

Homework 2

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Exercise 2

1. Consider the context-free grammar:

$$S \rightarrow SS+ \mid SS^* \mid a$$

and the string $aa + a^*$.

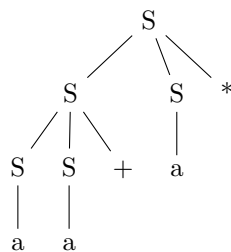
- (a) Give a leftmost derivation for the string.

$$\begin{aligned} S &\rightarrow SS^* \\ &\rightarrow SS + S^* \\ &\rightarrow aS + S^* \\ &\rightarrow aa + S^* \\ &\rightarrow aa + a^* \end{aligned}$$

- (b) Give a rightmost derivation for the string.

$$\begin{aligned} S &\rightarrow SS^* \\ &\rightarrow Sa^* \\ &\rightarrow SS + a^* \\ &\rightarrow aS + a^* \\ &\rightarrow aa + a^* \end{aligned}$$

- (c) Give a parse tree for the string.



- (d) Is the grammar ambiguous or unambiguous? Justify your answer.
: Unambiguous, leftmost and rightmost generate the same parse tree.

2. Consider the context-free grammar:

$$S \rightarrow S + S \mid SS \mid (S) \mid S^* \mid a$$

and the string $(a + a)^*a$.

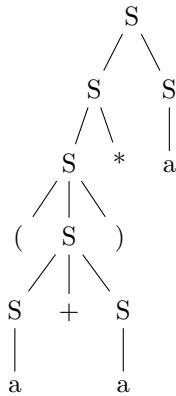
(a) Give a leftmost derivation for the string.

$$\begin{aligned} S &\rightarrow SS \\ &\rightarrow S^*S \\ &\rightarrow (S)^*S \\ &\rightarrow (S + S)^*S \\ &\rightarrow (a + S)^*S \\ &\rightarrow (a + a)^*S \\ &\rightarrow (a + a)^*a \end{aligned}$$

(b) Give a rightmost derivation for the string.

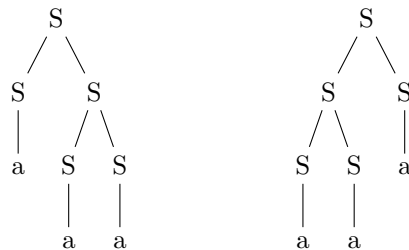
$$\begin{aligned} S &\rightarrow SS \\ &\rightarrow Sa \\ &\rightarrow S^*a \\ &\rightarrow (S)^*a \\ &\rightarrow (S + S)^*a \\ &\rightarrow (S + a)^*a \\ &\rightarrow (a + a)^*a \end{aligned}$$

(c) Give a parse tree for the string.



(d) Is the grammar ambiguous or unambiguous? Justify your answer.

: Ambiguous, the string 'aaa' can be generated by two different parse trees.



3. Design grammars for the following languages:

- (a) The set of all strings of 0s and 1s such that every 0 is immediately followed by at least one 1.

$$S \rightarrow AB$$

$$A \rightarrow 1A \mid \lambda$$

$$B \rightarrow CB \mid \lambda$$

$$C \rightarrow 01A$$

- (b) The set of all strings of 0s and 1s that are palindromes; that is, the string reads the same backward as forward.

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \lambda$$

- (c) The set of all strings of 0s and 1s with an equal number of 0s and 1s.

$$S \rightarrow 0S1S \mid 1S0S \mid \lambda$$

- (d) The set of all strings of 0s and 1s in which 011 does not appear as a substring.

$$S \rightarrow ABCB$$

$$A \rightarrow 1A \mid \lambda$$

$$B \rightarrow 0B \mid \lambda$$

$$C \rightarrow 01C \mid \lambda$$