

# <<< Only Problem 1.1 and 2.1 will be graded >>>

## Problem 1.1 (DFT)

Given the DFT spectrum  $X[k]$ , express the corresponding time-domain signal  $x[n]$  in terms of its constituent real sinusoids.

1.  $X[0]=3, X[1]=\frac{1}{\sqrt{2}}-j\frac{1}{\sqrt{2}}, X[2]=-2,$

$X[3]=\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$

2.  $X[0] = -2, X[1] = \sqrt{3} + j1, X[2] = 3, X[3] = \sqrt{3} - j1$

3.  $X[0] = 1, X[1] = 2 - j2\sqrt{3}, X[2] = -3, X[3] = 2 + j2\sqrt{3}$

## Problem 1.2 (DFT) (Optional)

Show that if  $x[n]$  is real sequence,  $X[N - k] = X^*[k]$

## Problem 2.1 (DTFT)

Use the properties of the discrete-time transform to determine  $X(e^{j\omega})$  for the following sequences

1.  $x[n] = \left(\frac{1}{3}\right)^{|n|}$

2.  $x[n] = a^n \cos(\Omega_0 n) \cdot u[n], |a| < 1$

3.  $x[n] = (n + 1)a^n \cdot u[n], |a| < 1$

## Problem 2.2 (DTFT)

Find the discrete-time sequence  $x[n]$  with transforms in range  $0 \leq \omega < 2\pi$  as follows

1.  $X(e^{j\omega}) = -j\pi\delta\left(\omega - \frac{\pi}{3}\right) + \pi\delta\left(\omega - \frac{2\pi}{3}\right) + \pi\delta\left(\omega - \frac{4\pi}{3}\right) + j\pi\delta\left(\omega - \frac{5\pi}{3}\right)$

2.  $X(e^{j\omega}) = \frac{1 - \frac{5}{6}e^{-j\omega}}{1 + \frac{1}{12}e^{-j\omega} - \frac{1}{12}e^{-j2\omega}}$

## Problem 2.3 (DTFT)

Using the DTFT, find the impulse response ( $h[n]$ ) of the system governed by the difference equation

$$1. y(n) = x[n] - 4x[n-1] + \frac{11}{12}y[n-1] + \frac{1}{12}y[n-2]$$

$$2. y(n) = x[n] - \frac{11}{15}y[n-1] + \frac{2}{15}y[n-2]$$

and find output with  $x[n] = (1/2)^n u[n]$