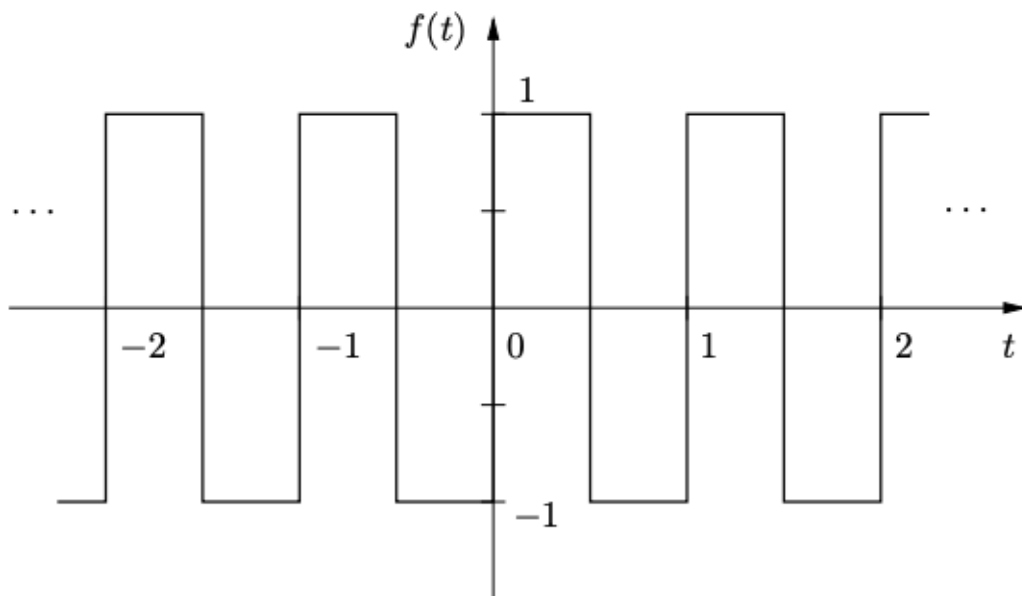


<<< Only Problem 2 and 4 will be graded >>>

```
In [ ]: import matplotlib.pyplot as plt
import numpy as np
import IPython.display as ipd
%matplotlib inline
import os
from scipy import signal, fftpack
from skimage.io import imread
import cv2
```

Problem 1

Find the Fourier series of the following periodic function \



Problem 2

Find the Fourier Series (FS) of the periodic function $x(t)$ which are provided as follows.

1. $x(t) = \frac{\pi t^3}{2}; -1 < t < 1$
2. $x(t) = \pi - t; -\pi \leq t \leq \pi$
3. $x(t) = t^2 + \sin^3(\pi t); -1 \leq t \leq 1$

Problem 3

Find the Fourier transform of the following signals in terms of $X(j\omega)$, the Fourier transform of $x(t)$ ($\mathcal{F}\{x(t)\} = X(j\omega)$)

1. $x(-t)$
2. $x_e(t) = \frac{x(t)+x(-t)}{2}$
3. $x_o(t) = \frac{x(t)-x(-t)}{2}$

Problem 4

Let $\mathcal{F}\{x(t)\} = X(j\omega) = \text{rect}[(\omega - 1)/2]$. Find Fourier tranform of

1. $x(-2t + 4)$
2. $(t - 1)x(t - 1)$
3. $t \frac{dx(t)}{dt}$
4. $x(2t - 1) \exp[-j2t]$
5. $x(t) * x(t - 1)$

Problem 5.1

Proof

$$\mathcal{F}\{e^{-|t|}\} = \mathcal{F}\{\exp(-|t|)\} = \frac{2}{\omega^2 + 1}$$

Problem 5.2

Using the outcome obtained in Problem 5.1, Find the Fourier Transform of the given equation.

1. $\frac{d}{dt}(e^{-|t|})$
2. $\exp(3t - |2t + 2|)$
3. $\frac{1}{2\pi t^2 + 1}$

Problem 6

For each of the following Fourier transform ($X(j\omega)$), use Fourier transform properties to determine the corresponding time-domain signal ($x(t)$)

1. $X(j\omega) = u(\omega) - u(\omega - 2)$
2. $X(j\omega) = \cos(2\omega) \sin\left(\frac{\omega}{2}\right)$

Problem 7

Proof

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

Problem 8

Determine a signal $x(t)$ which has its Fourier transform

$$X(j\omega) = \delta(\omega) - \delta(\omega - a_0\pi) - \delta(\omega - a_1)$$

and the impulse response in time-domain is $h(t) = u(t) - u(t - 2b_0)$

1. Show how to find $x(t)$ and determine whether it is periodic?
2. Show how to find $y(t) = x(t) * h(t)$ and determine whether it is periodic?

Verify your results by sketching $x(t)$, $h(t)$, $y(t)$, $X(j\omega)$, $H(j\omega)$ and $Y(j\omega)$. Note that for the complex number, plot only its magnitude.

In []: