

causality_03

February 10, 2025

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Conclusion for every questions (detail are provided below).

- Q1

QUESTION 1: Using PATH method, what is the expected causal effect of X on Y?

Answer-Q1: -0.42

Causal path: $X \rightarrow Z \rightarrow Y$

Total effect of $X \rightarrow Y$ = Effect of $X \rightarrow Z$ * Effect of $Z \rightarrow Y$

$$= (-0.6) * (0.7) = -0.42$$

- Q2

QUESTION 2: What can you identify from this result in combination with the graph above

Answer:

equation : $Y = 0.6589Z + 0.1159X + 0.4421$

both X and Z have a statistically significant(p-value < 0.001(lookup-table t-dist)) impact on Y, but the estimated effect of X (=0.1159) is biased due to confounding(U) and conditioning on the mediator Z.

From the causal graph, X affects Y only through Z, and using the Path Method (Q1), the true causal effect of X on Y = -0.42.

The discrepancy arises because OLS fails to account for the unobserved confounder U, which influences both X and Y, leading to an incorrect estimation of the direct effect of X.

- Q3

QUESTION 3: Does this match your expectation in Q1?

Answer: Yes

the estimated causal effect (-0.42) using the DoWhy frontdoor criterion with two-stage regression closely matches the expected causal effect (-0.42) calculated using the Path Method in Q1.

This confirms that the frontdoor adjustment method correctly accounts for the mediation effect of Z while handling the confounding from U.

The result validates that our previous theoretical expectation aligns with the causal estimation performed using DoWhy, reinforcing that the true causal effect of X on Y is indeed **-0.42**.

- Q4

QUESTION 4: How are the difference between OLS No Backdoor, OLS Backdoor, and DoWhy Backdoor

Answer:

OLS Backdoor and DoWhys Backdoor method produce similar results ($\beta_X = -0.3947$), correcting for confounding and aligning with theoretical expectations.

OLS No Backdoor control underestimates ($\beta_X = -0.3322$) the causal effect because it ignores confounding (only X as a predictor of Y without adjusting for any confounders).

```
[ ]: # !pip install dowhy econml
```

```
[1]: import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)

import networkx as nx
import numpy as np
import pandas as pd
from scipy import stats
import matplotlib.pyplot as plt
import statsmodels.api as sm

import dowhy
from dowhy import CausalModel
```

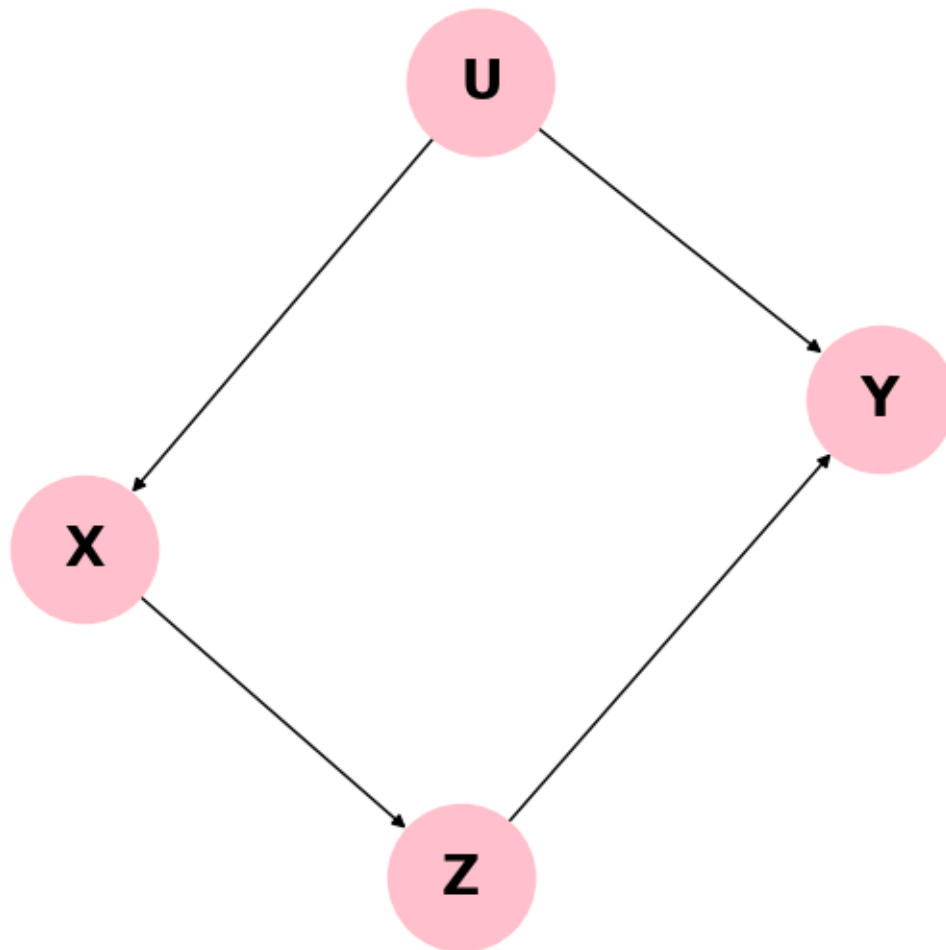
```
[2]: SAMPLES_SIZE = 1000
```

```
[3]: # A Simple Data Set
G = nx.DiGraph()
G.add_edges_from([
    ('U', 'X'),
    ('U', 'Y'),
    ('X', 'Z'),
    ('Z', 'Y')
])

U = stats.truncnorm(0, np.infty, scale=4).rvs(SAMPLES_SIZE)
X = 0.7*U + stats.norm(scale=2).rvs(SAMPLES_SIZE)
Z = -0.6*X + stats.norm(scale=2).rvs(SAMPLES_SIZE)
Y = 0.7*Z + 0.25*U + stats.norm(scale=2).rvs(SAMPLES_SIZE)

# QUESTION 1
# Using PATH method, what is the expected causal effect of X on Y?
```

```
[ ]: # TODO: plot
plt.figure(figsize=(5, 5))
pos = nx.spring_layout(G)
nx.draw(G, pos, with_labels=True, node_size=3000, node_color='pink',
        font_size=20, font_color='black', font_weight='bold')
plt.show()
```



QUESTION 1: Using PATH method, what is the expected causal effect of X on Y?

Answer-Q1: -0.42

Causal path: X -> Z -> Y

Total effect of X -> Y = Effect of X -> Z * Effect of Z -> Y

= (-0.6) * (0.7) = -0.42

```
[5]: # Try regular OLS
data = pd.DataFrame({'X': X, 'Z': Z})
data = sm.add_constant(data, prepend=True)
model = sm.OLS(Y, data) # Ordinary Linear Regression
results = model.fit()
results.summary()

# QUESTION 2
# What can you identify from this result in combination with the graph above
```

[5]:

Dep. Variable:	y	R-squared:	0.379
Model:	OLS	Adj. R-squared:	0.378
Method:	Least Squares	F-statistic:	304.7
Date:	Mon, 10 Feb 2025	Prob (F-statistic):	5.49e-104
Time:	14:22:23	Log-Likelihood:	-2107.0
No. Observations:	1000	AIC:	4220.
Df Residuals:	997	BIC:	4235.
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
const	0.5094	0.081	6.293	0.000	0.351	0.668
X	0.1445	0.031	4.686	0.000	0.084	0.205
Z	0.6857	0.032	21.745	0.000	0.624	0.748

Omnibus:	0.141	Durbin-Watson:	1.902
Prob(Omnibus):	0.932	Jarque-Bera (JB):	0.135
Skew:	-0.028	Prob(JB):	0.935
Kurtosis:	2.993	Cond. No.	5.45

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

QUESTION 2: What can you identify from this result in combination with the graph above

Answer:

equation : $Y = 0.6589Z + 0.1159X + 0.4421$

both X and Z have a statistically significant(p-value < 0.001(lookup-table t-dist)) impact on Y, but the estimated effect of X (=0.1159) is biased due to confounding(U) and conditioning on the mediator Z.

From the causal graph, X affects Y only through Z, and using the Path Method (Q1), the true causal effect of X on Y = -0.42.

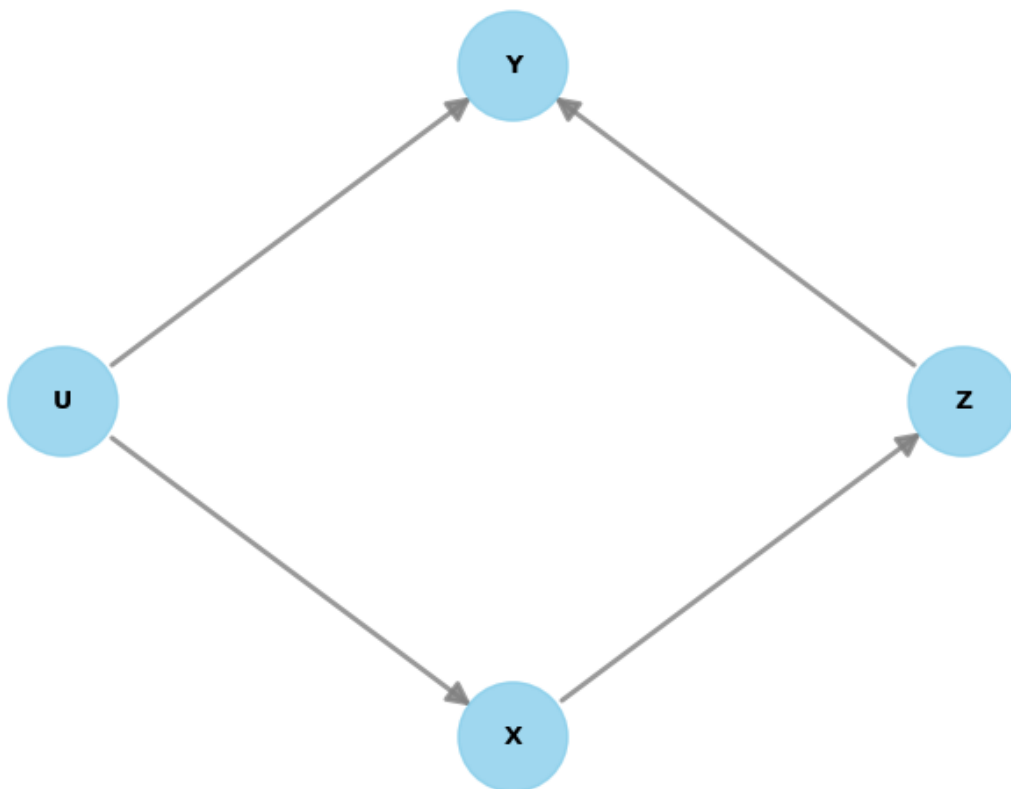
The discrepancy arises because OLS fails to account for the unobserved confounder U, which influences both X and Y, leading to an incorrect estimation of the direct effect of X.

```
[6]: # Use DoWhy Model
model = CausalModel(
    data=pd.DataFrame({'X': X, 'Y': Y, 'Z': Z}),
```

```
treatment='X',
outcome='Y',
graph="\n".join(nx.generate_gml(G))
)
```

```
/Users/pupipatsingkhorn/miniconda3/envs/datascience/lib/python3.11/site-
packages/dowhy/causal_model.py:583: UserWarning: 1 variables are assumed
unobserved because they are not in the dataset. Configure the logging level to
`logging.WARNING` or higher for additional details.
  warnings.warn(
```

```
[7]: model.view_model()
```



```
[8]: estimand = model.identify_effect()
      print(estimand)
```

```
Estimand type: EstimandType.NONPARAMETRIC_ATE
```

```
### Estimand : 1
Estimand name: backdoor
No such variable(s) found!
```

```

### Estimand : 2
Estimand name: iv
No such variable(s) found!

```

```

### Estimand : 3
Estimand name: frontdoor
Estimand expression:
      d      d
E  (Y)  ([Z])
    d[Z]  d[X]
Estimand assumption 1, Full-mediation: Z intercepts (blocks) all directed paths
from X to Y.
Estimand assumption 2, First-stage-unconfoundedness: If  $U \rightarrow \{X\}$  and  $U \rightarrow \{Z\}$  then
 $P(Z|X,U) = P(Z|X)$ 
Estimand assumption 3, Second-stage-unconfoundedness: If  $U \rightarrow \{Z\}$  and  $U \rightarrow Y$  then
 $P(Y|Z, X, U) = P(Y|Z, X)$ 

```

```

[9]: estimate = model.estimate_effect(
      identified_estimand=estimand,
      method_name='frontdoor.two_stage_regression')

print(f'Estimate of causal effect (linear regression): {estimate.value}')

# Question 3: Does this match your expectation in Q1?

```

Estimate of causal effect (linear regression): -0.42377304720809506

QUESTION 3: Does this match your expectation in Q1?

Answer: Yes

the estimated causal effect (-0.42) using the DoWhy frontdoor criterion with two-stage regression closely matches the expected causal effect (-0.42) calculated using the Path Method in Q1.

This confirms that the frontdoor adjustment method correctly accounts for the mediation effect of Z while handling the confounding from U.

The result validates that our previous theoretical expectation aligns with the causal estimation performed using DoWhy, reinforcing that the true causal effect of X on Y is indeed **-0.42**.

```

[10]: refute_subset = model.refute_estimate(
      estimand=estimand,
      estimate=estimate,
      method_name="data_subset_refuter",
      subset_fraction=0.4)

```

```

[11]: print(refute_subset)

```

Refute: Use a subset of data

Estimated effect:-0.42377304720809506
New effect:-0.4232292075982626
p value:0.9199999999999999

```
[12]: refute_placebo = model.refute_estimate(  
        estimand=estimand,  
        estimate=estimate,  
        method_name="placebo_treatment_refuter")
```

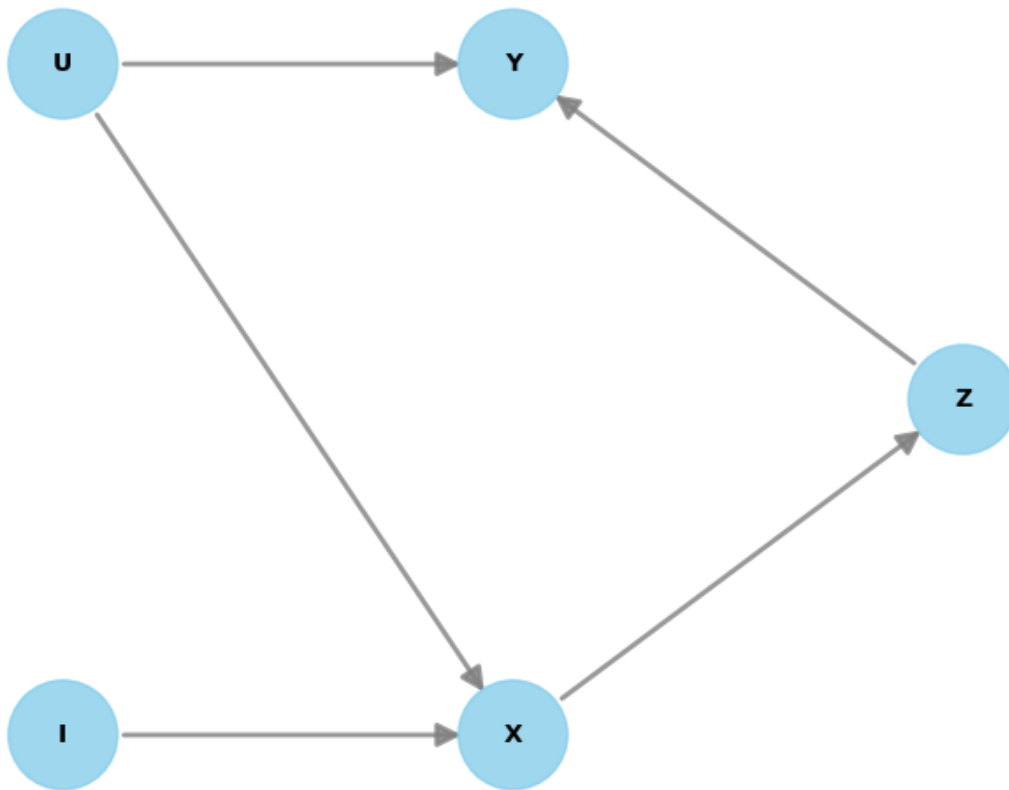
```
[13]: print(refute_placebo)
```

Refute: Use a Placebo Treatment
Estimated effect:-0.42377304720809506
New effect:-0.423773047208095
p value:0.0

```
[14]: # Another Data Set  
G = nx.DiGraph()  
G.add_edges_from([  
    ('U', 'X'),  
    ('U', 'Y'),  
    ('X', 'Z'),  
    ('Z', 'Y'),  
    ('I', 'X')  
)  
  
U = stats.truncnorm(0, np.infty, scale=4).rvs(SAMPLES_SIZE)  
I = stats.norm(scale=10).rvs(SAMPLES_SIZE)  
X = 0.7*U + 0.3*I + stats.norm(scale=2).rvs(SAMPLES_SIZE)  
Z = -0.6*X + stats.norm(scale=2).rvs(SAMPLES_SIZE)  
Y = 0.7*Z + 0.25*U + stats.norm(scale=2).rvs(SAMPLES_SIZE)
```

```
[15]: # Use DoWhy Model  
model = CausalModel(  
    data=pd.DataFrame({'X': X, 'Y': Y, 'Z': Z, 'U': U, 'I': I}),  
    treatment='X',  
    outcome='Y',  
    graph="\n".join(nx.generate_gml(G))  
)
```

```
[16]: model.view_model()
```



```
[17]: estimand = model.identify_effect()
      print(estimand)
```

Estimand type: EstimandType.NONPARAMETRIC_ATE

Estimand : 1

Estimand name: backdoor

Estimand expression:

$$d \left(\frac{E[Y|U]}{d[X]} \right)$$

Estimand assumption 1, Unconfoundedness: If $U \rightarrow \{X\}$ and $U \rightarrow Y$ then $P(Y|X, U, U) = P(Y|X, U)$

Estimand : 2

Estimand name: iv

Estimand expression:

$$\frac{d \left(\frac{E[Y]}{d[I]} \right)}{d \left(\frac{E[X]}{d[I]} \right)}$$

Estimand assumption 1, As-if-random: If $U \rightarrow Y$ then $\neg(U \rightarrow \{I\})$
 Estimand assumption 2, Exclusion: If we remove $\{I\} \rightarrow \{X\}$, then $\neg(\{I\} \rightarrow Y)$

```
### Estimand : 3
Estimand name: frontdoor
Estimand expression:
      d      d
E  (Y)  ([Z])
      d[Z]   d[X]
Estimand assumption 1, Full-mediation: Z intercepts (blocks) all directed paths
from X to Y.
Estimand assumption 2, First-stage-unconfoundedness: If  $U \rightarrow \{X\}$  and  $U \rightarrow \{Z\}$  then
 $P(Z|X, U) = P(Z|X)$ 
Estimand assumption 3, Second-stage-unconfoundedness: If  $U \rightarrow \{Z\}$  and  $U \rightarrow Y$  then
 $P(Y|Z, X, U) = P(Y|Z, X)$ 
```

```
[18]: # No backdoor
data = pd.DataFrame({'X': X})
data = sm.add_constant(data, prepend=True)
model_ols_no_backdoor = sm.OLS(Y, data) # Ordinary Linear Regression
results = model_ols_no_backdoor.fit()
results.summary()
```

```
[18]:
```

Dep. Variable:	y	R-squared:	0.232
Model:	OLS	Adj. R-squared:	0.231
Method:	Least Squares	F-statistic:	301.3
Date:	Mon, 10 Feb 2025	Prob (F-statistic):	3.49e-59
Time:	14:25:57	Log-Likelihood:	-2321.2
No. Observations:	1000	AIC:	4646.
Df Residuals:	998	BIC:	4656.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
const	0.7676	0.087	8.778	0.000	0.596	0.939
X	-0.3322	0.019	-17.358	0.000	-0.370	-0.295

Omnibus:	1.135	Durbin-Watson:	2.097
Prob(Omnibus):	0.567	Jarque-Bera (JB):	1.017
Skew:	-0.024	Prob(JB):	0.601
Kurtosis:	3.149	Cond. No.	5.17

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

\therefore OLS No Backdoor: $\beta_X = -0.3322$

```
[19]: # Homegrown backdoor
data = pd.DataFrame({'X': X, 'U': U})
data = sm.add_constant(data, prepend=True)
model_ols_backdoor = sm.OLS(Y, data) # Ordinary Linear Regression
results = model_ols_backdoor.fit()
results.summary()
```

[19]:

Dep. Variable:	y	R-squared:	0.273
Model:	OLS	Adj. R-squared:	0.271
Method:	Least Squares	F-statistic:	186.8
Date:	Mon, 10 Feb 2025	Prob (F-statistic):	1.21e-69
Time:	14:25:58	Log-Likelihood:	-2294.0
No. Observations:	1000	AIC:	4594.
Df Residuals:	997	BIC:	4609.
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P> t	[0.025	0.975]
const	0.0922	0.124	0.742	0.458	-0.151	0.336
X	-0.3947	0.020	-19.325	0.000	-0.435	-0.355
U	0.2451	0.033	7.473	0.000	0.181	0.309

Omnibus:	0.637	Durbin-Watson:	2.090
Prob(Omnibus):	0.727	Jarque-Bera (JB):	0.512
Skew:	-0.028	Prob(JB):	0.774
Kurtosis:	3.096	Cond. No.	9.22

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

∴ OLS Backdoor: $\beta_X = -0.3947, \beta_U = 0.2451$

```
[20]: estimate_backdoor = model.estimate_effect(
        identified_estimand=estimand,
        method_name='backdoor.linear_regression')

print(f'Estimate of causal effect via backdoor (linear regression):␣
      ↪{estimate_backdoor.value}')

# QUESTION 4 -- How are the difference between OLS No Backdoor, OLS Backdoor,␣
      ↪and DoWhy Backdoor
```

Estimate of causal effect via backdoor (linear regression): -0.3947006164963863

∴ DoWhy Backdoor: $\beta_X = -0.3947$

QUESTION 4: How are the difference between OLS No Backdoor, OLS Backdoor, and DoWhy Backdoor

Answer:

OLS Backdoor and DoWhys Backdoor method produce similar results ($\beta_X = -0.3947$), correcting

OLS No Backdoor control underestimates ($\beta_X = -0.3322$) the causal effect because it ignores confounding (only X as a predictor of Y without adjusting for any confounders).

[illegible]


```

    return np.sqrt(eigvals[0]/eigvals[-1])
/Users/pupipatsingkhorn/miniconda3/envs/datascience/lib/python3.11/site-
packages/statsmodels/regression/linear_model.py:1966: RuntimeWarning: divide by
zero encountered in scalar divide
    return np.sqrt(eigvals[0]/eigvals[-1])
/Users/pupipatsingkhorn/miniconda3/envs/datascience/lib/python3.11/site-
packages/statsmodels/regression/linear_model.py:1966: RuntimeWarning: divide by
zero encountered in scalar divide
    return np.sqrt(eigvals[0]/eigvals[-1])
/Users/pupipatsingkhorn/miniconda3/envs/datascience/lib/python3.11/site-
packages/statsmodels/regression/linear_model.py:1966: RuntimeWarning: divide by
zero encountered in scalar divide
    return np.sqrt(eigvals[0]/eigvals[-1])
/Users/pupipatsingkhorn/miniconda3/envs/datascience/lib/python3.11/site-
packages/statsmodels/regression/linear_model.py:1966: RuntimeWarning: divide by
zero encountered in scalar divide
    return np.sqrt(eigvals[0]/eigvals[-1])
/Users/pupipatsingkhorn/miniconda3/envs/datascience/lib/python3.11/site-
packages/statsmodels/regression/linear_model.py:1966: RuntimeWarning: divide by
zero encountered in scalar divide
    return np.sqrt(eigvals[0]/eigvals[-1])

```

```
[22]: print(refute_placebo)
```

```

Refute: Use a Placebo Treatment
Estimated effect:-0.3947006164963863
New effect:0.0
p value:1.0

```

```
[23]: estimate_iv = model.estimate_effect(
        identified_estimand=estimand,
        method_name='iv.instrumental_variable')

print(f'Estimate of causal effect via IV: {estimate_iv.value}')
```

```
Estimate of causal effect via IV: -0.3912598668999661
```

```
[24]: from sklearn.linear_model import LassoCV
from sklearn.ensemble import GradientBoostingRegressor

estimate = model.estimate_effect(
    identified_estimand=estimand,
    method_name='backdoor.econml.dml.DML',
```

```

method_params={
    'init_params': {
        'model_y': GradientBoostingRegressor(),
        'model_t': GradientBoostingRegressor(),
        'model_final': LassoCV(fit_intercept=False),
    },
    'fit_params': {}
}

print(f'Estimate of causal effect (DML): {estimate.value}')

```

Estimate of causal effect (DML): -0.3579060638957235

A column-vector y was passed when a 1d array was expected. Please change the shape of y to (n_samples,), for example using ravel().

A column-vector y was passed when a 1d array was expected. Please change the shape of y to (n_samples,), for example using ravel().