Chapter #7: Sequential Logic Case Studies

Contemporary Logic Design

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Motivation

- Flipflops: most primitive "packaged" sequential circuits
- More complex sequential building blocks:

Storage registers, Shift registers, Counters Available as components in the TTL Catalog

- How to represent and design simple sequential circuits: counters
- Problems and pitfalls when working with counters:

Start-up States
Asynchronous vs. Synchronous logic

Chapter Overview

Examine Real Sequential Logic Circuits Available as Components

- Registers for storage and shifting
- Random Access Memories
- Counters

Counter Design Procedure

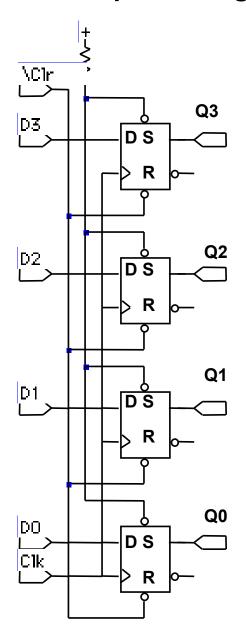
- Simple but useful finite state machine
- State Diagram, State Transition Table, Next State Functions
- Excitation Tables for implementation with alternative flipflop types

Synchronous vs. Asynchronous Counters

- Ripple vs. Synchronous Counters
- Asynchronous vs. Synchronous Clears and Loads

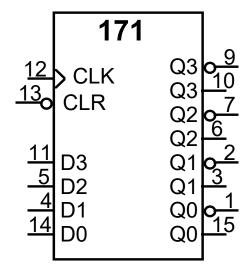
Storage Register

Group of storage elements read/written as a unit



4-bit register constructed from 4 D FFs Shared clock and clear lines

Schematic Shape



TTL 74171 Quad D-type FF with Clear (Small numbers represent pin #s on package)

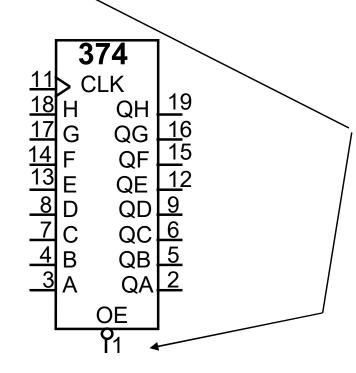
Input/Output Variations

Selective Load Capability
Tri-state or Open Collector Outputs
True and Complementary Outputs

377 CLK EN D7 D6 D5 D4 D3 D2 D1 D0	Q7 Q6 Q5 Q4 Q3 Q2 Q1 Q0	19 16 15 19 6 5 2
D0	Q0	2
	CLK EN D7 D6 D5 D4 D3 D2 D1	CLK EN D7 Q7 D6 Q6 D5 Q5 D4 Q4 D3 Q3 D2 Q2 D1 Q1

74377 Octal D-type FFs with input enable

EN enabled low and lo-to-hi clock transition to load new data into register

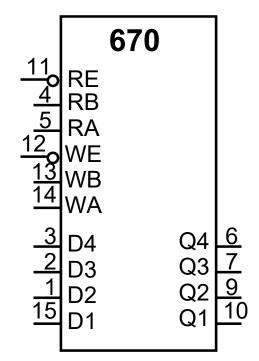


74374 Octal D-type FFs with output enable

OE asserted low presents FF state to output pins; otherwise high impedence

Register Files

Two dimensional array of flipflops
Address used as index to a particular word
Word contents read or written

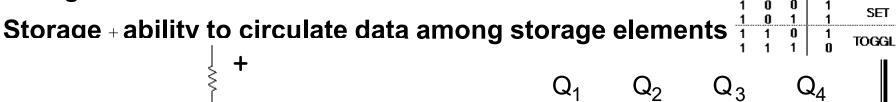


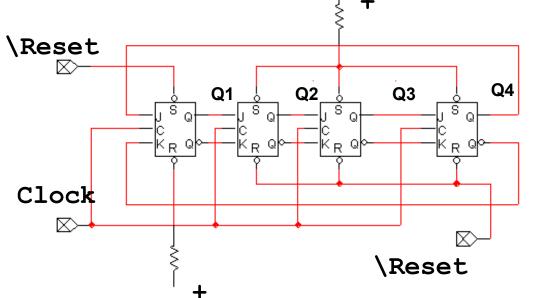
74670 4x4 Register File with Tri-state Outputs

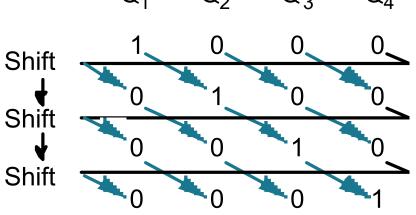
Separate Read and Write Enables Separate Read and Write Address Data Input, Q Outputs

Contains 16 D-ffs, organized as four rows (words) of four elements (bits)

Shift Registers







Contemporary J(t) K(t) Q(t) | Q(t+A)

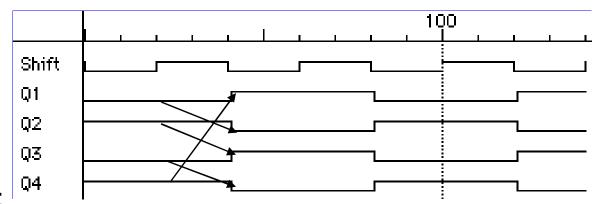
HOLD

RESET

Seguential Ca

Shift from left storage element to right neighbor on every lo-to-hi transition on shift signal

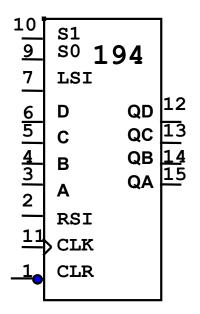
Wrap around from rightmost element to leftmost element



Master Slave FFs: sample inputs while clock is high; change outputs on falling edge

Shift Register I/O

Serial vs. Parallel Inputs Serial vs. Parallel Outputs Shift Direction: Left vs. Right



741944-bit Universal Shift Register

Serial Inputs: LSI, RSI
Parallel Inputs: D, C, B, A
Parallel Outputs: QD, QC, QB, QA
Clear Signal
Positive Edge Triggered Devices

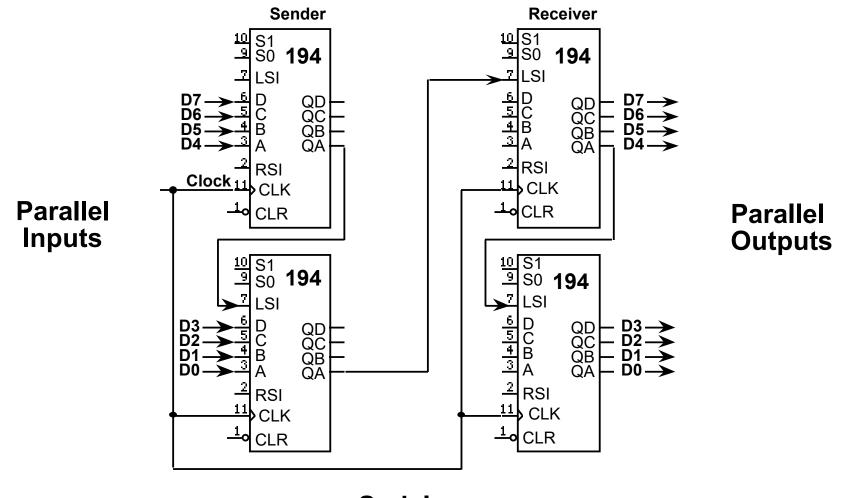
S1,S0 determine the shift function
S1=1, S0=1: Load on rising clk edge
synchronous load
S1=1, S0=0: shift left on rising clk edge
LSI replaces element D
S1=0, S0=1: shift right on rising clk edge
RSI replaces element A
S1=0, S0=0: hold state

Multiplexing logic on input to each FF:

Shifters well suited for serial-to-parallel conversions, such as terminal to computer communications

1. R.H. Katz Transparency No. 7-8

Shift Register Application: Parallel to Serial Conversion



Serial transmission

Counters

Proceed through a well-defined sequence of states in response to count signal

3 Bit Up-counter: 000, 001, 010, 011, 100, 101, 110, 111, 000, ...

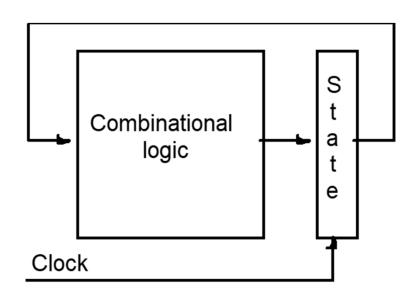
3 Bit Down-counter: 111, 110, 101, 100, 011, 010, 001, 000, 111, ...

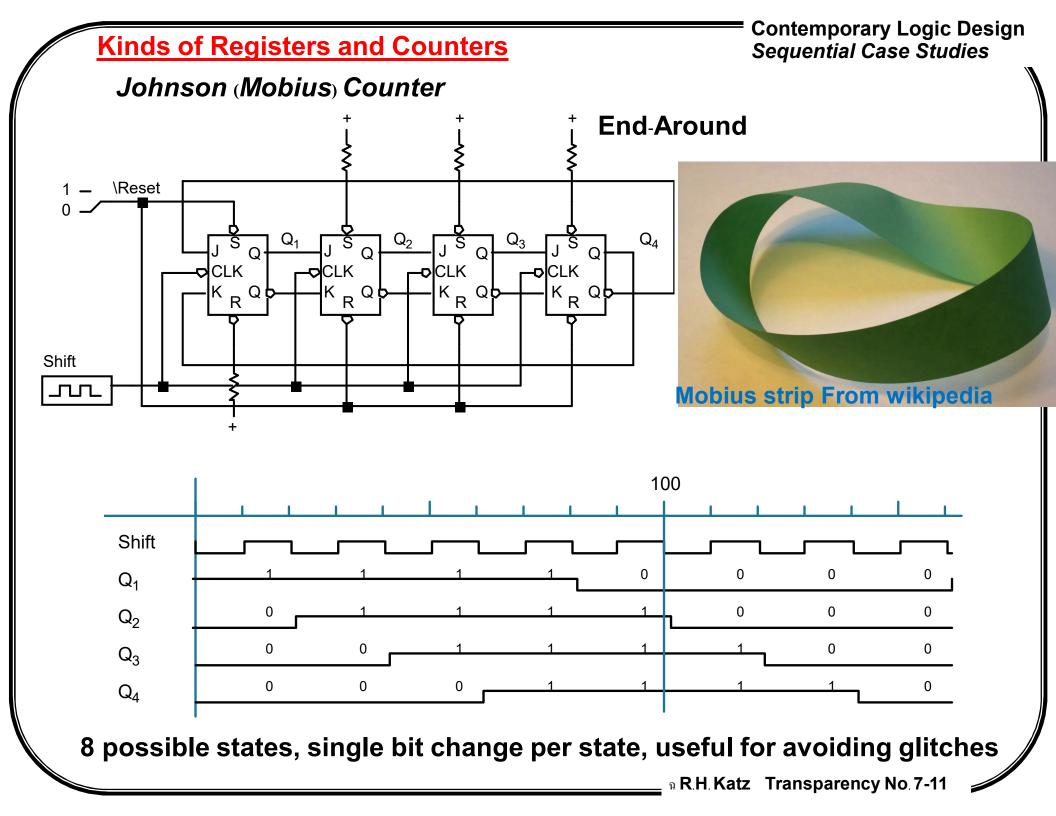
Binary vs. BCD vs. Gray Code Counters

A counter is a "degenerate" finite state machine/sequential circuit where the state *is* the only output

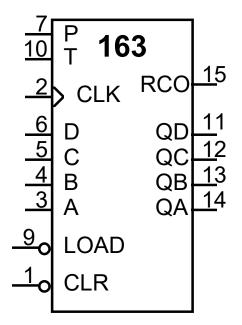
Generic Block Diagram for Clocked Sequential System

state implemented by latches or edge-triggered FFs





Catalog Counter



74163 Synchronous 4-Bit Upcounter **Synchronous Load and Clear Inputs**

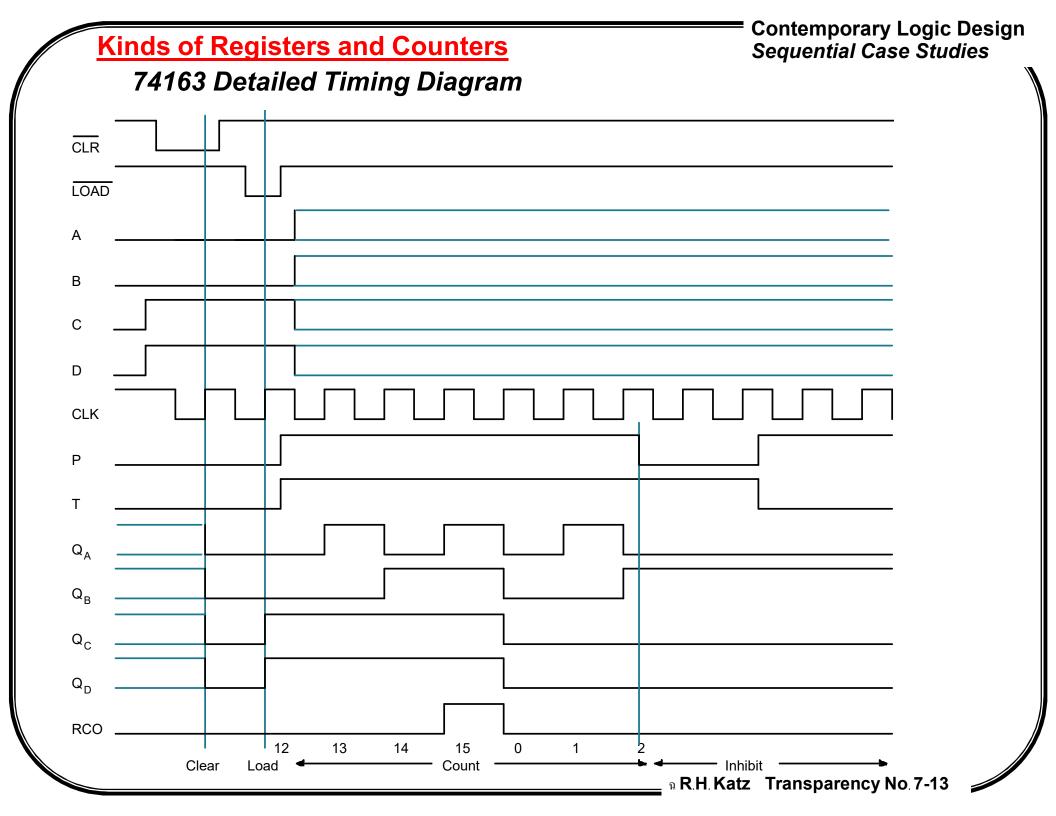
Positive Edge Triggered FFs

Parallel Load Data from D, C, B, A

P, T Enable Inputs: both must be asserted to enable counting

RCO: asserted when counter enters its highest state 1111, used for cascading counters "Ripple Carry Output"

74161: similar in function, asynchronous load and reset



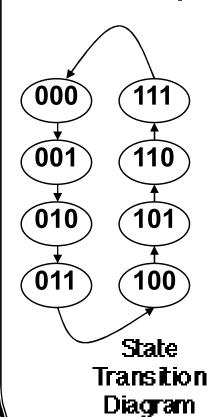
Introduction

This procedure can be generalized to implement ANY finite state machine

Counters are a very simple way to start: no decisions on what state to advance to next current state is the output

Q	Q ⁺	R	S	J	K	
0	0	X	0	0	Χ	0
0	1	0	1	1	X	1
1	0	1	0	X	1	1
1	1	0	X	X	0	0
		0 0	0 0 X 0 1 0 1 0 1	0 0 X 0 0 1 0 1 1 0 1 0	0 0 X 0 0 0 1 0 1 1 1 0 1 0 X	0 0 X 0 0 X 0 1 0 1 1 X 1 0 1 0 X 1

Example: 3-bit Binary Upcounter



Present State		Next State			Flip Flops Inputs			
C	В	A	C +	B+	A +	TC	TB	TA
0	0	0	0	0	1			
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

State Transition Flipflop Table Input Table

Decide to implement with Toggle Flipflops

What inputs must be presented to the T FFs to get them to change to the desired state bit?

This is called Remapping the Next State Function

Introduction

This procedure can be generalized to implement ANY finite state machine

O O+ | R S | J K |

Counters are a very simple way to start:

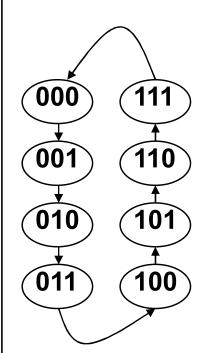
no decisions on what state to advance to next

current state is the output

1

Example: 3-bit Binary Upcounter

	×	Q	1 1	<u> </u>		1 \	•
ı	0	0	Х	0	0	Χ	0
χĺ	t 0	1	0	1	1	X	1
	1	0	1	0	Χ	1	1
	1	1	0	X	Χ	0	0
	1	1			Χ	0	0



	Present State		Next State				ip Flo Inputs	
C	В	A	C +	B+	A+	TC	TB	TA
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

State Transition Flipflop Table Input Table

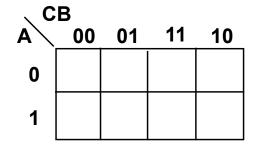
Decide to implement with Toggle Flipflops

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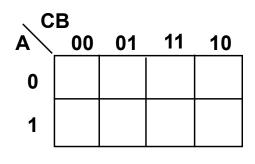
This is called Remapping the Next State Function

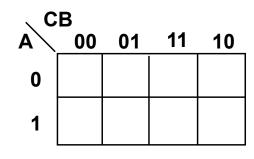
Example Continued

K-maps for Toggle Inputs:



$$TA =$$



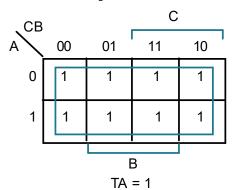


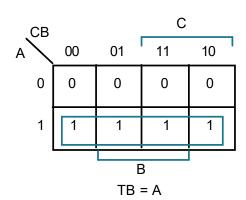
Resulting Logic Circuit

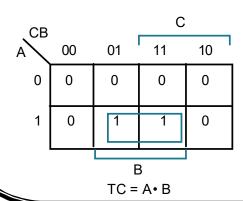
	Present		Next			Flip Flops		
	Stat	e		State		1	Inputs	
C	В	A	C+	B+	A +	TC	TB	TA
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

Example Continued

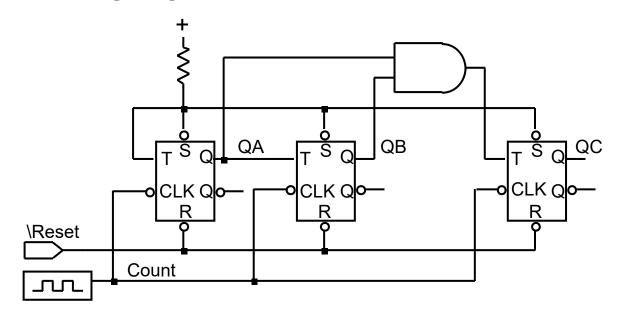
K-maps for Toggle Inputs:



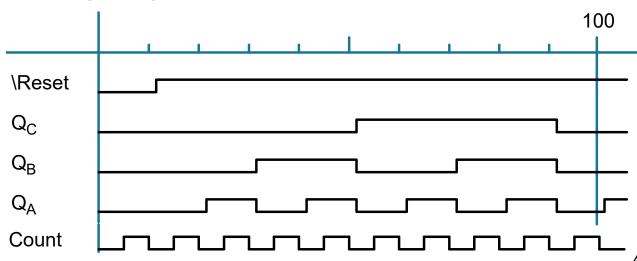


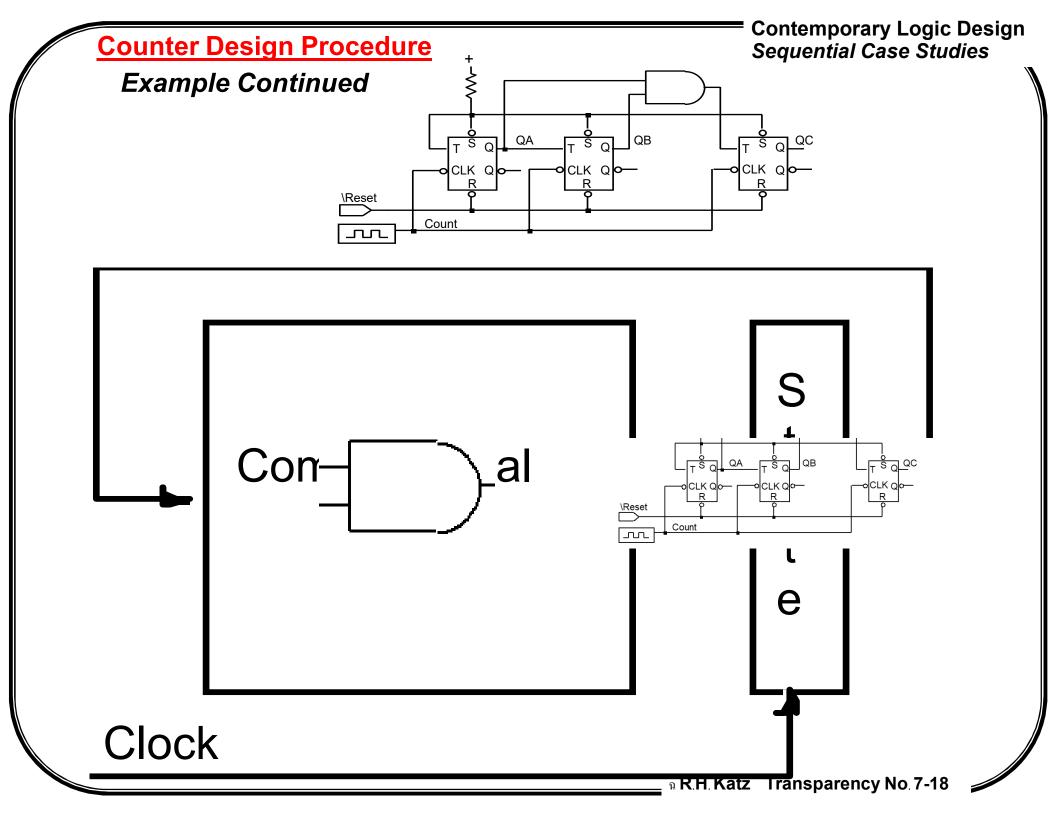


Resulting Logic Circuit:



Timing Diagram:

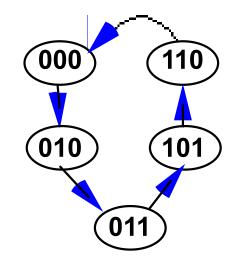




More Complex Count Sequence

Step 1: Derive the State Transition Diagram Count sequence: 000, 010, 011, 101, 110

Р	Present			Present Next			t
5	Stat	es		State			
С	В	Α	C+	B ⁺	A ⁺		
0	0	0	0	1	0		
0	0	1	×	×	×		
0	1	0	0	1	1		
0	1	1	1	0	1		
1	0	0	×	×	×		
1	0	1	1	1	0		
1	1	0	0	0	0		
1	1	1	*	×	×		

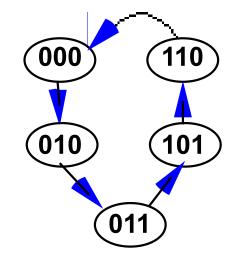


Step 2: State Transition Table

More Complex Count Sequence

Step 1: Derive the State Transition Diagram Count sequence: 000, 010, 011, 101, 110

Present States						
С	В	Α	C+	B ⁺	A ⁺	
0	0	0	0	1	0	
0	0	1	×	*	×	
0	1	0	0	1	1	
0	1	1	1	0	1	
1	0	0	×	*	*	
1	0	1	1	1	0	
1	1	0	0	0	0	
1	1	1	×	*	*	



Step 2: State Transition Table

Note the Don't Care conditions

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More Complex Count Sequence

Step 3: K-Maps for Next State Functions

、 C	В			
A	00	01	11	10
0				
1				

C + =

、 C	В			
A	00	01	11	10
0				
1				
'			•	

B + =

_							
	Present States			Next State			
С	В	Α	C+	B ⁺	A ⁺		
0	0	0	0	1	0		
0	0	1	×	×	×		
0	1	0	0	1	1		
0	1	1	1	0	1		
1	0	0	×	*	×		
1	0	1	1	1	0		
1	1	0	0	0	0		
1	1	1	×	×	×		

A C	B 00	01	11	10
0		01	••	-10
U				
1				
		$oldsymbol{A}_{+}$	- =	

More Complex Count Sequence

Step 3: K-Maps for Next State Functions

、 CB								
A	00	01	11	10				
0	0	0	0	×				
1	*	1	×	1				

、 C	В			
A	00	01	11	10
0	1	1	0	×
1	×	0	*	1
,				

し ⁺	=

\mathbf{H}_{\perp}	_
	_

	Present States		Next State		
С	В	Α	C+	B ⁺	A ⁺
0	0	0	0	1	0
0	0	1	×	×	×
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	×	×	×
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	×	*	×

、C	В			
A	00	01	11	10
0	0	1	0	*
1	×	1	*	0
'		Δ_+	=	

	Present State	Next State	Flip Flop input
-	0	0	0
	0	1	1
	1	0	1
	1	1	0

More Complex Counter Sequencing

Step 4: Choose Flipflop Type for Implementation Use Excitation Table to Remap Next State Functions

Q	Q+	T
0	0	0
0	1	1
1	0	1
1	1	0

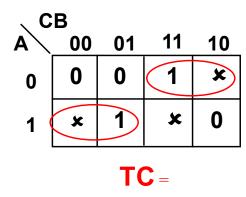
Toggle Excitation
Table

_					
P	res	ent	Next		
	Sta	tes		State)
С	В	Α	C+	B ⁺	A ⁺
0	0	0	0	1	0
0	0	1	×	×	×
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	×	×	×
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	×	×	×

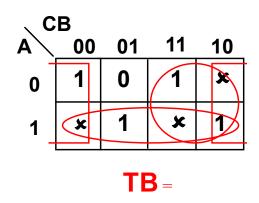
-	Toggle)
	Inputs	}
TC	ТВ	TA
0	1	0
*	×	*
0	0	1
1	1	0
×	×	×
0	1	1
1	1	0
×	×	×

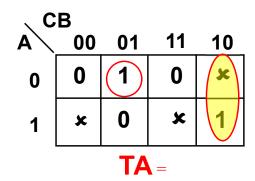
Remapped Next State Functions

More Complex Count Sequence



	res Sta	ent tes		Next State			Toggle Inputs	
С	В	Α	C+	B ⁺	A ⁺	TC	ТВ	TA
0	0	0	0	1	0	0	1	0
0	0	1	×	×	×	×	×	×
0	1	0	0	1	1	0	0	1
0	1	1	1	0	1	1	1	0
1	0	0	×	×	×	×	×	×
1	0	1	1	1	0	0	1	1
1	1	0	0	0	0	1	1	0
1	1	1	×	×	×	×	*	×





$$TC = \overline{A} C + \overline{A} \overline{C} = A xor C$$

$$TB = A + \overline{B} + C$$

$$TA = \overline{A} B \overline{C} + \overline{B} C$$

More Complex Counter Sequencing

Resulting Logic:

5 Gates

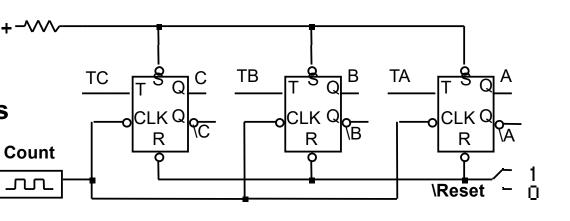
13 Input Literals + Flipflop connections

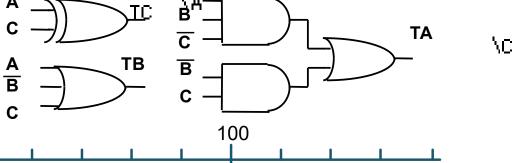
$$TC = \overline{A} C + A \overline{C} = A xor C$$

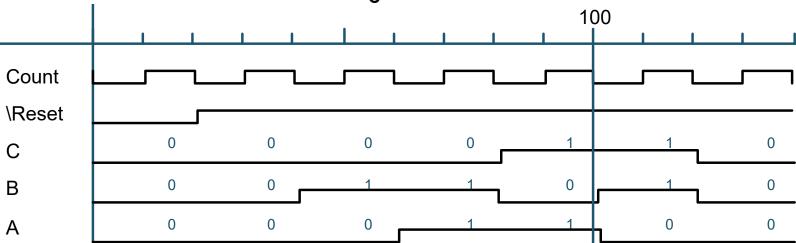
$$TB = A + B + C$$

$$TA = \overline{A} B \overline{C} + \overline{B} C$$

Timing Waveform:







Self-Starting Counters

Start-Up States

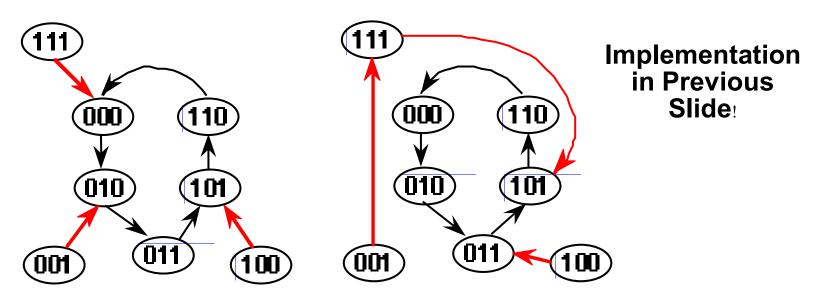
At power-up, counter may be in possible state

Designer must guarantee that it (eventually) enters a valid state

Especially a problem for counters that validly use a subset of states

Self-Starting Solution:

Design counter so that even the invalid states eventually transition to valid state



Two Self-Starting State Transition Diagrams for the Example Counter

Self-Starting Counters

Deriving State Transition Table from Don't Care Assignment

State Changes Inputs to Toggle Flip-flops CB 00 C 01 11 <u>10</u> ' <u>0</u>0 11 10 01 0 0 0 TC C+ **State Transition Table** 0 0 Present Next **State** State B В C+ B+ A+ В CB₀₀ CB 0 01 11 <u>10</u> ' 01 11 00 10 0 0 0 0 B+ TB 0 0 0 В В 0 C 0 1 CB Α· 11 00 01 10 00 01 11 10 0 0 TA **A**+ 0 В В

Implementation with Different Kinds of FFs

R-S Flipflops

Continuing with the 000, 010, 011, 101, 110, 000, ... counter example

Q	Q+	R	S			
0	0	Х	0			
0	1	0	1			
1	0	1	0			
1	1	0	X			
Q+=S+RQ						

RS Exitation Table

Present State		Next State			Remapped Next State			
C	В	Α	C+	B+	A+	RC SC	RB SB	RA SA
0	0	0	0	1	0		_	_
0	0	1	X	Х	Х			
0	1	0	0	1	1			
0	1	1	1	0	1			
1	0	0	X	Х	X			
1	0	1	1	1	0			
1	1	0	0	0	0			
1	1	1	X	Х	X			

Remapped Next State Functions

Implementation with Different Kinds of FFs

R-S Flipflops

Continuing with the 000, 010, 011, 101, 110, 000, ... counter example

Q	Q+	R	<u>S</u>				
0	0	X	0				
0	1	0	1				
1	0	1	0				
1	1 1 0 X						
Q+=S+RQ							

RS Exitation Table

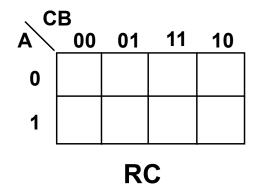
Present State			Next State			Remapped Next State					
C	В	A	C+	В+	A+	RC	SC	RB	SB	RA	SA
0	0	0	0	1	0	X	0	0	1	X	0
0	0	1	X	X	Х	Х	X	X	X	X	X
0	1	0	0	1	1	х	0	0	X	0	1
0	1	1	1	0	1	0	1	1	0	0	X
1	0	0	Х	Х	Х	х	X	X	X	X	X
1	0	1	1	1	0	0	X	0	1	1	0
1	1	0	Ó	0	Ō	1	0	1	0	X	0
1	4	1	X	X	X	X	X	Ιx	Х	Ιχ	X

Remapped Next State Functions

Implementation with Different Kinds of FFs

Contemporary Logic Design Sequential Case Studies

RS FFs Continued



SC

RC=

RB=

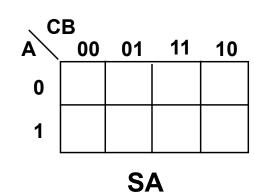
SC=

RB

SB

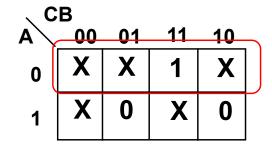
RA =

SB =



SA =

RS FFs Continued



、 C	В							
A	00	01	11	10				
0	0	0	0	X				
1	Х	1	X	X				
(SC							

RC	Δ

SC=A

AC	B 00	01	11	10
0	X	0	X	X
1	X	0	X	1

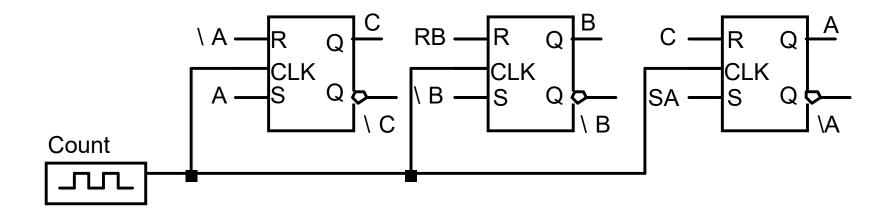
RB

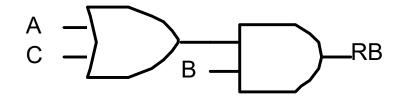
SB

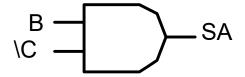
RA SA

Implementation With Different Kinds of FFs

RS FFs Continued







Resulting Logic Level Implementation: 3 Gates, 11 Input Literals + Flipflop connections

Implementation with Different FF Types

J-K FFs

		 _					
<u>Q</u> _	<u>Q+</u>	J	<u> K </u>				
0	0	0	X				
0	1	1	X				
1	0	X	1				
1	1	X	0				
$Q+=JQ^{-}KQ^{-}$							

J-K Excitation Table

Present State	Next State	Remapped Next State			
CBA	C+ B+ A+	JC KC JB KB JA KA			
0 0 0	0 1 0				
0 0 1	$X \times X$				
0 1 0	0 1 1				
0 1 1	1 0 1				
1 0 0	$x \times x$				
1 0 1	1 1 0				
1 1 0	0 0 0				
1 1 1	$X \times X$				

Remapped Next State Functions

Implementation with Different FF Types

J-K FFs

1		_	l <u>-</u>				
	$\underline{\mathbf{Q}}$	Q+	J	<u>_K</u> _			
	0	0	0	X			
	0	1	1	Х			
	1	0	X	1			
	1	1	X	0			
Q+=JQ+KQ							
J	J-K Excitation Table						

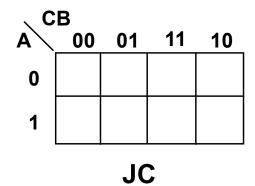
	Present State			Next State			Remapped Next State					
	C	В	A	C4	B+	A+	JC	KC	JB	KΒ	JA	KA
Ī	0	0	0	0	1	0	0	Х	1	Х	0	Х
	0	0	1	X	Х	X	X	X	X	Х	Х	X
	0	1	0	0	1	1	0	X	Х	0	1	X
	0	1	1	1	0	1	1	X	X	1	Х	0
	1	0	0	X	Х	X	X	X	X	X	Х	X
	1	0	1	1	1	0	X	0	1	X	Х	1
	1	1	0	0	0	0	Х	1	X	1	0	X
	1	1	1	X	X	X	X	X	X	X	X	X

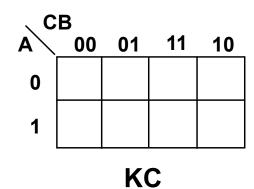
Remapped Next State Functions

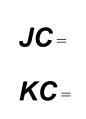
Contemporary Logic Design Sequential Case Studies

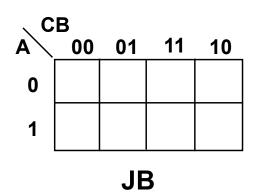
Implementation with Different FF Types

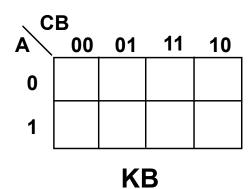
J-K FFs Continued



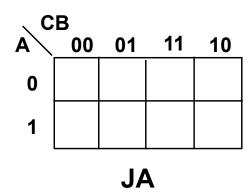








JB	=
KR	_

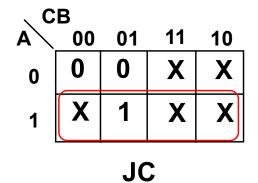


JA =

KA =

Implementation with Different FF Types

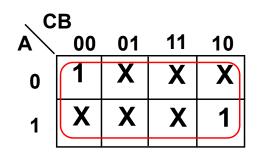
J-K FFs Continued



、C	В			
A	00	01	11	10
0	X	X	1	X
1	X	X	X	0
•	-		-	

KC





JD = I

$$KB = A + C$$

JB

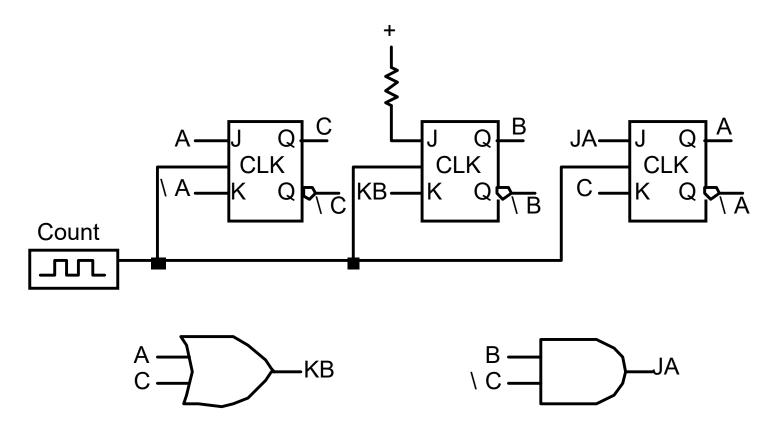
、 C	В			
A	00	01	11	10
0	X	X	X	X
1	X	0	X	1
			<u> </u>	

$$KA = C$$

JA KA

Implementation with Different FF Types

J-K FFs Continued

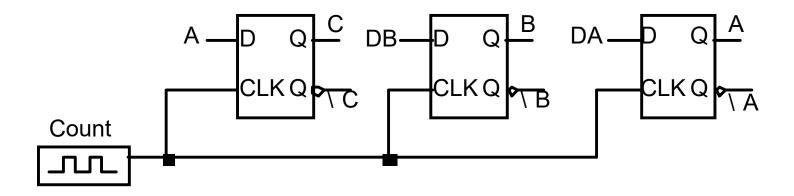


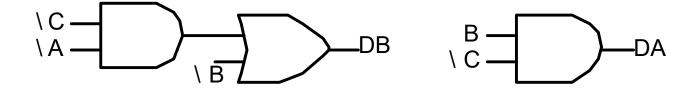
Resulting Logic Level Implementation: 2 Gates, 10 Input Literals + Flipflop Connections

Implementation with Different FF Types

D FFs

Simplest Design Procedure: No remapping needed:





Resulting Logic Level Implementation: 3 Gates, 8 Input Literals + Flipflop connections

Implementation with Different FF Types

Comparison

- T FFs well suited for straightforward binary counters

 But yielded worst gate and literal count for this example:
- No reason to choose R-S over J-K FFs: it is a proper subset of J-K

R-S FFs don't really exist anyway

J-K FFs yielded lowest gate count

Tend to yield best choice for packaged logic where gate count is key

D FFs yield simplest design procedure

Best literal count

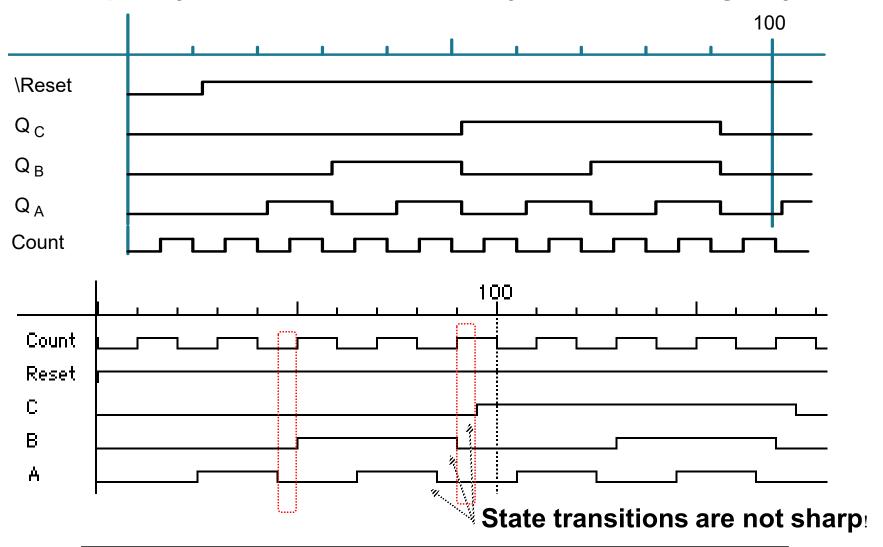
D storage devices very transistor efficient in VLSI

Best choice where area/literal count is the key

Asynchronous vs. Synchronous Counters



Deceptively attractive alternative to synchronous design style

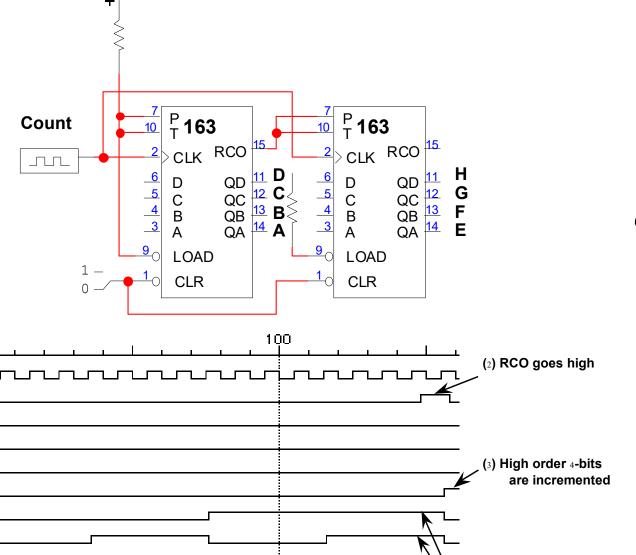


Can lead to "spiked outputs" from combinational logic decoding the counter's state



Contemporary Logic Design Sequential Case Studies

Cascaded Synchronous Counters with Ripple Carry Outputs



Count RCO

\Reset

Н

First stage RCO enables second stage for counting

RCO asserted soon after stage enters state 1111

also a function of the T Enable

Downstream stages lag in their 1111 to 0000 transitions

Affects Count period and decoding logic

n R.H. Katz Transparency No. 7-41

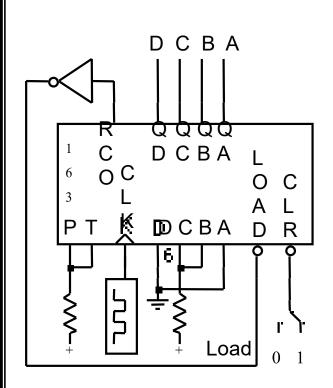
(1) Low order 4-bits = 1111

<u>Asynchronous vs. Synchronous Counters</u>

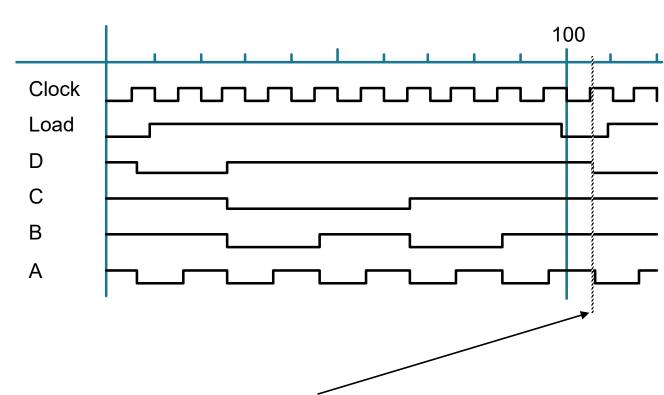
The Power of Synchronous Clear and Load

Starting Offset Counters:

e.g., 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1111, 0110, ...

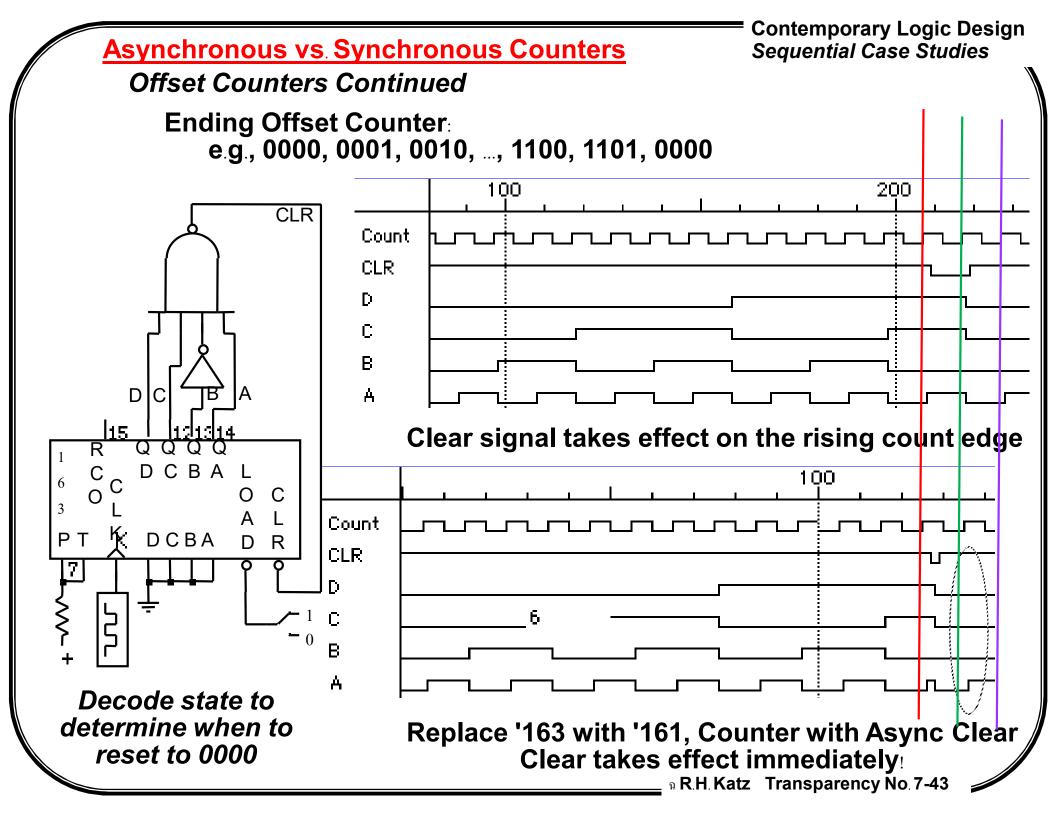


0110
is the state
to be loaded



Use RCO signal to trigger Load of a new state

Since 74163 Load is synchronous, state changes only on the next rising clock edge



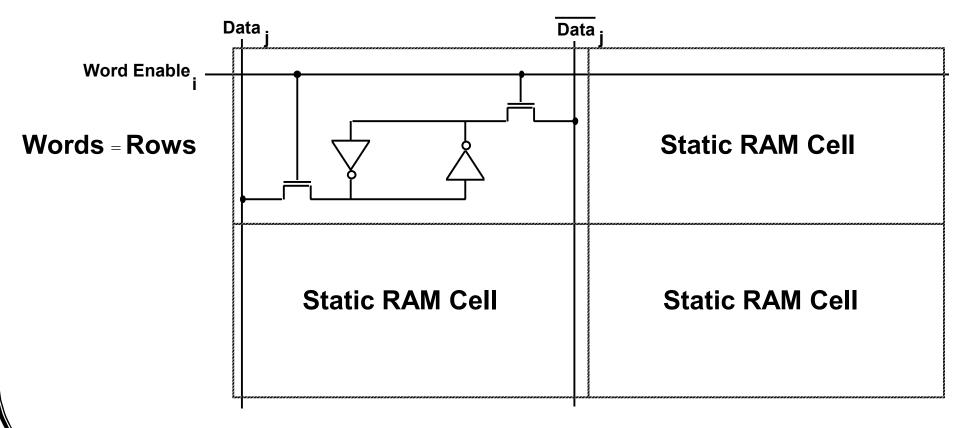
Static RAM

Transistor efficient methods for implementing storage elements

Small RAM: 256 words by 4-bit

Large RAM: 4 million words by 1-bit

We will discuss a 1024 x 4 organization



Columns = **Bits** (**Double Rail Encoded**)

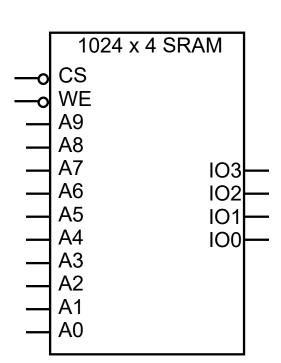
Static RAM Organization

Chip Select Line (active lo)

Write Enable Line (active lo)

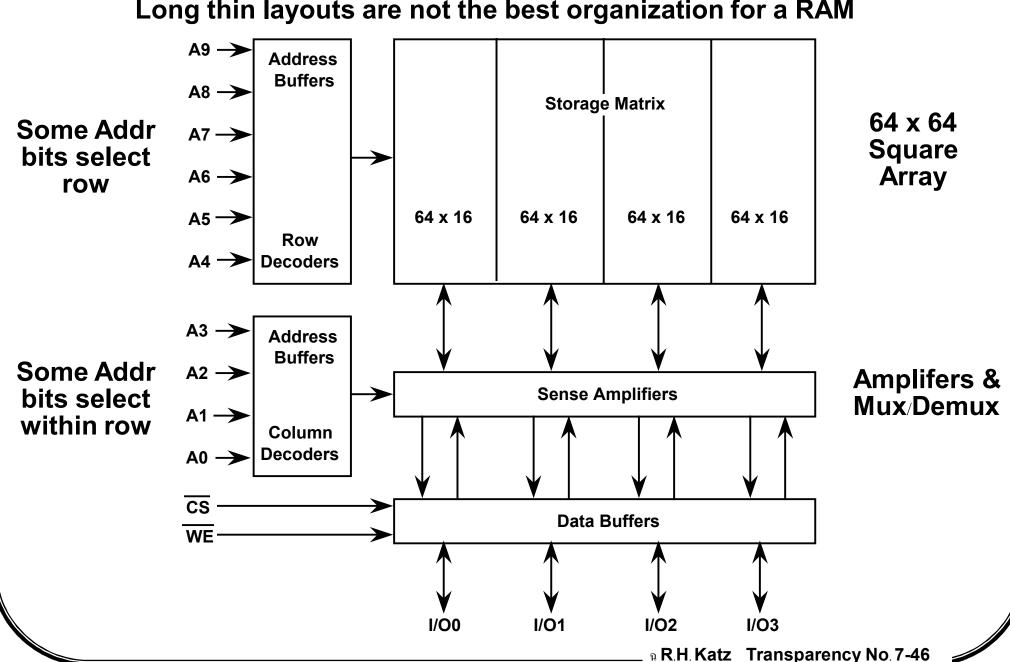
10 Address Lines

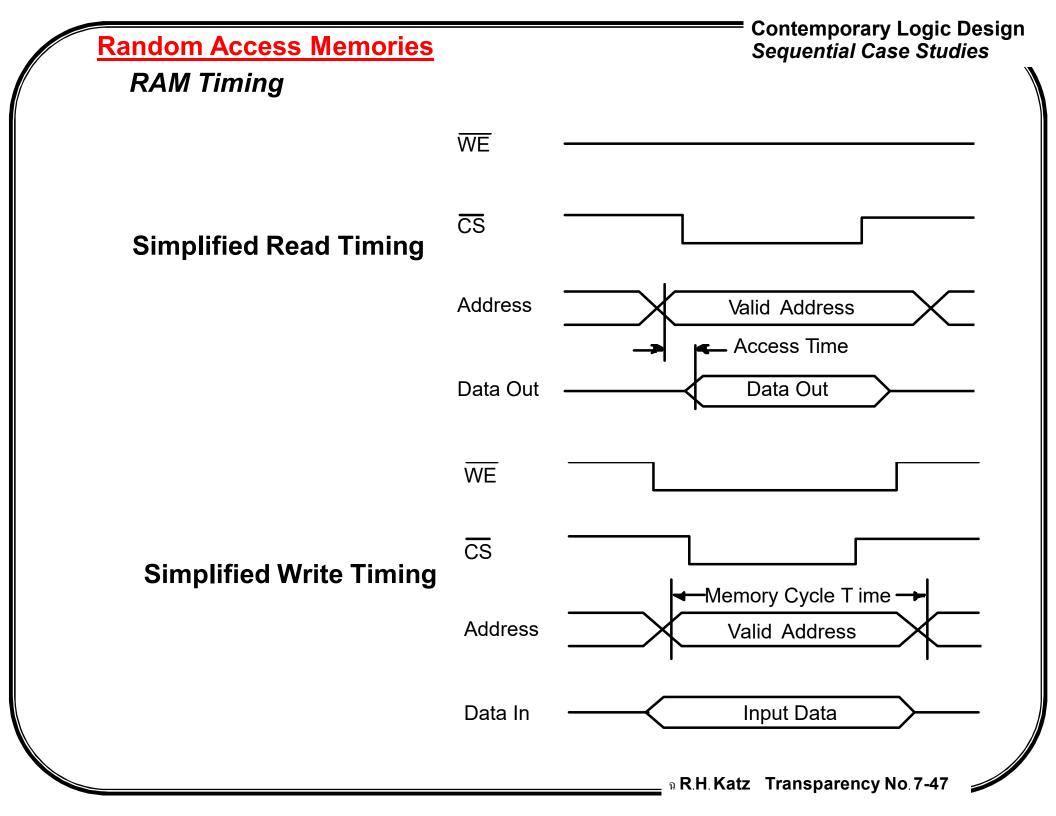
4 Bidirectional Data Lines



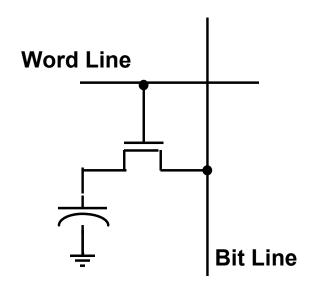
RAM Organization

Long thin layouts are not the best organization for a RAM





Dynamic RAMs



1 Transistor (+ capacitor) memory element

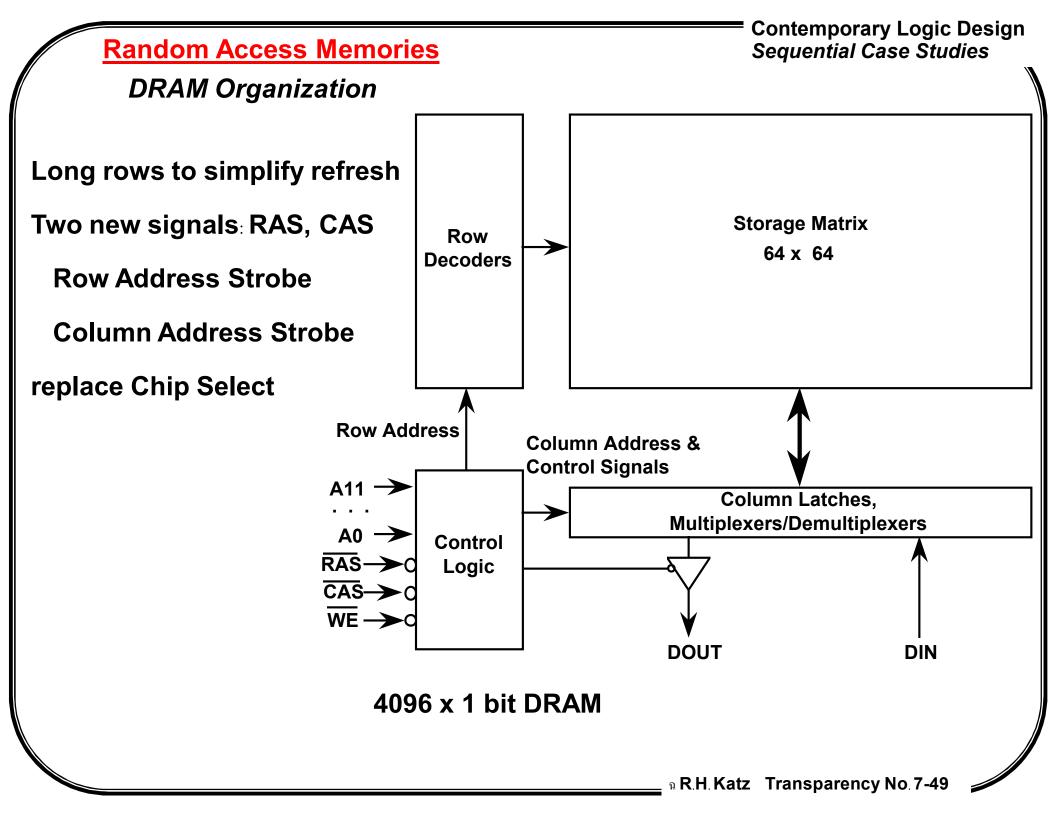
Read: Assert Word Line, Sense Bit Line

Write: Drive Bit Line, Assert Word Line

Destructive Read-Out

Need for Refresh Cycles: storage decay in ms

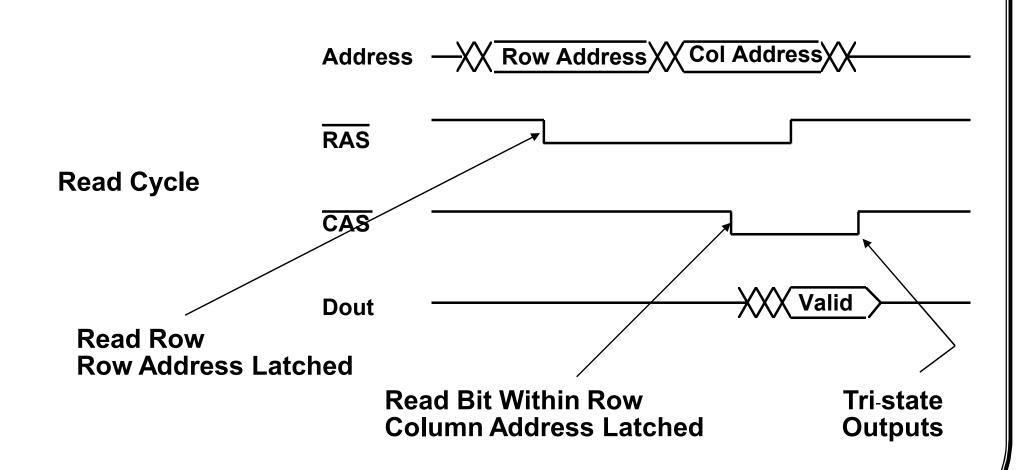
Internal circuits read word and write back



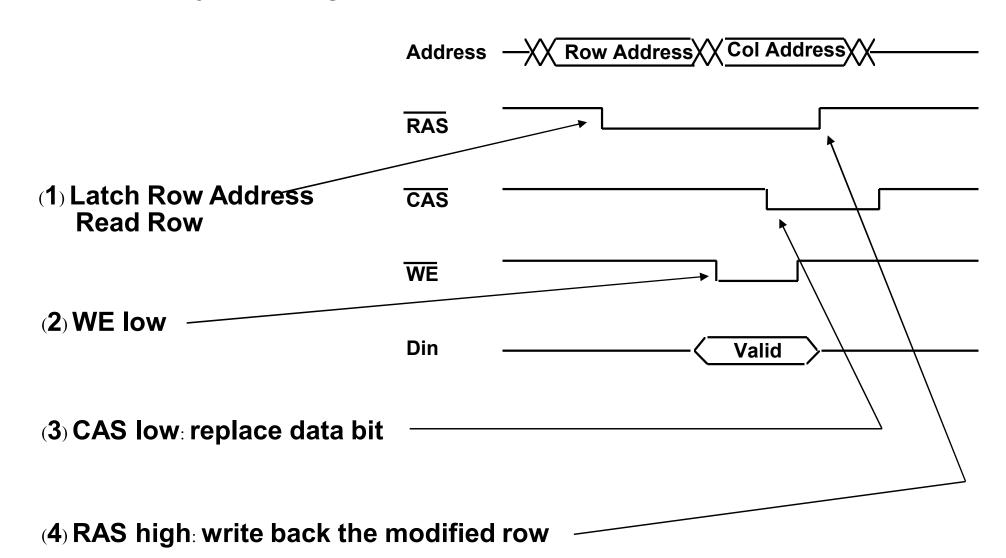
RAS, CAS Addressing

Even to read 1 bit, an entire 64-bit row is read:

Separate addressing into two cycles: Row Address, Column Address Saves on package pins, speeds RAM access for sequential bits:







(5) CAS high to complete the memory cycle

RAM Refresh

Refresh Frequency:

4096 word RAM - refresh each word once every 4 ms

Assume 120ns memory access cycle

This is one refresh cycle every 976 ns (1 in 8 DRAM accesses)!

But RAM is really organized into 64 rows

This is one refresh cycle every 62.5 µs (1 in 500 DRAM accesses)

Large capacity DRAMs have 256 rows, refresh once every 16 µs

RAS-only Refresh (RAS cycling, no CAS cycling)

External controller remembers last refreshed row

Some memory chips maintain refresh row pointer

CAS before RAS refresh: if CAS goes low before RAS, then refresh

DRAM Variations

Page Mode DRAM:

read/write bit within last accessed row without RAS cycle

RAS, CAS, CAS, ..., CAS, RAS, CAS, ...

New column address for each CAS cycle

Static Column DRAM:

like page mode, except address bit changes signal new cycles rather than CAS cycling

on writes, deselect chip or CAS while address lines are changing

Nibble Mode DRAM:

like page mode, except that CAS cycling implies next column address in sequence – no need to specify column address after first CAS

Chapter Summary

- The Variety of Sequential Circuit Packages Registers, Shifters, Counters, RAMs
- Counters as Simple Finite State Machines
- Counter Design Procedure
 - 1. Derive State Diagram
 - 2. Derive State Transition Table
 - 3. Determine Next State Functions
 - 4. Remap Next State Functions for Target FF Types Using Excitation Tables; Implement Logic
- Different FF Types in Counters
 J-K best for reducing gate count in packaged logic
 D is easiest design plus best for reducing wiring and area in VLSI
- Asynchronous vs. Synchronous Counters
 Avoid Ripple Counters: State transitions are not sharp
 Beware of potential problems when cascading synchronous counters

Offset counters: easy to design with synchronous load and clear Never use counters with asynchronous clear for this kind of application