Lesson 1: Algebraic Expressions

Concept 1: Definition of Terms in Algebra

In this lesson, we will learn related algebra terms and evaluate the algebraic expression. Mastering these concepts is fundamental to learning algebra. We are already familiar with the arithmetic statement 2 + 3 = 5, then how about this expression:

$$\mathbf{x + y = 5}$$

In the expression shown above, there is an infinite number of values that can take the places of x and y. These letters are called **variables** and the number 5 is the **constant** which is a number that has a known value and remains unchanged. If the expression consists of numbers, constants, variables, and operation symbols is called an **algebraic expression**.

Example:

Find the constants and variables in the following algebraic expression.

a.) x + 2 b.) 10 + a c.) b + z + 7

Answers:

a.) constant: **2**, variable: **x**

b.) constant: **10**, variable: **a**

c.) constant: **-7**, variable: **b** and **z**

Note:
The variables do not limit only to letters, but they can also be symbols or shapes. And unlike equations, algebraic expressions do not have equal signs.

Concept 2: Concept of Terms

A. Positive Integer Exponent (a^n, where a ≠ 0 and n is a positive integer)

The expression 2●2●2●2 can be written as 2^4

base ⟶ $\mathbf{2}^{\mathbf{4}}$ ⟶ exponent

Similarly, x●x●x can be written as x^3. The variable x is the base, and the number 3 is the exponent. The exponent tells how many times the base is used as a factor.

Example:

$(-4)^3$= (-4) (-4) (-4) = -64

$(\frac{1}{2})^2 = (\frac{1}{2})\ (\frac{1}{2}) = \frac{\mathbf{1}}{4}$

An exponent of 1 can be omitted in a term such as in 2x, zy, and -9x which means $2x^1$, $z^1\ y^1$, and $-9x^1$, respectively.

The following are read as follows:

5^2	five to the second power or five squared
3^3	three to the third power or three cube
y^4	y to the fourth power or y to the fourth

In general, an expression of the form $\mathbf{x^n}$**,** where x ≠ 0 and n is the degree of the expression or the exponent which is a nonnegative or a positive integer.

In $\mathbf{x^n}$ **=** x●x●x●x…●x in which there are **n** factors of a, a is called the base and n is the exponent.

base ⟶ $\mathbf{x}^{\mathbf{n}}$ ⟶ exponent

B. Term

Term is called when any algebraic expression separated from another algebraic expressions by a plus (+) or a minus (-).

There are three terms in the expression $3x^2$ + 2xy + 1, namely, $3x^2$, 2xy, and 1.

Example:

Count the number of terms in each expression.

A. $4x^3y^2$ + $3z^2$ – 4xy + 5 - There are four terms.

B. $\frac{3x+1}{6}$ - There is only one term.

C. 5 (y-2) + $\sqrt{2x+3}$ - There are two terms.

C. Literal Coefficients and Numerical Coefficients

The numerical factor of a term is called **numerical coefficient** or simply coefficient and the variable factor of a term is called **literal coefficient**.

In the term $2x^3$, 2 is called the **numerical coefficient** and x^3 is called the **literal coefficient.**

The term -y has a numerical coefficient which is -1 and literal coefficient which is y.

The term 4 is called the **constant**, which is usually referred to as the term without a variable or simply a constant term.

Number coefficient is the number part of a term. Literal coefficient is the variable including its exponent. The word coefficient alone is referred to as the numerical coefficient.

Concept 3: Concept of Degree

The **degree of a term** is the exponent of its variable while the **degree of the polynomial** is the highest degree appearing in any of the terms in that polynomial is the highest degree appearing in any of the terms in that polynomial. For example, in the polynomial $5x^4$ + $3x^2$ - 7x + 9, the degree of the terms are as follow:

$5x^4$ has degree 4, $3x^2$ has a degree 2, -7x has degree of 1, and 9 has degree 0.

Since 4 is the highest degree, the degree of the polynomial $5x^4$ + $3x^2$ - 7x + 9 is degree 4.

If a term consists of two or more variables, the degree of that term is the sum of the exponents of the variables. For example, in the polynomial xy^3 + $5x^2y^4$ - $6y^3$ we have the degree of each term as follow:

xy^3	has degree 4	(1 + 3 = 4)
$5x^2y^4$	has degree 6	(2 + 4 = 6)

$- 6y^3$ has degree 3

Since 6 is the highest sum of the exponent from the term $5x^2y^4$, the polynomial $xy^3 + 5x^2y^4 - 6y^3$ has a degree 6.

Concept 4: Polynomials and its Standard Form

A **polynomial** is an algebraic expression that represents a sum of one or more terms containing whole-number exponents on the variables.

Example:

4	3x	$-\frac{9}{7}x + y$
$\frac{y-x}{5}$	$x^2 + 2x - 6$	$\sqrt{9}x^3 + 5x^2y - xy^2 + y^2$

An expression is NOT a polynomial if:

1. Its exponent is not a whole number or a variable.

   Example: $5x^{-2} + 1$, $x^6y^3 + x^4y^2 + x^{1/2}$, 9^x
2. The variable is in the denominator.

   Example: $\frac{8}{x}$, $\frac{x}{3y}$, $\frac{z+y}{4x}$
3. The variable is under the radical.

   Example: $\sqrt{2x^3}$, $\sqrt{y}$, $\sqrt{x + 5y}$,

Concept 5: Kinds of Polynomials

Classifying polynomials based on the number of terms of: A polynomial with one term is called a **monomial.** A polynomial with two terms is called **binomial.** A polynomial with three terms is called **trinomial.**

Note: Polynomials with more than three terms have no special name and just simply called polynomial/ multinomial.

Kinds of Polynomial	No. of Terms	Examples
Monomial	One term	$-9x^2$, 62, 2x, x
Binomial	Two terms	3x - 8y, $\frac{x+y}{4}$, 5(x + z) *Note*: $\frac{x+y}{4}$ is the same as $\frac{x}{4} + \frac{y}{4}$, 5(x + z) is the same as 5x + 5z because of Distributive Property.
Trinomial	Three terms	$x^2 - 6x + 9$, 2(x – y + z) *Note*: 2(x – y + z) is equal to 2x – 2y + 2z because of Distributive Property.
Multinomial or Polynomial	Four or more terms	$x^3 + 2x^2 - 3x + 5$

Kind of Polynomial according to its degree

Kinds of Polynomial in terms of degree	Degree	Examples
Constant	Zero	1, 2, and any number
Linear	One	2x, x + 1, 2x – 3y + 4
Quadratic	Two	$2x^2$, $x^2 + 1$, $2x^2 - 3y + 4$
Cubic	Three	$2x^3$, $x^3 + 1$, $2x^3 - 2y^2 + 4$
Quartic	Four	$2x^4$, $x^4 + 1$, $2x^4 - 2y^3 + 4$
Quantic	Five	$2x^5$, $x^5 + 1$, $2x^5 - 2y^4 + 4$