HW2_SimpleBayesClassifier

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1 Simple Bayes Classifier

A student in Pattern Recognition course had finally built the ultimate classifier for cat emotions. He used one input features: the amount of food the cat ate that day, x (Being a good student he already normalized x to standard Normal). He proposed the following likelihood probabilities for class 1 (happy cat) and 2 (sad cat)

$P(x|w1) = N(4, 2)$
$P(x|w2) = N(0, 2)$

Normal Distribution: $\mathcal{N}(\mu, \sigma^2)$

$$P(w_i|x) = \frac{P(x|w_i)P(w_i)}{P(x)}$$

$$Posterior = \frac{likelihood * prior}{evidence}$$

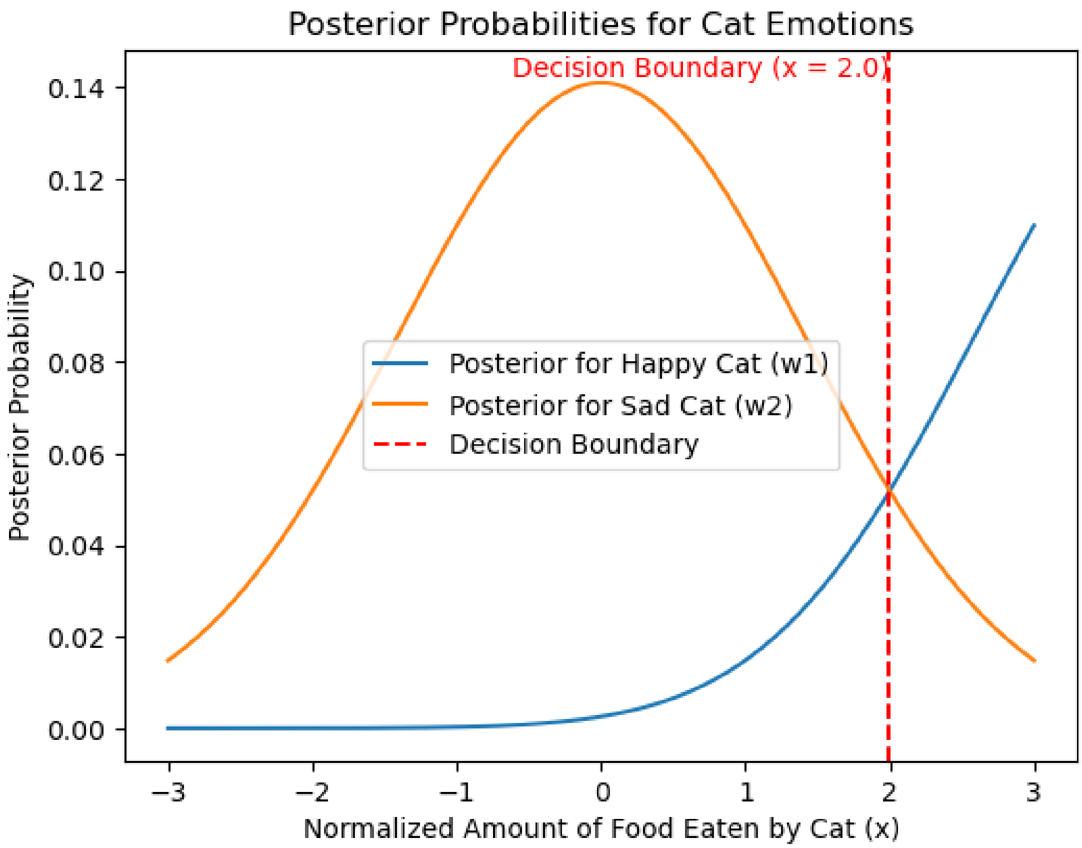
1.1 T2.

Plot the posteriors values of the two classes on the same axis. Using the likelihood ratio test, what is the decision boundary for this classifier? Assume equal prior probabilities.

[ ]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm

class SimpleBayesClassifier:
    def __init__(self, mu1, var1, mu2, var2, prior_w1=0.5, prior_w2=0.5):
        self.mu1 = mu1
        self.var1 = var1
        self.sd1 = var1**0.5
        self.mu2 = mu2
        self.var2 = var2
        self.sd2 = var2**0.5
        self.prior_w1 = prior_w1
        self.prior_w2 = prior_w2
```

```python
        self.decision_boundary = None

    def calculate_posteriors(self, x_values):
        likelihood_w1 = norm.pdf(x_values, loc=self.mu1, scale=self.sd1)
        likelihood_w2 = norm.pdf(x_values, loc=self.mu2, scale=self.sd2)

        posterior_w1 = likelihood_w1 * self.prior_w1
        posterior_w2 = likelihood_w2 * self.prior_w2

        return posterior_w1, posterior_w2

    def find_decision_boundary(self, x_values):
        self.decision_boundary = x_values[np.argmin(np.abs(self.
 ↪calculate_posteriors(x_values)[0] - self.calculate_posteriors(x_values)[1]))]

    def plot_posteriors(self, x_values):
        if self.decision_boundary is None:
            self.find_decision_boundary(x_values)

        posterior_w1, posterior_w2 = self.calculate_posteriors(x_values)

        # Plot posteriors
        plt.plot(x_values, posterior_w1, label='Posterior for Happy Cat (w1)')
        plt.plot(x_values, posterior_w2, label='Posterior for Sad Cat (w2)')

        # Plot decision boundary
        plt.axvline(x=self.decision_boundary, color='r', linestyle='--',␣
 ↪label='Decision Boundary')

        # Add text annotation for decision boundary value
        plt.text(self.decision_boundary, max(max(posterior_w1),␣
 ↪max(posterior_w2)),
                 f'Decision Boundary (x = {round(self.decision_boundary, 2)})',
                 verticalalignment='bottom', horizontalalignment='right', color='r')

        # Add labels and legend
        plt.xlabel('Normalized Amount of Food Eaten by Cat (x)')
        plt.ylabel('Posterior Probability')
        plt.title('Posterior Probabilities for Cat Emotions')
        plt.legend()
        plt.show()

if __name__ == "__main__":
    # Generate standard normalized x values
    x_values = np.linspace(-3, 3, 1000)

    # Create the classifier instance
```

```python
classifier = SimpleBayesClassifier(mu1=4, var1=2, mu2=0, var2=2)

# Plot posteriors graph
classifier.plot_posteriors(x_values)

print(f'Decision Boundary: x = {classifier.decision_boundary}')
```



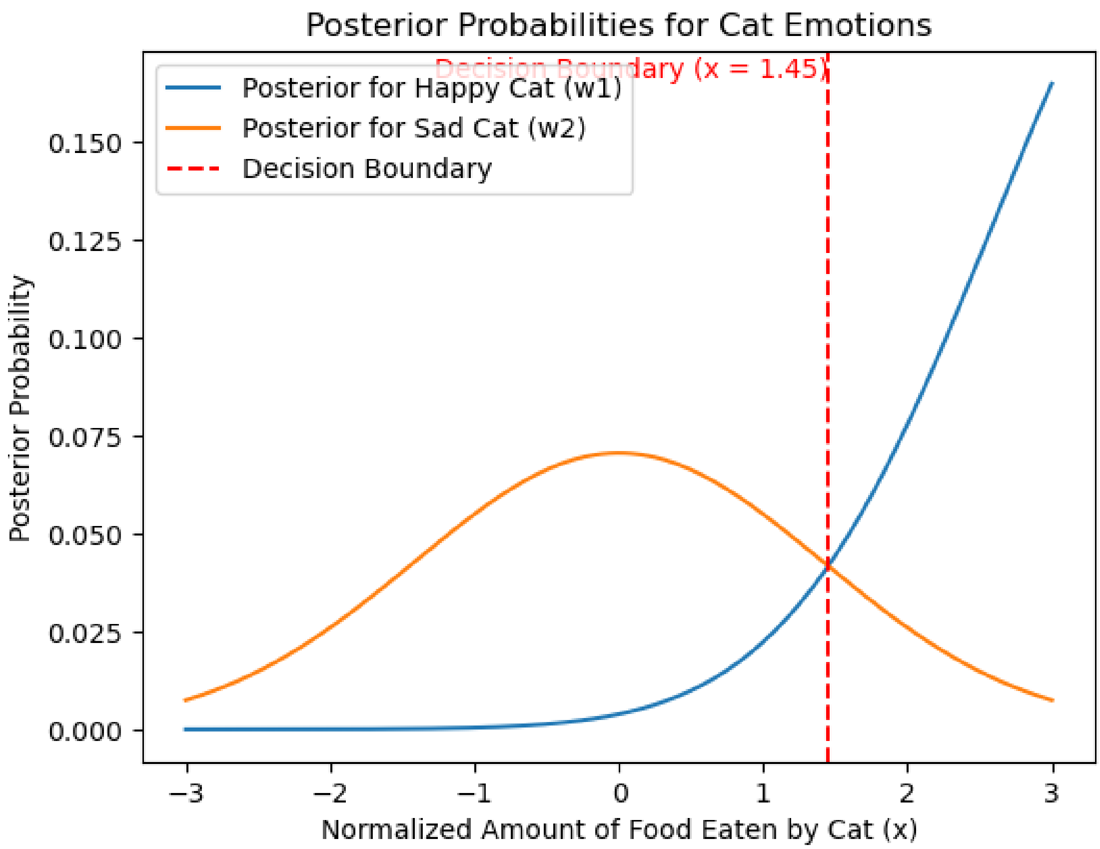

```
Decision Boundary: x = 1.9969969969969972
```

1.2 T3.

What happen to the decision boundary if the cat is happy with a prior of 0.75?

[ ]:
```python
# Generate standard normalized x values
x_values = np.linspace(-3, 3, 1000)

# Create the classifier instance with a different prior for the happy cat
classifier = SimpleBayesClassifier(mu1=4, var1=2, mu2=0, var2=2, prior_w1=0.75,
    prior_w2=0.25)
```

```python
# Plot posteriors graph
classifier.plot_posteriors(x_values)

print(f'Decision Boundary: x = {classifier.decision_boundary}')
```




```
Decision Boundary: x = 1.4504504504504503
```

1.3 OT2.

$$P(x|w_1) = \mathcal{N}(\mu_1, \sigma^2), \quad P(x|w_2) = \mathcal{N}(\mu_2, \sigma^2), \quad p(w_1) = p(w_2) = 0.5$$

The decision boundary is where the posterior is the same

$$P(w_1|x) = P(w_2|x)$$

$$\frac{P(x|w_1)P(w_1)}{P(x)} = \frac{P(x|w_2)P(w_2)}{P(x)}$$

$$P(x|w_1) = P(x|w_2)$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\frac{(x-\mu_1)^2}{\sigma^2}} = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\frac{(x-\mu_2)^2}{\sigma^2}}$$

$$(x-\mu_1)^2 = (x-\mu_2)^2$$

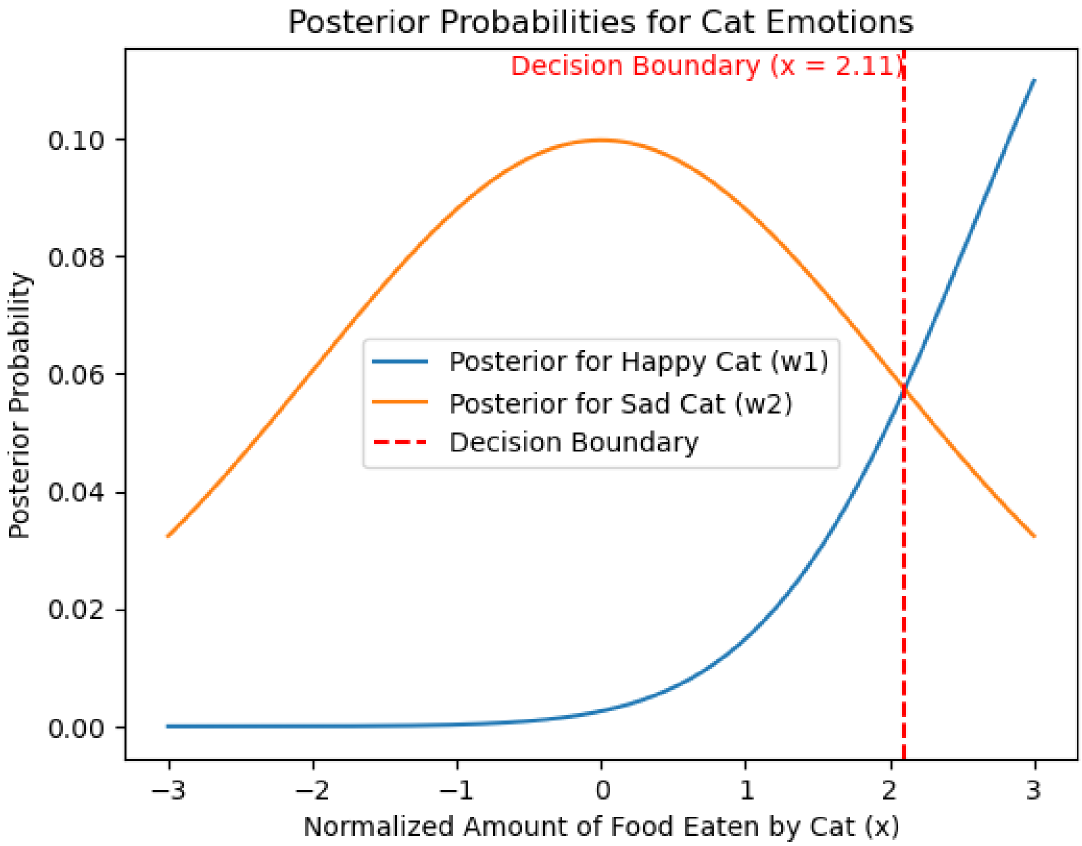
$$x^2 - 2\mu_1 x + \mu_1^2 = x^2 - 2\mu_2 x + \mu_2^2$$

$$\therefore x = \frac{\mu_1 + \mu_2}{2}$$

1.4 OT3.

If the student changed his model to P(x|w1) = N(4,2) P(x|w2) = N(0,4) Plot the posteriors values of the two classes on the same axis. What is the decision boundary for this classifier? Assume equal prior probabilities.

[ ]:
```
# Generate standard normalized x values
x_values = np.linspace(-3, 3, 1000)

# Create the classifier instance with a different prior for the happy cat
classifier = SimpleBayesClassifier(mu1=4, var1=2, mu2=0, var2=4, prior_w1=0.5,
 ↪prior_w2=0.5)

# Plot posteriors graph
classifier.plot_posteriors(x_values)

print(f'Decision Boundary: x = {classifier.decision_boundary}')
```




```
Decision Boundary: x = 2.105105105105105
```