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CS 161 Homework 5

1.

a. $P \Rightarrow \neg Q, Q \Rightarrow \neg P$

P	Q	**$P \Rightarrow \neg Q$**	**$Q \Rightarrow \neg P$**
T	T	**F**	**F**
T	F	**T**	**T**
F	F	**T**	**T**
F	T	**T**	**T**

b. $P \Leftrightarrow \neg Q, ((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	Q	**$P \Leftrightarrow \neg Q$**	**$((P \wedge \neg Q) \vee (\neg P \wedge Q))$**
T	T	**F**	**F**
T	F	**T**	**T**
F	F	**F**	**F**
F	T	**T**	**T**

2.

a. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

Smoke	Fire	$Smoke \Rightarrow Fire$	$\neg Smoke \Rightarrow \neg Fire$	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
T	T	T	T	**T**
T	F	F	T	**T**
F	F	T	T	**T**
F	T	T	F	**F**

Based on the last column of the truth table, this sentence is neither valid nor unsatisfiable. Not all cases are true so $(Smoke \Rightarrow Fire)$ does not imply $(\neg Smoke \Rightarrow \neg Fire)$ but since some are, then it is satisfiable in some worlds.

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b. (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

Smoke	Fire	Heat	Smoke ⇒ Fire	(Smoke ∨ Heat) ⇒ Fire	(Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)
T	T	T	T	T	**T**
T	F	T	F	F	**T**
F	F	T	T	F	**F**
F	T	T	T	T	**T**
F	F	F	T	T	**T**
F	T	F	T	T	**T**
T	T	F	T	T	**T**
T	F	F	F	F	**T**

Based on the above table, this sentence is also neither valid nor unsatisfiable. It is unsatisfiable in only 1 out of 8 possible worlds.

c. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Smoke	Fire	Heat	(Smoke ∧ Heat) ⇒ Fire	(Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)	((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
T	T	T	T	T	**T**
T	F	T	F	F	**T**
F	F	T	T	T	**T**
F	T	T	T	T	**T**
F	F	F	T	T	**T**
F	T	F	T	T	**T**
T	T	F	T	T	**T**
T	F	F	T	T	**T**

Based on the truth table above, this sentence is valid and satisfiable.

3. *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.*
   If the unicorn is either immortal or a mammal, then it is horned.
   The unicorn is magical if it is horned.
   a. Mythical ⇒ ¬Mortal
      ¬Mythical ⇒ (Mortal ∧ Mammal)
      (¬Mortal ∨ Mammal) ⇒ Horned
      Horned ⇒ Magical
   b. (¬Mythical ∨ ¬Mortal) ∧ (Mythical ∨ Mortal) ∧ (Mythical ∨ Mammal) ∧ (Mortal ∨ Horned) ∧ (¬Mammal ∨ Horned) ∧ (¬Horned ∨ Magical)
   c. Unicorn is mythical?

1	¬Mythical ∨ ¬Mortal	
2	Mythical ∨ Mortal	
3	Mythical ∨ Mammal	
4	Mortal ∨ Horned	
5	¬Mammal ∨ Horned	
6	¬Horned ∨ Magical	
7	**Assume** ¬Mythical	
8	Mortal	2, 7
9	Mammal	3, 7
10	Horned	5, 9
11	Magical	6, 10

We cannot move further from this and thus cannot prove that the unicorn is mythical.

Magical?

1	¬Mythical ∨ ¬Mortal	
2	Mythical ∨ Mortal	
3	Mythical ∨ Mammal	
4	Mortal ∨ Horned	
5	¬Mammal ∨ Horned	
6	¬Horned ∨ Magical	
7	**Assume** ¬Magical	
8	¬Horned	6, 7
9	Mortal	8, 4
10	¬Mythical	9, 1
11	Mammal	3, 10
12	Horned	5, 11

Since there is a contradiction created from clauses 12 and 8, our assumption is wrong and thus it is proven that the unicorn is magical.

Horned?

1	¬Mythical ∨ ¬Mortal	
2	Mythical ∨ Mortal	
3	Mythical ∨ Mammal	
4	Mortal ∨ Horned	
5	¬Mammal ∨ Horned	
6	¬Horned ∨ Magical	
7	**Assume** ¬Horned	
8	Mortal	4, 7
9	¬Mythical	8, 1
10	Mammal	9, 3
11	¬Mammal	7, 5

Again, there is a contradiction here, between clauses 11 and 10 and so we can prove that the unicorn is horned.