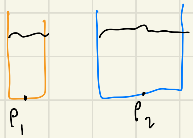
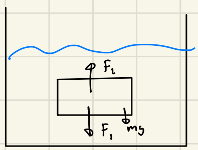
Fluids — static
└ dynamic

Density $\rho = \frac{m}{V}$ $\frac{kg}{m^3}$

Pressure $P = \frac{F}{A}$ $\frac{N}{m^2}$ (Pa)



$P_1 = P_2$ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

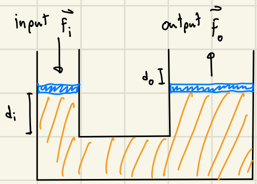


$F_2 = F_1 + \overset{m=\rho A h}{mg}$

$P_2 \cancel{A} = P_1 \cancel{A} + \rho \cancel{A} h g$

$P_2 = P_1 + \rho h g$ ความดันเกจ = ผลต่างจาก P_{atm}

ρ water

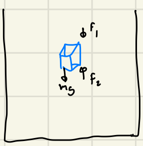


$P_i = P_o$

$\frac{F_i}{A_i} = \frac{F_o}{A_o}$; $W_{F_i} = F_i d_i = W_{F_o} = F_o d_o$

$\frac{d_i}{d_o} = \frac{F_o}{F_i} = \frac{A_o}{A_i}$

แรงลอยตัว



$mg + F_1 = F_2$

$F_2 - F_1 = mg = \rho_w V g$

$F_b = \rho_w V g$

เขื่อน $F = \frac{1}{2}\rho g w H^2$

$\tau = \frac{FH}{3}$

$PV = nRT$ barometric equation

$PVM = nMRT$

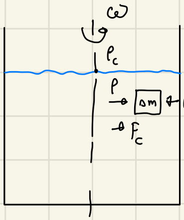
$PM = \rho RT$

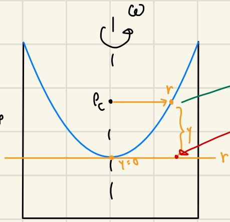
$\rho = \frac{PM}{RT}$ $dP = -\rho g dy$

$dP = -\frac{PM}{RT} g\, dy$

$\int_{P_{atm}}^{P(y)} \frac{1}{P} dP = \int_0^y -\frac{Mg}{RT} dy$ $\ln \frac{P(y)}{P_{atm}} = -\frac{Mg}{RT} y$ $P(y) = P_{atm} e^{-\frac{Mgy}{RT}}$

มีการหมุนภาชนะ





$P = P_c + \frac{1}{2}\omega^2 r^2 \rho - \rho g y = P_c = P_{atm}$

$\frac{1}{2}\omega^2 r^2 \rho = \rho g y$

$y = \frac{1}{2}\frac{\omega^2 r^2}{g}$ หมุนเร็วขึ้น น้ำยิ่งลึก

$(P+\Delta P)A = PA + \omega^2 r \Delta m$

$(P+\Delta P)A = PA + \omega^2 r \rho A \Delta r$

$\Delta P = \omega^2 r \rho \Delta r$

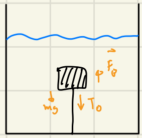
$\frac{\Delta P}{\Delta r} = \omega^2 r \rho$

$\int_{P_c}^{P} dP = \int_0^{r_2} \omega^2 r \rho \, dr$

$P - P_c = \frac{\omega^2 r^2 \rho}{2}$

$P = \frac{\omega^2 r^2 \rho}{2} + P_c$ ไม่ใช่ที่ผิวน้ำ

ระบบมี $\vec{a}$



$\vec{a} = 0$

$T_0 = mg = F_B = \rho g V$

$T_0 = (\rho V - m) g$

$\vec{a} \neq 0$ $\quad F_B - mg - T = ma$

$\rho(a+g)V - mg - T = ma$

$T = (\rho V - m)(g+a)$

Fluid Dynamic

condition

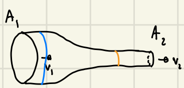
① laminar flow = steady flow

② Incompressible flow

③ non viscosity

④ Irrotational flow

Continuity equation



$V_1 = V_2$

$v_1 A_1 t = v_2 A_2 t$

$Q = A_1 v_1 = A_2 v_2 = R_v$ (volume flow rate)

ρR_v = mass flow rate

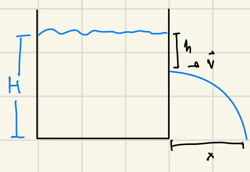
$\left(\frac{kg}{s}\right)$

Bernoulli's equation

(ณ ตำแหน่งใดๆ ในท่อน้ำ)

$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$

สังเกตได้ว่า v มาก P น้อย



$\vec{v} = \sqrt{2gh}$

$x = \vec{v} t$

$x^2 = \vec{v}^2 t^2$

$x^2 = 2gh \cdot t^2$ $\quad s = ut + \frac{1}{2}at^2$

$x^2 = 2gh \cdot \frac{2(H-h)}{g}$

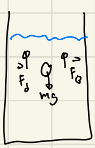
$(x(h))^2 = 4h(H-h)$

ถ้า $\frac{dx}{dh} = 0$ $\quad 2x\frac{dx}{dh} = 4h(-1) + (H-h)\cdot 4$

$h = \frac{H}{2}$ ✱

Motion with Linear Drag

Viscosity of fluids



$\vec{F_d} \propto -\vec{v}$

$\vec{F_d} = -6\pi a \eta \vec{v}$ (r of sphere; viscosity of fluid)

$mg - F_d - F_B = ma$

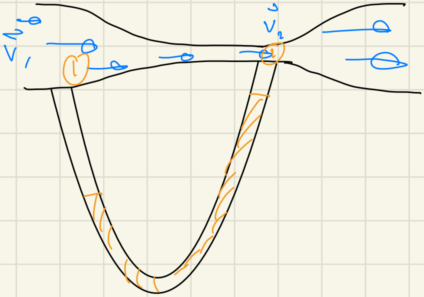
$m\frac{dv}{dt} = mg - 6\pi a \eta v - \rho_l V g$

$m\frac{dv}{dt} = (\rho_0 - \rho_l) V g - 6\pi a \eta v$

$\int_0^v \frac{m\,dv}{(\rho_0-\rho_l)Vg - 6\pi a \eta v} = \int_0^t dt$

$t = \frac{-m}{6\pi a \eta}\left[\ln[(\rho_0-\rho_l)Vg - 6\pi a\eta v] - \ln[(\rho_0-\rho_l)Vg]\right]$

$v(t) = \frac{(\rho_0-\rho_l)Vg}{6\pi a \eta}\left(1 - e^{\frac{-6\pi a \eta t}{m}}\right)$



$① : P_1 + \frac{1}{2}\rho v^2 + \cancel{\rho g h}$

$② : P_2 + \frac{1}{2}\rho v^2 + \cancel{\rho g h}$

$① = ② \quad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$A_1 v_1 = A_2 v_2$

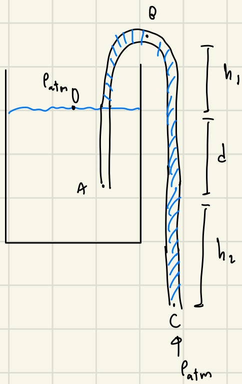
$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$

$v_2 = \frac{A_1 v_1}{A_2}$

$\Delta P = \frac{1}{2}\rho\left(\frac{A_1 v_1}{A_2}\right)^2 - \frac{1}{2}\rho v_1^2$

$v_1^2 = \frac{2\Delta P A_2}{\rho(A_1^2 - A_2^2)}$

$v_1 = \sqrt{\frac{2 \cdot \Delta P \cdot A_2}{\rho(A_1^2 - A_2^2)}}$



$v_A = v_B = v_C$

$Ⓐ \; P_A + \frac{1}{2}\rho v_A^2 + \rho g h_2$

$Ⓑ \; P_B + \frac{1}{2}\rho v_B^2 + \rho g(h_1 + h_2 + d)$

$Ⓒ \; P_C + \frac{1}{2}\rho v_C^2 + \rho g \cdot 0$ ✓

$Ⓓ \; P_D + \frac{1}{2}\rho v_D^2 + \rho g(d + h_2)$ ✓

$Ⓒ = Ⓓ \quad \cancel{P_C}^{P_{atm}} + \frac{1}{2}\rho v_C^2 = \cancel{P_D}^{P_{atm}} + \frac{1}{2}\rho v_D^2 + \rho g(d + h_2)$

สมมติ พ.ท. หน้าตัดท่อน้อยมาก

$A v_D = a v_C \qquad A >> a$

$v_D << v_C$

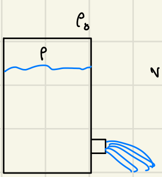
$\frac{1}{2}\cancel{\rho} v_C^2 = \cancel{\frac{1}{2}\rho v_D^2}^{0} + \cancel{\rho} g(d + h_2)$

$\frac{1}{2} v_C^2 = g(d + h_2)$

$v_C = \sqrt{2g(d + h_2)}$

$P_{atm} + \rho g(h_1 + h_2 + d) = P_B$

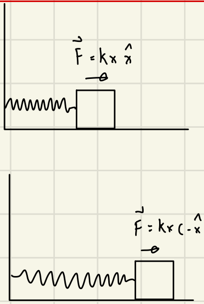
Torricelli



$v = \sqrt{2\left(\frac{P - P_0}{\rho} + gh\right)}$

$P = P_0 \; ; \; v = \sqrt{2gh}$

SHM



สปริง

$F(x) = -kx$

$F(x) = -\dfrac{\partial U(x)}{\partial(x)}$

Intregrated

$U = \frac{1}{2}kx^2$

$= \frac{1}{2}m\omega^2[x_m\cos(\omega t+\phi)]^2$

$E_K = \frac{1}{2}mv^2$

$= \frac{1}{2}m(x_m\omega\sin(\omega t+\phi))^2$

$E_K + U = \frac{1}{2}m\omega^2x_m^2\,\cancel{(\sin^2(\omega t+\phi)+\cos^2(\omega t+\phi))}$

$= \frac{1}{2}m\omega^2x_m^2$

$x(t) = x_m\cos(\omega t+\phi)$

$v(t) = -x_m\omega\sin(\omega t+\phi)$

$a(t) = -x_m\omega^2\cos(\omega t+\phi) = -\omega^2x$

$\omega = \sqrt{\dfrac{k}{m}}$

SHM equation

$\ddot{x} = -\omega^2x + C$

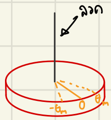
$x(t) = A\cos(\omega t+\phi)$

$v(t) = -A\omega\sin(\omega t+\phi)$

$a(t) = -A\omega^2\cos(\omega t+\phi)$

$\omega = 2\pi f = \dfrac{2\pi}{T}$

Torsional Pendulum (สปริงแบบเชิงมุม)



torsional constant

$\tau = -\kappa\theta = I\alpha$

$-\kappa\theta = I\dfrac{d^2\theta}{dt^2}$

$\underbrace{I\dfrac{d^2\theta}{dt^2} + \kappa\theta = 0}_{\text{เป็น SHM}}$

$\theta(t) = \theta_m\sin(\omega t+\phi)$

$\omega = \sqrt{\dfrac{\kappa}{I}}$

Simple pandulum

$x = \theta L$

$F = m\dfrac{d^2x}{dt^2} = -mg\sin\theta$

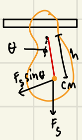
$-mg\sin\theta = mL\dfrac{d^2\theta}{dt^2}$; $\sin\theta \approx \theta$ ถ้า θ น้อยมากๆ

$\underbrace{L\dfrac{d^2\theta}{dt} + g\theta = 0}_{\text{เป็น SHM}}$

$\theta(t) = \theta_m\sin(\omega t+\phi)$

; $\omega = \sqrt{\dfrac{g}{l}}$

Physical Pandulum (วัตถุแข็งเกร็งแกว่ง)



$\tau = mg\sin\theta\, h$

$-mg\sin\theta\, h = I\alpha = I\dfrac{d^2\theta}{dt^2}$; θ น้อยๆ

$\underbrace{I\dfrac{d^2\theta}{dt^2} + mgh\theta = 0}_{\text{เป็น SHM}}$

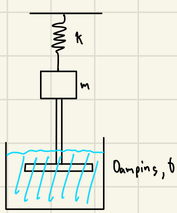
$\theta(t) = \theta_m\sin(\omega t+\phi)$

; $\omega = \sqrt{\dfrac{mgh}{I}}$

$= \sqrt{\dfrac{mgh}{mh^2}} = \sqrt{\dfrac{g}{h}}$

$I_{จุดหมุน} = I_{cm} + md^2$

Damped Harmonic Oscillator



$\vec{F}_d$: damping force

$\vec{F}_d \propto v \quad F_d = -bv$

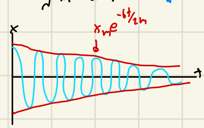
$-kx - bv = m\dfrac{d^2x}{dt^2}$

$m\dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + kx = 0$

$x(t) = x_m e^{-bt/2m}\cos(\omega' t+\phi')$

$= x_m e^{-bt/2m}\cdot e^{\pm\sqrt{\frac{b^2-4mk}{2m}}\,t}$

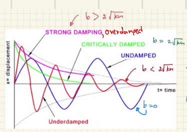
; $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2}$ $\quad \omega' = \sqrt{\omega_{s:uu}^2 - \left(\dfrac{b}{2m}\right)^2}$



$\tau = \dfrac{2m}{b}$ damping constant

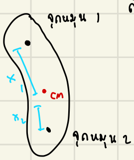
ถ้า $t = \tau$

$x(\tau) = \dfrac{x_m}{e}\cos(\omega'\tau + \phi)$



energy of Damped oscillation

$E = \frac{1}{2}kx_m^2 e^{-2t/\tau}$



ถ้าจุดหมุน 1,2 แกว่งด้วยคาบ T เท่ากัน

$$T = 2\pi\sqrt{\frac{I}{mgx}}$$

$$T^2 = 4\pi^2\frac{I}{mgx} = \frac{4\pi^2(I_c + mx^2)}{mgx}$$

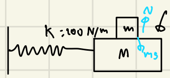
$$mgxT^2 = 4\pi^2 I_c + 4\pi^2 mx^2$$

$$4\pi^2 mx^2 - mgxT^2 + 4\pi^2 I_c = 0$$

$$x = \frac{mgT^2 \pm \sqrt{m^2g^2T^4 - 4(4\pi^2)^2 m I_c}}{8\pi^2 m}$$

$$x_1 + x_2 = l = \frac{\cancel{2m}gT^2}{\cancel{8}\pi^2\cancel{m}}_{4} \quad ; \quad g = \frac{4\pi^2 l}{T^2}$$

หา amplitude ที่ทำให้กล่องยังคงอยู่ด้านบนได้



$mg = 18$

$f = 18 \times 0.4 = 7.2N = ma$

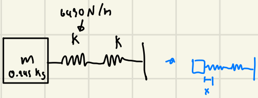
$a = 4$

$x(t) = x_m\cos(\omega t + \phi)$

$\frac{dx}{dt} = v \quad \frac{d^2x}{dt^2} = a = x_m\omega^2 = 4$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{10+1.8}}$

$x_{cm} = \frac{4}{\omega^2} = \frac{4(10+1.8)}{200}$



หา $f = ?$

$\omega = 2\pi f$

$f = \sqrt{\frac{k}{m}}\,2\pi$

$= \sqrt{\frac{\frac{k}{2}}{m}}\,2\pi$

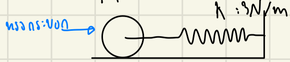
$= \sqrt{\frac{3215}{0.245}}\,2\pi$

$x_L + x_R = x$

$F_L = k_L x_L \quad F_R = k_R x_R$

$k_L x_L = k_R x_R$

$k_L = k_R \; ; \; x_L = x_R$



หาพลังงานจลน์จากการเลื่อนตำแหน่ง และพลังงานจลน์จากการหมุน

$E_{spring} = \frac{1}{2}kx^2 = 0.09375\ J$

$E_s = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ ที่ $x = 0$

$\frac{1}{2}kx^2 = \frac{1}{2}\cdot\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$

$0.09375 = \left(\frac{1}{4} + \frac{1}{2}\right)mv^2$

$\frac{1}{2}mv^2 = \frac{0.09375\cdot 4}{2\cdot 3} = 0.0625\ J$

หา E_K หมุนต่อได้

พิสูจน์ $T = 2\pi\sqrt{\frac{3M}{2K}}$

$E = \frac{1}{2}kx^2 + \frac{3}{4}mv^2$

$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{1}{2}kx^2 + \frac{3}{4}mv^2\right] = 0$

$kx\,\cancel{\frac{dx}{dt}}^{\,v} + \frac{3}{2}mv\frac{dv}{dt} = 0$

$kx + \frac{3}{2}m\frac{d^2x}{dt^2} = 0$

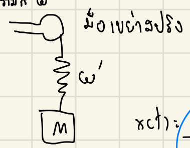
$\omega = \sqrt{\frac{2k}{3m}} \; ; \; \frac{2\pi}{T} = \sqrt{\frac{2k}{3M}}$

$T = 2\pi\sqrt{\frac{3M}{2k}}$

$\boxed{\text{สมการการเคลื่อนที่ของการสั่น} \quad m\frac{d^2x}{dt^2} + kx = 0 \quad \text{จะได้} \quad \omega = \sqrt{\frac{k}{m}}}$

Forced Oscillation

สั่นด้วยความถี่ ω''



$$\ddot{x} + \frac{k}{m}x = \frac{f_0}{m}\cos(\underbrace{\omega_d}_{\text{ออกแรง}} t)$$

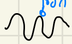
$$x(t) = \underbrace{\frac{F_m}{G}}_{\text{amplitude}}\cos(\omega'' t + \beta) \qquad \beta = \cos^{-1}\frac{b\omega''}{G}$$

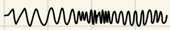
$$G = \sqrt{m^2(\omega''^2 - \omega^2)^2 + b^2\omega''^2}$$

if $\omega'' = \omega$ G จะน้อยสุด amplitude มากสุด (เกิด resonance)

Waves

- Mechanical wave
- Electro magnetic wave
- Matter wave

transverse wave  เชือก

longitudinal wave  สปริง

$Y(x,t) = Y_m \sin(kx - \omega t + \phi)$, $k = \frac{2\pi}{\lambda}$

$v = f\lambda$ $\quad \omega = 2\pi f = \frac{2\pi}{T}$

คลื่นนิ่งในเส้นเชือก $v = \sqrt{\frac{T}{\mu}}$

กำลังในการส่งพลังงาน $P = \vec{F} \cdot \vec{v}$ ใดๆ

$P(x,t) = Fk\omega\,(A\cos(kx - \omega t))(A\cos(kx - \omega t))$

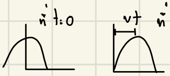
$P(x,t) = Fk\omega A^2 \cos^2(kx - \omega t)$

$P(x,t) = \mu \omega^2 A^2 v \cos^2(kx - \omega t) \therefore P_{av} = \frac{1}{2}\mu\omega^2 A^2 v$

wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

การเคลื่อนที่ของคลื่น

ที่ t=0 $\quad$ vt ที่ t



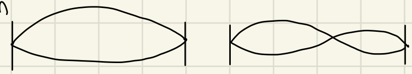
แทรกสอด

$|S_1P - S_2P| = n\lambda$ antinode

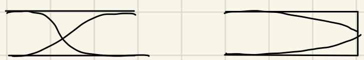
$|S_1P - S_2P| = (n+\frac{1}{2})\lambda$ node

node - antinode

ในเชือก



ในท่อ



1^{st} harmonic $\qquad 1^{st}$

2^{nd} harmonic 1^{st} overtone $\qquad 3^{rd}$

3^{rd} harmonic $\qquad 5^{th}$

Sound

การกระจัด

$S(x,t) = S_{max}\cos(kx - \omega t)$

$\Delta P(x,t) = \Delta P_{max}\sin(kx - \omega t)$

bulk modulus $= \frac{\Delta P}{\Delta V / V_0}$

adiabatic + ideal gas

$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma R T}{\mu}}$

Intensity

$I = \frac{P}{A} = \frac{P}{4\pi r^2}$

$I = \frac{1}{2}\rho v \omega^2 s_m^2$

$\beta = 10\log\frac{I}{I_0}$

I_0 ค.เข้มที่ต่ำสุดที่ได้ยิน $= 10^{-12}\ W/m^2$

Doppler effect

$f' = f\frac{[V \pm V_0]}{[V \pm V_s]}$, $v = f\lambda$

V_0 ผู้ฟัง, V_s แหล่งกำเนิด

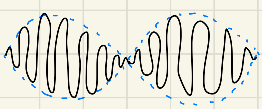
ความถี่บีตส์

$T_{beat} = \frac{T_{mod}}{2}$

$f_{beat} = f_1 - f_2 = 2 f_{mod}$

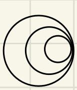
$\frac{|f_1 - f_2|}{2}$ = ความถี่ที่ได้ยิน 1 node

$\frac{f_1 + f_2}{2}$ = ความถี่เฉลี่ย (ความถี่ลัพธ์)

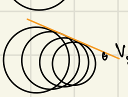


$y = 2A\cos\left(2\pi\left(\frac{f_1 - f_2}{2}\right)t\right)\sin\left(2\pi\left(\frac{f_1 + f_2}{2}\right)t\right)$

Shock wave



$V_s = V$



$V_s > V$

$\sin\theta = \frac{vt}{V_s t} = \frac{V}{V_s}$

•••23 Figure 17-38 shows two point sources S_1 and S_2 that emit sound of wavelength $\lambda = 2.00$ m. The emissions are isotropic and in phase, and the separation between the sources is $d = 16.0$ m. At any point P on the x axis, the wave from S_1 and the wave from S_2 interfere. When P is very far away ($x \approx \infty$), what are (a) the phase difference between the arriving waves from S_1 and S_2 and (b) the type of interference they produce? Now move point P along the x axis toward S_1. (c) Does the phase difference between the waves increase or decrease? At what distance x do the waves have a phase difference of (d) 0.50λ, (e) 1.00λ, and (f) 1.50λ?

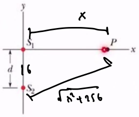


Fig. 17-38 Problem 23.

(a) เฟสจะตรงกัน

(b) antinode

$(1000 - 2000\ Hz)$

•43 In Fig. 17-40, S is a small loudspeaker driven by an audio oscillator with a frequency that is varied from 1000 Hz to 2000 Hz, and D is a cylindrical pipe with two open ends and a length of 45.7 cm. The speed of sound in the air-filled pipe is 344 m/s. (a) At how many frequencies does the sound from the loudspeaker set up resonance in the pipe? What are the (b) lowest and (c) second lowest frequencies at which resonance occurs?

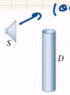


Fig. 17-40 Problem 43.

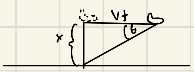
$l = \frac{n\lambda}{2}$ $\qquad v = f\lambda$

$45.7 = n \cdot \frac{v}{f \cdot 2}$

$f = \frac{n}{2} \cdot \frac{v}{45.7\,cm} = 376.36\,n \quad ; n = 1, 2, 3, 4, \ldots$

$1000 < f < 2000 \quad \therefore n = 3, 4, 5$

••70 A plane flies at 1.25 times the speed of sound. Its sonic boom reaches a man on the ground 1.00 min after the plane passes directly overhead. What is the altitude of the plane? Assume the speed of sound to be 330 m/s.



$\sin\theta = \frac{v}{v_s} = \frac{1}{1.25}$

$\theta = 53^\circ$

$s = vt = 330 \cdot 1.25 \cdot 60 = 24750\ m$

$\tan 53^\circ = \frac{x}{24750}$

$x = 32844\ m$

Thermal expansion

- Linear $\Delta L = L\alpha\Delta T$ — α: coefficient of linear expansion
- Volume $\Delta V = V\beta\Delta T$ — β: Coefficient of volume expansion

$\beta = 3\alpha$

Heat Capacity, C

$$\frac{Q}{\Delta T} = C$$

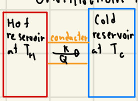
การถ่ายเทความร้อน

- นำ - พา - แผ่รังสี

การนำความร้อน

- การถ่ายเทตลอดสภาวะคงที่ (T ไม่เปลี่ยน)

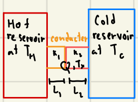
- อัตราการถ่ายเท Heat = $P = \frac{Q}{t}$



$$P_{cond} = \frac{Q}{t} = \frac{kA(T_H - T_C)}{L}$$

k: thermal conductivity
A: area surface
L: Lenth

- Thermal resistance of conduction $= R = \frac{L}{k}$; $P = \frac{A(T_H - T_C)}{R}$
- ถ้าตัวกลางมีมากกว่า 1 ชั้น



$$P = \frac{A(T_H - T_C)}{R}$$

$$\left.\begin{aligned} P_1 &= \frac{k_1 A(T_H - T_x)}{L_1} \\ P_2 &= \frac{k_2 A(T_x - T_C)}{L_2} \end{aligned}\right\} P_1 = P_2 = P$$

$$T_x = \frac{k_1 L_2 T_C + k_2 L_1 T_H}{k_1 L_2 + k_2 L_1}$$

$$P = \frac{A(T_H - T_C)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

$$P = \frac{A(T_H - T_C)}{\sum \frac{L_i}{k_i}} \#$$

**62 Leidenfrost effect. A water drop that is slung onto a skillet with a temperature between 100°C and about 200°C will last about 1 s. However, if the skillet is much hotter, the drop can last several minutes, an effect named after an early investigator. The longer lifetime is due to the support of a thin layer of air and water vapor that separates the drop from the metal (by distance L in Fig. 18-47). Let L = 0.100 mm, and assume that the drop is flat with height h = 1.50 mm and bottom face area $A = 4.00 \times 10^{-6}$ m^2. Also assume that the skillet has a constant temperature T_s = 300°C and the drop has a temperature of 100°C. Water has density ρ = 1000 kg/m^3, and the supporting layer has thermal conductivity k = 0.026 W/m·K. (a) At what rate is energy conducted from the skillet to the drop through the drop's bottom surface? (b) If conduction is the primary way energy moves from the skillet to the drop, how long will the drop last?

Fig. 18-46 Problem 61.

Fig. 18-47 Problem 62.

การแผ่รังสี

- $P_{rad} = \sigma\varepsilon A T^4$

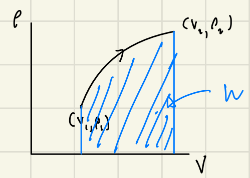
ε: emissivity $0 \to 1$ (0: ไม่แผ่เลย, 1: แผ่ 100% (black body))

σ: Stefan-Boltzmann constant

- $P_{abs} = \sigma\varepsilon A T_{env}^4$

1st Law of Thermodynamics

$$\Delta U = W + Q$$



$W = -\int P\,dV$.- area

kinetic energy of gases

Ideal gas Law

$$PV = Nk_B T = nRT$$

k_B: Boltzman's constant $= 1.38 \times 10^{-23}$ J/K

N: จำนวนอนุภาค

แก๊สขยายตัวใน Isothermal Process

$$PV = Nk_B T$$

$$W = -\int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{Nk_B T}{V}\,dV$$

$$W = -Nk_B T[\ln V]_{V_i}^{V_f}$$

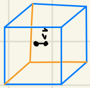
$$W = -Nk_B T \ln\left(\frac{V_f}{V_i}\right) \#$$

Constant Volume , Constant Pressure

$$W = -\int_{V_i}^{V_f} P\,dV = 0 \qquad W = -\int_{V_i}^{V_f} P\,dV = -P\Delta V$$

Microscopic view of gases

อนุภาคแก๊สชนกับผนัง



momentum

$\Delta P_y = \Delta P_z = 0$

$\Delta P_x = P_{after} - P_{before} = -m\vec{v}_x - m\vec{v}_x = -2m\vec{v}_x$

โมเมนตัมถ่ายโอนไปให้กำแพง $+2m\vec{v}_x$

$\frac{\Delta P_x}{\Delta t} = \frac{2m\vec{v}_x}{\frac{2L}{v_x}} = \frac{m}{L} v_x^2$ 1 อนุภาค

$P = \frac{F}{A} = \frac{F}{L^2}$ $\quad F = \frac{dp}{dt}$

$P = \frac{\sum_i \frac{m_i v_{x_i}^2}{L}}{L^2}$ สมมติ m อนุภาคแก๊สเท่ากัน

$P = \frac{m}{L^3} \sum_i v_{x_i}^2 = \frac{m}{L^3}\left(v_{x_1}^2 + \dots + v_{x_n}^2\right)$

จน.อนุภาค

$P = \frac{m}{L^3} N [\overline{v_x^2}]$ $\quad v^2 = v_x^2 + v_y^2 + v_z^2$ ← Isotropic ทุกแกนเหมือนกัน

$v_x^2 = v_y^2 = v_z^2 = \frac{1}{3} v^2$

$P = \frac{mN\overline{v^2}}{3L^3} = \frac{\rho}{3}\overline{v^2}$ (L^3 = V)

$$\boxed{U_{1อนุภาค} = E_{k\,1อนุภาค} = \frac{f}{2} k_B T}$$

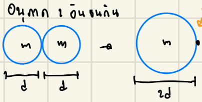
$\overline{v^2} = \frac{3PV}{mN} = \frac{3}{m} k_B T$

$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3}{m} k_B T} = \sqrt{\frac{3P}{\rho}}$ #

$$\boxed{\begin{aligned} \bar{E}_K &= \frac{1}{2} m\overline{v^2} = \frac{3}{2} k_B T \\ U &= N\bar{E}_K = \frac{3}{2} N k_B T \end{aligned}}$$ 1 อนุภาค

Mean Free path : ระยะทางก่อนที่จะชนเฉลี่ย

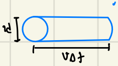
อนุภาค 1 อันชนกัน



ประมาณให้ดูง่ายขึ้น

m อยู่นิ่ง

สนใจแค่วงกลมใหญ่ เคลื่อนที่



อนุภาคเคลื่อนที่เป็นเส้นตรง

$V_{cylinder} = \pi d^2 v\Delta t$

ความหนาแน่น : $\frac{N}{V} = \frac{N_c}{V_c}$ จำนวนอนุภาค • ที่อยู่ในทรงกระบอก = จำนวนครั้งที่ชน

ระยะทางที่อนุภาคชน 1 ครั้ง

$N_c = \frac{N}{V} V_c = \frac{N}{V} \pi d^2 v\Delta t$

$$\boxed{\lambda_{\text{mean free path}} = \frac{s}{\text{number of collision}} = \frac{v\Delta t}{N/V(\pi d^2 v\Delta t)} = \frac{V}{N\pi d^2}}$$

$t_{mean} = \frac{\lambda}{v_{rms}}$

สำหรับ ideal gas

$PV = N k_B T$

$\frac{V}{N} = \frac{k_B T}{P}$ $\quad \boxed{\lambda = \frac{1}{\sqrt{2}\pi d^2 (N/V)}}$

$\boxed{\lambda = \frac{k_B T}{P\pi d^2}}$ => mean free path ของโมเลกุลใหญ่ ชนโมเลกุล จุด ล้วน ot

ถ้า อนุภาคทั้ง 2 เคลื่อนที่ชนกัน

$\boxed{\lambda = \frac{k_B T}{\sqrt{2} P \pi d^2}}$; อัตราการชน $R = \frac{v_{av}}{\lambda}$ $\left(\frac{1}{s}\right)$

The distribution of molecular speed $dN = N(v)dv$

Maxwell distribution $N(v) = 4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{\frac{-mv^2}{2k_B T}}$

อัตราส่วน, มวล 1 อนุภาค

$P(v) = \frac{N(v)}{N} = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} v^2 e^{\frac{-mv^2}{2k_B T}}$ $\quad \int_0^\infty P(v)dv = 1$

จน.อนุภาคที่มี ค.เร็ว v

$d\frac{N(v)}{N}$

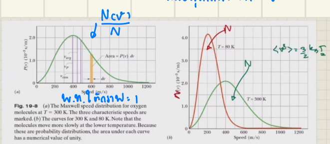


Fig. 19-8 (a) The Maxwell speed distribution for oxygen molecules at T = 300 K. The three characteristic speeds are marked. (b) The curves for 300 K and 80 K. Note that the molecules move more slowly at the lower temperature. Because these are probability distributions, the area under each curve has a numerical value of unity.

v_p : the most probable speed $\quad$ เมื่อ dv น้อยๆ

$\frac{d\frac{N(v)}{N}}{dv}\Big|_{v=v_p} = 0$ $\quad dN = Nf(v)\,dv$

ฐานนิยม

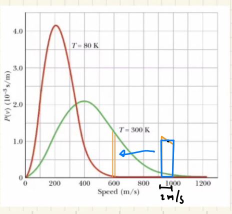
$v_p = \sqrt{\frac{2k_B T}{m}}$ มวล 1 อนุภาค

$v_{avg} = \frac{1}{N}\int_0^\infty N(v)v\,dv = \int_0^\infty vP(v)dv = \sqrt{\frac{8k_B T}{\pi m}}$

คิดแค่ผลบวก (วาดได้) ทั้ง di, poly ใช้เหมือนกัน

$v_{rms} = \sqrt{(v^2)_{avg}} = \sqrt{\frac{1}{N}\int_0^\infty N(v)v^2 dv} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}}$

A container is filled with oxygen gas maintained at room temperature (300 K). What fraction of the molecules have speeds in the interval 599 to 601 m/s? The molar mass M of oxygen is 0.0320 kg/mol.



$\frac{N'(v)}{N} = \int_{599}^{601} 4\pi\left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} v^2 e^{\frac{-mv^2}{2k_B T}} dv$

อินทริเกตยากไป

$\frac{N'(v)}{N} = \frac{N(600)}{N} \times 2\,m/s$

$= 4\pi\left(\frac{m}{2\pi k_B T}\right) v^2 e^{\frac{-mv^2}{2k_B T}}$

•34 The speeds of 22 particles are as follows (N_i represents the number of particles that have speed v_i):

N_i	2	4	6	8	2
v_i (cm/s)	1.0	2.0	3.0	4.0	5.0
v_i^2	1	4	9	16	25

What are (a) v_{avg}, (b) v_{rms}, and (c) v_P?

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•35 Ten particles are moving with the following speeds: four 200 m/s, two at 500 m/s, and four at 600 m/s. Calculate their (a) average and (b) rms speeds. (c) Is $v_{rms} > v_{avg}$?

••36 It is found that the most probable speed of molecules in a gas when it has (uniform) temperature T_2 is the same as the rms speed of the molecules in this gas when it has (uniform) temperature T_1. Calculate T_2/T_1.

••37 SSM WWW Figure 19-23 shows a hypothetical speed distribution for a sample of N gas particles (note that $P(v) = 0$ for speed $v > 2v_0$). What are the values of (a) av_0, (b) v_{avg}/v_0, and (c) v_{rms}/v_0? (d) What fraction of the particles has a speed between $1.5v_0$ and $2.0v_0$?

$P(v) = \frac{a v}{v_0}$

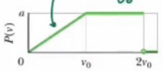


Fig. 19-23 Problem 37.

34. $$v_{avg} = \frac{1\cdot2 + 2\cdot4 + 6\cdot3 + 8\cdot4 + 5\cdot2}{22} = \frac{70}{22} \text{ m/s}$$

$$v_{rms} = \sqrt{(v^2)_{avg}} = \sqrt{\frac{1\cdot2 + 4\cdot4 + 9\cdot6 + 16\cdot8 + 25\cdot2}{32}}$$

$$= \sqrt{\frac{250}{22}} = \sqrt{\frac{125}{16}} = \frac{5\sqrt{5}}{4} \text{ m/s}$$

$$v_p = 4 \text{ m/s}$$

37. a) พ.ท.ใต้กราฟ = 1

$$\frac{1}{2} \times (3v_0) \times a = 1$$

$$av_0 = \frac{2}{3}$$

b) $$v_{avg} = \int_0^{\infty} v P(v)\, dv$$

$$= \int_0^{v_0} v \cdot \frac{a}{v_0} v\, dv + \int_{v_0}^{2v_0} v \cdot a\, dv$$

$$= \frac{a}{v_0}\left[\frac{1}{3}v^3\right]_0^{v_0} + a\left[\frac{v^2}{2}\right]_{v_0}^{2v_0}$$

$$= \frac{a}{3}v_0^2 + 2av_0^2 - \frac{a}{2}v_0^2$$

$$= \frac{11}{6} a v_0^2$$

$$v_{avg}/v_0 = \frac{11}{6} a v_0 = \frac{11}{6} \cdot \frac{2}{3} = \frac{11}{9}$$

c) $$v_{rms} = \sqrt{(v^2)_{avg}}$$

$$(v^2)_{avg} = \int_0^{2v_0} v^2 P(v)\, dv$$

$$= \int_0^{v_0} v^2 \cdot \frac{a}{v_0} v\, dv + \int_{v_0}^{2v_0} v^2 \cdot a\, dv$$

$$= \frac{a}{v_0}\left[\frac{1}{4}v^4\right]_0^{v_0} + a\left[\frac{v^3}{3}\right]_{v_0}^{2v_0}$$

$$= (1.31 v_0)^2$$

$$v_{rms} = 1.31 v_0$$

$$\frac{v_{rms}}{v_0} = 1.31$$

d) $$\int_{1.5v_0}^{2v_0} P(v)\, dv = a \times 0.5v_0 = \frac{1}{3}$$

การขยายตัว เชิงเส้น $\frac{\Delta L}{L_0} = \alpha \Delta T$

$$\frac{\Delta A}{A_0} = \beta \Delta T = 2\alpha \Delta T$$

$$\frac{\Delta V}{V_0} = \gamma \Delta T = 3\alpha \Delta T$$

Internal Energy of gas

interaction btw particle + evironment

$$E = \sum_i \frac{1}{2} m v_i^2 + \sum U_i$$ ≈ สมมติเป็น 0 เนื่องจาก ρ ต่ำมาก

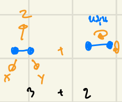
$$\bar{E} = N \frac{1}{2} m \bar{v}^2 = \frac{N}{2} m v_{rms}^2$$

$$= \frac{N}{2} m \frac{3}{m} k_B T = \frac{3}{2} N k_B T = 3\underbrace{\left(\frac{1}{2} N k_B T\right)}_{\text{พลังงานจาก 1 ทิศทาง}}$$

3 ทิศทาง x, y, z (degree of freedom)

Degree of freedom

monoatomic  3 degree of freedom $E = \frac{3}{2} N k_B T$

diatomic  5 degree of freedom $E = \left(\frac{3}{2} k_B T + k_B T\right) N = \frac{5}{2} N k_B T$

polyatomic  เคลื่อนที่ 3 แกน + หมุน 3 แกน $E = \left(\frac{3}{2} k_B T + \frac{3}{2} k_B T\right) N = 3 N k_B T$

$$E = n C_v' dT$$

Specific heat

- Constant Volume, C_v

$$Q = \underset{kg}{m} \underset{J/kg \cdot K}{C_v} \Delta T = \underset{mol}{n} \underset{J/mol \cdot K}{C_v'} \Delta T$$

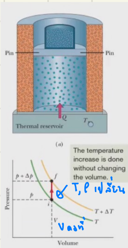


Fig. 19-9 (a) The temperature of an ideal gas is raised from T to $T + \Delta T$ in a constant-volume process. Heat is added, but no work is done. (b) The process on a p-V diagram.

$$\Delta U = Q + W \qquad W = 0$$

$$\Delta U = Q = m C_v \Delta T = n C_v' \Delta T$$

สำหรับ monoatomic

$$\Delta U = U_f - U_i = \frac{3}{2} N k_B (T + \Delta T) - \frac{3}{2} N k_B T$$

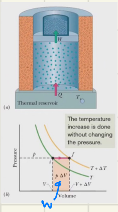
$$\boxed{\Delta U = \frac{3}{2} N k_B T}$$

จะได้ $\frac{3}{2} N k_B T = m C_v \Delta T$

$$\boxed{C_v = \frac{3}{2} \frac{N}{m} k_B}, \quad \boxed{C_v' = \frac{3}{2} R} \; J/mol \cdot K$$

$$\boxed{\therefore C_v' = \frac{f}{2} R}$$

- constant Pressure



$Q = nC_p'\Delta T$

$\Delta E = Q + W \qquad W = -P\Delta V = -nR\Delta T$

สำหรับ Mono atonic

$\frac{3}{2}nR\Delta T = Q - P\Delta V$

$\frac{3}{2}nR\Delta T = nC_p'\Delta T - nR\Delta T$

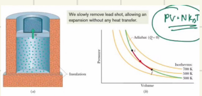
$C_p' = \frac{3}{2}R + R \qquad C_v' = \frac{3}{2}R$

มีงานมาเกี่ยวข้อง C_p' จึงเยอะกว่า C_v'

$\therefore C_p' = C_v' + R \qquad C_v' = \frac{f}{2}R$

Process ที่เกี่ยวข้องกับ ideal gas

Adiabatic Process (ไม่มี Heat ถ่ายเท)



ระบบทำงาน อุณหภูมิลด

$Q = 0 \qquad \Delta E = W = -PdV \qquad PV = nK_BT$

$\Delta E = nC_v dT = -\frac{nK_BT}{V}dV$

$\int_{T_i}^{T_f}\frac{dT}{T} = -\frac{NK_B}{mC_V}\int_{V_i}^{V_f}\frac{1}{V}dV$

$\ln\left(\frac{T_f}{T_i}\right) = \underbrace{\frac{-NK_B}{mC_V}}_{\text{ค่าคงที่} = A}\ln\frac{V_f}{V_i}$

$\boxed{T_f V_f^{A} = T_i V_i^{A}} \quad ; A = -\frac{NK_B}{mC_V}$

mol $\boxed{T_f V_f^{\gamma-1} = T_i V_f^{\gamma-1} \quad ; \gamma - 1 = \frac{R}{C_V'}, \ \gamma = \frac{C_p'}{C_V'}}$

$NK_B = nR$

	ΔU	W	Q	ΔS_{sys}	note
isothermal	0	$-nRT\ln\frac{V_f}{V_i}$	$nRT\ln\frac{V_f}{V_i}$	$nR\ln\left(\frac{V_f}{V_i}\right)$	$PV = nRT$ $\downarrow$ $W = \int PdV$; $PV = PV$
isovolumic	$\frac{f}{2}NK_B\Delta T$	0	$\frac{f}{2}NK_B\Delta T = nC_v'\Delta T$	$nC_v'\ln\frac{T_f}{T_i}$	$Q = C_vK_B\Delta T$, $C_v = \frac{f}{2}NK_B$
isobaric	$\frac{f}{2}nR\Delta T = \frac{f}{2}P\Delta V$	$-P\Delta V$	$(\frac{f}{2}+1)NK_B\Delta T = nC_p'\Delta T$	$nC_p'\ln\frac{T_f}{T_i}$	$\Delta P = 0$, $C_p = (\frac{f}{2}+1)NK_B$
Adiabatic $Q = 0$	$\frac{f}{2}NK_B\Delta T = \frac{f}{2}nR\Delta T$	$\frac{f}{2}NK_B\Delta T = \frac{f}{2}nR\Delta T$	0	0	$PV^{\gamma} = PV^{\gamma}$
free expansion	0	0	0	$nR\ln\left(\frac{V_f}{V_i}\right)$	inreversible process

$PV = nRT$
$= NK_BT$

$$\boxed{\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}}$$

$$\boxed{\text{Isothermal} \quad \Delta S = \frac{Q_{if}}{T}}$$

$dU = dQ + dw \qquad P = \frac{nRT}{V}$

$dQ = dU - dw = nC_v'dT + PdV$

$\frac{dQ}{T} = \frac{nC_v'dT}{T} + \frac{nRT}{TV}dV$

$\Delta S = \int_i^f\frac{dQ}{T} = \int_{T_i}^{T_f}\frac{nC_v'dT}{T} + \int_{V_i}^{V_f} nR\frac{dV}{V}$

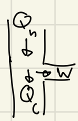
$$\boxed{\Delta S = nR\ln\frac{V_f}{V_i} + nC_v'\ln\frac{T_f}{T_i}}$$

2nd Law of Thermodynamics

$\Delta S_{uni} = S_f - S_i \geq 0$

Heat engine $\Delta S_{system} \leq 0$ (ไม่สร้าง entropy เพิ่ม)

engine

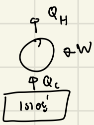


$\eta = \frac{|W|}{Q_{in}}$

in = out

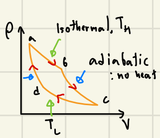
$Q_H = Q_L + W$

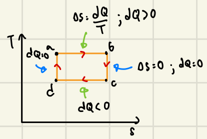
ตู้เย็น



$\mu = \frac{Q_c}{W}$

carnot engine, reversible $\Delta S = 0$





work a→b→c→d→a

$\Delta U = 0\,J = Q + W$

$Q = -W$

entropy

$$\left.\begin{aligned} S_b - S_a &= \frac{dQ}{T} = \frac{Q_H}{T_H} \\ S_c - S_b &= 0 \;;\; Q = 0 \\ S_d - S_c &= \frac{dQ}{T} = -\frac{Q_L}{T_L} \\ S_a - S_d &= 0 \end{aligned}\right\} \Delta S = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$$

$$\frac{|Q_H|}{T_H} = \frac{|Q_L|}{T_L}$$

Efficiency of carnot engine

$$\varepsilon = \frac{\text{Output}}{\text{input}} = \frac{|W|}{|Q_H|} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

ถ้า $\varepsilon = 1 \rightarrow T_L = 0,\ T_H = \infty$ เป็นไปไม่ได้

reversible

$\Delta S_{sys}, \Delta S_{surr} = 0$

inreversible

เป็น cycle

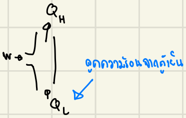
$\Delta S_{sys} = 0$,

ไม่เป็น cycle

$\Delta S_{sys} \leq 0$,

$\}\ \Delta S_{surr} > 0,\ \Delta S_{uni} > 0$

ตู้เย็น



coefficient of Performance

$$K_c = \frac{Q_{out}}{W} = \frac{Q_{out}}{Q_{in} - Q_{out}} = \frac{T_c}{T_H - T_L}$$

$K_{c\,pump} = 1 + K_{c\ refrigerator}$

Heat pump $K_c = \dfrac{T_H}{T_H - T_C}$