$$\text{Objective}: \max(3x+4y)$$

$$\text{s.t.} \quad \begin{aligned} x+2y &\le 7 \\ 3x-y &\ge 0 \\ x-y &\le 2 \\ x,y &\ge 0 \end{aligned}$$

→

$z - 3x - 4y = 0$

$x + 2y + s_1 = 7$

$3x - y - s_2 + a = 0$

$x - y + s_3 = 2$

Phase I

Obj min(a) → W − a = 0

W	x	y	s_1	s_2	s_3	a	RHS	
1	-3	1	0	1	0	0	0	
0	1	2	1	0	0	0	7	7
0	3	-1	0	-1	0	1	0	0
0	1	-1	0	0	1	0	2	2

W − a = 0

W	x	y	s_1	s_2	s_3	a	RHS
1	0	0	0	0	0	1	0
0	0	$\frac{7}{3}$	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	7
0	1	$-\frac{1}{3}$	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0
0	0	$\frac{2}{3}$	0	$\frac{1}{3}$	1	$-\frac{1}{3}$	2

W = 0 → feasible solution

phase II W − a = 0

z	x	y	s_1	s_2	s_3	RHS	
1	-3	-4	0	0	0	0	
0	0	$\frac{7}{3}$	1	$\frac{1}{3}$	0	7	3
0	1	$-\frac{1}{3}$	0	$-\frac{1}{3}$	0	0	
0	0	$-\frac{2}{3}$	0	$\frac{1}{3}$	1	2	

z	x	y	s_1	s_2	s_3	RHS	
1	0	0	$\frac{15}{7}$	$-\frac{2}{7}$	0	15	
0	0	1	$\frac{3}{7}$	$\frac{1}{7}$	0	3	21
0	1	0	$\frac{1}{7}$	$-\frac{2}{7}$	0	1	
0	0	0	$\frac{2}{7}$	$\frac{3}{7}$	1	4	$\frac{28}{3} \approx 9.33$

z	x	y	s_1	s_2	s_3	RHS
1	0	0	$\frac{49}{21}$	0	0	$\frac{53}{3}$ ✓
0	0	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{5}{3}$
0	1	0	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{11}{3}$
0	0	0	$\frac{2}{3}$	1	$\frac{7}{3}$	$\frac{28}{3}$

$\max(3x+4y) = \frac{53}{3}$

$x = \frac{11}{3}, \; y = \frac{5}{3}$

Problem 6: Hamtaro factory (part 2)

After finding the recipe for the Hamtaro snack, he then starts hiring a worker to work for his sweatshop. Initially, he has 50 hamster workers in the factory. However, due to substandard working conditions, 10% of the worker ~~die~~ resign every month. Despite that, Hamtaro does not care about this problem and just hire new workers to fulfill the factory's demand. Before working in the factory, the newly hired hamster has to undergo training for one month to become a skilled worker, of which 40% of the hamsters dropped out before the training finishes as they realize how terrible the Hamtaro factory is. The salary for each hamster worker is 8,000 THB per month, and it cost 500 THB to train each hamster. As Hamtaro predicted the number of required workers each month, how many hamsters should he hire each month to satisfy the factory's demand? Formulate the problem as a linear program and solve for an optimal solution.

Month	1	2	3	4	5	6
Amount of required factory worker	40	60	80	40	100	90

Note : The optimal solution does not have to be an integer.

Sets and Parameters

I : set of month $= \{1,2,3,4,5,6\}$

m_i : # required factory worker in month i ; $i \in I$

Decision variables

x_i = # hiring in month i ; $i \in I$

เดือน	1	2	3	4	5	6
จน.พนักงาน	50	$50 \cdot 0.9 + 0.6 \cdot x_1$	$50 \cdot 0.9^2 + 0.6x_1 \cdot 0.9 + 0.6x_2$	$50 \cdot 0.9^3 + 0.6 \cdot 0.9^2 x_1 + 0.6 \cdot 0.9 x_2 + 0.6x_3$	$50 \cdot 0.9^4 + 0.6\ 0.9^3 x_1 + 0.6 \cdot 0.9^2 x_2 + 0.6 \cdot 0.9 x_3 + 0.6x_4$	〃
สมัคร	x_1	x_2	x_3	x_4	x_5	

ค่าใช้จ่าย $\underbrace{[50 \cdot 8000 + 500x_1]}_{1} + \underbrace{[8000(50 \cdot 0.9 + 0.6x_1) + 500x_2]}_{2} + \underbrace{[8000(50 \cdot 0.9^2 + 0.6 \cdot 0.9 \cdot x_1 + 0.6x_2) + 500x_3]}_{3}$

$+ \underbrace{[8000(50 \cdot 0.9^3 + 0.6x_1 \cdot 0.9^2 + 0.6x_2 \cdot 0.9 + 0.6x_3) + 500x_4]}_{4} + \underbrace{[8000(50 \cdot 0.9^4 + 0.6\ 0.9^3 x_1 + 0.6 \cdot 0.9^2 x_2 + 0.6 \cdot 0.9 x_3 + 0.6x_4) + 500x_5]}_{5}$

$+ \underbrace{[8000(50 \cdot 0.9^5 + 0.6 \cdot 0.9^4 x_1 + 0.6 \cdot 0.9^3 x_2 + 0.6 \cdot 0.9^2 x_3 + 0.6 \cdot 0.9 x_4 + 0.6x_5)]}_{6}$

Obj $\rightarrow$ $\text{Min}(20156.48x_1 + 17007.2x_2 + 13508x_3 + 9620x_4 + 5300x_5) + 1874236$

S.T.

$50 \cdot 0.9 + 0.6x_1 \geq 60$

$50 \cdot 0.9^2 + 0.6 \cdot 0.9x_1 + 0.6x_2 \geq 80$

$50 \cdot 0.9^3 + 0.6 \cdot 0.9^2 x_1 + 0.6 \cdot 0.9x_2 + 0.6x_3 \geq 40$

$50\ 0.9^4 + 0.6 \cdot 0.9^3 x_1 + 0.6 \cdot 0.9^2 x_2 + 0.6 \cdot 0.9x_3 + 0.6x_4 \geq 100$

$50\ 0.9^5 + 0.6 \cdot 0.9^4 x_1 + 0.6 \cdot 0.9^3 x_2 + 0.6 \cdot 0.9^2 x_3 + 0.6 \cdot 0.9 \cdot x_4 + 0.6x_5 \geq 90$

$\rightarrow$

$0.6x_1 - S_1 = 15$

$0.6 \cdot 0.9x_1 + 0.6x_2 - S_2 = 39.5$

$0.6 \cdot 0.9^2 x_1 + 0.6 \cdot 0.9x_2 + 0.6x_3 - S_3 = 3.55$

$0.6 \cdot 0.9^3 x_1 + 0.6 \cdot 0.9^2 x_2 + 0.6 \cdot 0.9x_3 + 0.6x_4 - S_4 = 67.195$

$0.6 \cdot 0.9^4 x_1 + 0.6 \cdot 0.9^3 x_2 + 0.6 \cdot 0.9^2 x_3 + 0.6 \cdot 0.9 \cdot x_4 + 0.6x_5 - S_5 = 60.4755$

$S_i, x_i \geq 0 \ , \ \forall i \in I$

From colab, the optimal soln is 3679500