

<<< Only Problem 2 and 4 will be graded >>>

```
In [ ]: import matplotlib.pyplot as plt
import numpy as np
import IPython.display as ipd
%matplotlib inline
import os
from scipy import signal, fftpack
from skimage.io import imread
import cv2
```

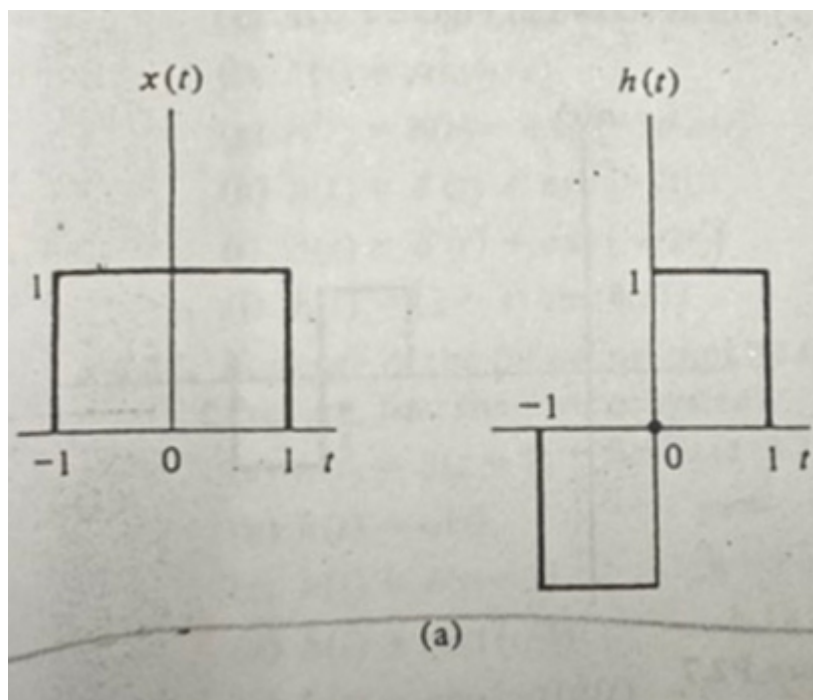
Problem 1

Evaluate the convolution of the following signals

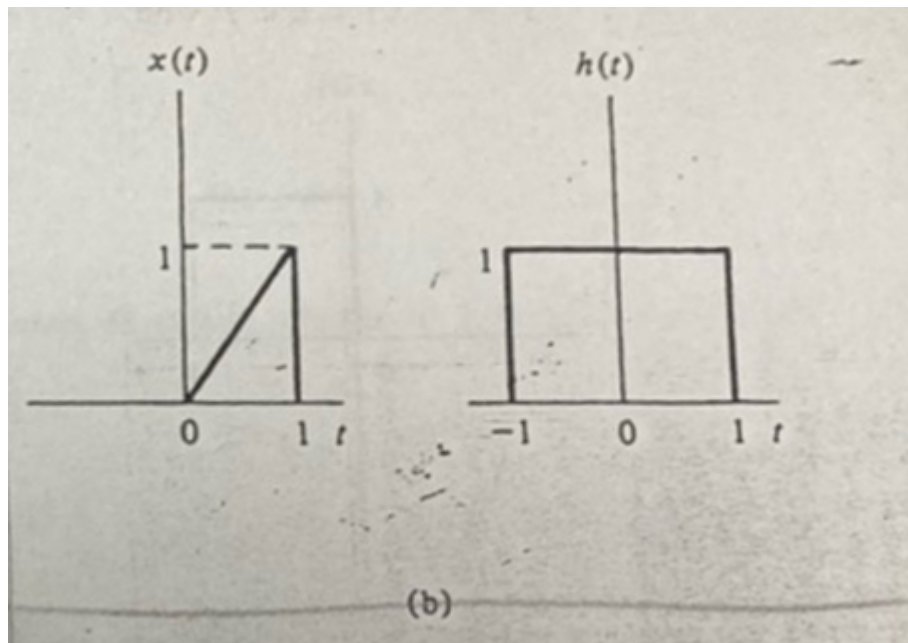
1. $\text{rect}\left(\frac{t-a}{a}\right) * \delta(t-b)$
2. $\text{rect}\left(\frac{t}{a}\right) * \text{rect}\left(\frac{t}{a}\right)$
3. $t[u(t) - u(t-1)] * u(t)$

Problem 2

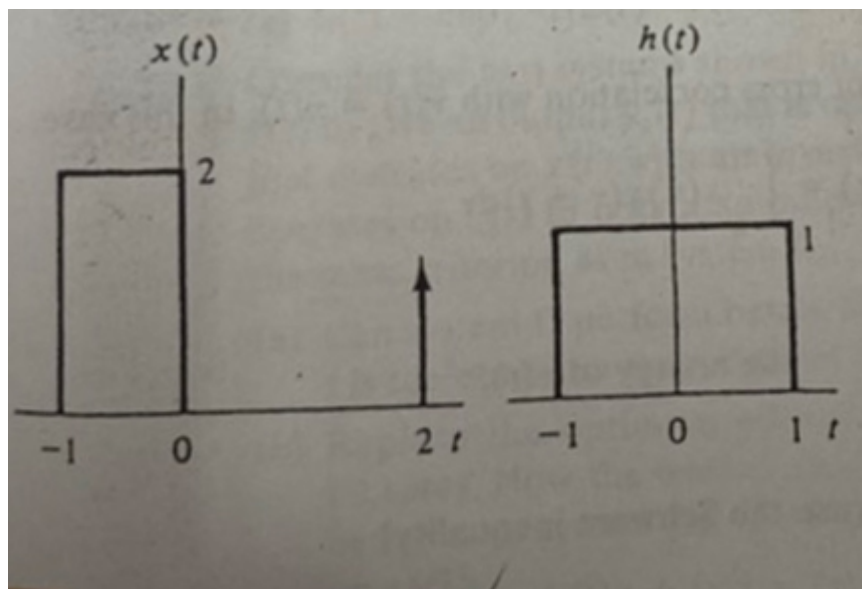
Determine the convolution $y(t) = h(t) * x(t)$ using Graphical Interpretation of the pairs of the signals shown



1.

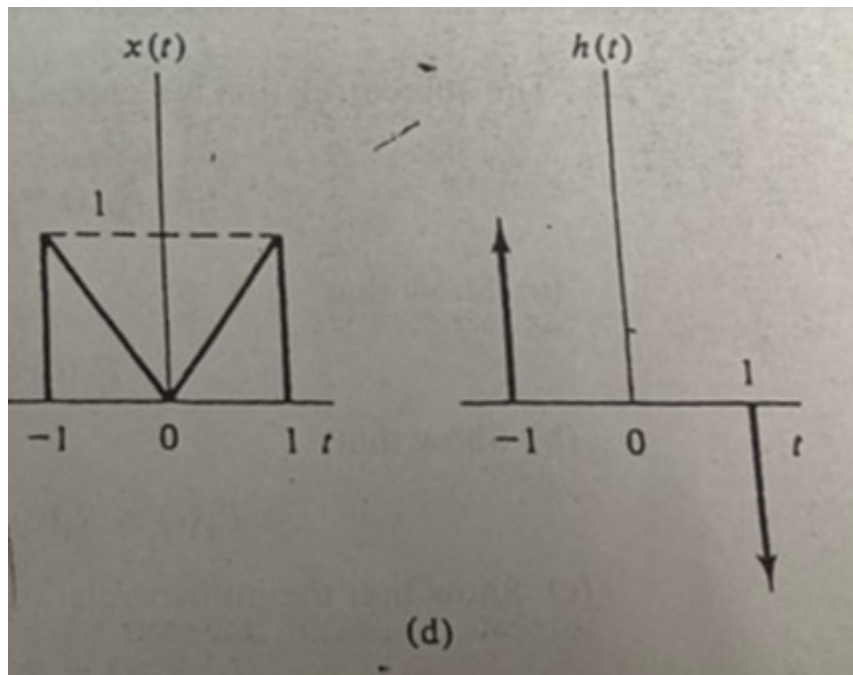


2.



3.

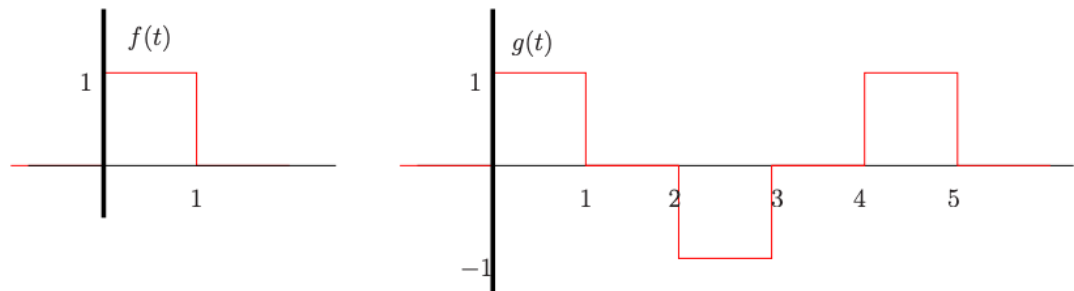
[optional]



4.

Problem 3

Let $f(t)$ and $g(t)$ be given as follows:



1. sketch the function : $x(t) = f(t) * g(t)$
2. show that if $a(t) = b(t) * c(t)$, then $(Mb(t)) * c(t) = Ma(t)$, for any real number M (hint: use the convolution integral formula)

Problem 4

Find the convolution $y[n] = h[n] * x[n]$ of the following signals:

1.

$$x[n] = \begin{cases} -1, & -5 \leq n \leq -1 \\ 1, & 0 \leq n \leq 4 \end{cases} \quad h[n] = 2u[n]$$

2.

$$x[n] = \left(\frac{1}{2}\right)^n u[n], \quad h[n] = \delta[n] + \delta[n-1] + \left(\frac{1}{3}\right)^n u[n]$$

3.

$$x[n] = u[n], h[n] = 1; 0 \leq n \leq 9$$

4.

$$x[n] = \left(\frac{1}{3}\right)^n u[n], h[n] = \delta[n] + \left(\frac{1}{2}\right)^n u[n]$$

Problem 5

Find the convolution $y[n] = h[n] * x[n]$ of the following signals

1.

$$x[n] = \left\{1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}\right\}, h[n] = \{1, -1, 1, -1\}$$

1.

$$x[n] = \{1, 2, 3, 0, -1, \}, h[n] = \{2, -1, 3, 1, -2\}$$

1.

$$x[n] = \left\{3, \frac{1}{2}, -\frac{1}{4}, 1, 4\right\}, h[n] = \left\{2, -1, \frac{1}{2}, -\frac{1}{2}\right\}$$

1.

$$x[n] = \left\{-1, \frac{1}{2}, \frac{3}{4}, -\frac{1}{5}, 1\right\}, h[n] = \{1, 1, 1, 1, 1\}$$

Problem 6

Problem 6.1 : Convolution - 1D

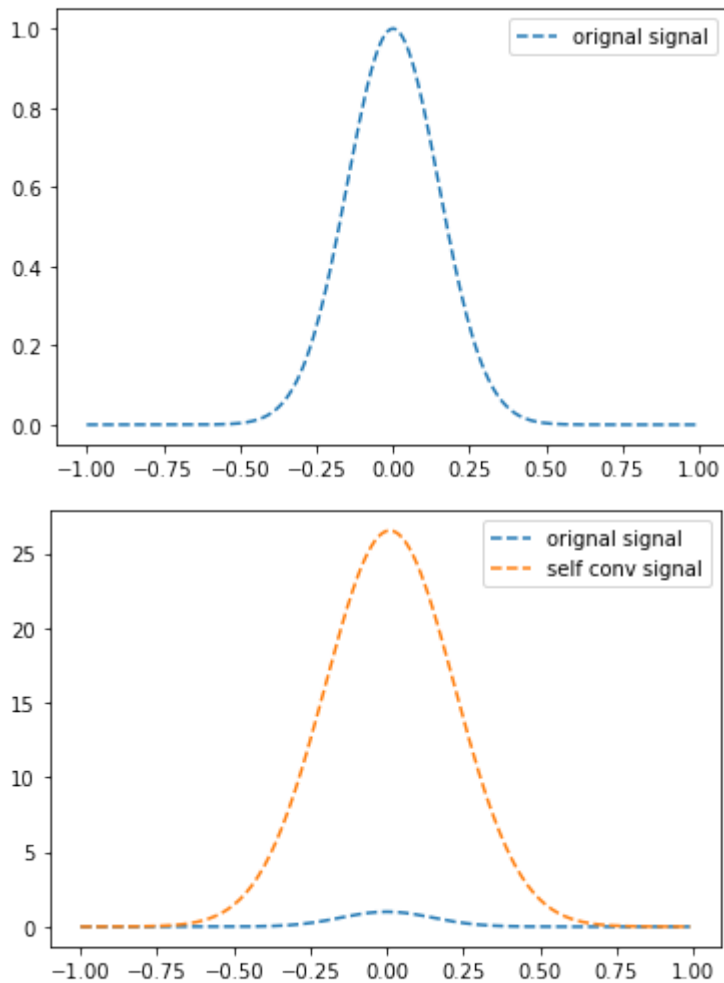
The following code creates a gaussian pulse and its self convolutions. Study and apply the convolution between signal e and another signal e with noise (e_noise) and write the report to analyze the results.

```
In [ ]: t = np.linspace(-1, 1, 2 * 100, endpoint=False)
i, q, e = signal.gausspulse(t, fc=5, retquad=True, retenv=True)
plt.plot(t, e, '--', label = 'original signal')
plt.legend(loc='upper right')
plt.show()

conv_e = np.convolve(e,e,'same')
plt.plot(t, e, '--', label = 'original signal')
plt.plot(t, conv_e, '--', label = 'self conv signal')
plt.legend(loc='upper right')
plt.show()

e_noise = e + np.random.randn(len(e))*2.5
conv_e_noise = np.convolve(e,e_noise,'same')
```

TODO : Apply the convolution between signal e and another signal e with



Problem 6.2

From the self convolution below, when increasing the number of self convolution (now is 8), what is noticeable from the final shape resulted from the convolution?

(HINT 01: Central limit theorem)

(HINT 02: What is Probability Density Function (PDF) of z if $z = x + y$?)

```
In [ ]: from scipy.stats import uniform

x = np.linspace(-5,5, 1000)
plt.plot(x, uniform.pdf(x),
         'r-', lw=5, alpha=0.6, label='uniform pdf')

plt.show()

x = np.linspace(-15,15, 10000)
pdf_1 = uniform.pdf(x)
pdf_2 = uniform.pdf(x)

for i in range(8):
    pdf_2 = np.convolve(pdf_2,pdf_1, 'same')
```

```
pdf_2 = pdf_2/np.max(pdf_2)
plt.plot(x, pdf_2, 'r-', lw=5, alpha=0.6, label='conv uniform')
```

Problem 7

2D (image) signal convolution:

The following code show the 2D signal (image $f(x,y)$) and a kernel (diag_line). Study the convolution of the kernel and the image. Apply with "circuits.png" image and analyze the results.

TODO : Apply diag_line to the "circuits.png image" and analyse the results

```
In [ ]: !wget https://drive.google.com/uc?id=1hQ8uKocLTjaKmrJm-04BfZxvLLM1Cfa- -O
!wget https://drive.google.com/uc?id=1WoISJ6-FECt-gt60vjlfmz89oGH812GM -O
```

```
In [ ]: image_path = 'hamtaro0.jpg'

diag_line = np.array([[ 2, -1, -1],
                      [-1, 2, -1],
                      [-1, -1, 2]])

ham = cv2.imread(image_path,0)
plt.figure(figsize=(10,10))
plt.imshow(ham, cmap='gray')
plt.show()

grad = signal.convolve2d(ham,diag_line,boundary='symm',mode='same')
plt.figure(figsize=(10,10))
plt.imshow(grad, cmap='gray')
plt.show()

# TODO : Apply diag_line to the "circuits.png image" and analyse the resu
```

Problem 8

Are the following systems linear or time invariant?

1. $x(t) \rightarrow \text{System(a)} \rightarrow 7x(t-1)$
2. $x(t) \rightarrow \text{System(b)} \rightarrow \cos(2x(t))$
3. $x(t) \rightarrow \text{System(c)} \rightarrow t$
4. $x(t) \rightarrow \text{System(d)} \rightarrow x(t) + t$

```
In [ ]:
```