Homework 1
M1522.001300 Probabilistic Graphical Models (2017 Fall)

2014-31117 Hyun Gi Ahn

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1 Change of Random Variables [5 points]

If $a > 0$, then cdf $F_Y(y) = P\left[x \leq \frac{Y-b}{a}\right] = F_X\left(\frac{Y-b}{a}\right)$

On the other hand, if $a < 0$, then cdf $F_Y(y) = P\left[x \geq \frac{Y-b}{a}\right] = 1 - F_X\left(\frac{Y-b}{a}\right)$

We can get the pdf of Y by differentiating with repect to y

If $a > 1$, then $f_Y(y) = \frac{1}{a} f_x\left(\frac{y-b}{a}\right)$ and If $a < 1$, then $f_Y(y) = -\frac{1}{a} f_x\left(\frac{y-b}{a}\right)$
As a result,

$$f_Y(y) = \frac{1}{|a|} f_x\left(\frac{y-b}{a}\right) \rightarrow (1)$$

Gaussian pdf with mean m and standard deviation σ

$$f_X(x) = \frac{1}{\sqrt{2\pi\,|a\sigma|}} e^{\frac{-(x-m)^2}{2\sigma^2}} \rightarrow (2)$$

Substitution of Eq. (2) into Eq. (1) yields

$$f_Y(y) = \frac{1}{\sqrt{2\pi\,|a\sigma|}} e^{\frac{-(y-b-am)^2}{2a\sigma^2}}$$

Therfore $N(am + b; a^2\sigma^2)$

2 Conditional and Total Probability [10 points]

1.

$$P(Department\ A) = 0.62 \times \frac{825}{825 + 108} + 0.82 \times \frac{108}{825 + 108} = 0.64$$

$$P(Department\ C) = 0.33 \times \frac{417}{417 + 375} + 0.35 \times \frac{375}{417 + 375} = 0.33$$

2.

$$P(Men) = \frac{0.62 \times 825 + 0.63 \times 560 + 0.33 \times 417 + 0.06 \times 272}{825 + 108 + 560 + 25 + 417 + 375 + 272 + 341} = 0.34$$

$$P(Women) = \frac{0.82 \times 108 + 0.68 \times 25 + 0.35 \times 375 + 0.07 \times 341}{825 + 108 + 560 + 25 + 417 + 375 + 272 + 341} = 0.08$$

3. Because When Department C and D is given, the probability of Woman is low. In case of Woman Applicants of Department C, D is Higher than A, B. It means that the number of admitted students in Department C and D can affect to admission rate a lot

3 Independence Properties [20 points]

1. $(X, W \perp Y \mid Z) \xrightarrow{symmetry} (Y \perp X, W \mid Z) \xrightarrow{Decomposition} (Y \perp X \mid Z)$ and $(Y \perp W \mid Z) \xrightarrow{symmetry} (\mathbf{X} \perp \mathbf{Y} \mid \mathbf{Z})$ and $(W \perp Y \mid Z)$

2. $(X, W \perp Y \mid Z) \xrightarrow{symmetry} (Y \perp X, W \mid Z) \xrightarrow{WeakUnion} (Y \perp X \mid Z, W)$ and $(Y \perp W \mid Z) \xrightarrow{symmetry} (\mathbf{X} \perp \mathbf{Y} \mid \mathbf{W}, \mathbf{Z})$ and $(W \perp Y \mid Z)$

3.

4. $(X \perp Y \mid Z, W)$ and $(Y \perp W \mid Z, X) \xrightarrow{symmetry} (Y \perp X \mid Z, W)$ and $(Y \perp W \mid X, Z) \xrightarrow{Intersection} (Y \perp X, W \mid Z) \xrightarrow{symmetry} (\mathbf{X}, \mathbf{W} \perp \mathbf{Y} \mid \mathbf{Z})$

4 Naive Bayes Example [15 points]

1. By Chain Rules

$$\begin{aligned}
P(C, X_1, ..., X_n) &= P(C)P(X_1, ..., X_n \mid C) && (1)\\
&= P(C)P(X_1 \mid C)P(X_2, ..., X_n \mid C, X_1)\\
&= P(C)P(X_1 \mid C)P(X_2 \mid C, X_1)P(X_3, ..., X_n \mid C, X_1, X_2) && (2)\\
&= P(C)P(X_1 \mid C)P(X_2 \mid C, X_1) \ ... \ P(X_n \mid C, X_1, X_2, \ ... \ X_{n-1}) && (3)
\end{aligned}$$

You can insert your figure by using `\begin{figure}`. You can refer your figure by using `\ref{fig:example_figure}`.

You can enumerate sub questions like this.

1. Enumerate item 1.
2. Enumerate item 2.
3. Enumerate item 3.

You can write your equations by using `\begin{aligned}`. We highly recommend you to use `\newcommand` to simplify your equation in LaTeX.

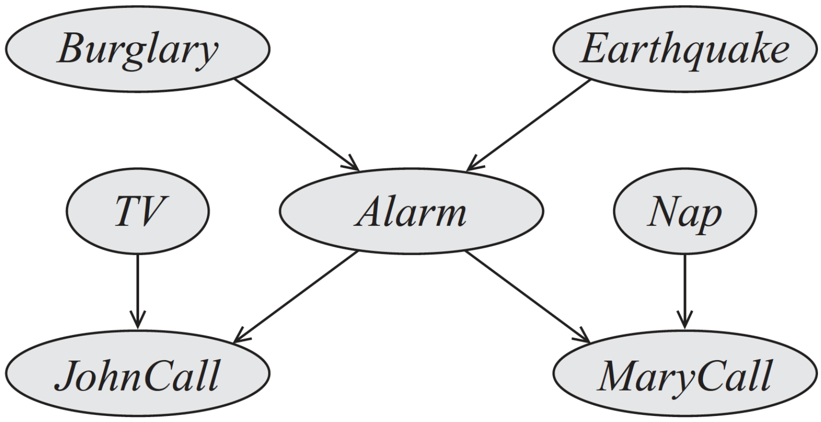

Figure 1: Example Figure

$$\boldsymbol{A} = 12 \tag{4}$$
$$\alpha_{12}^{35} = 1234 \tag{5}$$
$$\beta_1 = 10 \tag{6}$$

You can cite your reference by using `\cite{reference_name}`. For example, cite Koller's PGM book [1] like this.

5 Some materials for HW2 [5 points]

Step	Variable eliminated	Factors used	Variables involved	New factor
1				
2				
3				
4				
5				
6				
7				
8				

Table 1: A run of variable elimination for the query $P(J)$

References

[1] Daphne Koller and Nir Friedman. *Probabilistic Graphical Models: Principles and Techniques - Adaptive Computation and Machine Learning*. The MIT Press, 2009.