C	0.054722	0.9602034	1						1968	5.21%	-0.26%	2.57%
													1969	6.58%	-5.07%	-8.09%
													1970	6.52%	12.11%	18.37%
													1971	4.39%	13.23%	11.01%
				COVAR									1972	3.84%	5.69%	7.26%
					A	B	C						1973	6.93%	-1.11%	1.14%
				A	0.000671	0.000222	0.000165						1974	8%	4.35%	-3.06%
				B	0.000222	0.014400	0.013401						1975	5.80%	9.20%	14.64%
				C	0.000165	0.013401	0.013526						1976	5.08%	16.75%	18.65%
													1977	5.12%	-0.69%	1.71%
													1978	7.18%	-1.18%	-0.07%
													1979	10.38%	-1.23%	-4.18%
													1980	11.24%	-3.95%	-2.76%
				Weight	20.20%	38.39%	41.41%	100%					1981	14.71%	1.86%	-1.24%
													1982	10.54%	40.36%	42.56%
				Portfolio St	9.38%								1983	8.80%	0.65%	6.26%
				Return	8.00%								1984	9.85%	15.48%	16.86%
				Risk Free R _f	5.95%								1985	7.72%	30.97%	30.09%
				Sharpe Ratio	0.22								1986	6.16%	24.53%	19.85%
													1987	5.47%	-2.71%	-0.27%
													1988	6.35%	9.67%	10.70%
													1989	8.37%	18.11%	16.23%
				Inflation Cl	4.610%								1990	7.81%	6.18%	6.78%
													1991	5.60%	19.30%	19.89%
													1992	3.51%	8.05%	9.39%
													1993	2.90%	18.24%	13.19%
													1994	3.90%	-7.77%	-5.76%
													1995	5.60%	31.67%	27.20%
													1996	5.21%	-0.93%	1.40%
													1997	5.26%	15.88%	12.95%

Exhibit B

	a	a+	a-	a+*a-		Dollar Value
Bank of America	0.8726%	0.87%	0.00%	0.00%		\$ 152,703,472.68
Apple	0.3298%	0.33%	0.00%	0.00%		\$ 57,706,874.99
Visa	0.0000%	0.00%	0.00%	0.00%		\$ -
McDonalds	0.0000%	0.00%	0.00%	0.00%		\$ -
Caterpillar	0.6425%	0.64%	0.00%	0.00%		\$ 112,430,728.41
UnitedHealth Group	0.3752%	0.38%	0.00%	0.00%		\$ 65,666,303.88
Nike	0.0000%	0.00%	0.00%	0.00%		\$ -
Microsoft	0.3439%	0.34%	0.00%	0.00%		\$ 60,189,768.67
Wal-Mart Stores	0.0000%	0.00%	0.00%	0.00%		\$ -
3M	0.0000%	0.00%	0.00%	0.00%		\$ -
American Express	1.4796%	1.48%	0.00%	0.00%		\$ 258,928,565.24
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%		\$ -
Home Depot	0.0000%	0.00%	0.00%	0.00%		\$ -
DowDuPont	0.5547%	0.55%	0.00%	0.00%		\$ 97,063,821.18
United Technologies	0.3601%	0.36%	0.00%	0.00%		\$ 63,026,066.74
Coca-Cola	0.0000%	0.00%	0.00%	0.00%		\$ -
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%		\$ -
JPMorgan Chase	23.8336%	23.83%	0.00%	0.00%		\$ 4,170,875,336.76
Merck & Co	0.3299%	0.33%	0.00%	0.00%		\$ 57,726,555.46
Cisco Systems	0.6524%	0.65%	0.00%	0.00%		\$ 114,164,724.23
Travelers Companies	0.4519%	0.45%	0.00%	0.00%		\$ 79,079,062.47
Pfizer	0.3408%	0.34%	0.00%	0.00%		\$ 59,646,051.35
Disney	0.3440%	0.34%	0.00%	0.00%		\$ 60,197,010.89
Intel	0.5888%	0.59%	0.00%	0.00%		\$ 103,048,606.25
Goldman Sachs	2.4963%	2.50%	0.00%	0.00%		\$ 436,844,156.41
Chevron	0.4389%	0.44%	0.00%	0.00%		\$ 76,813,516.03
Verizon Communications	0.0000%	0.00%	0.00%	0.00%		\$ -
Exxon Mobil	0.3315%	0.33%	0.00%	0.00%		\$ 58,011,810.99
IBM	0.3324%	0.33%	0.00%	0.00%		\$ 58,164,856.16
GE	0.4556%	0.46%	0.00%	0.00%		\$ 79,726,519.33
Total	35.55%	35.55%	0.00%	0.00%		
Risk-Free Investment	64.446%					

Exhibit C

	a	a+	a-	a+*a-	Dollar Value
Bank of America	0.6934%	0.69%	0.00%	0.00%	\$ 121,346,542.25
Apple	0.3079%	0.31%	0.00%	0.00%	\$ 53,877,156.05
Visa	-0.8384%	0.00%	0.84%	0.00%	\$ (146,726,262.18)
McDonalds	-0.4422%	0.00%	0.44%	0.00%	\$ (77,382,894.27)
Caterpillar	0.5232%	0.52%	0.00%	0.00%	\$ 91,551,692.86
UnitedHealth Group	0.3468%	0.35%	0.00%	0.00%	\$ 60,682,166.62
Nike	-0.5742%	0.00%	0.57%	0.00%	\$ (100,481,493.37)
Microsoft	0.3353%	0.34%	0.00%	0.00%	\$ 58,670,263.03
Wal-Mart Stores	0.0168%	0.02%	0.00%	0.00%	\$ 2,937,271.08
3M	-2.3221%	0.00%	2.32%	0.00%	\$ (406,375,645.40)
American Express	0.8578%	0.86%	0.00%	0.00%	\$ 150,110,429.51
Johnson & Johnson	-1.4903%	0.00%	1.49%	0.00%	\$ (260,799,366.26)
Home Depot	-0.7275%	0.00%	0.73%	0.00%	\$ (127,318,700.60)
DowDuPont	0.4992%	0.50%	0.00%	0.00%	\$ 87,355,446.70
United Technologies	0.3149%	0.31%	0.00%	0.00%	\$ 55,108,912.06
Coca-Cola	-0.7813%	0.00%	0.78%	0.00%	\$ (136,733,086.55)
Procter & Gamble	-0.3212%	0.00%	0.32%	0.00%	\$ (56,214,300.14)
JPMorgan Chase	-1.3390%	0.00%	1.34%	0.00%	\$ (234,333,405.18)
Merck & Co	0.3092%	0.31%	0.00%	0.00%	\$ 54,101,638.32
Cisco Systems	0.5722%	0.57%	0.00%	0.00%	\$ 100,131,296.11
Travelers Companies	0.4258%	0.43%	0.00%	0.00%	\$ 74,513,883.70
Pfizer	0.3269%	0.33%	0.00%	0.00%	\$ 57,199,290.04
Disney	0.3184%	0.32%	0.00%	0.00%	\$ 55,712,518.29
Intel	0.4106%	0.41%	0.00%	0.00%	\$ 71,859,168.78
Goldman Sachs	0.6644%	0.66%	0.00%	0.00%	\$ 116,273,226.05
Chevron	0.3913%	0.39%	0.00%	0.00%	\$ 68,473,979.85
Verizon Communications	-0.5894%	0.00%	0.59%	0.00%	\$ (103,148,144.84)
Exxon Mobil	0.3144%	0.31%	0.00%	0.00%	\$ 55,012,758.66
IBM	0.3169%	0.32%	0.00%	0.00%	\$ 55,457,254.07
GE	0.5274%	0.53%	0.00%	0.00%	\$ 92,301,056.73
Total	-0.95%	8.47%	9.43%	0.00%	
Risk-Free Investment	86.815%				

Exhibit D

Portfolio growth	5.3276%	5.3276%
a'G	0.0008	0.0007
(a+)'G	0.0046	0.0041
(a-)'G	0.0038	0.0034
Portfolio variance	0.01%	0.01%
Portfolio volatility	0.77%	
Added value	5.31%	
Sharpe ratio	-0.811609728	
Probability	1.00%	
VAR	3.54%	
Inflation 1998	2.51%	
Inflation Adjusted Return	2.8176%	

Exhibit E

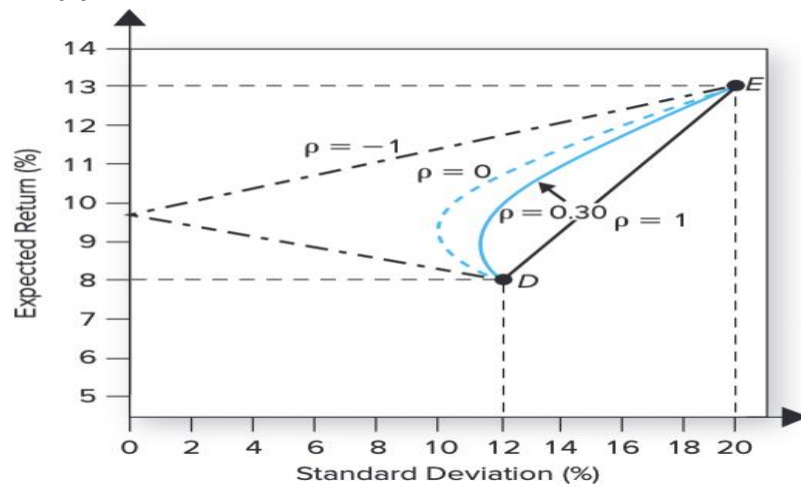


Exhibit F

	a	a+	a-	a+*a-	Dollar Value
Bank of America	5.7165%	5.72%	0.00%	0.00%	\$ 1,000,388,304.31
Apple	0.3138%	0.31%	0.00%	0.00%	\$ 54,910,634.88
Visa	0.0000%	0.00%	0.00%	0.00%	\$ -
McDonalds	0.0000%	0.00%	0.00%	0.00%	\$ -
Caterpillar	1.3449%	1.34%	0.00%	0.00%	\$ 235,356,742.50
UnitedHealth Group	0.7442%	0.74%	0.00%	0.00%	\$ 130,243,169.70
Nike	0.0000%	0.00%	0.00%	0.00%	\$ -
Microsoft	0.0000%	0.00%	0.00%	0.00%	\$ -
Wal-Mart Stores	0.0174%	0.02%	0.00%	0.00%	\$ 3,045,799.30
3M	0.0000%	0.00%	0.00%	0.00%	\$ -
American Express	0.7568%	0.76%	0.00%	0.00%	\$ 132,446,479.76
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%	\$ -
Home Depot	0.0000%	0.00%	0.00%	0.00%	\$ -
DowDuPont	1.5482%	1.55%	0.00%	0.00%	\$ 270,942,797.92
United Technologies	0.6430%	0.64%	0.00%	0.00%	\$ 112,529,819.17
Coca-Cola	0.0000%	0.00%	0.00%	0.00%	\$ -
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%	\$ -
JPMorgan Chase	1.6022%	1.60%	0.00%	0.00%	\$ 280,385,800.96
Merck & Co	0.7748%	0.77%	0.00%	0.00%	\$ 135,592,150.84
Cisco Systems	3.0489%	3.05%	0.00%	0.00%	\$ 533,557,512.05
Travelers Companies	1.0763%	1.08%	0.00%	0.00%	\$ 188,360,934.05
Pfizer	0.6034%	0.60%	0.00%	0.00%	\$ 105,603,715.15
Disney	0.2553%	0.26%	0.00%	0.00%	\$ 44,676,910.34
Intel	1.7714%	1.77%	0.00%	0.00%	\$ 309,998,341.40
Goldman Sachs	2.9137%	2.91%	0.00%	0.00%	\$ 509,904,572.36
Chevron	1.1969%	1.20%	0.00%	0.00%	\$ 209,449,264.46
Verizon Communications	0.0000%	0.00%	0.00%	0.00%	\$ -
Exxon Mobil	0.6083%	0.61%	0.00%	0.00%	\$ 106,458,744.25
IBM	0.9115%	0.91%	0.00%	0.00%	\$ 159,514,416.16
GE	0.5306%	0.53%	0.00%	0.00%	\$ 92,846,612.90
Total	26.38%	26.38%	0.00%	0.00%	
Risk-Free Investment	73.622%				

Exhibit G

Portfolio growth	8.2413%	8.2413%
a'G	0.0158	0.0125
(a+)'G	0.0158	0.0125
(a-)'G	0.0000	0.0000
Portfolio variance	0.42%	0.42%
Portfolio volatility	6.48%	
Added value	7.09%	
Sharpe ratio	0.353532511	
Probability	1.00%	
VAR	-6.84%	
Inflation 1998	2.51%	
Inflation Adjusted Return	5.7313%	

Exhibit H

	a	a+	a-	a+*a-	Dollar Value
Bank of America	7.4705%	7.47%	0.00%	0.00%	\$ 1,307,334,766.88
Apple	0.0000%	0.00%	0.00%	0.00%	\$ -
Visa	0.0000%	0.00%	0.00%	0.00%	\$ -
McDonalds	0.0000%	0.00%	0.00%	0.00%	\$ -
Caterpillar	0.0000%	0.00%	0.00%	0.00%	\$ -
UnitedHealth Group	0.0000%	0.00%	0.00%	0.00%	\$ -
Nike	0.0000%	0.00%	0.00%	0.00%	\$ -
Microsoft	0.0000%	0.00%	0.00%	0.00%	\$ -
Wal-Mart Stores	0.0169%	0.02%	0.00%	0.00%	\$ 2,961,655.54
3M	0.0000%	0.00%	0.00%	0.00%	\$ -
American Express	0.0000%	0.00%	0.00%	0.00%	\$ -
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%	\$ -
Home Depot	0.0000%	0.00%	0.00%	0.00%	\$ -
DowDuPont	0.0000%	0.00%	0.00%	0.00%	\$ -
United Technologies	0.0000%	0.00%	0.00%	0.00%	\$ -
Coca-Cola	0.0000%	0.00%	0.00%	0.00%	\$ -
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%	\$ -
JPMorgan Chase	1.9736%	1.97%	0.00%	0.00%	\$ 345,375,172.76
Merck & Co	1.5654%	1.57%	0.00%	0.00%	\$ 273,938,467.52
Cisco Systems	2.9321%	2.93%	0.00%	0.00%	\$ 513,115,696.57
Travelers Companies	0.7137%	0.71%	0.00%	0.00%	\$ 124,895,542.95
Pfizer	0.0000%	0.00%	0.00%	0.00%	\$ -
Disney	0.0000%	0.00%	0.00%	0.00%	\$ -
Intel	3.6513%	3.65%	0.00%	0.00%	\$ 638,973,105.60
Goldman Sachs	2.5862%	2.59%	0.00%	0.00%	\$ 452,590,563.29
Chevron	0.7209%	0.72%	0.00%	0.00%	\$ 126,162,996.66
Verizon Communications	0.0000%	0.00%	0.00%	0.00%	\$ -
Exxon Mobil	0.0000%	0.00%	0.00%	0.00%	\$ -
IBM	3.4986%	3.50%	0.00%	0.00%	\$ 612,246,576.52
GE	0.0000%	0.00%	0.00%	0.00%	\$ -
Total	25.13%	25.13%	0.00%	0.00%	
Risk-Free Investment	74.871%				

Exhibit I

Portfolio growth	8.1419%	8.1419%
a'G	0.0151	0.0114
(a+)'G	0.0151	0.0114
(a-)'G	0.0000	0.0000
Portfolio variance	0.36%	0.36%
Portfolio volatility	6.04%	
Added value	7.14%	
Sharpe ratio	0.363012509	
Probability	1.00%	
VAR	-5.91%	
Inflation 1998	2.51%	
Inflation Adjusted Return	5.6319%	

Exhibit J

	a	a+	a-	a+*a-	Dollar Value
Bank of America	2.9655%	2.97%	0.00%	0.00%	\$ 518,960,451.40
Apple	0.0000%	0.00%	0.00%	0.00%	\$ -
Visa	0.0000%	0.00%	0.00%	0.00%	\$ -
McDonalds	0.0000%	0.00%	0.00%	0.00%	\$ -
Caterpillar	0.0000%	0.00%	0.00%	0.00%	\$ -
UnitedHealth Group	0.0000%	0.00%	0.00%	0.00%	\$ -
Nike	0.0000%	0.00%	0.00%	0.00%	\$ -
Microsoft	0.0000%	0.00%	0.00%	0.00%	\$ -
Wal-Mart Stores	0.0165%	0.02%	0.00%	0.00%	\$ 2,893,880.00
3M	0.0000%	0.00%	0.00%	0.00%	\$ -
American Express	0.0000%	0.00%	0.00%	0.00%	\$ -
Johnson & Johnson	0.0000%	0.00%	0.00%	0.00%	\$ -
Home Depot	0.0000%	0.00%	0.00%	0.00%	\$ -
DowDuPont	0.0000%	0.00%	0.00%	0.00%	\$ -
United Technologies	0.0000%	0.00%	0.00%	0.00%	\$ -
Coca-Cola	0.0000%	0.00%	0.00%	0.00%	\$ -
Procter & Gamble	0.0000%	0.00%	0.00%	0.00%	\$ -
JPMorgan Chase	0.7609%	0.76%	0.00%	0.00%	\$ 133,152,980.14
Merck & Co	0.6113%	0.61%	0.00%	0.00%	\$ 106,982,255.77
Cisco Systems	1.1684%	1.17%	0.00%	0.00%	\$ 204,472,542.76
Travelers Companies	0.2882%	0.29%	0.00%	0.00%	\$ 50,442,157.04
Pfizer	0.0000%	0.00%	0.00%	0.00%	\$ -
Disney	0.0000%	0.00%	0.00%	0.00%	\$ -
Intel	1.4392%	1.44%	0.00%	0.00%	\$ 251,853,964.67
Goldman Sachs	1.0420%	1.04%	0.00%	0.00%	\$ 182,343,072.33
Chevron	0.2903%	0.29%	0.00%	0.00%	\$ 50,807,147.97
Verizon Communications	0.0000%	0.00%	0.00%	0.00%	\$ -
Exxon Mobil	0.0000%	0.00%	0.00%	0.00%	\$ -
IBM	1.4177%	1.42%	0.00%	0.00%	\$ 248,091,547.94
GE	0.0000%	0.00%	0.00%	0.00%	\$ -
Total	10.00%	10.00%	0.00%	0.00%	
Risk-Free Investment	90.000%				

Exhibit K

Portfolio growth	6.8208%	6.8208%
a'G	0.0060	0.0045
(a+)'G	0.0060	0.0045
(a-)'G	0.0000	0.0000
Portfolio variance	0.06%	0.06%
Portfolio volatility	2.40%	
Added value	6.66%	
Sharpe ratio	0.363009761	
Probability	1.00%	
VAR	1.24%	
Inflation 1998	2.51%	
Inflation Adjusted Return	4.3108%	