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Assignment II

1. **Normalization by Hand**

   Normalize vector $|\overrightarrow{v_1}\rangle$ = $2|\uparrow\rangle + 1|\downarrow\rangle$ by hand.

2. **Normalization Using Python**

   Check using Google Colab. You can use the scikit-learn library to do so:

   ```
   import numpy as np
   v1=np.array([[2],[1]])
   print(v1)
   from sklearn.preprocessing import normalize
   v2 = v1/np.linalg.norm(v1)
   print(v2)
   ```

3. **Normalization by Hand with Complex Coefficient**

   Normalize vector $|\overrightarrow{v_1}\rangle$ = $2|\uparrow\rangle + (1+i)|\downarrow\rangle$ by hand.

4. **Normalization Using Python with Complex Coefficient**

   Verify with Google Colab. Remember you need to use 1 + 1*j* to represent 1 + *i*.

5. **Vectors on a 2D Plane**

   On a 2D plane, draw the vectors $|\overrightarrow{x'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle + |\hat{y}\rangle)$ and $|\overrightarrow{y'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle - |\hat{y}\rangle)$

   Write the column forms of $|\overrightarrow{x'}\rangle$ and $|\overrightarrow{y'}\rangle$. Show that $|\overrightarrow{x'}\rangle$ and $|\overrightarrow{y'}\rangle$ are orthonormal, i.e. $\left\langle \overrightarrow{x'} \middle| \overrightarrow{x'} \right\rangle = \left\langle \overrightarrow{y'} \middle| \overrightarrow{y'} \right\rangle = 1$ and $\left\langle \overrightarrow{x'} \middle| \overrightarrow{y'} \right\rangle = \left\langle \overrightarrow{y'} \middle| \overrightarrow{x'} \right\rangle = 0$. You have derived a new orthonormal basis for a 2D plane. But this is not surprising, right? As $|\overrightarrow{x'}\rangle$ and $|\overrightarrow{y'}\rangle$ can be obtained by just rotating $|\vec{x}\rangle$ and $|\vec{y}\rangle$ by $45°$.

6. **Basis Change by Hand**

   Vector $\overrightarrow{v_1} = \binom{1}{2}$ is in the $|+\rangle/|-\rangle$ basis. Represent it in the $|0\rangle/|1\rangle$ basis try to $|+\rangle$ and $|-\rangle$ in terms of $|0\rangle$ and $|1\rangle$ and perform the substitution. To check your answer, convert it back to $|+\rangle/|-\rangle$ using the equations introduced in this chapter and you should get back $\binom{1}{2}$.

7. **Vector Normalization by Hand and Using Python**

   Normalize $|\overrightarrow{v_1}\rangle = \binom{1}{2}$ (which is in the $|0\rangle/|1\rangle$ basis) in the $|+\rangle/|-\rangle$ basis. Use Google Colab to verify your answer.

1. Normalization by Hand

Normalize vector $|\overrightarrow{v_1}\rangle = 2|\uparrow\rangle + 1|\downarrow\rangle$ by hand.

$$\therefore \ |\vec{v}_1{}'\rangle = \frac{|\vec{v}_1\rangle}{\sqrt{\sum_{i=0}^{n-1} |a_i|^2}} = \frac{|\vec{v}_1\rangle}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}}|\uparrow\rangle + \frac{1}{\sqrt{5}}|\downarrow\rangle$$

2. Normalization Using Python

Check using Google Colab. You can use the scikit-learn library to do so:

```python
import numpy as np

v1 = np.array([[2], [1]])
print(f"v1: {v1}")

v2 = v1 / np.linalg.norm(v1)
print(f"v2: {v2}")
```

```
(quantum-computing) (base)
veloper/repositories/quantu
computing/assignments-02/q2
v1: [[2]
 [1]]
v2: [[0.89442719]
 [0.4472136 ]]
(quantum-computing) (base)
```

3. Normalization by Hand with Complex Coefficient

Normalize vector $|\vec{v_1}\rangle = 2|\uparrow\rangle + (1+i)|\downarrow\rangle$ by hand.

$$|\vec{v}_1'\rangle = \frac{|\vec{v}_1\rangle}{\sqrt{\sum_{i=0}^{n-1} |a_i|^2}} = \frac{|\vec{v}_1\rangle}{\sqrt{2^2 + |a_2|^2}} = \frac{|\vec{v}_1\rangle}{\sqrt{4 + a_2^* a_2}} = \frac{|\vec{v}_1\rangle}{\sqrt{4 + (1-i)(1+i)}}$$

$$= \frac{|\vec{v}_1\rangle}{\sqrt{4 + 1^2 - i^2}} = \frac{|\vec{v}_1\rangle}{\sqrt{6}}$$

$$\therefore |\vec{v}_1'\rangle = \frac{2}{\sqrt{6}}|\uparrow\rangle + \frac{1+i}{\sqrt{6}}|\downarrow\rangle \quad *$$

4. Normalization Using Python with Complex Coefficient

Verify with Google Colab. Remember you need to use 1 + 1*j* to represent 1 + *i*.

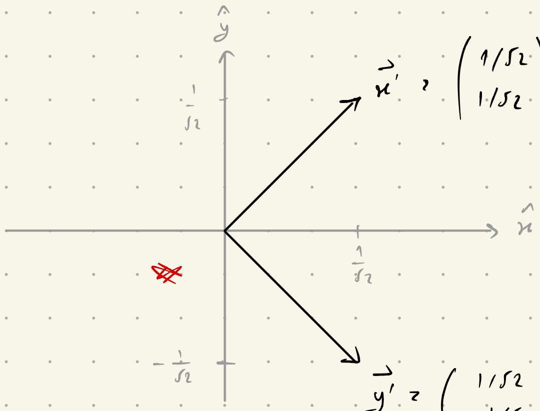
```python
import numpy as np

v1 = np.array([[2], [1 + 1j]])
print(f"v1: {v1}")

v2 = v1 / np.linalg.norm(v1)
print(f"v2: {v2}")
```

```
(quantum-computing) (base) pupi
veloper/repositories/quantum-co
computing/assignments-02/c3.py
v1: [[2.+0.j]
 [1.+1.j]]
v2: [[0.81649658+0.j          ]
 [0.40824829+0.40824829j]]
(quantum-computing) (base) pupi
```

5. Vectors on a 2D Plane

On a 2D plane, draw the vectors $|\vec{x'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle + |\hat{y}\rangle)$ and $|\vec{y'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle - |\hat{y}\rangle)$

Write the column forms of $|\vec{x'}\rangle$ and $|\vec{y'}\rangle$. Show that $|\vec{x'}\rangle$ and $|\vec{y'}\rangle$ are orthonormal, i.e. $\langle\vec{x'}|\vec{x'}\rangle = \langle\vec{y'}|\vec{y'}\rangle = 1$ and $\langle\vec{x'}|\vec{y'}\rangle = \langle\vec{y'}|\vec{x'}\rangle = 0$. You have derived a new orthonormal basis for a 2D plane. But this is not surprising, right? As $|\vec{x'}\rangle$ and $|\vec{y'}\rangle$ can be obtained by just rotating $|\vec{x}\rangle$ and $|\vec{y}\rangle$ by 45°.



$$\vec{x'} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\vec{y'} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$\langle \vec{x'} | \vec{x'} \rangle = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = 1/\sqrt{2} \times 1/\sqrt{2} + 1/\sqrt{2} \times 1/\sqrt{2} = 1$$

$$\langle \vec{y'} | \vec{y'} \rangle = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = 1/\sqrt{2} \times 1/\sqrt{2} + (-1/\sqrt{2}) \times (-1/\sqrt{2}) = 1$$

$$\therefore \langle \vec{x'} | \vec{x'} \rangle = \langle \vec{y'} | \vec{y'} \rangle = 1$$

$$\langle \vec{x'} | \vec{y'} \rangle = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0.$$

$$\langle \vec{y'} | \vec{x'} \rangle = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$$

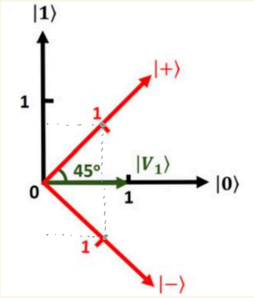
$$\therefore \langle \vec{x'} | \vec{y'} \rangle = \langle \vec{y'} | \vec{x'} \rangle = 0$$

$\therefore$ $|\vec{x'}\rangle$ and $|\vec{y'}\rangle$ are Orthonormal

6. Basis Change by Hand

Vector $\overrightarrow{v_1} = \binom{1}{2}$ is in the $|+\rangle/|-\rangle$ basis. Represent it in the $|0\rangle/|1\rangle$ basis try to $|+\rangle$ and $|-\rangle$ in terms of $|0\rangle$ and $|1\rangle$ and perform the substitution. To check your answer, convert it back to $|+\rangle/|-\rangle$ using the equations introduced in this chapter and you should get back $\binom{1}{2}$.

$$|\vec{v}_1\rangle = 1\,|+\rangle + 2\,|-\rangle$$



$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|-\rangle = +\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$|\vec{v}_1\rangle = 1 \cdot \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) + 2 \cdot \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$\therefore |\vec{v}_1\rangle = \frac{3}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

from the chapter

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \text{ and } |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

$$|\vec{v}_1\rangle = \frac{3}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$$\therefore |\vec{v}_1\rangle = |+\rangle + 2\,|-\rangle$$

7. Vector Normalization by Hand and Using Python

Normalize $|\vec{v_1}\rangle = \binom{1}{2}$ (which is in the $|0\rangle/|1\rangle$ basis) in the $|+\rangle/|-\rangle$ basis. Use Google Colab to verify your answer.

$$|\vec{v}_1\rangle = 1\,|0\rangle + 2\,|1\rangle$$

$$|0\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right)$$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$$= 1 \cdot \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right) + 2 \cdot \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$$|\vec{v}_1\rangle = \frac{3}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

Normalize.

$$|\vec{v}_1'\rangle = \frac{|\vec{v}_1\rangle}{\sqrt{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}} = \frac{3}{\sqrt{10}}|+\rangle - \frac{1}{\sqrt{10}}|-\rangle$$

```python
import numpy as np

v1 = np.array([[3 / 2**0.5], [-1 / 2**0.5]])
print(f"v1: {v1}")

v2 = v1 / np.linalg.norm(v1)
print(f"v2: {v2}")
```

```
(quantum-computing)
veloper/repositories
computing/assignment
v1: [[ 2.12132034]
 [-0.70710678]]
v2: [[ 0.9486833 ]
 [-0.31622777]]
(quantum-computing)
```

8. **Measurement Probability**

   For the $|\overrightarrow{v_1}\rangle$ in the last question, what is the probability to find $|+\rangle$ in the $|+\rangle/|-\rangle$ basis measurement?

9. **Vector Normalization by Hand and Using Python 2**

   Normalize $|\overrightarrow{v_1}\rangle$ in the $|0\rangle/|1\rangle$ basis. What is the probability to find $|0\rangle$ in the $|0\rangle/|1\rangle$ basis measurement? Use Google Colab to verify your answer.

10. **Basis Change and Measurement**

    If I get $|1\rangle$ in the measurement in Problem 9, what is the probability of getting $|+\rangle$ if I measure again in the $|+\rangle/|-\rangle$ basis measurement? (hint: after getting $|1\rangle$, how to represent it in the $|+\rangle/|-\rangle$ basis?)

8. **Measurement Probability**

For the $|\vec{v_1}\rangle$ in the last question, what is the probability to find $|+\rangle$ in the $|+\rangle/|-\rangle$ basis measurement?

From $|\vec{v}_1{}'\rangle = \frac{3}{\sqrt{10}}|+\rangle - \frac{1}{\sqrt{10}}|-\rangle$

$\therefore P(+) = \left|\frac{3}{\sqrt{10}}\right|^2$: Born rule

$= \frac{9}{10}$

$= 90\%$

9. **Vector Normalization by Hand and Using Python 2**

Normalize $|\vec{v_1}\rangle$ in the $|0\rangle/|1\rangle$ basis. What is the probability to find $|0\rangle$ in the $|0\rangle/|1\rangle$ basis measurement? Use Google Colab to verify your answer.

From $|\vec{v}_1\rangle = 1|0\rangle + 2|1\rangle$

$|\vec{v}_1{}'\rangle = \frac{|\vec{v}_1\rangle}{\sqrt{1^2+2^2}}$

$= \frac{1}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$

$\therefore P(0) = \left|\frac{1}{\sqrt{5}}\right|^2$: Born rule

$= \frac{1}{5}$

$= 20\%$

```
import numpy as np

# Original vector in |0>,|1> basis
v1 = np.array([[1], [2]])

# Normalize
v1_norm = v1 / np.linalg.norm(v1)

# Probability of measuring |0>
prob_0 = abs(v1_norm[0, 0]) ** 2

print("Normalized vector:\n", v1_norm)
print("Probability of |0>:", prob_0 * 100, "%")
```

```
(quantum-computing) (base) pupi
veloper/repositories/quantum-co
computing/assignments-02/q9.py
Normalized vector:
 [[0.4472136 ]
 [0.89442719]]
Probability of |0>: 20.0 %
(quantum-computing) (base) pupi
```

10. Basis Change and Measurement

If I get $|1\rangle$ in the measurement in Problem 9, what is the probability of getting $|+\rangle$ if I measure again in the $|+\rangle/|-\rangle$ basis measurement? (hint: after getting $|1\rangle$, how to represent it in the $|+\rangle/|-\rangle$ basis?)

After Q.9, state collapse to:

$$|\psi\rangle = |1\rangle$$

$$|1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

$$= \left(\frac{1}{\sqrt{2}}\right)|+\rangle - \left(\frac{1}{\sqrt{2}}\right)|-\rangle$$

Born rule: $P(+) = \left|\frac{1}{\sqrt{2}}\right|^2$

$$= \frac{1}{2}$$

$$= 50\%$$