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Quiz 4

The following table is a result from observing the behavior of a person whether he went out or stayed home given the two weather conditions (sunny or rainy) and the two options regarding his car status (car-broken or car-working)

- $y_i \in \{go - out, stayhome\}$
- $x_i^1 \in \{sunny, rainy\}$
- $x_i^2 \in \{car - broken, car - working\}$

i	x_i^1	x_i^2	y_i
1	sunny	car-broken	go-out
2	rainy	car-working	go-out
3	sunny	car-broken	go-out
4	sunny	car-broken	go-out
5	sunny	car-broken	go-out
6	sunny	car-working	stay home
7	rainy	car-working	stay home
8	rainy	car-broken	stay home
9	sunny	car-working	stay home
10	rainy	car-working	stay home

Assume that we are using Binomial distribution as the modeling distribution. You are to demonstrate solutions to the following questions.

1. Estimate P(y=go-out). $P(y=go\text{-}out) = \frac{1}{n}\sum_{i=1}^{n} I(y_i = y) = \frac{5}{10} = \frac{1}{2}$ #
2. Estimate P(y=stay home). $P(y = stay\ home) = \frac{5}{10} = \frac{1}{2}$ #
3. What is the estimate of P(y)? ก.น่าจะเป็นของเหตุการณ์ y
4. What is the estimate of P(x)? ก.น่าจะเป็นของเหตุการณ์ x
5. Estimate P(x = (rainy, car-working) and y=go-out). $P(x^{(i)} = j \mid Y = y) = \frac{\sum_{k=1}^{n} I(y_k = y)(x_k^{(i)} = j)}{\sum_{k=1}^{n} I(y_k = y)} = \frac{1}{5}$
6. Estimate P(y=go-out | x = (rainy, car-working)) directly. $\rightarrow \frac{1}{3}\sum_{k=1}^{n} \frac{I(x_k = x)\cdot(y_k = y)}{I(x_k = x)}$
7. Estimate P(x = (rainy, car-working) | y=go-out) using Naive Bayes assumption.
8. By using Naive Bayes assumption, what would be the return of h(x = (sunny, car-broken))?

6. $P(y = \text{go-out} \mid x = \text{(rainy, car working)})$

the bays rule $P(y \mid x) = \dfrac{P(x \mid y)\, P(y)}{P(x)}$

from previous $P(y) = \frac{1}{2}$ | $P(x) = \frac{1}{n} \sum_{i=1}^{n} (x_i = x) = \frac{3}{10}$

$P(x \mid y) = \frac{1}{5}$

$\therefore P(y \mid x) = \left(\frac{1}{5}\right)\left(\frac{1}{2}\right)\left(\frac{10}{3}\right) = \frac{1}{3}$ # Verify from data ✓

7. $P(x = \text{(raining, car working)} \mid y = \text{go-out})$ using Naive Bays Assumption !

by the Naive bays Assumption

$\rightarrow h(x) = \underset{y}{\operatorname{argmax}}\; P(y \mid x)$

$= \underset{y}{\operatorname{argmax}}\; \dfrac{P(x \mid y)\, P(y)}{P(x)}$ (original bays rule)

$= \underset{y}{\operatorname{argmax}}\; P(x \mid y)\, P(y)$ (Naive bays assumption)

$P(A \cap B)$ all feature're independent

So $= P(A)\, P(B)$

$\therefore P(x \mid y) = \prod_{\alpha=1}^{d} P(x_\alpha \mid y)$

$P(x = \text{(rainy, car working)} \mid y = \text{go-out})$

$= P(x = \text{rainy} \mid y = \text{go-out})\, P(x = \text{car working} \mid y = \text{go-out})$

$= \sum_{k=1}^{n} \dfrac{I(y_k = y)(x_k = x_i)}{I(y_k = y)} \cdot \sum_{k=1}^{n} \dfrac{I(y_k = y)(x_k = x)}{I(y_k = y)}$

$= \left(\frac{1}{5}\right)\left(\frac{1}{5}\right) = \frac{1}{25} = 0.04$ #

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8. $h(x = (\overset{x_1}{\text{sunny}}, \overset{x_2}{\text{car broken}})$ by Naiive bays assumption

$$h(x) = \arg\max_y P(y \mid x)$$

$$= \arg\max_y P(x \mid y)\, P(y) \quad \text{(naiive bays assump.)}$$

$$= \arg\max_y \underbrace{\prod_{\alpha=1}^{d} P(x_\alpha \mid y)\, P(y)}_{} \rightarrow [\theta_{j,y}]^\alpha$$

let $y =$ go-out

$$[\theta_{j,y}]^\alpha = P(x_1 \mid y)\, P(x_2 \mid y)\, P(y)$$

$$= \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{2}\right) = 0.32$$

$y =$ stay home

$$= \left(\frac{2}{5}\right)\left(\frac{1}{5}\right)\left(\frac{1}{2}\right) = 0.04$$

$h(x)$ will return $\arg\max_y P(y \mid x)$

Ans $y =$ go-out #

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