Problem 2 : Two-phased simplex method

Solve the following program using a two-phased simplex method by hand :

$$Objective : max(3x + 4y)$$

\begin{equation*} s.t.

$$\begin{aligned} x + 2y &\leq 7 \\ 3x - y &\geq 0 \\ x - y &\leq 2 \\ x, y &\geq 0 \end{aligned}$$

\end{equation*}

(a) Standard form

max $3x + 4y$

s.t. $x + 2y + s_1 = 7$ ✓

$3x - y - e_2 = 0$ a_2

$x - y + s_3 = 2$ ✓

$x, y, s_1, e_2, s_3 \geq 0$

(b) Auxiliary Problem

min a_2 $\leftrightarrow$ max $W = -a_2$

$W + a_2 = 0$

s.t. $x + 2y + s_1 = 7$

$3x - y - e_2 + a_2 = 0$

$x - y + s_3 = 2$

$x, y, s_1, e_2, s_3, a_2 \geq 0$

(c) Phase I

W	x	y	s_1	e_2	s_3	a_2	RHS	BV
1	0	0	0	0	0	1	0	W
0	1	2	1	0	0	0	7	s_1
0	3	-1	0	-1	0	1	0	a_2
0	1	-1	0	0	1	0	2	s_3

W	x	y	s_1	e_2	s_3	a_2	RHS	BV	
1	-3	1	0	1	0	0	0	W	$R_0 - R_2$
0	1	2	1	0	0	0	7	s_1	
0	3	-1	0	-1	0	1	0	a_2	
0	1	-1	0	0	1	0	2	s_3	

W	x	y	s_1	e_2	s_3	a_2	RHS	BV	ratio
1	-3	1	0	1	0	0	0	W	
0	1	2	1	0	0	0	7	s_1	$7/1 = 7$
0	3	-1	0	-1	0	1	0	a_2	$0/3 = 0$ ← min
0	1	-1	0	0	1	0	2	s_3	$2/1 = 2$

W	x	y	s_1	e_2	s_3	a_2	RHS	BV	
1	0	0	0	0	0	1	0	W	$R_0 + R_2$
0	0	7/3	1	1/3	0	-1/3	7	s_1	$R_1 - R_2/3$
0	1	-1/3	0	-1/3	0	1/3	0	x	$R_2/3$
0	0	-2/3	0	1/3	1	-1/3	2	s_3	$R_3 - R_2/3$

Phase II.

W	x	y	s_1	e_2	s_3	a_2	RHS	BV
1	0	0	0	0	0	1	0	W
0	0	7/3	1	1/3	0	-1/3	7	s_1
0	1	-1/3	0	-1/3	0	1/3	0	x
0	0	-2/3	0	1/3	1	-1/3	2	s_3

$\max Z = 3x + 4y \quad \leftrightarrow \quad Z - 3x - 4y = 0$

Z	x	y	s_1	e_2	s_3	RHS	BV
1	-3	-4	0	0	0	0	Z
0	0	7/3	1	1/3	0	7	s_1
0	1	-1/3	0	-1/3	0	0	x
0	0	-2/3	0	1/3	1	2	s_3

z	x	y	s_1	e_2	s_3	RHS	BV		ratio
1	0	−5	0	−1	0	0	z	$R_0 + 3R_2$	
0	0	7/3	1	1/3	0	7	s_1		7/7/3 = 7 ← min
0	1	−1/3	0	−1/3	0	0	x		0/−1/3 = −0 (∞)
0	0	−2/3	0	1/3	1	2	s_3		1/−2/3 (∞)

z	x	y	s_1	e_2	s_3	RHS	BV	
1	0	−5	0	−1	0	0	z	
0	0	1	3/7	1/7	0	3	y	$\frac{3}{7}R_1$
0	1	−1/3	0	−1/3	0	0	x	
0	0	−2/3	0	1/3	1	2	s_3	

z	x	y	s_1	e_2	s_3	RHS	BV	
1	0	0	15/7	−2/7	0	15	z	$R_0 + 5R_1$
0	0	1	3/7	1/7	0	3	y	
0	1	0	1/7	−6/21	0	1	x	$R_2 + \frac{R_1}{3}$
0	0	0	2/7	9/21	1	4	s_3	$R_3 + \frac{2}{3}R_1$

z	x	y	s_1	e_2	s_3	RHS	BV	ratio
1	0	0	15/7	−2/7	0	15	z	
0	0	1	3/7	1/7	0	3	y	3/1/7 = 21
0	1	0	1/7	−6/21	0	1	x	1/−6/21 (∞)
0	0	0	2/7	9/21	1	4	s_3	4/9/21 = 21×4/9 ← min

z	x	y	s_1	e_2	s_3	RHS	BV	
1	0	0	15/7	-2/7	0	15	z	
0	0	1	3/7	1/7	0	3	y	
0	1	0	1/7	-6/21	0	1	x	
0	0	0	2/3	1	$\frac{21}{9}$	$\frac{84}{9}$	e_2	$\frac{21}{9} R_3$

z	x	y	s_1	e_2	s_3	RHS	BV	
1	0	0	7/3	0	2/3	53/3	z	$R_0 + \frac{2}{7} R_3$
0	0	1	1/3	0	-1/3	5/3	y	$R_1 - \frac{1}{7} R_3$
0	1	0	1/3	0	2/3	11/3	x	$R_2 + \frac{6}{21} R_3$
0	0	0	2/3	1	$\frac{7}{3}$	$\frac{28}{3}$	e_2	

No neg $\therefore$ Optimal.

$\therefore z = \frac{53}{3}$, $y = \frac{5}{3}$, $x = \frac{11}{3}$, $e_2 = \frac{28}{3}$ ✱