Quiz 3

A.1 6610402230 ศิริสุข ทุ่นธรรม

from $\theta_{MLE} = \underset{\theta}{\text{argmax}} \prod_{i=1}^{n} f(X_i, \theta)$; $\theta = (\mu, \sigma)$

สำหรับ การ Maximize $D = \{150\}$, $n = 1$

θ_1 ($\mu_1 = 165$, $\sigma_1 = 15$) ทำให้ D อยู่ในช่วง $\mu - \sigma$

θ_2 ($\mu_2 = 166$, $\sigma_2 = 10$) ทำให้ D อยู่ในช่วง $\mu - 2\sigma$

จาก การ estimat distribution จะเห็นว่า θ_1 ทำให้ข้อมูลอยู่ใน ช่วง dist. ที่มี $P(X)$ สูงกว่า

∴ Answer θ_1

- **Problem A:** Suppose that we have modeled $P(X)$ by using Normal distribution $\mathrm{f}(\mathrm{x};\theta) = \mathcal{N}\big(\theta = (\mu, \sigma)\big)$. Given two parameters $\theta_1 = (\mu_1 = 165,\ \sigma = 15\ ), \theta_2 = (u_2 = 166, \sigma_2 = 10))$, you are to decide which parameter is the most appropriate regarding the MLE principle, given that the data we have observed is as follows:

  $\mu = 150$ SD = 0

  - **Problem A.1:** $\mathrm{n} = 1; \mathrm{D} = \{\mathrm{X} = 150\}$

  $\mu = 165$ SD = 11.75

  - **Problem A.2:** $\mathrm{n} = 10; \mathrm{D} = \{\mathrm{X}_1 = 145, \mathrm{X}_2 = 170, \mathrm{X}_3 = 174, \mathrm{X}_4 = 168, \mathrm{X}_5 = 180, \mathrm{X}_6 = 155, \mathrm{X}_7 = 162, \mathrm{X}_8 = 166, \mathrm{X}_9 = 180\ , \mathrm{X}_{10} = 152\ \}$

For each subproblem, demonstrate your solution and describe reasons in details to support your answer

A.2 เลือก θ_1 เพราะ ค่า μ มีค่าใกล้เคียงมากที่สุด และ θ_1 θ_2 มี ค่า μ ใกล้เคียงกันมาก เมื่อดูที่ σ θ_1 มี σ_1 กว้างกว่า θ_2 ทำให้ค่า P(ช่วง μ D A.2) กว้างกว่า

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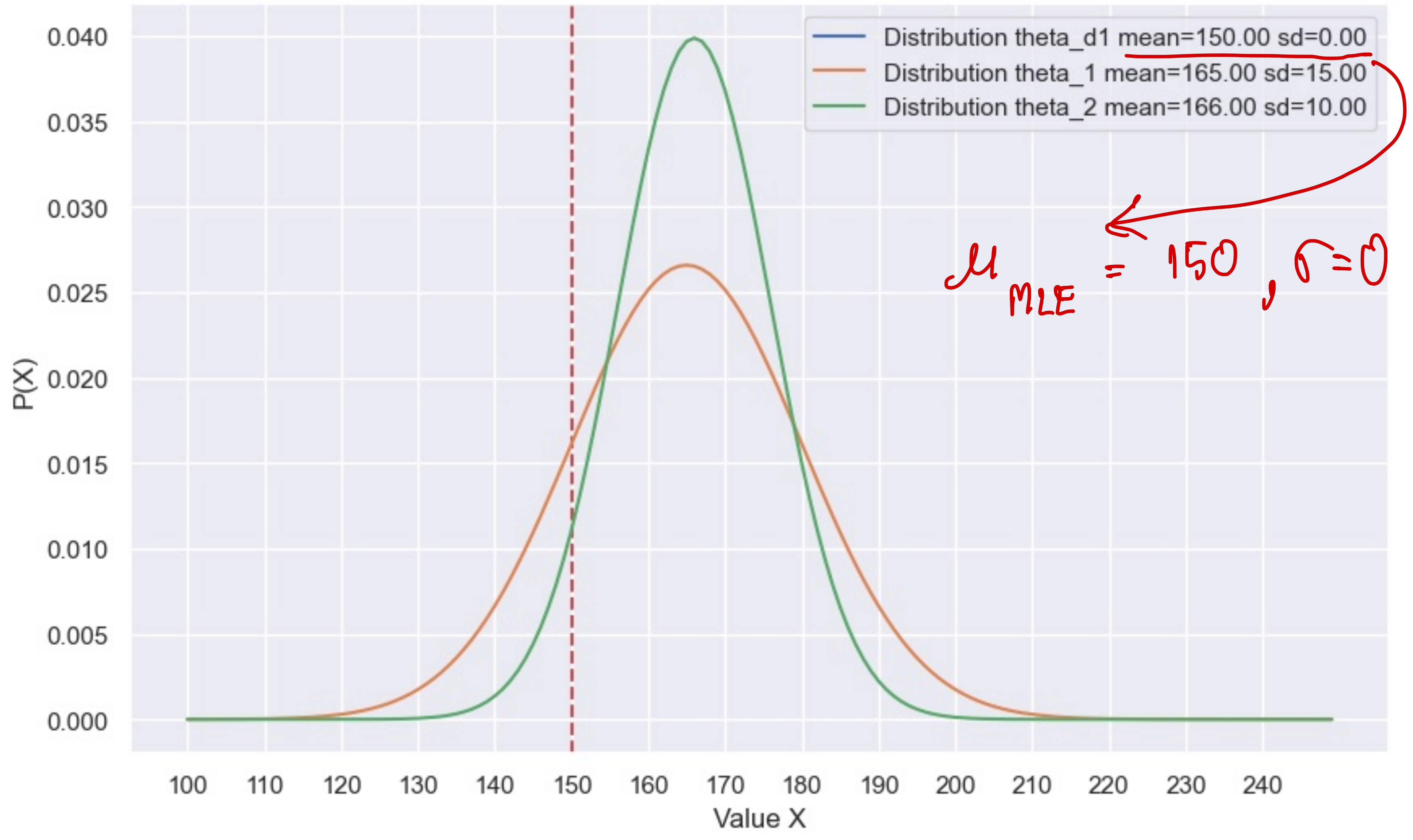
Quiz 3

- **Problem B:** Suppose that we have modeled $P(X)$ by using Normal distribution $\mathrm{f}(\mathrm{x};\theta) = \mathcal{N}\big(\theta = (\mu, \sigma)\big)$. You are to find the optimal parameter θ_{MLE} regarding the MLE principle, given that the data we have observed is as follows:
  - **Problem B.1:** $\mathrm{n} = 1; \mathrm{D} = \{\mathrm{X} = 150\}$
  - **Problem B.2:** $\mathrm{n} = 10; \mathrm{D} = \{\mathrm{X}_1 = 145, \mathrm{X}_2 = 170, \mathrm{X}_3 = 174, \mathrm{X}_4 = 168, \mathrm{X}_5 = 180, \mathrm{X}_6 = 155, \mathrm{X}_7 = 162, \mathrm{X}_8 = 166, \mathrm{X}_9 = 180, \mathrm{X}_{10} = 152\}$

For each subproblem, demonstrate your solution and describe reasons in details to support your answer

$$\theta_{MLE} = \begin{cases} \mu_{MLE} = \dfrac{1}{n}\sum_{i=1}^{n} x_i \\ \sigma_{MLE} = \sqrt{\dfrac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2} \end{cases}$$

B.1

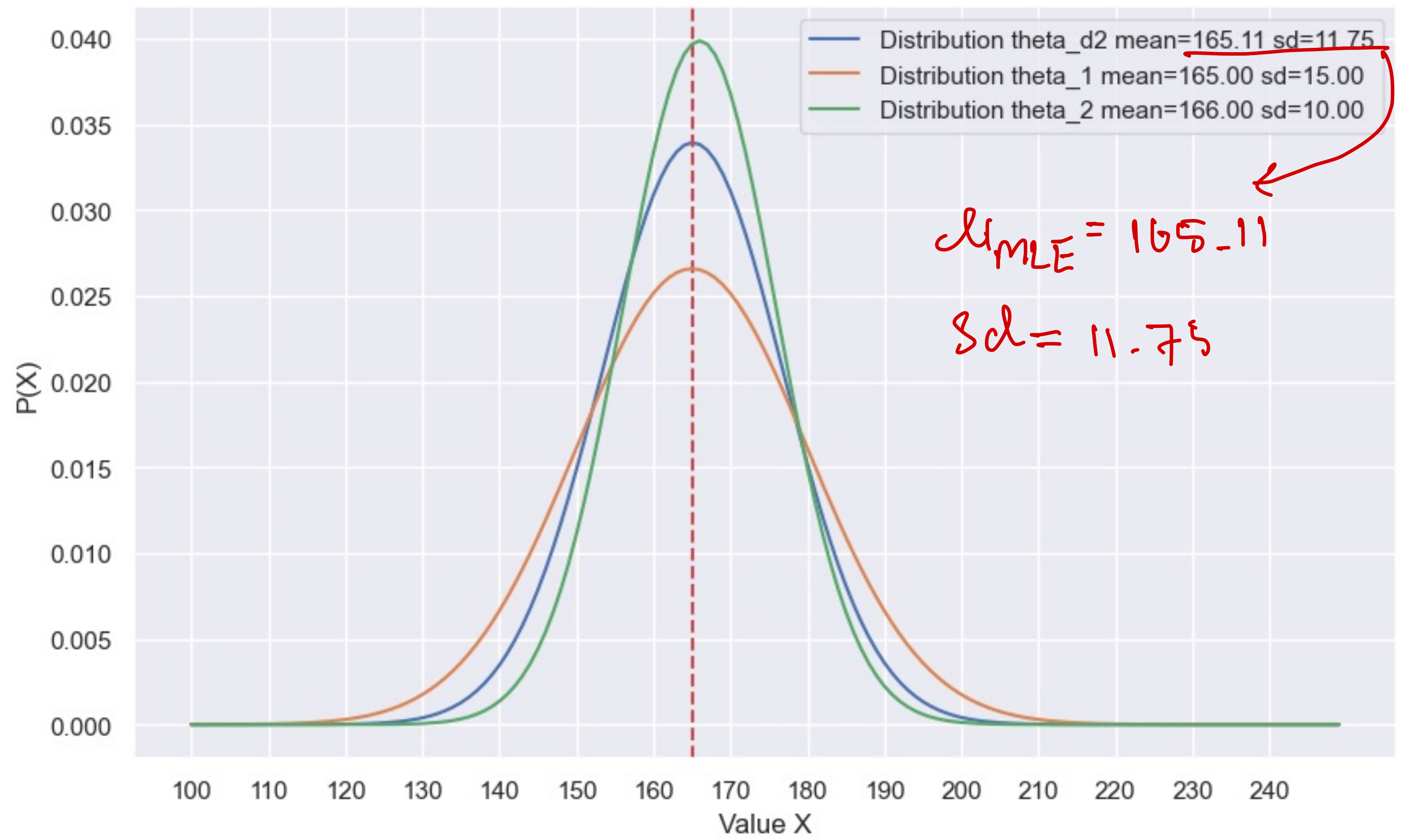


$\mu_{MLE} = 150 , \sigma = 0$

```
Normal Distribution D2 with theta_d1,1,2 (SUM) [nan, 0.016131381634609556, 0.011092083467945555]
```

```python
d1 = np.array([150]).astype(float)
x_test = np.arange(100,250,1)
prob_x = []
mean = np.mean(d1)
sd = np.std(d1)
prob_x.append(norm.pdf(d1, mean, sd).sum())
sns.lineplot(x=x_test, y=norm.pdf(x_test, mean, sd), label=f'Distribution theta_d1 mean={mean:.2f} sd={sd:.2f}')
plt.axvline(x=mean, color='r', linestyle='--')
mean = 165
sd = 15
prob_x.append(norm.pdf(d1, mean, sd).sum())
sns.lineplot(x=x_test, y=norm.pdf(x_test, mean, sd), label=f'Distribution theta_1 mean={mean:.2f} sd={sd:.2f}')
mean = 166
sd = 10
prob_x.append(norm.pdf(d1, mean, sd).sum())
sns.lineplot(x=x_test, y=norm.pdf(x_test, mean, sd), label=f'Distribution theta_2 mean={mean:.2f} sd={sd:.2f}')
plt.xticks(np.arange(min(x_test),max(x_test),10))
sns.set(rc={'figure.figsize':(10,6)})
plt.xlabel('Value X')
plt.ylabel('P(X)')
plt.legend()
print("Normal Distribution D2 with theta_d1,1,2 (SUM)",prob_x)
plt.show()
```

B.2



$\mu_{MLE} = 165.11$

$sd = 11.75$

```
Normal Distribution D2 with theta_d2,1,2 (SUM) [0.20229455776586355, 0.18227528101129103, 0.21282926972637978]
```

```python
d2 = np.array([145,170,174,168,180,155,162,180,152]).astype(float)
x_test = np.arange(100,250,1)
prob_x = []

mean = np.mean(d2)
sd = np.std(d2)
prob_x.append(norm.pdf(d2, mean, sd).sum())
sns.lineplot(x=x_test, y=norm.pdf(x_test, mean, sd), label=f'Distribution theta_d2 mean={mean:.2f} sd={sd:.2f}')
plt.axvline(x=mean, color='r', linestyle='--')
mean = 165
sd = 15
prob_x.append(norm.pdf(d2, mean, sd).sum())
sns.lineplot(x=x_test, y=norm.pdf(x_test, mean, sd), label=f'Distribution theta_1 mean={mean:.2f} sd={sd:.2f}')
mean = 166
sd = 10
prob_x.append(norm.pdf(d2, mean, sd).sum())
sns.lineplot(x=x_test, y=norm.pdf(x_test, mean, sd), label=f'Distribution theta_2 mean={mean:.2f} sd={sd:.2f}')
plt.xticks(np.arange(min(x_test),max(x_test),10))
sns.set(rc={'figure.figsize':(10,6)})
plt.xlabel('Value X')
plt.ylabel('P(X)')
plt.legend()
print("Normal Distribution D2 with theta_d2,1,2 (SUM)",prob_x)
plt.show()
```

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from $\theta_{MLE} = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{n} f(x_i, \theta) \quad ; \quad \theta = (\mu, \sigma)$

$$= \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} \right)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{n} \left(\log_e \frac{1}{(2\pi\sigma^2)^{1/2}} - \frac{(x_i-\mu)^2}{2\sigma^2} \right)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{n} \left(-\frac{1}{2} \ln 2\pi\sigma^2 - \left(\frac{1}{2\sigma^2} \cdot (x_i-\mu)^2 \right) \right)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{n} \left(-\frac{1}{2} \left(\ln 2\pi + \ln \sigma^2 \right) - \left(\frac{1}{2\sigma^2} \cdot (x_i-\mu)^2 \right) \right)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^{n} \left(-\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma^2 - \left(\frac{1}{2\sigma^2} (x_i-\mu)^2 \right) \right)$$

$$= \underset{\theta}{\operatorname{argmax}} \left(-\frac{n}{2} \ln \pi - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i-\mu)^2 \right) = \underline{l(\mu, \sigma)}$$

log likelyhood

maximize $\mu_{MLE} = \frac{\partial l}{\partial \mu} = \frac{\partial}{\partial \mu}\left(-\frac{n}{2}\ln \pi - \frac{n}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right)$

$$= -\frac{\cancel{2}}{\cancel{2}\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = -\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu)$$

Set $\frac{\partial l}{\partial \mu} = 0 \longrightarrow$

$$-\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\sum_{i=1}^{n}x_i - \sum_{i=1}^{n}\mu = 0$$

$$\sum_{i=1}^{n}x_i = n\mu$$

$$\mu = \frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\therefore \mu_{MLE} = \frac{1}{n}\sum_{i=1}^{n}x_i$$

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maximize σ

$$\frac{\partial l}{\partial \sigma^2} = \frac{\partial}{\partial \sigma^2}\left(-\frac{n}{2}\ln \pi - \frac{n}{2}\ln \sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 \right)$$

$$= -\frac{n}{2}\cdot\frac{1}{\sigma^2} - \frac{1}{(2\sigma^2)^2}\left(\cancel{2\sigma^2(0)} - 2\right)\sum_{i=1}^{n}(x_i-\mu)^2$$

$$= -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2$$

Set $\frac{\partial l}{\partial \sigma^2} = 0$ $\longrightarrow$

$$-\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

$$\sum_{i=1}^{n}(x_i-\mu)^2 = \frac{n}{\cancel{2\sigma^2}}\,\cancel{(2\sigma^4)}$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\therefore\ \sigma_{MLE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}\ \#$$

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