

ภาคการศึกษาที่ 3

ภาคการศึกษาที่ 4

2110202	DISCRETE STRUC COM	4
	โครงสร้างดีสครีต และภาวะคำนวณได้	
2110211	INTRO DATA STRUCT	3
	ความรู้เบื้องต้นเกี่ยวกับโครงสร้างข้อมูล	
2110251	DIG COMP LOGIC	3
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2110263	DIG LOGIC LAB I	1
	การปฏิบัติการทางตรรกศาสตร์ของดิจิทัลคอมพิวเตอร์ 1	
2603284	STAT PHYS SCIENCE	3
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21102xx	COMP ENG MATH II	3
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21102xx	COMP ENG MATH LAB	1
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2110322	DB SYS	3
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	วิศวกรรมซอฟต์แวร์ 1	
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รวม 19

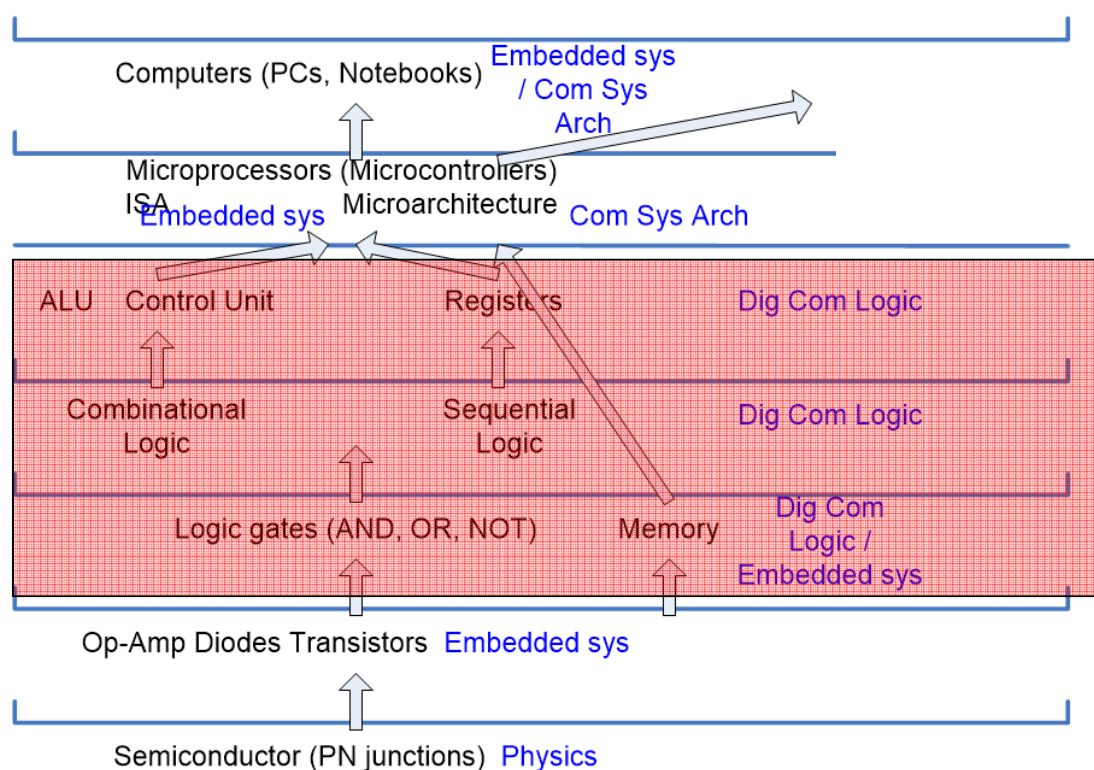
I - Introc

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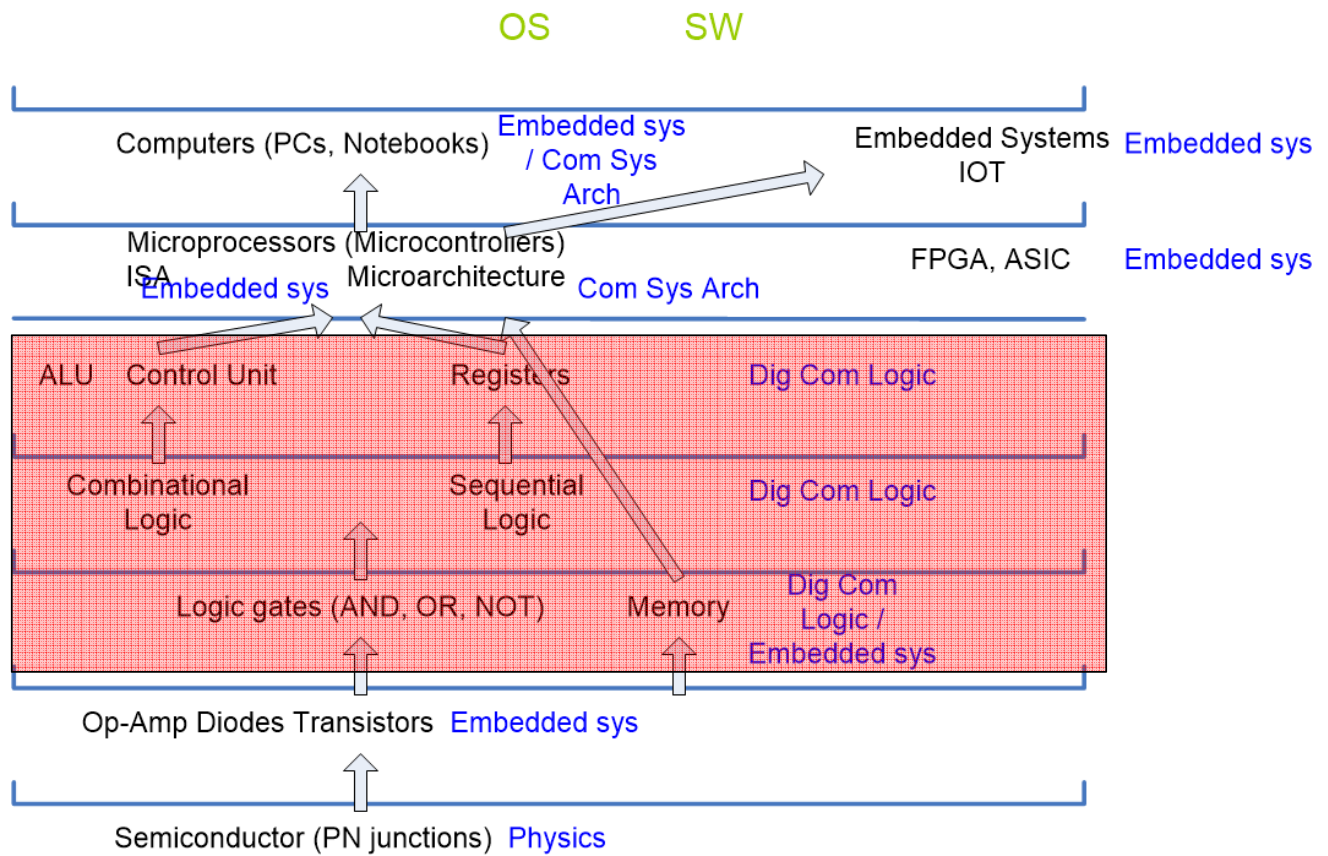
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HW classes

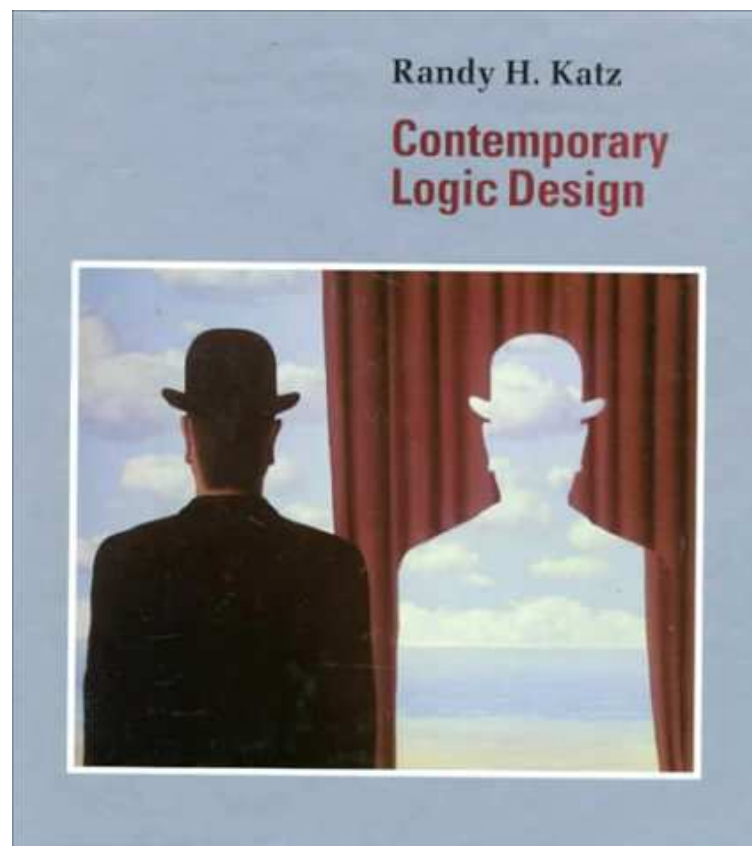
OS SW



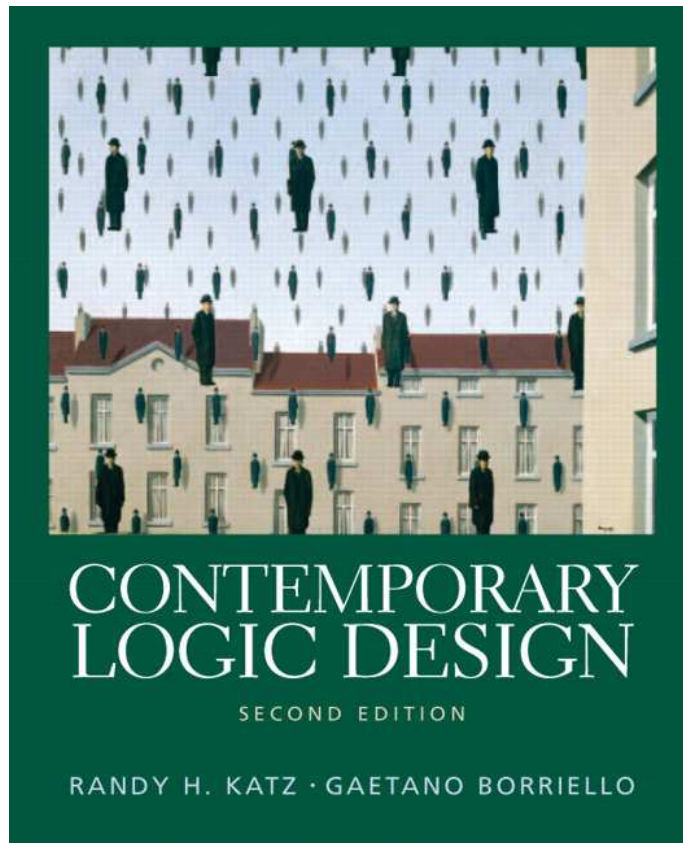
HW classes



Contemporary logic design 1st edition cover



Contemporary logic design 2nd edition cover



I - Introduction

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Why study logic design?

- Obvious reasons
 - this course is part of the CS/CompE requirements
 - it is the implementation basis for all modern computing devices
 - building large things from small components
 - provide a model of how a computer works
- More important reasons
 - the inherent parallelism in hardware is often our first exposure to parallel computation
 - it offers an interesting counterpoint to software design and is therefore useful in furthering our understanding of computation, in general

I - Introduction

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What will we learn in this class?

- The language of logic design
 - Boolean algebra, logic minimization, state, timing, CAD tools
- The concept of state in digital systems
 - analogous to variables and program counters in software systems
- Contrast with software design
 - sequential and parallel implementations
 - specify algorithm as well as computing/storage resources it will use

Applications of logic design

- Conventional computer design
 - CPUs, busses, peripherals
- Networking and communications
 - phones, modems, routers
- Embedded products
 - in cars, toys, appliances, entertainment devices
- Scientific equipment
 - testing, sensing, reporting
- The world of computing is much much bigger than just PCs!

A quick history lesson

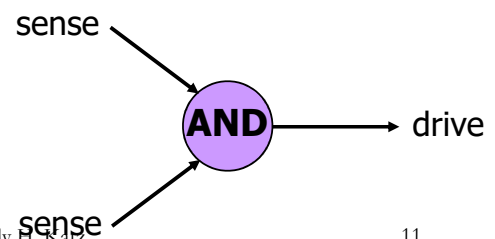
- 1850: George Boole invents Boolean algebra
 - maps logical propositions to symbols
 - permits manipulation of logic statements using mathematics
- 1938: Claude Shannon links Boolean algebra to switches
 - his Masters' thesis
- 1945: John von Neumann develops the first stored program computer
 - its switching elements are vacuum tubes (a big advance from relays)
- 1946: ENIAC . . . The world's first completely electronic computer
 - 18,000 vacuum tubes
 - several hundred multiplications per minute
- 1947: Shockley, Brittain, and Bardeen invent the transistor
 - replaces vacuum tubes
 - enable integration of multiple devices into one package
 - gateway to modern electronics

What is logic design?

- What is design?
 - given a specification of a problem, come up with a way of solving it choosing appropriately from a collection of available components
 - while meeting some criteria for size, cost, power, beauty, elegance, etc.
- What is logic design?
 - determining the collection of digital logic components to perform a specified control and/or data manipulation and/or communication function and the interconnections between them
 - which logic components to choose? – there are many implementation technologies (e.g., off-the-shelf fixed-function components, programmable devices, transistors on a chip, etc.)
 - the design may need to be optimized and/or transformed to meet design constraints

What is digital hardware?

- Collection of devices that sense and/or control wires that carry a digital value (i.e., a physical quantity that can be interpreted as a “0” or “1”)
 - example: digital logic where voltage < 0.8v is a “0” and > 2.0v is a “1”
 - example: pair of transmission wires where a “0” or “1” is distinguished by which wire has a higher voltage (differential)
 - example: orientation of magnetization signifies a “0” or a “1”
- Primitive digital hardware devices
 - logic computation devices (sense and drive)
 - are two wires both “1” - make another be “1” (AND)
 - is at least one of two wires “1” - make another be “1” (OR)
 - is a wire “1” - then make another be “0” (NOT)
 - memory devices (store)
 - store a value
 - recall a previously stored value



What is happening now in digital design?

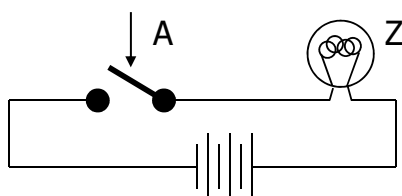
- Important trends in how industry does hardware design
 - larger and larger designs
 - shorter and shorter time to market
 - cheaper and cheaper products
- Scale
 - pervasive use of computer-aided design tools over hand methods
 - multiple levels of design representation
- Time
 - emphasis on abstract design representations
 - programmable rather than fixed function components
 - automatic synthesis techniques
 - importance of sound design methodologies
- Cost
 - higher levels of integration
 - use of simulation to debug designs
 - simulate and verify before you build

Computation: abstract vs. implementation

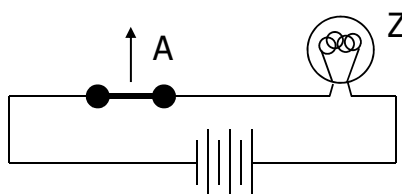
- Up to now, computation has been a mental exercise (paper, programs)
- This class is about physically implementing computation using physical devices that use voltages to represent logical values
- Basic units of computation are:
 - representation: "0", "1" on a wire
set of wires (e.g., for binary ints)
 - assignment: $x = y$
 - data operations: $x + y - 5$
 - control:
 - sequential statements: A; B; C
 - conditionals: if $x == 1$ then y
 - loops: for ($i = 1$; $i == 10$, $i++$)
 - procedures: A; proc(...); B;
- We will study how each of these are implemented in hardware and composed into computational structures

Switches: basic element of physical implementations

- Implementing a simple circuit (arrow shows action if wire changes to "1"):



close switch (if A is "1" or asserted)
and turn on light bulb (Z)

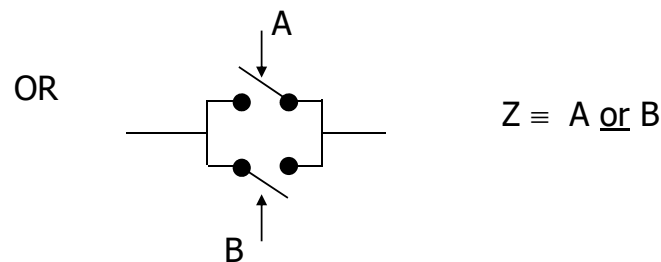
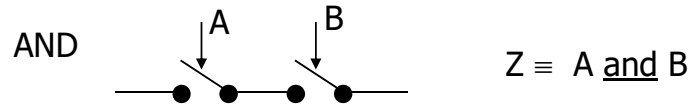


open switch (if A is "0" or unasserted)
and turn off light bulb (Z)

$$Z \equiv A$$

Switches (cont'd)

- Compose switches into more complex ones (Boolean functions):

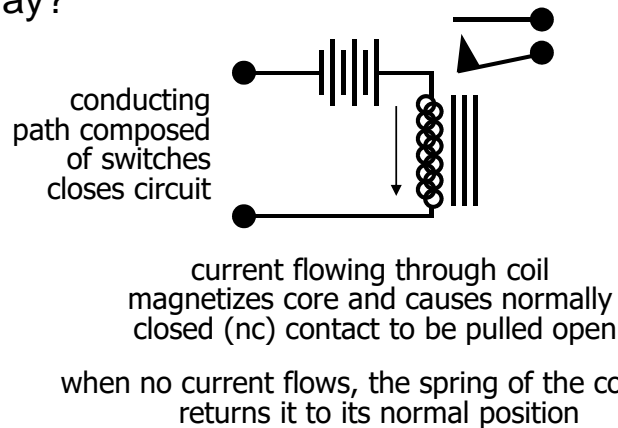


Switching networks

- Switch settings
 - determine whether or not a conducting path exists to light the light bulb
- To build larger computations
 - use a light bulb (output of the network) to set other switches (inputs to another network).
- Connect together switching networks
 - to construct larger switching networks, i.e., there is a way to connect outputs of one network to the inputs of the next.

Relay networks

- A simple way to convert between conducting paths and switch settings is to use (electro-mechanical) relays.
- What is a relay?



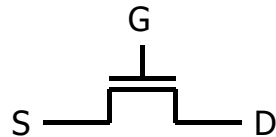
What determines the switching speed of a relay network?

Transistor networks

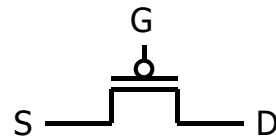
- Relays aren't used much anymore
 - ❑ some traffic light controllers are still electro-mechanical
- Modern digital systems are designed in CMOS technology
 - ❑ MOS stands for Metal-Oxide on Semiconductor
 - ❑ C is for complementary because there are both normally-open and normally-closed switches
- MOS transistors act as voltage-controlled switches
 - ❑ similar, though easier to work with than relays.

MOS transistors

- MOS transistors have three terminals: drain, gate, and source
 - they act as switches in the following way:
if the voltage on the gate terminal is (some amount) higher/lower than the source terminal then a conducting path will be established between the drain and source terminals

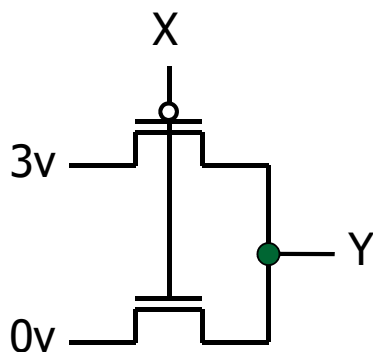


n-channel
open when voltage at G is low
closes when:
 $\text{voltage}(G) > \text{voltage}(S) + \varepsilon$



p-channel
closed when voltage at G is low
opens when:
 $\text{voltage}(G) < \text{voltage}(S) - \varepsilon$

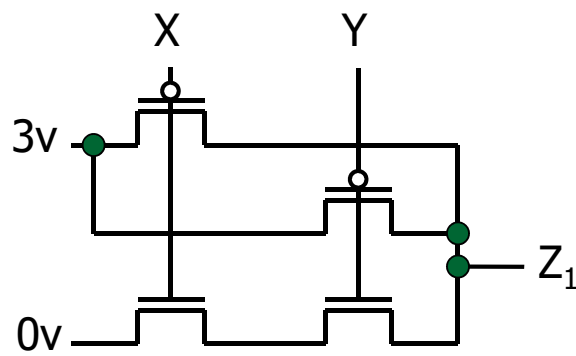
MOS networks



what is the
relationship
between x and y?

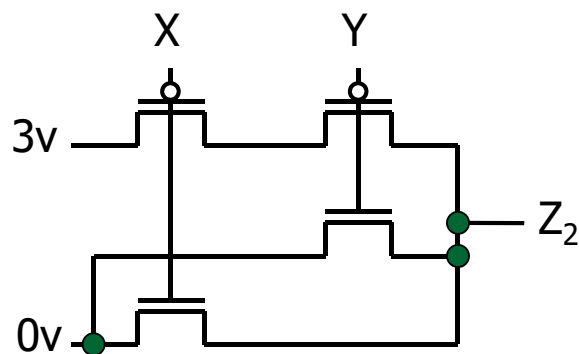
x	y
0 volts	
3 volts	

Two input networks



what is the relationship between x, y and z?

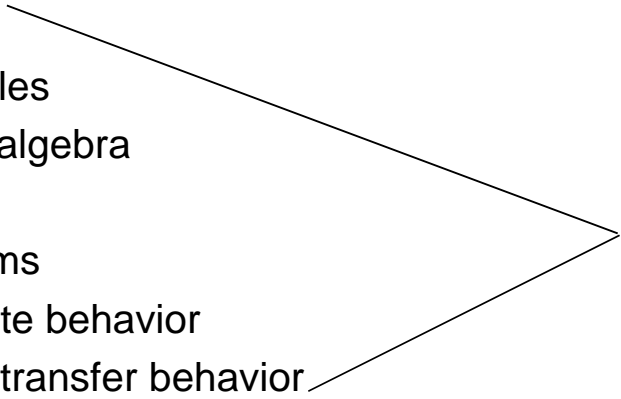
x	y	z1	z2
0 volts	0 volts		
0 volts	3 volts		
3 volts	0 volts		
3 volts	3 volts		



Speed of MOS networks

- What influences the speed of CMOS networks?
 - charging and discharging of voltages on wires and gates of transistors
- Capacitors hold charge
 - capacitance is at gates of transistors and wire material
- Resistors slow movement of electrons
 - resistance mostly due to transistors

Representation of digital designs

- Physical devices (transistors, relays)
 - Switches
 - Truth tables
 - Boolean algebra
 - Gates
 - Waveforms
 - Finite state behavior
 - Register-transfer behavior
 - Concurrent abstract specifications
- 
- scope of CSE 370

Digital vs. analog

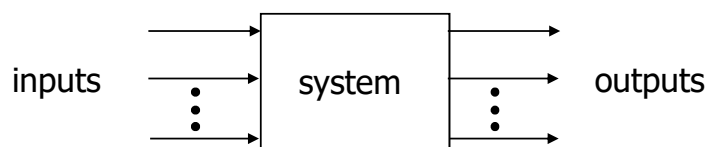
- Convenient to think of digital systems as having only discrete, digital, input/output values
- In reality, real electronic components exhibit continuous, analog, behavior
- Why do we make the digital abstraction anyway?
 - switches operate this way
 - easier to think about a small number of discrete values
- Why does it work?
 - does not propagate small errors in values
 - always resets to 0 or 1

Mapping from physical world to binary world

Technology	State 0	State 1
Relay logic	Circuit Open	Circuit Closed
CMOS logic	0.0-1.0 volts	2.0-3.0 volts
Transistor transistor logic (TTL)	0.0-0.8 volts	2.0-5.0 volts
Fiber Optics	Light off	Light on
Dynamic RAM	Discharged capacitor	Charged capacitor
Nonvolatile memory (erasable)	Trapped electrons	No trapped electrons
Programmable ROM	Fuse blown	Fuse intact
Bubble memory	No magnetic bubble	Bubble present
Magnetic disk	No flux reversal	Flux reversal
Compact disc	No pit	Pit

Combinational vs. sequential digital circuits

- A simple model of a digital system is a unit with inputs and outputs:



- Combinational means "memory-less"
 - a digital circuit is combinational if its output values only depend on its input values

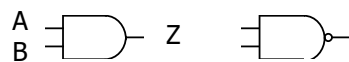
Combinational logic symbols

- Common combinational logic systems have standard symbols called logic gates

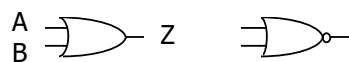
- Buffer, NOT



- AND, NAND



- OR, NOR



easy to implement
with CMOS transistors
(the switches we have
available and use most)

Sequential logic

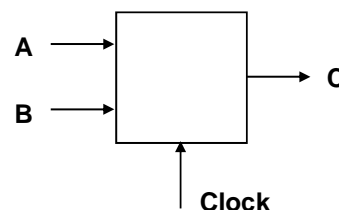
- Sequential systems
 - exhibit behaviors (output values) that depend not only on the current input values, but also on previous input values
- In reality, all real circuits are sequential
 - because the outputs do not change instantaneously after an input change
 - why not, and why is it then sequential?
- A fundamental abstraction of digital design is to reason (mostly) about steady-state behaviors
 - look at the outputs only after sufficient time has elapsed for the system to make its required changes and settle down

Synchronous sequential digital systems

- Outputs of a combinational circuit depend only on current inputs
 - ❑ after sufficient time has elapsed
- Sequential circuits have memory
 - ❑ even after waiting for the transient activity to finish
- The steady-state abstraction is so useful that most designers use a form of it when constructing sequential circuits:
 - ❑ the memory of a system is represented as its state
 - ❑ changes in system state are only allowed to occur at specific times controlled by an external periodic clock
 - ❑ the clock period is the time that elapses between state changes it must be sufficiently long so that the system reaches a steady-state before the next state change at the end of the period

Example of combinational and sequential logic

- Combinational:
 - ❑ input A, B
 - ❑ wait for clock edge
 - ❑ observe C
 - ❑ wait for another clock edge
 - ❑ observe C again: will stay the same
- Sequential:
 - ❑ input A, B
 - ❑ wait for clock edge
 - ❑ observe C
 - ❑ wait for another clock edge
 - ❑ observe C again: may be different



Abstractions

- Some we've seen already
 - ❑ digital interpretation of analog values
 - ❑ transistors as switches
 - ❑ switches as logic gates
 - ❑ use of a clock to realize a synchronous sequential circuit
- Some others we will see
 - ❑ truth tables and Boolean algebra to represent combinational logic
 - ❑ encoding of signals with more than two logical values into binary form
 - ❑ state diagrams to represent sequential logic
 - ❑ hardware description languages to represent digital logic
 - ❑ waveforms to represent temporal behavior

An example

- Calendar subsystem: number of days in a month (to control watch display)
 - ❑ used in controlling the display of a wrist-watch LCD screen
 - ❑ inputs: month, leap year flag
 - ❑ outputs: number of days

Implementation in software

```
integer number_of_days ( month, leap_year_flag)
{
  switch (month) {
    case 1: return (31);
    case 2: if (leap_year_flag == 1) then return (29)
            else return (28);

    case 3: return (31);
    ...
    case 12: return (31);
    default: return (0);
  }
}
```

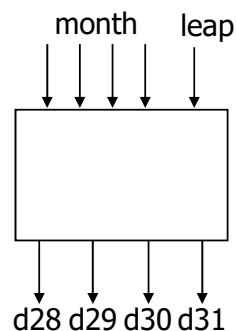
Implementation as a combinational digital system

■ Encoding:

- how many bits for each input/output?
- binary number for month
- four wires for 28, 29, 30, and 31

■ Behavior:

- combinational
- truth table specification



month	leap	d28	d29	d30	d31
0000	—	—	—	—	—
0001	—	0	0	0	1
0010	0	1	0	0	0
0010	1	0	1	0	0
0011	—	0	0	0	1
0100	—	0	0	1	0
0101	—	0	0	0	1
0110	—	0	0	1	0
0111	—	0	0	0	1
1000	—	0	0	0	1
1001	—	0	0	1	0
1010	—	0	0	0	1
1011	—	0	0	1	0
1100	—	0	0	0	1
1101	—	—	—	—	—
111—	—	—	—	—	—

Combinational example (cont'd)

- Truth-table to logic to switches to gates

- $d_{28} = 1$ when month=0010 and leap=0

- $d_{28} = m_8' \cdot m_4' \cdot m_2 \cdot m_1' \cdot \text{leap}'$

- $d_{31} = 1$ when month=0001 or month=0011 or ... month=1100

- $d_{31} = (m_8' \cdot m_4' \cdot m_2' \cdot m_1) + (m_8' \cdot m_4' \cdot m_2 \cdot m_1) + \dots$
 $(m_8 \cdot m_4 \cdot m_2' \cdot m_1')$

- $d_{31} =$ can we simplify more?

symbol
for and

symbol
for or

symbol
for not

month	leap	d28	d29	d30	d31
0001	–	0	0	0	1
0010	0	1	0	0	0
0010	1	0	1	0	0
0011	–	0	0	0	1
0100	–	0	0	1	0
...					
1100	–	0	0	0	1
1101	–	–	–	–	–
111–	–	–	–	–	–
0000	–	–	–	–	–

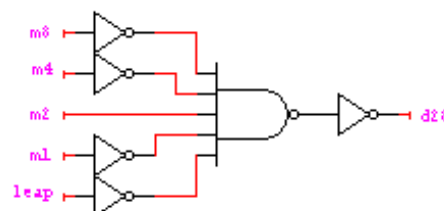
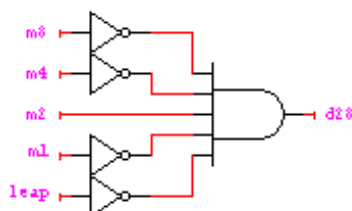
Combinational example (cont'd)

- $d_{28} = m_8' \cdot m_4' \cdot m_2 \cdot m_1' \cdot \text{leap}'$

- $d_{29} = m_8' \cdot m_4' \cdot m_2 \cdot m_1' \cdot \text{leap}$

- $d_{30} = (m_8' \cdot m_4 \cdot m_2' \cdot m_1') + (m_8' \cdot m_4 \cdot m_2 \cdot m_1') +$
 $(m_8 \cdot m_4' \cdot m_2' \cdot m_1) + (m_8 \cdot m_4' \cdot m_2 \cdot m_1)$
 $= (m_8' \cdot m_4 \cdot m_1') + (m_8 \cdot m_4' \cdot m_1)$

- $d_{31} = (m_8' \cdot m_4' \cdot m_2' \cdot m_1) + (m_8' \cdot m_4' \cdot m_2 \cdot m_1) +$
 $(m_8' \cdot m_4 \cdot m_2' \cdot m_1) + (m_8' \cdot m_4 \cdot m_2 \cdot m_1) +$
 $(m_8 \cdot m_4' \cdot m_2' \cdot m_1') + (m_8 \cdot m_4' \cdot m_2 \cdot m_1') +$
 $(m_8 \cdot m_4 \cdot m_2' \cdot m_1')$

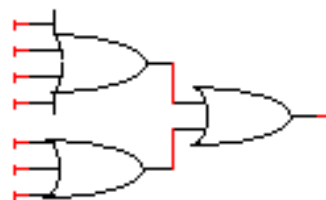
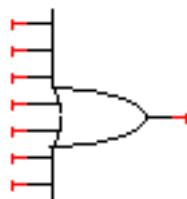


Activity

- How much can we simplify d31?
- What if we started the months with 0 instead of 1?
(i.e., January is 0000 and December is 1011)

Combinational example (cont'd)

- $d28 = m8' \bullet m4' \bullet m2 \bullet m1' \bullet \text{leap}'$
- $d29 = m8' \bullet m4' \bullet m2 \bullet m1' \bullet \text{leap}$
- $d30 = (m8' \bullet m4 \bullet m2' \bullet m1') + (m8' \bullet m4 \bullet m2 \bullet m1') + (m8 \bullet m4' \bullet m2' \bullet m1) + (m8 \bullet m4' \bullet m2 \bullet m1)$
- $d31 = (m8' \bullet m4' \bullet m2' \bullet m1) + (m8' \bullet m4' \bullet m2 \bullet m1) + (m8' \bullet m4 \bullet m2' \bullet m1) + (m8' \bullet m4 \bullet m2 \bullet m1) + (m8 \bullet m4' \bullet m2' \bullet m4') + (m8 \bullet m4' \bullet m2 \bullet m1') + (m8 \bullet m4 \bullet m2' \bullet m1')$



Another example

- Door combination lock:
 - punch in 3 values in sequence and the door opens; if there is an error the lock must be reset; once the door opens the lock must be reset
 - inputs: sequence of input values, reset
 - outputs: door open/close
 - memory: must remember combination
or always have it available as an input

Implementation in software

```
integer combination_lock ( ) {
    integer v1, v2, v3;
    integer error = 0;
    static integer c[3] = 3, 4, 2;

    while (!new_value( ));
    v1 = read_value( );
    if (v1 != c[1]) then error = 1;

    while (!new_value( ));
    v2 = read_value( );
    if (v2 != c[2]) then error = 1;

    while (!new_value( ));
    v3 = read_value( );
    if (v3 != c[3]) then error = 1;

    if (error == 1) then return(0); else return (1);
}
```

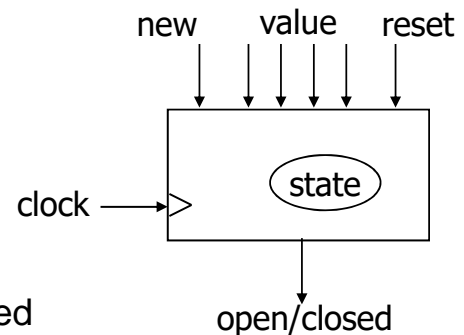
Implementation as a sequential digital system

■ Encoding:

- how many bits per input value?
- how many values in sequence?
- how do we know a new input value is entered?
- how do we represent the states of the system?

■ Behavior:

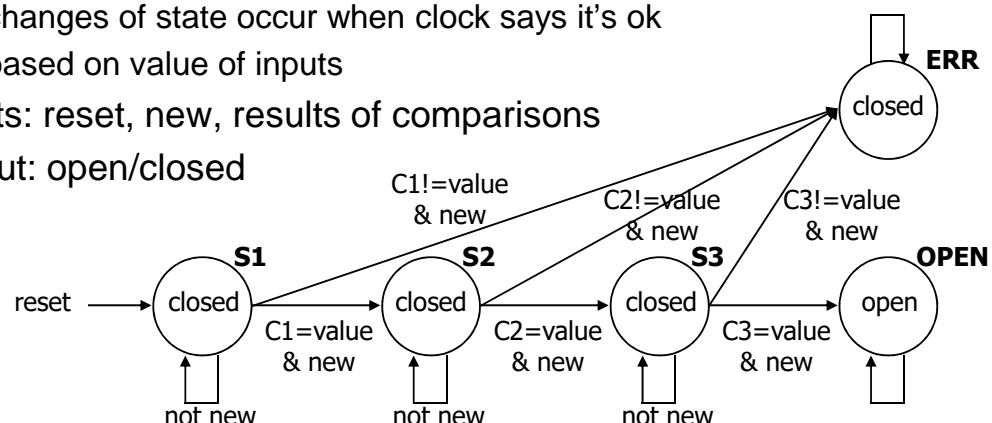
- clock wire tells us when it's ok to look at inputs (i.e., they have settled after change)
- sequential: sequence of values must be entered
- sequential: remember if an error occurred
- finite-state specification



Sequential example (cont'd): abstract control

■ Finite-state diagram

- states: 5 states
 - represent point in execution of machine
 - each state has outputs
- transitions: 6 from state to state, 5 self transitions, 1 global
 - changes of state occur when clock says it's ok
 - based on value of inputs
- inputs: reset, new, results of comparisons
- output: open/closed



Sequential example (cont'd): data-path vs. control

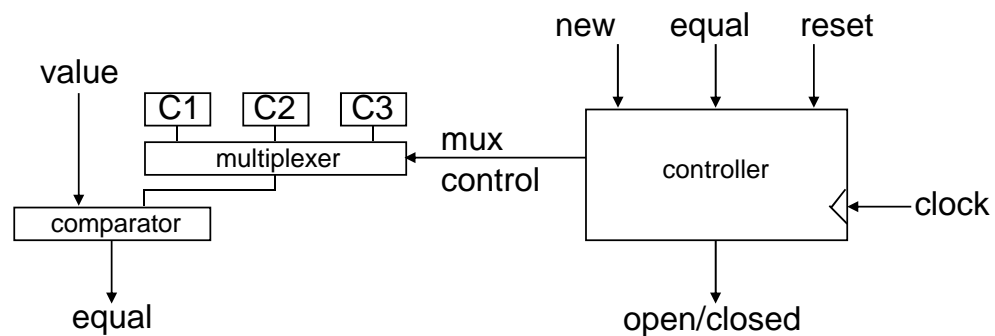
■ Internal structure

□ data-path

- storage for combination
- comparators

□ control

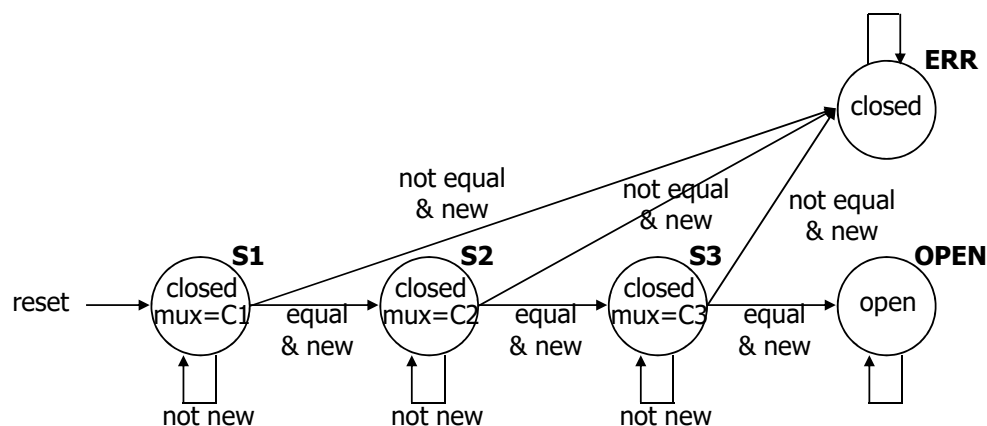
- finite-state machine controller
- control for data-path
- state changes controlled by clock



Sequential example (cont'd): finite-state machine

■ Finite-state machine

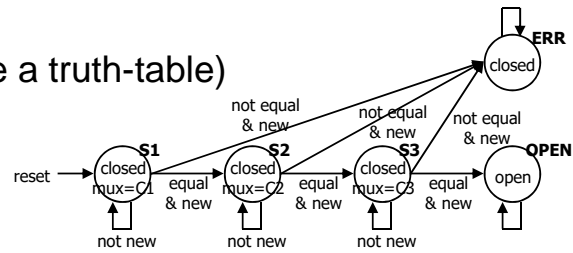
- refine state diagram to include internal structure



Sequential example (cont'd): finite-state machine

■ Finite-state machine

- generate state table (much like a truth-table)



reset	new	equal	state	next state	mux	open/closed
1	—	—	—	S1	C1	closed
0	0	—	S1	S1	C1	closed
0	1	0	S1	ERR	—	closed
0	1	1	S1	S2	C2	closed
0	0	—	S2	S2	C2	closed
0	1	0	S2	ERR	—	closed
0	1	1	S2	S3	C3	closed
0	0	—	S3	S3	C3	closed
0	1	0	S3	ERR	—	closed
0	1	1	S3	OPEN	—	open
0	—	—	OPEN	OPEN	—	open
0	—	—	ERR	ERR	—	closed

Sequential example (cont'd): encoding

■ Encode state table

- state can be: S1, S2, S3, OPEN, or ERR
 - needs at least 3 bits to encode: 000, 001, 010, 011, 100
 - and as many as 5: 00001, 00010, 00100, 01000, 10000
 - choose 4 bits: 0001, 0010, 0100, 1000, 0000
- output mux can be: C1, C2, or C3
 - needs 2 to 3 bits to encode
 - choose 3 bits: 001, 010, 100
- output open/closed can be: open or closed
 - needs 1 or 2 bits to encode
 - choose 1 bits: 1, 0

Sequential example (cont'd): encoding

■ Encode state table

- state can be: S1, S2, S3, OPEN, or ERR
 - choose 4 bits: 0001, 0010, 0100, 1000, 0000
- output mux can be: C1, C2, or C3
 - choose 3 bits: 001, 010, 100
- output open/closed can be: open or closed
 - choose 1 bits: 1, 0

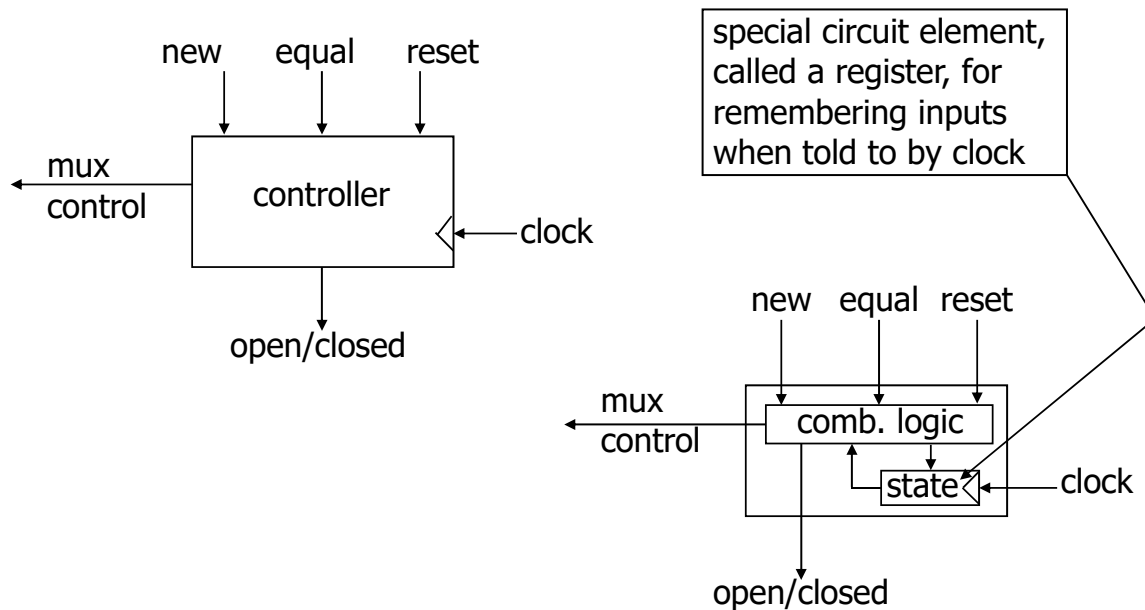
reset	new	equal	state	next state	mux	open/closed	
1	—	—	—	0001	001	0	
0	0	—	0001	0001	001	0	
0	1	0	0001	0000	—	0	good choice of encoding!
0	1	1	0001	0010	010	0	
0	0	—	0010	0010	010	0	
0	1	0	0010	0000	—	0	mux is identical to last 3 bits of state
0	1	1	0010	0100	100	0	
0	0	—	0100	0100	100	0	
0	1	0	0100	0000	—	0	open/closed is identical to first bit of state
0	1	1	0100	1000	—	1	
0	—	—	1000	1000	—	1	
0	—	—	0000	0000	—	0	

Activity

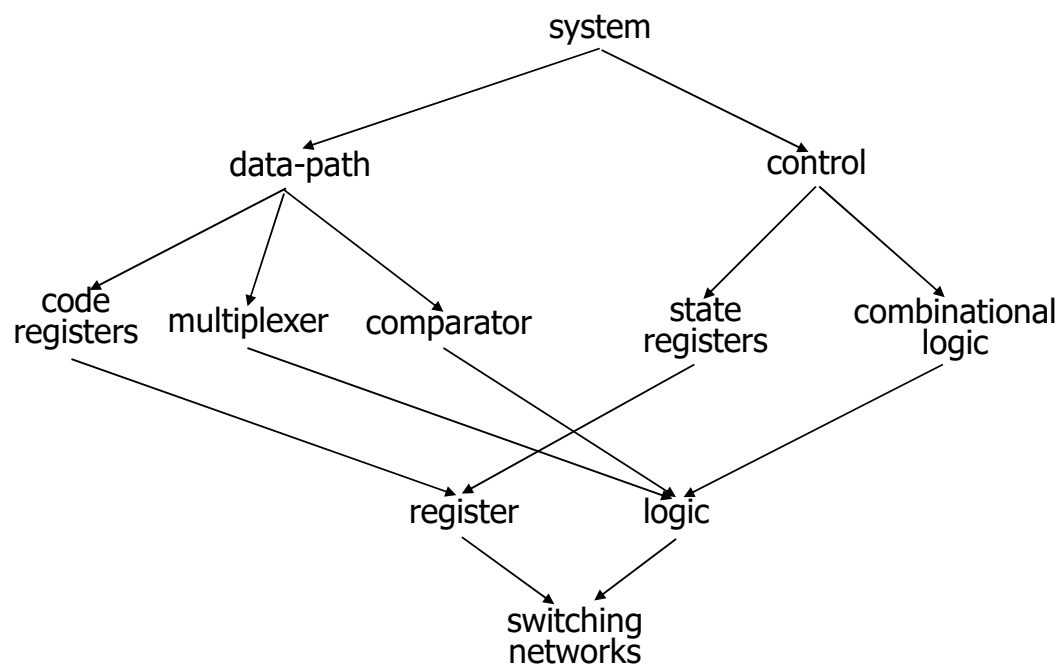
- Have lock always wait for 3 key presses exactly before making a decision

Sequential example (cont'd): controller implementation

■ Implementation of the controller



Design hierarchy



Summary

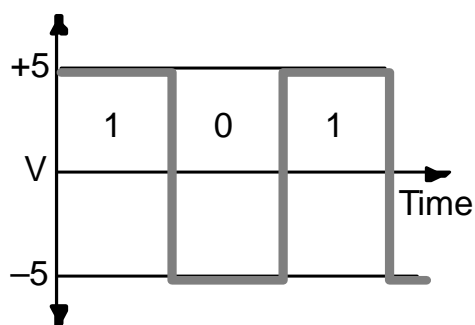
- That was what the entire course is about
 - converting solutions to problems into combinational and sequential networks effectively organizing the design hierarchically
 - doing so with a modern set of design tools that lets us handle large designs effectively
 - taking advantage of optimization opportunities
- Now lets do it again
 - this time we'll take nine weeks instead of one

Digital Hardware Systems

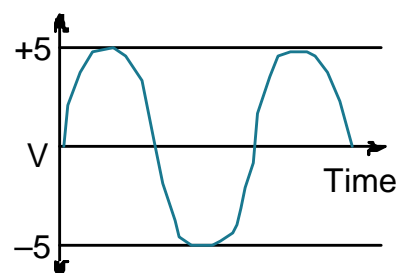
Contemporary Logic Design
Introduction

Digital Systems

Digital vs. Analog Waveforms



Digital:
only assumes discrete values



Analog:
values vary over a broad range
continuously

Digital Hardware Systems

Advantages of Digital Systems

Analog systems: slight error in input yields large error in output

Digital systems more accurate and reliable

Readily available as self-contained, easy to cascade building blocks

Computers use digital circuits internally

Interface circuits (i.e., sensors & actuators) often analog

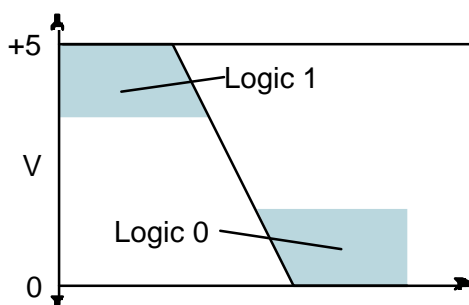
This course is about logic design, not system design (processor architecture), not circuit design (transistor level)

Digital Hardware Systems

The Real World

Physical electronic components are continuous, not discrete!

These are the building blocks of all digital components!



Transition from logic 1 to logic 0 does not take place instantaneously in real digital systems

Intermediate values may be visible for an instant

Boolean algebra useful for describing the steady state behavior of digital systems

Be aware of the dynamic, time varying behavior too!

Digital Circuit Technologies

Integrated circuit technology

choice of conducting, non-conducting, sometimes conducting ("semiconductor") materials

whether or not their interaction allows electrons to flow forms the basis for electrically controlled switches

Main technologies

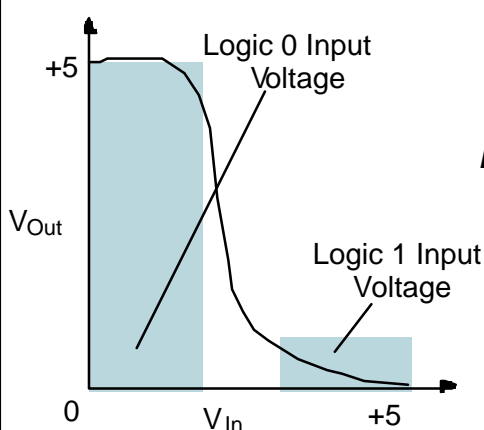
MOS: Metal-Oxide-Silicon

Bipolar

Transistor-Transistor Logic
Emitter Coupled Logic

Digital Hardware Systems

Circuit that implements logical negation (NOT)



1 at input yields 0 at output
0 at input yields 1 at output

Inverter behavior as a function of input voltage
input ramps from 0V to 5V
output holds at 5V for some range of small input voltages
then changes rapidly, but not instantaneously!

remember distinction between
steady state and dynamic behavior

Representations of a Digital Design

Truth Tables

tabulate all possible input combinations and their associated output values

Example: half adder
adds two binary digits
to form Sum and Carry

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

NOTE: 1 plus 1 is 0 with a carry of 1 in binary

Example: full adder
adds two binary digits and
Carry in to form Sum and
Carry Out

A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Representations of a Digital Design

Boolean Algebra

values: 0, 1

variables: A, B, C, . . . , X, Y, Z

operations: NOT, AND, OR, . . .

NOT X is written as \bar{X}

X AND Y is written as X & Y, or sometimes X Y

X OR Y is written as X + Y

Deriving Boolean equations from truth tables:

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$\text{Sum} = \bar{A} B + A \bar{B}$$

OR'd together *product* terms
for each truth table
row where the function is 1

if input variable is 0, it appears in
complemented form;
if 1, it appears uncomplemented

$$\text{Carry} = A B$$

Representations of a Digital Design: Boolean Algebra

Another example:

A	B	Cin	Sum	Cout	
0	0	0	0	0	
0	0	1	1	0	
0	1	0	1	0	
0	1	1	0	1	
1	0	0	1	0	
1	0	1	0	1	
1	1	0	0	1	
1	1	1	1	1	

$$\text{Sum} = \bar{A} \bar{B} \text{Cin} + \bar{A} B \bar{\text{Cin}} + A \bar{B} \bar{\text{Cin}} + A B \text{Cin}$$

$$\text{Cout} = \bar{A} B \text{Cin} + A \bar{B} \text{Cin} + A B \bar{\text{Cin}} + A B \text{Cin}$$

Representations of a Digital Design: Boolean Algebra

Reducing the complexity of Boolean equations

Laws of Boolean algebra can be applied to full adder's carry out function to derive the following simplified expression:

$$\text{Cout} = A \text{Cin} + B \text{Cin} + A B$$

	A	B	Cin	Cout
	0	0	0	0
	0	0	1	0
	0	1	0	0
	0	1	1	1
B Cin	1	0	0	0
	1	0	1	1
A Cin	1	1	0	1
	1	1	1	1
A B	1	1	1	1

Verify equivalence with the original Carry Out truth table:

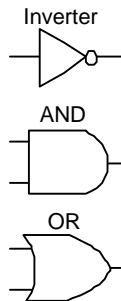
place a 1 in each truth table row where the product term is true

each product term in the above equation covers exactly two rows in the truth table; several rows are "covered" by more than one term

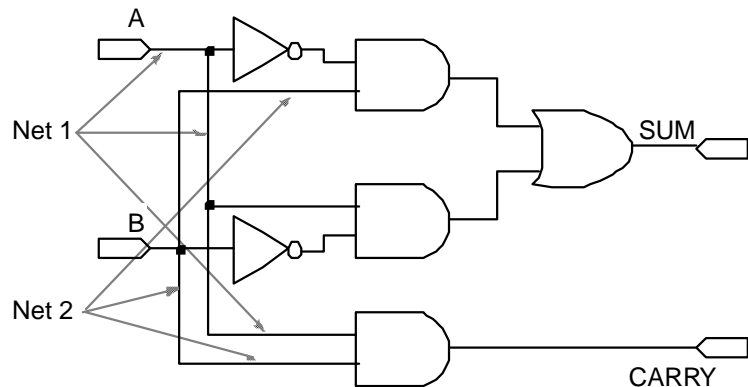
Gates

most widely used primitive building block in digital system design

Standard Logic Gate Representation



Half Adder Schematic

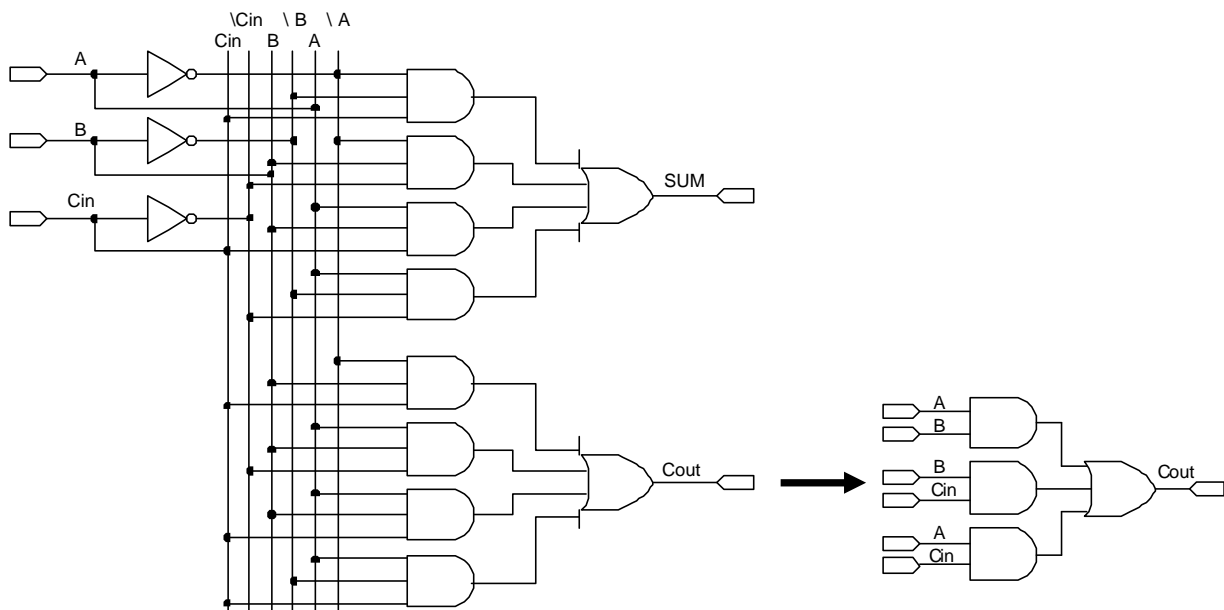


Net: electrically connected collection of wires

Netlist: tabulation of gate inputs & outputs and the nets they are connected to

Representations of a Digital Design: Gates

Full Adder Schematic



Fan-in: number of inputs to a gate

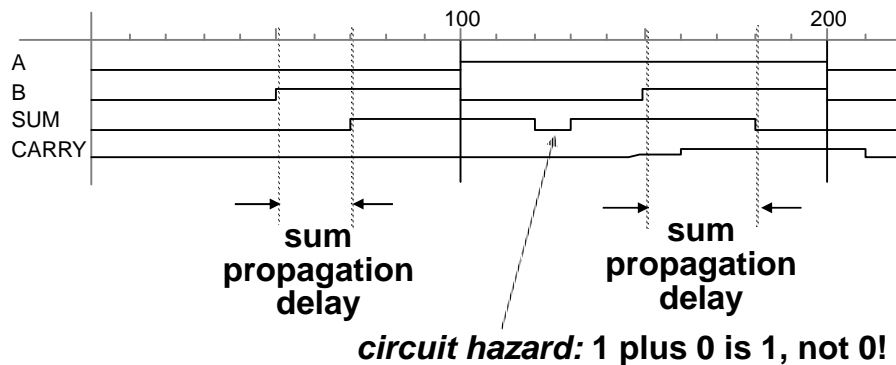
Fan-out: number of gate inputs an output is connected to

Technology "Rules of Composition" place limits on fan-in/fan-out

Waveforms

dynamic behavior of a circuit
real circuits have non-zero delays

Timing Diagram of the Half Adder



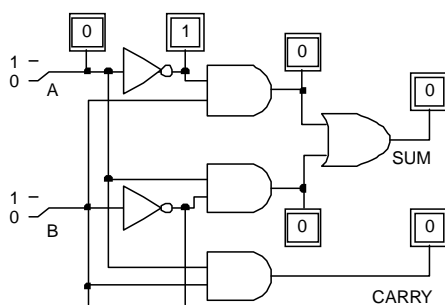
Output changes are delayed from input changes

The propagation delay is sensitive to paths in the circuit

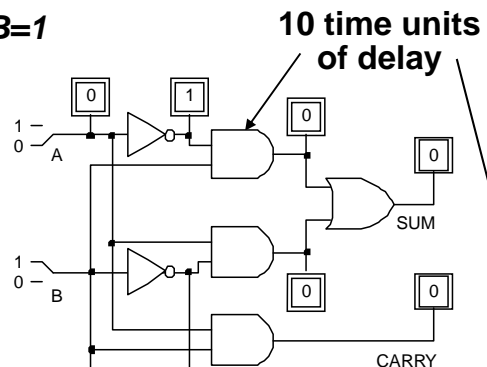
Outputs may temporarily change from the correct value to the wrong value back again to the correct value: this is called a *glitch* or *hazard*

Representations of a Digital Design: Waveforms

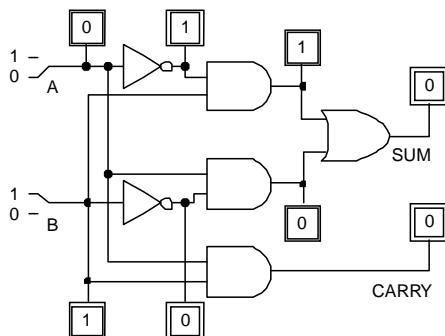
Tracing the Delays: A=0,B=0 to A=0,B=1



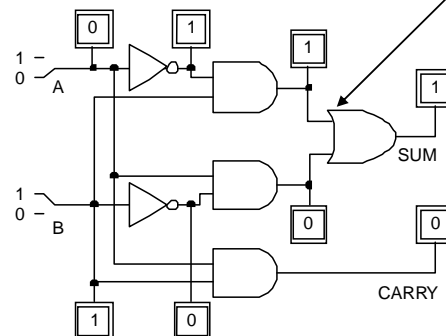
(i) Initial conditions



(ii) Y changes from 0 to 1



(iii) Output of top AND gate changes after 10 time units



(iv) Output of OR gate changes after 10 time units

Representations of a Digital Design

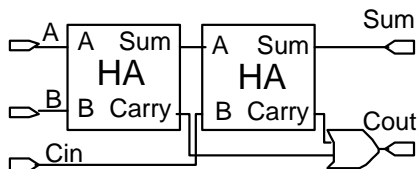
Blocks

structural organization of the design

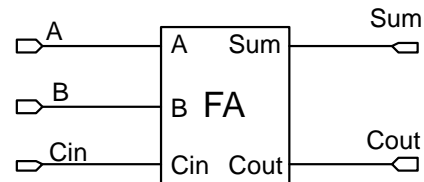
black boxes with input and output connections

corresponds to well defined functions

concentrates on how the components are composed by wiring



Full Adder realized in terms of composition of half adder blocks

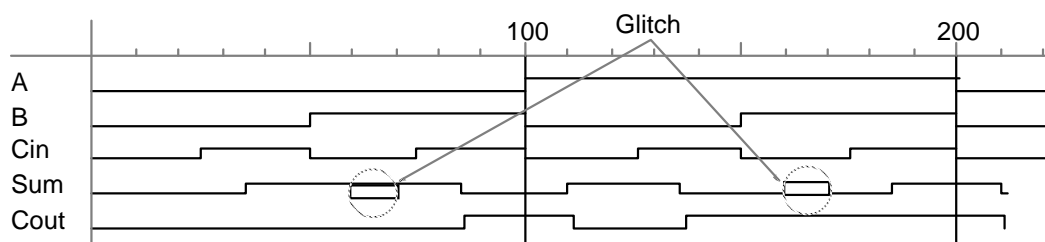


Block diagram representation of the Full Adder

Representations of a Digital Design

Waveform Verification

Does the composed full adder behave the same as the full gate implementation?



Sum, Cout waveforms lag input changes in time

How many time units after input change is it safe to examine the outputs?

Representations of a Digital Design

Behaviors

Hardware description languages
structure and function of the digital design

Example: Half Adder in VHDL

```
-- ***** inverter gate model *****
-- external ports
ENTITY inverter_gate;
  PORT (a: IN BIT; z: OUT BIT);
END inverter_gate;
```

Black Box View
as seen by outside
world

```
-- internal behavior
ARCHITECTURE behavioral OF inverter_gate IS
BEGIN
  z <= NOT a AFTER 10 ns;
END behavioral;
```

Internal Behavior
Note delay statement

```
-- ***** and gate model *****
-- external ports
ENTITY and_gate;
  PORT (a, b: IN BIT; z: OUT BIT);
END and_gate;
```

```
-- internal behavior
ARCHITECTURE behavioral OF and_gate IS
BEGIN
  z <= a AND b AFTER 10 ns;
END behavioral;
```

Representation of a Digital Design: Behaviors

```
-- ***** or gate model *****
-- external ports
ENTITY or_gate;
  PORT (a, b: IN BIT; z: OUT BIT);
END or_gate;
```

AND, OR, NOT models
typically included in a
library

```
-- internal behavior
ARCHITECTURE behavioral OF or_gate IS
BEGIN
  z <= a OR b AFTER 10 ns;
END behavioral;
```

```
-- ***** half adder model *****
-- external ports
ENTITY half_adder;
  PORT (a_in, b_in: INPUT; sum, c_out: OUTPUT);
END half_adder;
```

```
-- internal structure
ARCHITECTURE structural of half_adder IS
  -- component types to use
  COMPONENT inverter_gate
    PORT (a: IN BIT; z: OUT BIT); END COMPONENT;
  COMPONENT and_gate
    PORT (a, b: IN BIT; z: OUT BIT); END COMPONENT;
  COMPONENT or_gate
    PORT (a, b: IN BIT; z: OUT BIT); END COMPONENT;
```

Particular components
to be used within the
model of the half adder

```
-- internal signal wires
SIGNAL s1, s2, s3, s4: BIT;
```

Representation of a Digital Design: Behaviors

BEGIN

-- one line for each gate, describing its type and connections

i1: inverter_gate PORT MAP (a_in, s1);

i2: inverter_gate PORT MAP (b_in, s2);

a1: and_gate PORT MAP (b_in, s1, s3);

a2: and_gate PORT MAP (a_in, s2, s4);

o1: or_gate PORT MAP (s3, s4, sum);

END structural;

**Textual description
of the netlist**

This VHDL specification corresponds to the following labeled schematic

