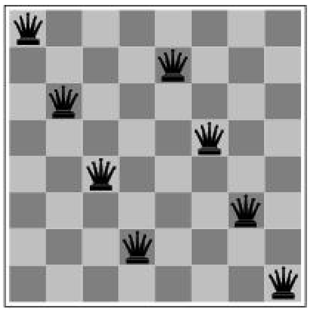
Local Search

Note: this material was originated from the slides provided by Prof. Padhraic Smyth

Local search and optimization

- Previously: systematic exploration of search space.
  - Path to goal is solution to problem
- But, for some problems path is irrelevant.
  - E.g 8-queens
- Different algorithms can be used
  - Local search
    ex) Hill-climbing



Local search and optimization

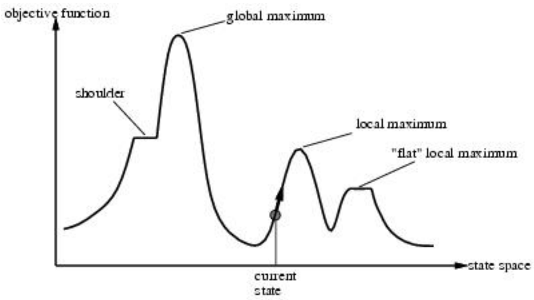
- Local search
  - Keep track of single current state
  - Move only to neighboring states
  - Ignore paths
- Advantages:
  - Use very little memory
  - Can often find reasonable solutions in large or infinite (continuous) state spaces.
- "Pure optimization" problems
  - Property 1: All states have an objective or heuristic function
  - Property 2: Goal is to find state with max (or min) objective value
  - Property 3: Does not quite fit into path-cost/goal-state formulation
  - Property 4: Local search can do quite well on these problems.

Hill-climbing search

- Very simple algorithm**:**
  - **continuously moves in the direction of increasing value**
  - terminates when a peak is reached
  - A kind of "greedy local best search"
- Value can be
  - Objective or heuristic function value
- Hill climbing does not look ahead of the immediate neighbors of the current state.
- Can randomly choose among the set of best successors, if multiple have the best value
- Characterized as "trying to find the top of Mount Everest while in a thick fog"

Hill climbing and local maxima

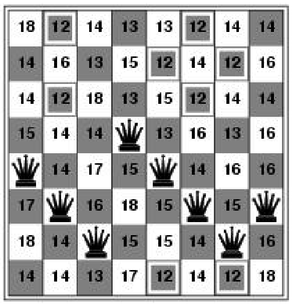
- When local maxima exist, hill climbing is suboptimal
- Simple (often effective) solution
  - Multiple random restarts



Hill-climbing example

- 8-queens problem, complete-state formulation
  - All 8 queens on the board in some configuration
- Successor function:
  - move a single queen to another square in the same column.
- Example of a heuristic function *h(n)*:
  - the number of pairs of queens that are attacking each other (directly or indirectly)
  - (so we want to minimize this)

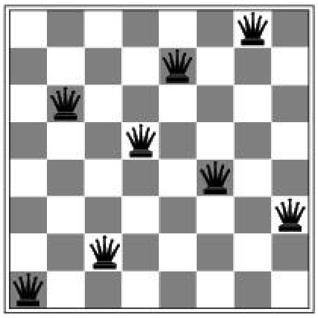
Hill-climbing example



Current state: h=17

Shown is the h-value for each possible successor in each column

A local minimum for 8-queens



A local minimum in the 8-queens state space (h=1)

Performance of hill-climbing on 8-queens

- Randomly generated 8-queens starting states…
- 14% the time it solves the problem
- 86% of the time it get stuck at a local minimum
- However…
  - Takes only 4 steps on average when it succeeds
  - And 3 on average when it gets stuck
  - (for a state space with ~17 million states)

Hill-climbing with random restarts

- Different variations
  - Run a fixed number N of restarts
  - For each restart: run until termination
- Analysis
  - Say each search has probability p of success
    - E.g., for 8-queens, p = 0.14 of success
  - Expected number n of restarts for over 99% success?

    Very Simple Interpretation (It may not be real !)

    $(0.86)^n < 0.01 \rightarrow n > \log_{0.86} 0.01 = 30.53\ldots$
  - Expected maximum number of steps taken?

    4 X 31 = 124 (for a state space with ~17 million states)