Uninformed Search

Note: this material was originated from the slides provided by Prof. Padhraic Smyth

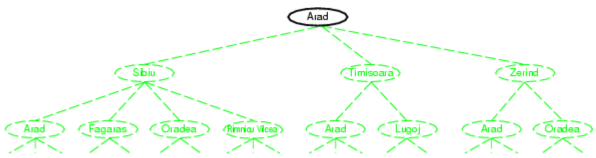
Search Algorithms

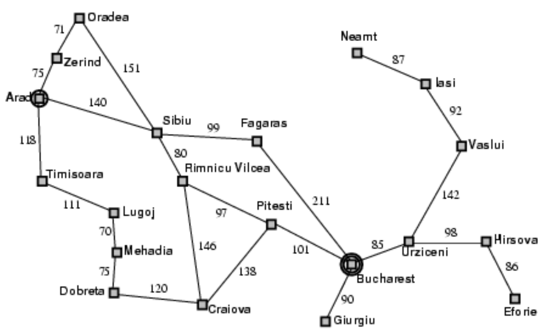
- Uninformed Blind search
  - Breadth-first
  - depth-first
  - Iterative deepening depth-first
  - uniform cost
- Informed Heuristic search
  - Greedy search, Heuristics, hill climbing,
- Important concepts:
  - Completeness
  - Time complexity
  - Space complexity
  - Quality of solution

Tree-based Search

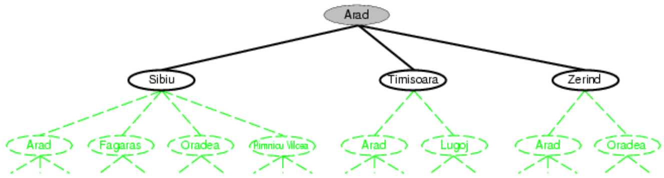
- Basic idea:
  - Exploration of state space by generating successors of already-explored states (a.k.a. expanding states).
  - Every state is evaluated: *is it a goal state*?

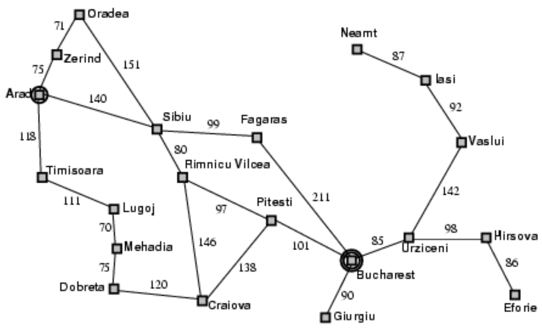
Tree search example





Tree search example





Search Tree for the 8 puzzle problem

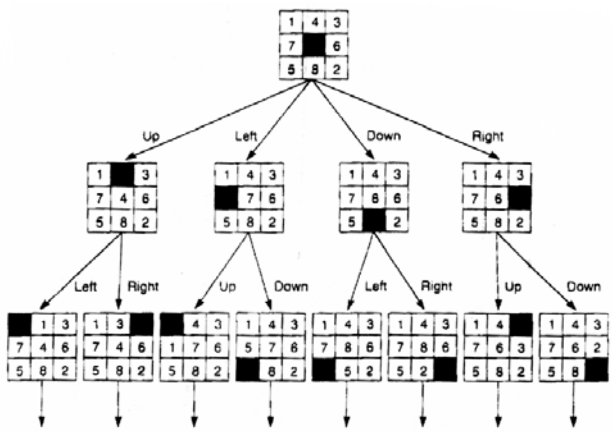


Figure 3.6 State space of the 8-puzzle generated by "move blank" operations.

Search Strategies

- A search strategy is defined by picking the order of node expansion

- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?

- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the least-cost solution
  - m: maximum depth of the state space (may be ∞)

Breadth-First Search (BFS)

- Expand shallowest unexpanded node

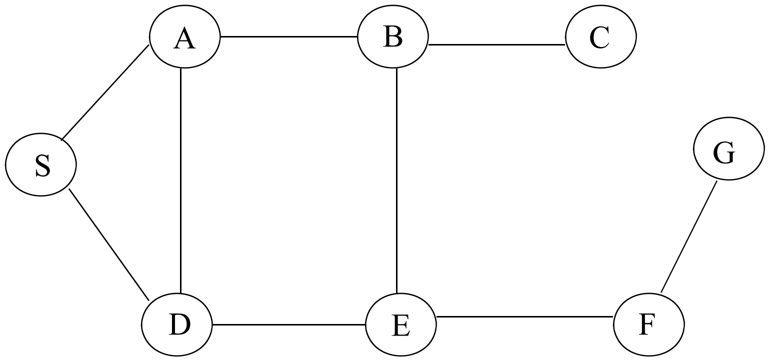
- Fringe: nodes waiting in a queue to be explored, also called OPEN

- Implementation:
  - For BFS, *fringe* is a first-in-first-out (FIFO) queue
  - new successors go at end of the queue

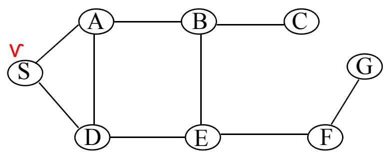
- Repeated states?
  - Simple strategy: do not add an already-expanded node to the queue
    do not expand an already-expanded node

Example: Map Navigation

State Space:
S = start, G = goal, other nodes = intermediate states, links = legal transitions



BFS Search Tree



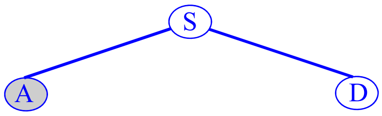
Queue = {S}

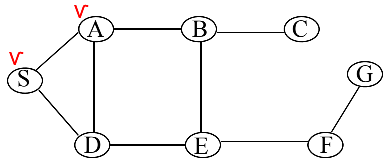
Select S

Goal(S) = true?

If not, Expand(S)

BFS Search Tree





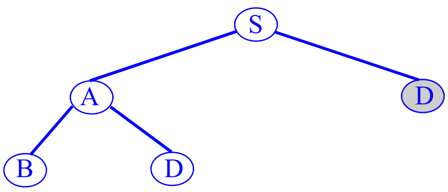
Queue = {A, D}

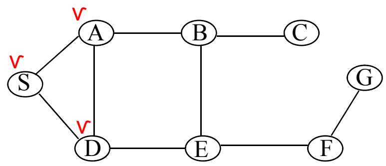
Select A

Goal(A) = true?

If not, Expand(A)

BFS Search Tree





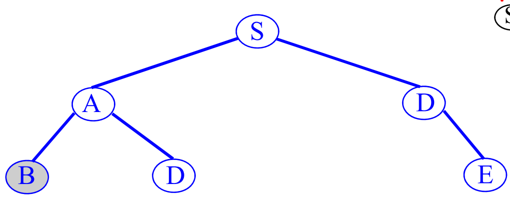
Queue = {D, B, D}

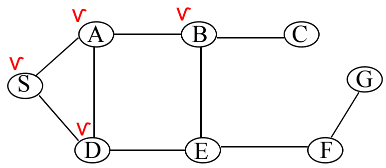
Select D

Goal(D) = true?

If not, expand(D)

BFS Search Tree





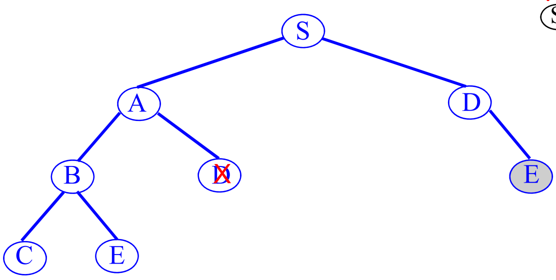
Queue = {B, D, E}

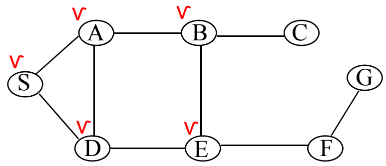
Select B

Goal(B) = true?

If not, expand(B)

BFS Search Tree





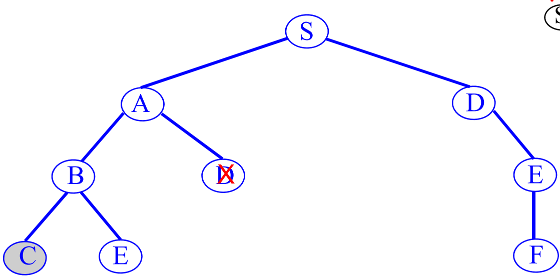
Queue = {~~D~~, E, C, E}

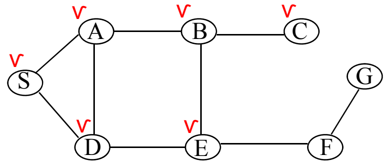
Select E (D already expanded)

Goal(E) = true?

If not, expand(E)

BFS Search Tree





Queue = {C, E, F}

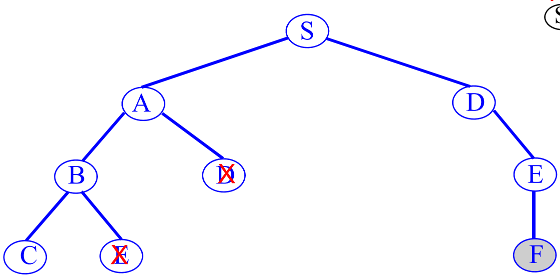
Select C

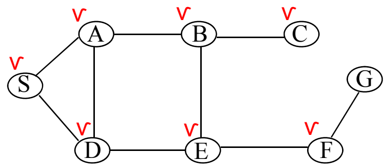
Goal(C) = true?

If not, expand(C)

(but nothing expanded)

BFS Search Tree





Queue = {~~E~~, F}

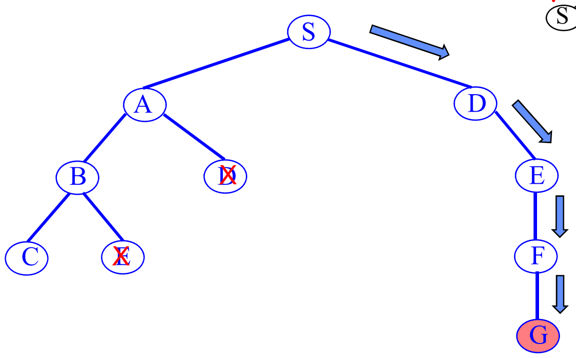
Select F (E already expanded)

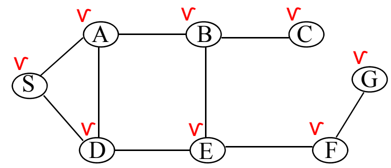
Goal(F) = true?

If not, expand(F)

(but nothing expanded)

BFS Search Tree





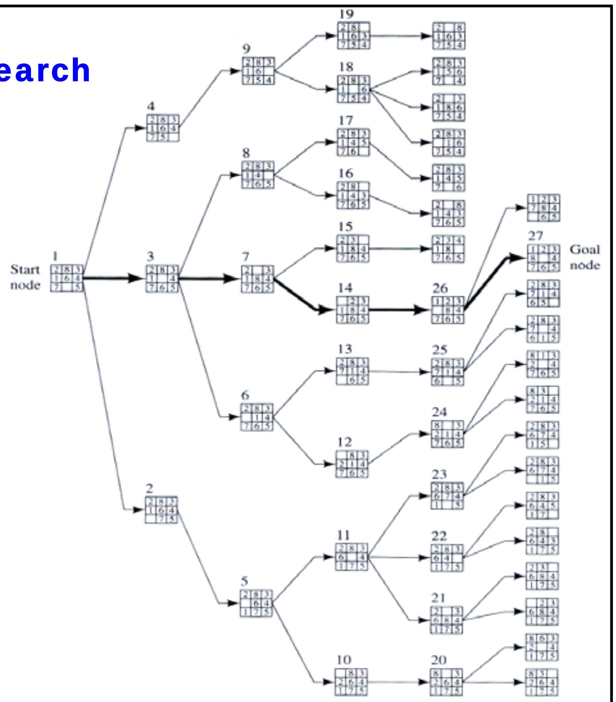
Queue = {G}

Select G

Goal(G) = true?

If yes, finish with goal G

Breadth-First Search



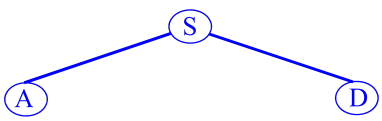
Depth-First Search (DFS)

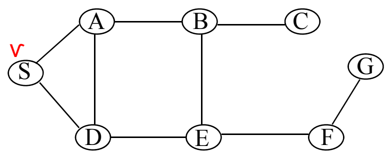
- Expand deepest unexpanded node

- Implementation:
  - For DFS, *fringe* is a Last-in-first-out (LIFO) stack
  - new successors go at beginning of the stack

- Repeated nodes?
  - Simple strategy: do not add an already-expanded node to the stack
    do not expand an already-expanded node

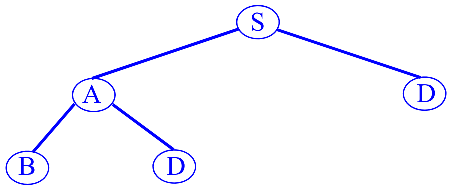
DFS Search Tree

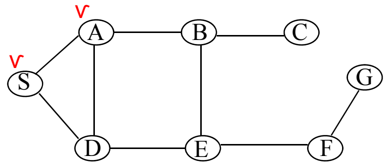




Stack = {A,D}

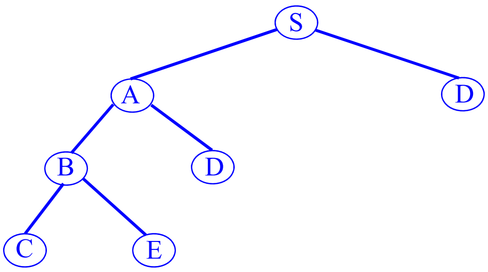
DFS Search Tree

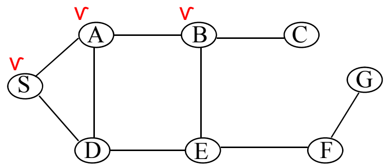




Stack = {B,D,D}

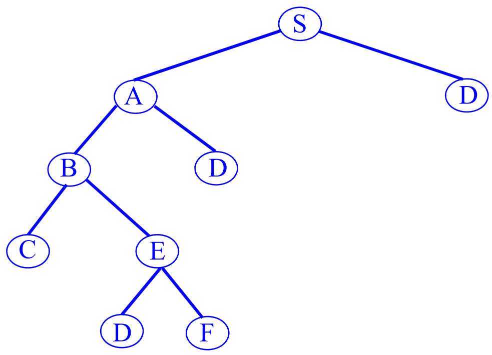
DFS Search Tree

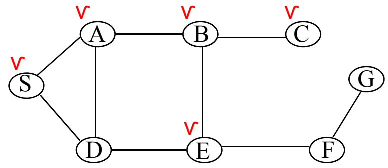




Stack = {C,E,D,D}

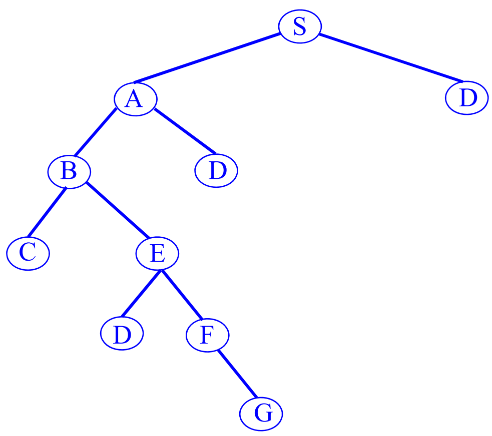
DFS Search Tree

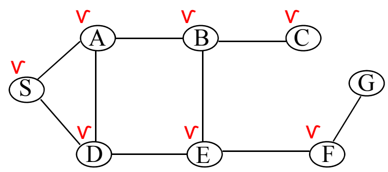




Stack = {D,F,D,D}

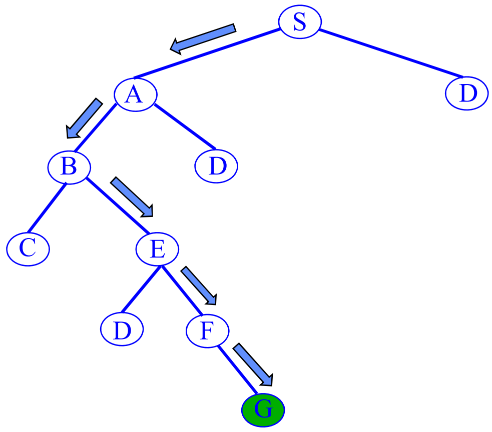
DFS Search Tree

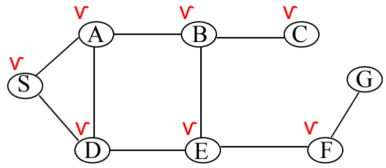




Stack = {G,D,D}

DFS Search Tree





Stack = {G,D,D}

Select G

Goal(G) = true?

If yes, finish with goal G

Evaluation of Search Algorithms

- Completeness
  - does it always find a solution if one exists?
- Optimality
  - does it always find a least-cost (or min depth) solution?
- Time complexity
  - number of nodes generated (worst case)
- Space complexity
  - number of nodes in memory (worst case)

- Time and space complexity are measured in terms of
  - *b*: maximum branching factor of the search tree
  - *d*: depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Breadth-First Search (BFS) Properties

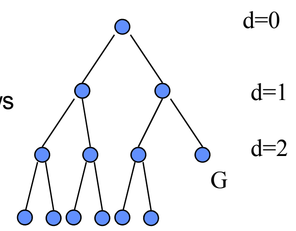
- Complete? Yes
- Optimal? Yes
- Time complexity $O(b^d)$
- Space complexity $O(b^d)$
- Main practical drawback? exponential space complexity

Complexity of Breadth-First Search

- **Time Complexity**
  - assume (worst case) that there is 1 goal leaf at the RHS at depth d
  - so BFS will generate nodes as follows

    $= b + b^2 + \; ..... + b^d + b^{d+1} - b$

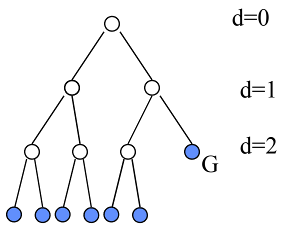
    $= \mathbf{O\ (b^{d+1})} = O\ (b \cdot b^d) = O\ (b^d)$



- **Space Complexity**
  - how many nodes can be in the queue (worst-case)?
  - at depth d there are b^{d+1} unexpanded nodes in the Q as follows

    $= b^{d+1} - b$

    $= \mathbf{O\ (b^{d+1})}$



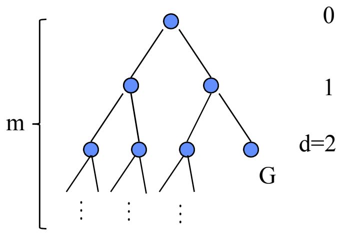
Examples of Time and Memory Requirements for Breadth-First Search

Assuming b=10, speed =10000 nodes/sec, node_size=1kbyte/node

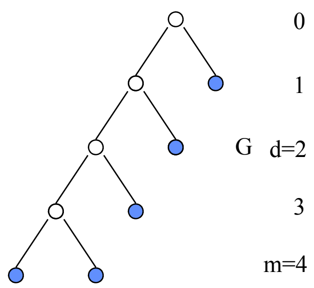
Depth of Solution	Nodes Generated	Time	Memory
2	1100	0.11 seconds	1 MB
4	111,100	11 seconds	106 MB
8	$\approx 10^9$	$\approx$ 31 hours	1 TB
12	$\approx 10^{13}$	$\approx$ 35 years	10 PB

What is the Complexity of Depth-First Search?

- Time Complexity
  - maximum tree depth = m
  - assume (worst case) that there is 1 goal leaf at the RHS at depth d
  - so DFS will generate **O (b^m)**



- Space Complexity
  - how many nodes can be in the queue (worst-case)?
  - at depth m we have b nodes
  - and b-1 nodes at earlier depths
  - total = b + (m-1)*(b-1) = **O(bm)**



Examples of Time and Memory Requirements for Depth-First Search

Assuming b=10, m = 12, speed=10000 nodes/sec, node_size=1kbyte/node

Depth of Solution	Nodes Generated	Time	Memory
2	$\approx 10^{12}$	$\approx$ 3 years	120kb
4	$\approx 10^{12}$	$\approx$ 3 years	120kb
8	$\approx 10^{12}$	$\approx$ 3 years	120kb
12	$\approx 10^{12}$	$\approx$ 3 years	120kb

Depth-First Search (DFS) Properties

- Complete?
  - Not complete if tree has unbounded depth
- Optimal?
  - No
- Time complexity?
  - Exponential
- Space complexity?
  - Linear

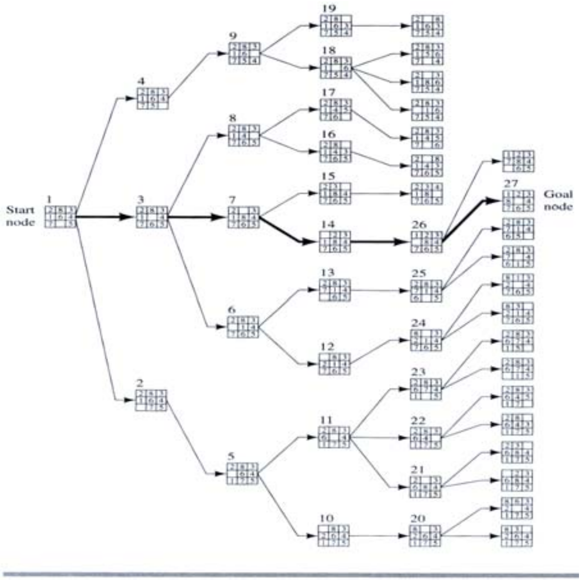
Comparing DFS and BFS

- Time complexity: same, but
  - In the worst-case, BFS is generally better than DFS
  - Sometime, on the average DFS is better if:
    - many goals, no loops and no infinite paths
- BFS is much worse memory-wise
    - DFS is linear space
    - BFS may store the order of the whole search space.
- In general
    - BFS is better if goal is not deep, if infinite paths, if many loops, if small search space
    - DFS is better if many goals, not many loops, no infinite paths
    - DFS is much better in terms of memory

DFS with a depth-limit L

- Standard DFS, but tree is not explored below some depth-limit L
- Solves problem of infinitely deep paths with no solutions
  - But will be incomplete if solution is below depth-limit
- Depth-limit L can be selected based on problem knowledge
  - E.g., diameter of state-space:
    - E.g., max number of steps between 2 cities
  - But typically not known ahead of time in practice

Depth-First Search with a depth-limit, L = 5



Iterative Deepening Search (IDS)

- Run multiple DFS searches with increasing depth-limits

Iterative deepening search

- L = 1
- While no solution, do
  - DFS from initial state S_0 with cutoff L
  - If found goal,
  - stop and return solution,
  - else, increment depth limit L

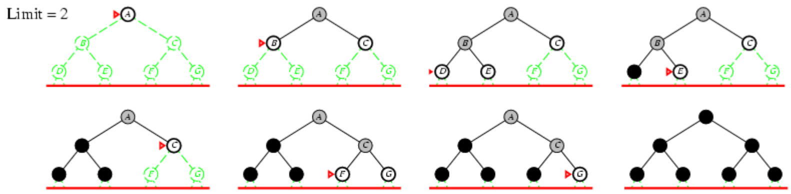
Iterative deepening search $L = 0$

Limit = 0

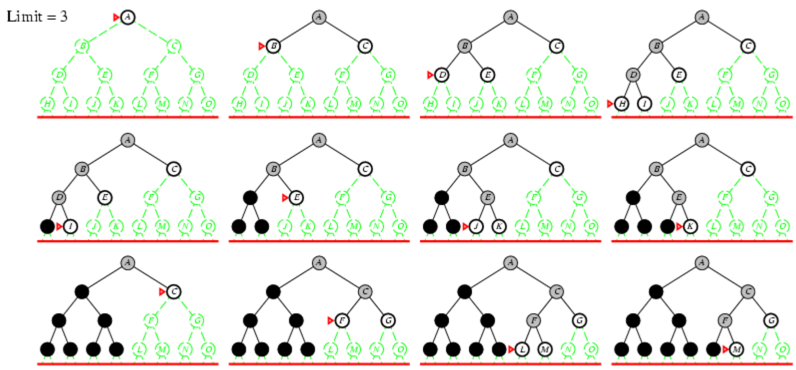
Iterative deepening search $L = 1$

Limit = 1

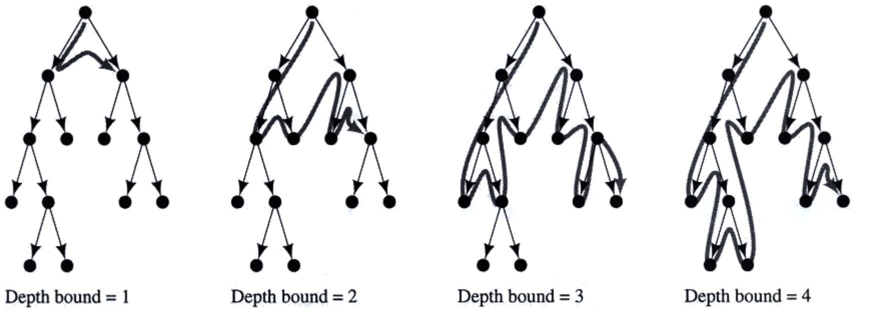
Iterative deepening search $L=2$



Iterative Deepening Search $L=3$



Iterative deepening search



Stages in Iterative-Deepening Search

Properties of Iterative Deepening Search

- Space complexity = O(bd)
    - (since its like depth first search run different times, with maximum depth limit d)
- Time Complexity
    - b + (b+b^2) + ...(b+....b^d) = O(b^d)
      (i.e., asymptotically the same as BFS or DFS to limited depth d in the worst case)
- Complete?
    - Yes
- Optimal
    - Yes as long as path cost is a non-decreasing function of depth
- IDS combines the small memory footprint of DFS, and has the completeness guarantee of BFS

IDS in Practice

- Isn't IDS wasteful?
  - Repeated searches on different iterations
  - Compare IDS and BFS:
    - E.g., b = 10 and d = 5
    - N(IDS) ~ $db + (d-1)b^2 + \ldots\ldots b^d = 123{,}450 \approx b/b\text{-}1$ times of N(BFS)
    - N(BFS) ~ $b + b^2 + \ldots\ldots b^d = 111{,}110$
    - Difference is only about 11%
      - Most of the time is spent at depth d, which is the same amount of time in both algorithms
- In practice, IDS is the preferred uniform search method with a large search space and unknown solution depth

Uniform Cost Search

- Optimality: path found = lowest cost
  - Algorithms so far are only optimal under restricted circumstances
- Let g(n) = cost from start state S to node n
- Uniform Cost Search:
  - Always expand the node on the fringe with minimum cost g(n)
  - Note that if costs are equal (or almost equal) will behave similarly to BFS

Uniform Cost Search

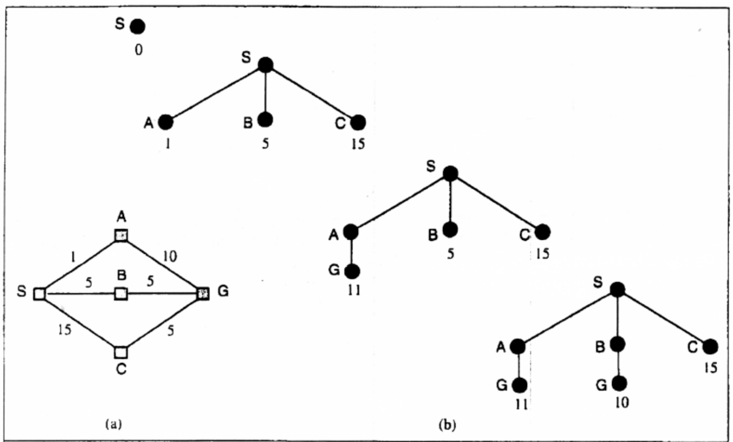


Figure 3.13 A route-finding problem. (a) The state space, showing the cost for each operator. (b) Progression of the search. Each node is labelled with $g(n)$. At the next step, the goal node with $g = 10$ will be selected.

Optimality of Uniform Cost Search?

- Assume that every step costs at least $\varepsilon > 0$
- Proof of Completeness:

  Given that every step will cost more than 0, and assuming a finite branching factor, there is a finite number of expansions required before the total path cost is equal to the path cost of the goal state. Hence, we will reach it in a finite number of steps.

- Proof of Optimality given Completeness:
  - Assume UCS is not optimal.
  - Then, there must be a goal state with path cost smaller than the goal state which was found (invoking completeness)
  - However, this is impossible because UCS would have expanded that node first by definition.
  - Contradiction.

Complexity of Uniform Cost

- Let C* be the cost of the optimal solution
- Assume that every step costs at least $\varepsilon > 0$
- Worst-case time and space complexity is:

$$O(\ b^{[1 + \text{floor}(C^*/\varepsilon)]}\ )$$

Why?
floor(C*/ε) ~ depth of solution if all costs are approximately equal

Comparison of Uninformed Search Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^{d+1})$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Summary

- A review of search
  - a search space consists of states and operators: it is a graph
  - a search tree represents a particular exploration of search space
- There are various strategies for “uninformed search”
  - breadth-first
  - depth-first
  - iterative deepening
  - Uniform cost search
- Various trade-offs among these algorithms
  - “best” algorithm will depend on the nature of the search problem
- Next up – heuristic search methods