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Turbo Equalization

Turbo Equalization

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Turbo Equalization

Outline

- The discrete model of the channel with ISI.
- The turbo principle applied to iterative equalization and decoding: *Turbo Equalization*.
- The BCJR algorithm applied to equalization and convolutional decoding.
- Example of *Turbo Equalization.*
- Results and discussion.
- *Turbo Equalization* using the "*Interference* Canceler (IC)".

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Introduction

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Turbo Equalization

- Turbo codes were invented by Berrou, Glavieux and Thitimajshima in 1993 [Ber93].
- Turbo Equalization was proposed first by Douillard, Jezequel, Berrou, Picart, Didier, and Glavieux in 1995.
- A simplified Turbo Equalizer was proposed by Glavieux, Laot, and Labat in 1997.

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Transmission System

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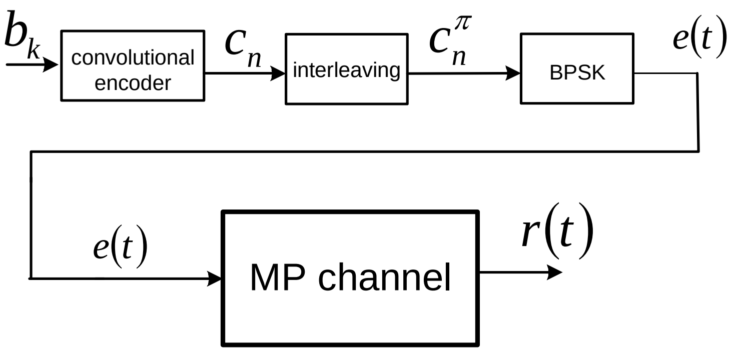

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Multipath Channel (MP)

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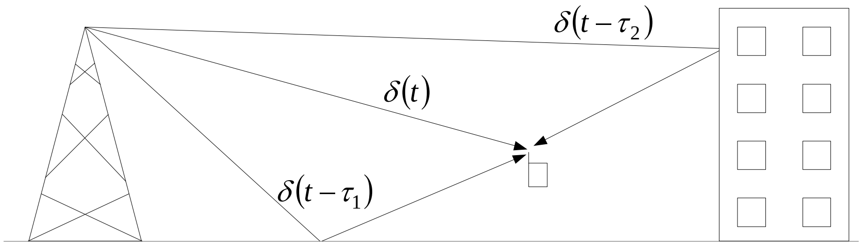

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Multipath (MP) channel

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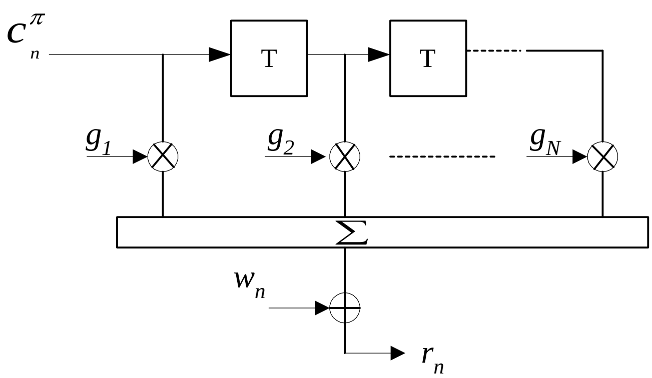

$g_n \rightarrow$ taps of the channel

$w_n \rightarrow$ aditive noise

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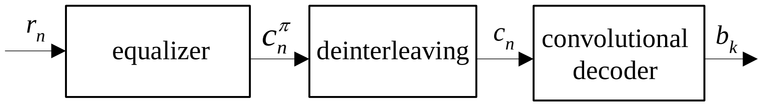
Conventional system

- Equalization and decoding separately.
  - Equalization: Filtering (LMS, DFE)
    MLSE (Viterbi Algorithm)



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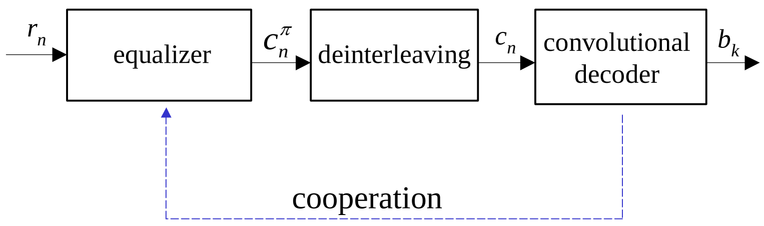

MP channel as a Markov chain

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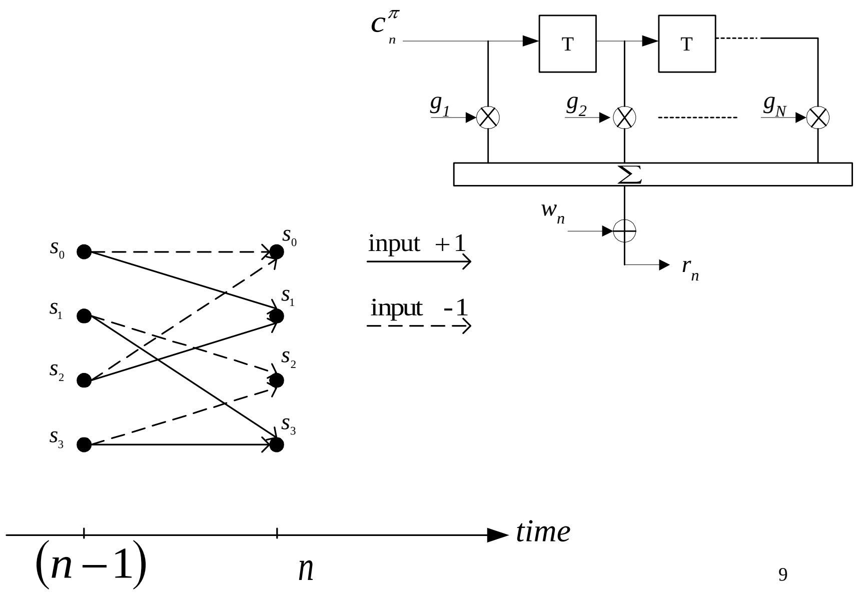

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Transmission model with MP channel

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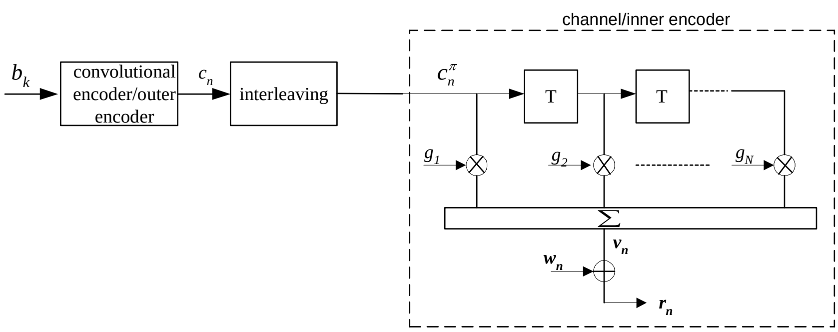

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SISO device

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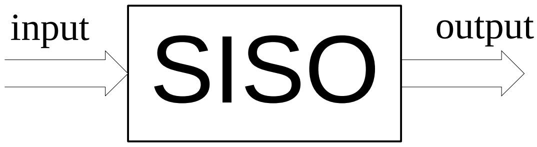

$$\textit{Log-Likelihood Ratio}\ (\text{LLR}) = \ln\left(\frac{P(x=+1)}{P(x=-1)}\right)$$

- A hard decision can be done based on the signal of the LLR
- The reliability of the decision is related to by the magnitude of the LLR.

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Maximum A Posteriori (MAP) Equalizer

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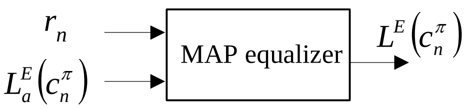

$$L^E(c_n^\pi) \equiv \ln\left(\frac{P(c_n^\pi = +1 \mid \mathbf{r})}{P(c_n^\pi = -1 \mid \mathbf{r})}\right)$$

$$L^E(c_n^\pi) \equiv \ln\left(\frac{P(c_n^\pi = +1 \mid \mathbf{r})}{P(c_n^\pi = -1 \mid \mathbf{r})}\right), \text{ using Bayes' Rule}:$$

$$= \ln\left(\frac{p(\mathbf{r} \mid c_n^\pi = +1)}{p(\mathbf{r} \mid c_n^\pi = -1)}\right) + \ln\left(\frac{P(c_n^\pi = +1)}{P(c_n^\pi = -1)}\right) \equiv L_e^E(c_n^\pi) + L_a^E(c_n^\pi)$$

$L_e^E(c_n^\pi) \rightarrow$ Extrinsic Information

$L_a^E(c_n^\pi) \rightarrow$ A Priori Information

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MAP Decoder

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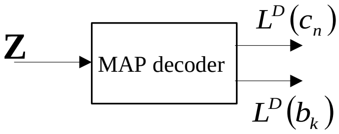
$$\mathbf{Z} \equiv [P(c_1 \,|\, \mathbf{r})\, P(c_2 \,|\, \mathbf{r}) \ldots P(c_N \,|\, \mathbf{r})]$$



$$L^D(c_n) \equiv \ln\left(\frac{P(c_n = +1 \,|\, \mathbf{Z})}{P(c_n = -1 \,|\, \mathbf{Z})}\right), \text{ using Bayes Rule:}$$

$$= \ln\left(\frac{p(\mathbf{Z} \,|\, c_n = +1)}{p(\mathbf{Z} \,|\, c_n = -1)}\right) + \ln\left(\frac{P(c_n = +1)}{P(c_n = -1)}\right) \equiv L_e^D(c_n) + L_a^D(c_n)$$

Also, the MAP decoder computes an estimate $\hat{b}_k$ of the transmitted data as the most likely bit given **Z**.

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Turbo Equalization (TuEqu)

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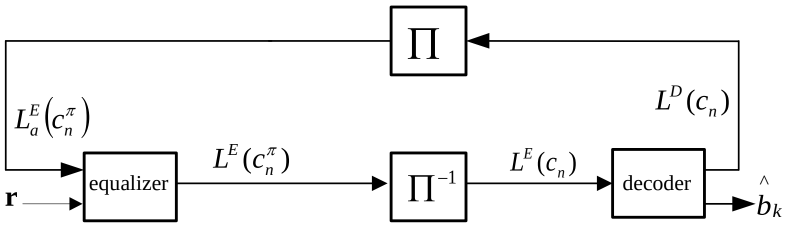

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Turbo Equalization (TuEqu)

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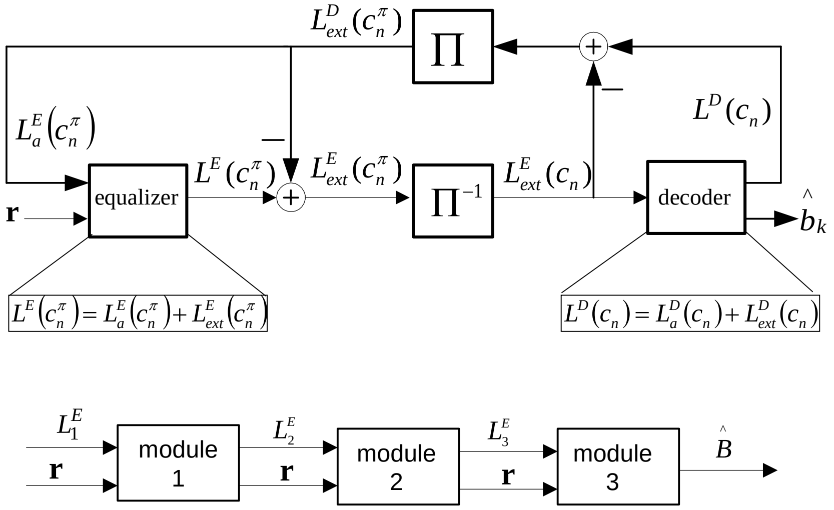
Turbo Equalization



Questions???

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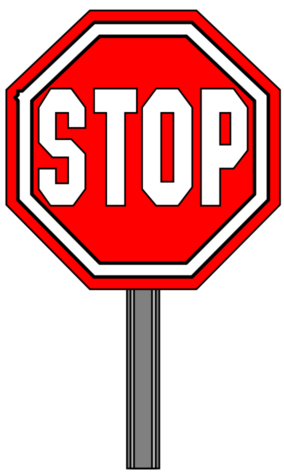

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MAP algorithm (BCJR)

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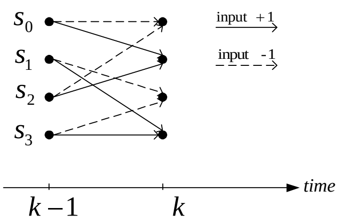
$$\boxed{L(b_k) \equiv \ln\left(\frac{P(b_k = +1 \mid \mathbf{r})}{P(b_k = -1 \mid \mathbf{r})}\right)}$$



using Bayes' rule : $P(a,b) = P(a \mid b)P(b)$

$$L(b_k) = \ln\frac{p(b_k = +1, \mathbf{r})}{p(b_k = -1, \mathbf{r})}, \quad L(b_k) = \ln\frac{\left(\sum\limits_{b_k=+1} p(S_{k-1}, S_k, \mathbf{r})\right)}{\left(\sum\limits_{b_k=-1} p(S_{k-1}, S_k, \mathbf{r})\right)}$$

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MAP algorithm

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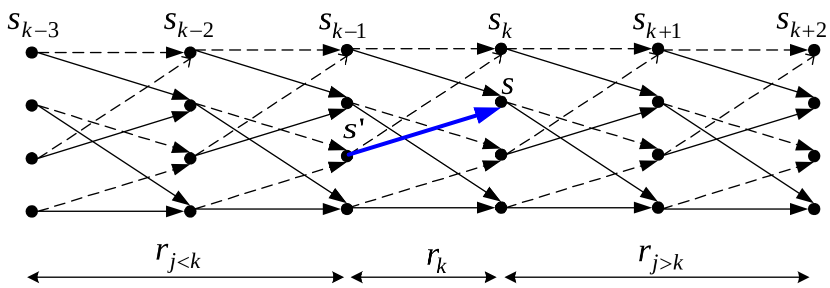

$$p(s', s, \mathbf{r}) = p(s', s, r_{j<k}, r_k, r_{j>k})$$

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MAP algorithm

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$$p(s',s,R) = p(s',s,r_{j<k},r_k,r_{j>k})$$

$$\text{Using Baye's rule}: P(a,b) = P(a \mid b)P(b)$$

$$p(s',s,\mathbf{r}) = p\left(r_{j>k} \mid \{s',s,r_{j<k},r_k\}\right)p(s',s,r_{j<k},r_k)$$

Using the assumption that the channel is memorryless, the future received sequence $r_{j>k}$ will only depend on the present state s:

$$P(s',s,\mathbf{r}) = p(r_{j>k} \mid s)\,p(s',s,r_{j<k},r_k)$$
$$= p(r_{j>k} \mid s)\,p(\{r_k,s\} \mid \{s',r_{j<k}\})\,p(s',r_{j<k})$$
$$= p(r_{j>k} \mid s)\,p(\{r_k,s\} \mid s')\,p(s',r_{j<k})$$

$$\boxed{P(s',s,R) = \beta_k(s)\gamma_k(s',s)\alpha_{k-1}(s')}$$

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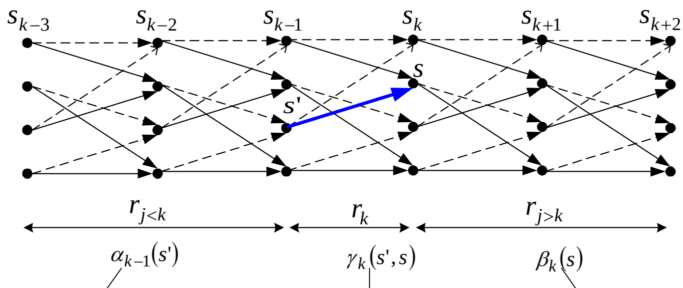

$$\alpha_{k-1}(s') = p(S_{k-1} = s', r_{j<k})$$

$$\beta_k(s) = p(r_{j>k} \mid S_k = s)$$

$$\gamma_k(s',s) = p(\{r_k, S_k = s\} \mid S_{k-1} = s')$$

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Forward recursive computation of α

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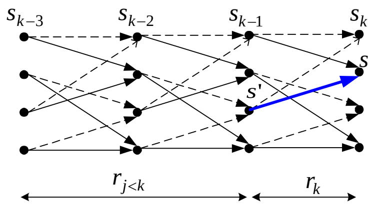
$$\alpha_k(s) = p\left(S_k = s, r_{j<k+1}\right)$$



$$\alpha_k(s) = p\left(s, r_{j<k}, r_k\right) = \sum_{all\ s'} p\left(s', s, r_{j<k}, r_k\right)$$

Using Bayes' rule and the assumption that the channel is memoryless:

$$\alpha_k(s) = \sum_{all\ s'} p\left(s', s, r_{j<k}, r_k\right) = \sum_{all\ s'} p\left(\{s, r_k\} \mid \{s', r_{j<k}\}\right) p\left(s', r_{j<k}\right)$$

$$\alpha_k(s) = \sum_{all\ s'} p\left(\{s, r_k\} \mid s'\right) p\left(s', r_{j<k}\right)$$

Baye's rule:
$P(a,b) = P(a \mid b) P(b)$

$$\boxed{\alpha_k(s) = \sum_{all\ s'} \gamma_k\left(s', s\right) \alpha_{k-1}\left(s'\right)}$$

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Forward recursive computation of α

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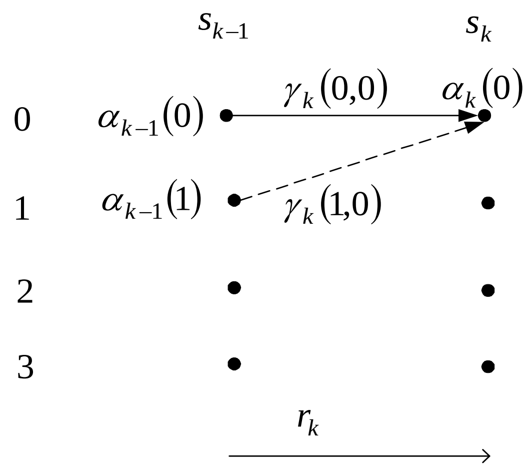

$$\alpha_k(0) = \alpha_{k-1}(0).\gamma_k(0,0) + \alpha_{k-1}(1).\gamma_k(1,0)$$

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Backward recursive computation of β

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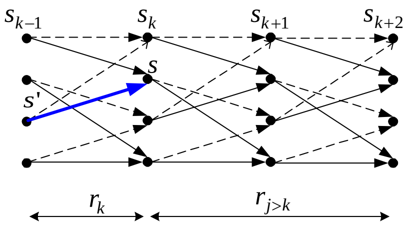
$$\boxed{\beta_{k-1}(s') = p\left(r_{j>k-1} \mid S_{k-1} = s'\right)}$$



$$\begin{aligned}
\beta_{k-1}(s') &= p\left(r_{j>k-1} \mid s'\right) \\
&= \sum_{\text{all } s} p\left(\left\{r_{j>k-1}, s\right\} \mid s'\right) \\
&= \sum_{\text{all } s} p\left(\left\{r_{j>k}, r_k, s\right\} \mid s'\right) \\
&= \sum_{\text{all } s} p\left(r_{j>k} \mid \left\{s, s', r_k\right\}\right) p\left(\left\{r_k, s\right\} \mid s'\right) \\
&= \sum_{\text{all } s} p\left(r_{j>k} \mid \left\{s\right\}\right) p\left(\left\{r_k, s\right\} \mid s'\right)
\end{aligned}$$

Baye's rule:
$P(a,b) = P(a \mid b)P(b)$
$P(\{a,b\} \mid c) = P(a \mid (b,c))P(b \mid c)$

$$\boxed{\beta_{k-1}(s') = \sum_{\text{all } s} \beta_k(s)\gamma_k(s',s)}$$

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Backward recursive computation of β

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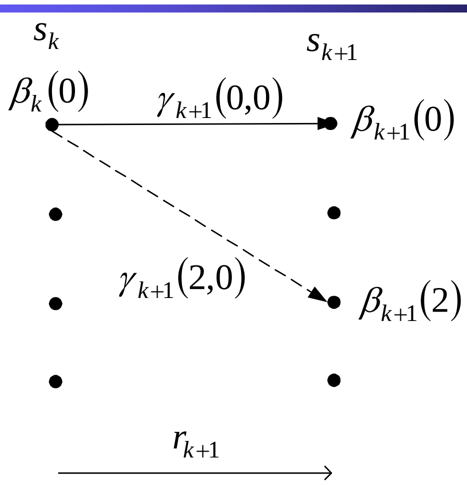

$$\beta_k(0) = \beta_{k+1}(0).\gamma_{k+1}(0,0) + \beta_{k+1}(2).\gamma_{k+1}(0,2)$$

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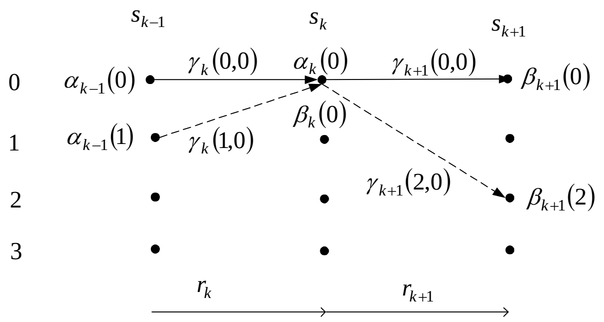
$$\alpha_k(s)=\sum_{all\,s'}\gamma_k(s,s').\alpha_{k-1}(s') \quad , \quad \beta_{k-1}(s')=\sum_{all\,s}\gamma_k(s',s).\beta_k(s)$$



$$\alpha_k(0)=\alpha_{k-1}(0).\gamma_k(0,0)+\alpha_{k-1}(1).\gamma_k(1,0)$$
$$\beta_k(0)=\beta_{k+1}(0).\gamma_{k+1}(0,0)+\beta_{k+1}(2).\gamma_{k+1}(0,2)$$

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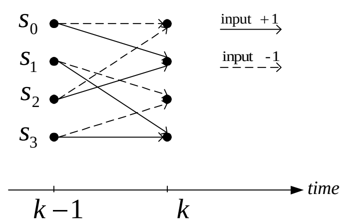
$$L(b_k)\equiv\ln\left(\frac{P(b_k=+1|\mathbf{r})}{P(b_k=-1|\mathbf{r})}\right)$$



$$L(b_k)=\ln\frac{\left(\sum_{b_k=+1}p(S_{k-1},S_k,\mathbf{r})\right)}{\left(\sum_{b_k=-1}p(S_{k-1},S_k,\mathbf{r})\right)}=\ln\frac{\left(\sum_{b_k=+1}\alpha_{k-1}(s')\gamma_k(s',s)\beta_k(s)\right)}{\left(\sum_{b_k=-1}\alpha_{k-1}(s')\gamma_k(s',s)\beta_k(s)\right)}$$

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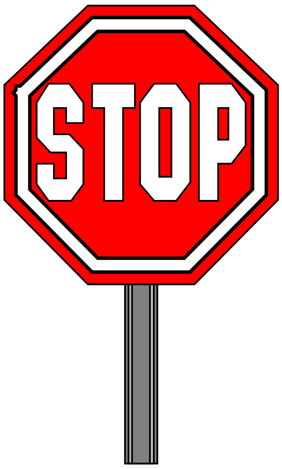
Questions???



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MAP Equalizer

$$L^E\left(c_n^\pi\right) \equiv \ln\left(\frac{P\left(c_n^\pi = +1 \mid \mathbf{r}\right)}{P\left(c_n^\pi = -1 \mid \mathbf{r}\right)}\right)$$

$$\begin{array}{c} r_n \longrightarrow \\ L_a^E\left(c_n^\pi\right) \longrightarrow \end{array} \boxed{\text{MAP equalizer}} \longrightarrow L^E\left(c_n^\pi\right)$$

$$\gamma_n\left(s', s\right) = p\left(\{r_n, S_n = s\} \mid S_{n-1} = s'\right)$$

Baye's rule:

$P(a,b) = P(a \mid b).P(b)$

$P(\{a,b\} \mid c) = P(a \mid (b,c)).P(b \mid c)$

Using Baye's rule:

$$\gamma_n\left(s', s\right) = p\left(r_n \mid \{s, s'\}\right).P(s \mid s')$$

is governed by the output symbol

is governed by the input symbol

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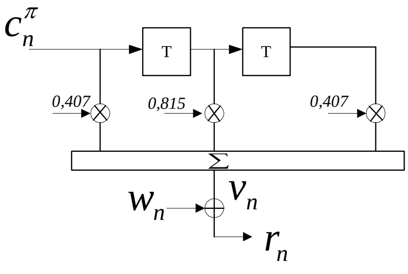
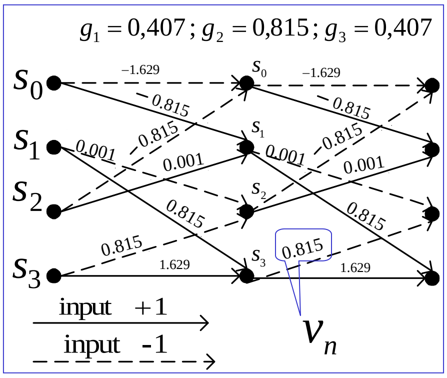
MAP Equalizer

Turbo Equalization





$$\gamma_n(s',s) = p(r_n \mid \{s,s'\}).P(s \mid s')$$

$$\gamma_n(s',s) = p(r_n \mid v_n).P(c_n^\pi)$$

$$p(r_n \mid v_n) = \exp\left(-(r_n - v_n)^2 / 2\sigma^2\right) \Big/ \sqrt{2\pi\sigma^2}$$

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MAP Equalizer

Turbo Equalization

$$\gamma(s',s) = P(c_n^\pi)\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-(r_n - v_n)^2/2\sigma^2\right)$$

obtained from the extrinsic information $L_{ext}^D(c_n)$ of the decoder :

$$L_{ext}^D(c_n) \cong \ln\frac{P(c_n^\pi = +1)}{P(c_n^\pi = -1)}$$

$$L_{ext}^D(c_n^\pi) \cong \ln\frac{P(c_n^\pi = +1)}{1 - P(c_n^\pi = +1)}$$

$$P(c_n^\pi = +1) \cong \frac{\exp\left(L_{ext}^D(c_n^\pi)\right)}{1 + \exp\left(L_{ext}^D(c_n^\pi)\right)}$$

$$L_{ext}^D(c_n^\pi) \cong \ln\frac{1 - P(c_n^\pi = -1)}{P(c_n^\pi = +1)}$$

$$P(c_n^\pi = -1) \cong \frac{1}{1 + \exp\left(L_{ext}^D(c_n^\pi)\right)}$$

$$P(c_n^\pi = c) \cong \frac{\exp\left(c.L_{ext}^D(c_n^\pi)\right)}{1 + \exp\left(L_{ext}^D(c_n^\pi)\right)} \qquad c \in \{0,1\}$$

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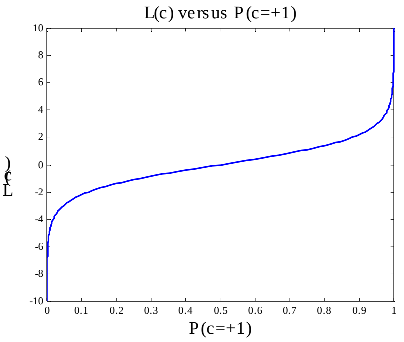
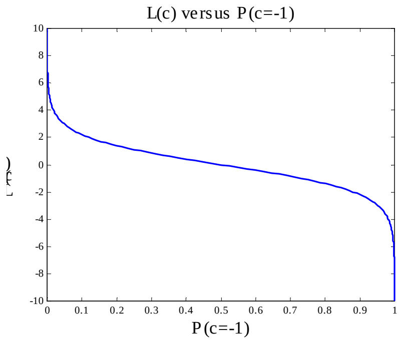
MAP Equalizer

$$\gamma(s',s)=\frac{\exp\left(c.L_{ext}^{D}\left(c_n^{\pi}\right)\right)}{1+\exp\left(L_{ext}^{D}\left(c_n^{\pi}\right)\right)}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\left(r_n-v_n\right)^2/2\sigma^2\right)\quad c\in\{0,1\}$$





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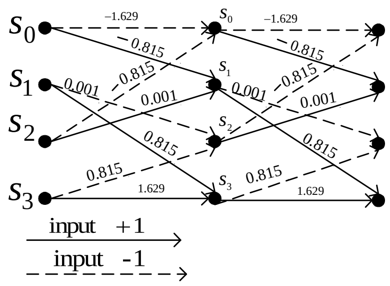
MAP Equalizer implementation [Koetter]



$$\mathbf{A}(+1)=\begin{bmatrix}0&1&0&0\\0&0&0&1\\0&1&0&0\\0&0&0&1\end{bmatrix},\quad \mathbf{A}(-1)=\begin{bmatrix}1&0&0&0\\0&0&1&0\\1&0&0&0\\0&0&1&0\end{bmatrix}$$

$$\{\mathbf{P}\}_{i,j,n}=\gamma_n\left(s_i,s_j,n\right)$$

$$\{\mathbf{B}\}_{i,j,n}\rightarrow \textit{componentwise}\text{ product of }A\text{ and }P$$

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MAP Equalizer implementation

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Input : Matrices $\mathbf{P}_n$, $\mathbf{A}(+1)$, $\mathbf{A}(-1)$, $\mathbf{B}_n(+1)$, $\mathbf{B}_n(-1)$, $\mathbf{f}_n$ e $\mathbf{b}_n$.

Initialization: the first column of vector **f** and the last of vector **b** are initialized as 1 for every lines.

Recursively compute of **f** and **b** :

$$\mathbf{f}_n = \mathbf{P}_{n-1}^{\mathbf{T}}\,\mathbf{f}_{n-1} \qquad , n = 1,\dots,N$$

$$\mathbf{b}_n = \mathbf{P}_n\,\mathbf{b}_{n+1} \qquad , n = N\text{-}1,\dots,1$$

Output : for $n = 1,\dots,N$

$$L^E\left(c_n^\pi \mid \mathbf{r}\right) = \ln \frac{\mathbf{f}_n^{\mathbf{T}}\,\mathbf{B}_n(+1)\mathbf{b}_{n+1}}{\mathbf{f}_n^{\mathbf{T}}\,\mathbf{B}_n(-1)\mathbf{b}_{n+1}}$$

$$L^E\left(c_n^\pi \mid \mathbf{r}\right) = \ln \frac{\left(\sum_{c_n^\pi=+1} \alpha_{n-1}(s')\gamma_n(s',s)\beta_n(s)\right)}{\left(\sum_{c_n^\pi=-1} \alpha_{n-1}(s')\gamma_n(s',s)\beta_n(s)\right)}$$

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MAP Decoder

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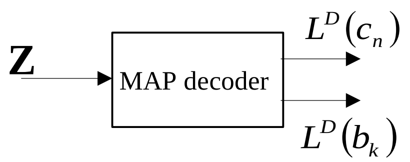


$$L^D(c_n) \equiv \ln\left(\frac{P(c_n=+1|\mathbf{Z})}{P(c_n=-1|\mathbf{Z})}\right) = L_e^D(c_n) + L_a^D(c_n)$$

$$P(c_n \mid \mathbf{r}) \cong \frac{\exp\left(c.L_{ext}^E(c_n)\right)}{1+\exp\left(L_{ext}^E(c_n)\right)} \qquad c \in \{0,1\}$$

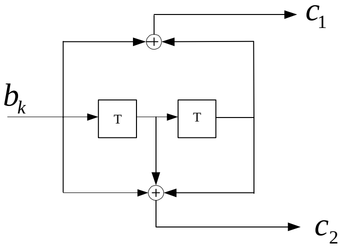
$$\mathbf{Z} = \left[P(c_1 \mid \mathbf{r})\,P(c_2 \mid \mathbf{r})\dots P(c_N \mid \mathbf{r})\right]^{\mathrm{T}}$$

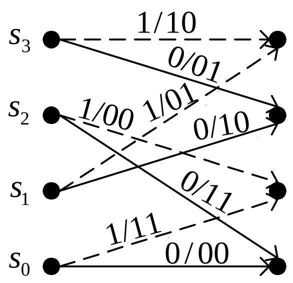
Turbo Equalization

IWT 2004

MAP Decoder

Turbo Equalization





$$\gamma_n(s',s) = p(\{r_n, S_n = s\} \mid S_{n-1} = s')$$

Using Baye's rule:

$$\gamma_n(s',s) = \boxed{p(r_n \mid \{s,s'\})}\ \boxed{P(s \mid s')}$$

is governed by the output symbol

is governed by the input symbol

$$\gamma(s_i, s_j) = \boxed{P(b_k)}\, P(c_1 = c_{1,i,j} \mid \mathbf{r})\, P(c_2 = c_{2,i,j} \mid \mathbf{r})$$

there is no a priori information of the information bits

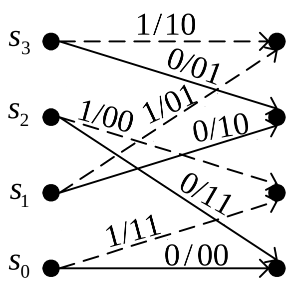
MAP Decoder implementation [Koetter]

Turbo Equalization

$$\mathbf{A}_b(1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{A}_b(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{A}_{c_1}(\mathbf{1}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{A}_{c_1}(\mathbf{0}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{A}_{c_2}(\mathbf{1}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{A}_{c_2}(\mathbf{0}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



MAP Decoder implementation

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Turbo Equalization

Input : Matrices $\mathbf{P}_n, \boxed{\mathbf{A}(+1), \mathbf{A}(0), \mathbf{B}_n(+1), \mathbf{B}_n(0)}, \mathbf{f}_n$ e $\mathbf{b}_n$.

for b, c_1 and c_2

Initialization: the first column of vector **f** and the last of vector **b** are initialized as 1 for every lines.

Recursively compute of **f** and **b** :

$$\mathbf{f_n} = \mathbf{P_{n-1}^T}\,\mathbf{f_{n-1}} \qquad , n = 1,\ldots,N$$

$$\mathbf{b_n} = \mathbf{P_n}\,\mathbf{b_{n+1}} \qquad , n = N\text{-}1,\ldots,1$$

Output : for $n = 1,\ldots,N$

$$L^D(c_n\,|\,\mathbf{r}) = \ln\frac{\mathbf{f}_n^{\mathbf{T}}\,\mathbf{B}_n(+1)\mathbf{b}_{n+1}}{\mathbf{f}_n^{\mathbf{T}}\,\mathbf{B}_n(-1)\mathbf{b}_{n+1}}$$

$$\boxed{L^D(c_n\,|\,\mathbf{r}) = \ln\frac{\left(\sum_{c_n=+1}\alpha_{n-1}(s')\gamma_n(s',s)\beta_n(s)\right)}{\left(\sum_{c_n=-1}\alpha_{n-1}(s')\gamma_n(s',s)\beta_n(s)\right)}}$$

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MAP implementation

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Turbo Equalization

- For a practical implementation, the vectors *forward* and *backward* need to be normalized to avoid underflow.

- The *MAP algorithm* can be implemented in the log domain (Log-MAP-algorithm) for computational simplicity.

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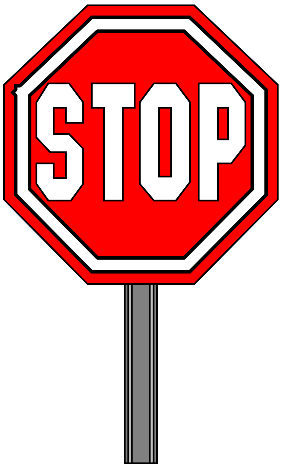
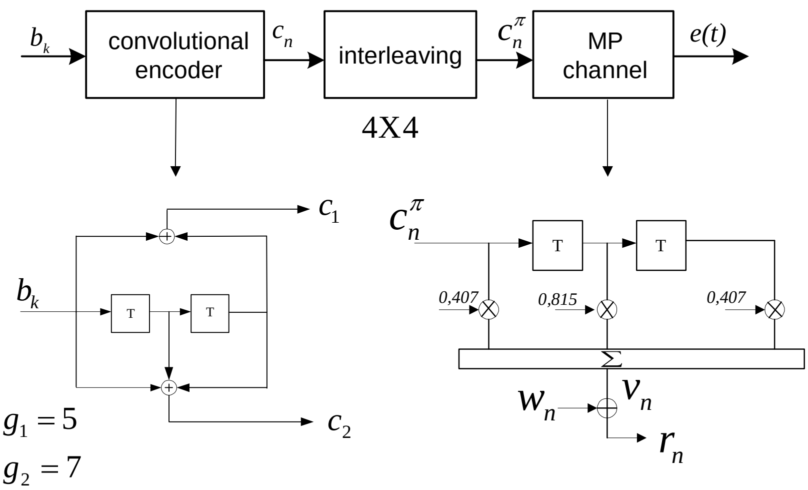
Questions???



TuEqu example



TuEqu example

Turbo Equalization

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$$b_k = [1 \;\; 1 \;\; 0 \;\; 1 \;\; 0 \;\; 0 \;\; 1 \;\; 0]$$

$$c_n = [1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; 1 \;\; 1 \;\; 1 \;\; 0]$$

$$c_n^\pi = [1 \;\; -1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; -1 \;\; 1 \;\; 1 \;\; 1 \;\; -1 \;\; 1 \;\; -1]$$

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TuEqu example

Turbo Equalization

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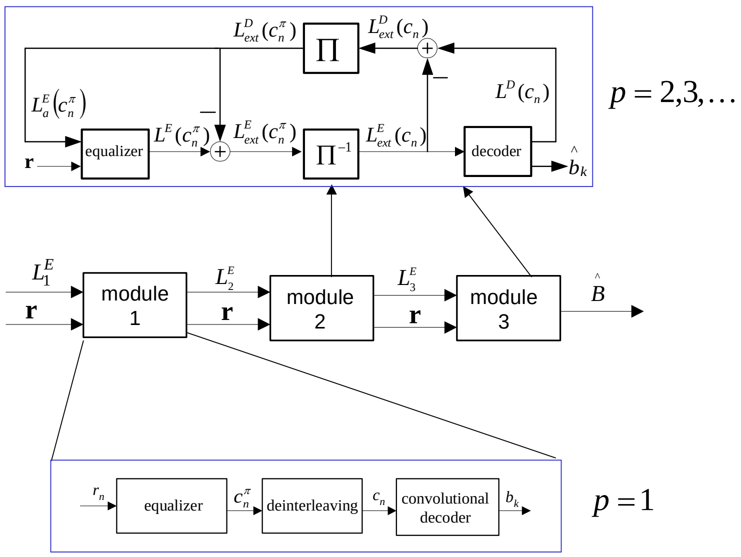

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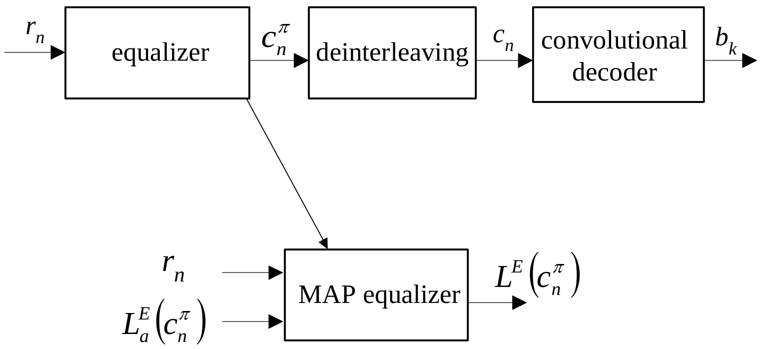
TuEqu example (p=1)

Turbo Equalization



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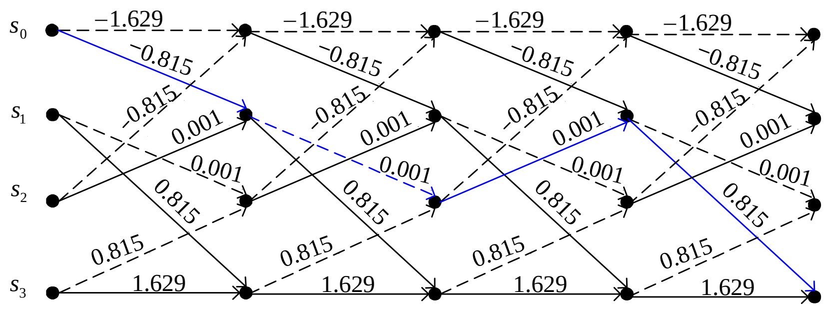
TuEqu example (p=1)

Turbo Equalization



$r_1 = -1{,}21$ $r_2 = -1{,}53$ $r_3 = 0{,}11$ $r_4 = 1{,}08$

$$c_n^\pi = [1 \;\; -1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; -1 \;\; 1 \;\; -1 \;\; -1 \;\; 1 \;\; 1 \;\; 1 \;\; -1 \;\; 1 \;\; -1]$$

$$\gamma(s',s) = P(c_n^\pi)\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-(r_n - v_n)^2 / 2\sigma^2\right)$$

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TuEqu example (p=1)

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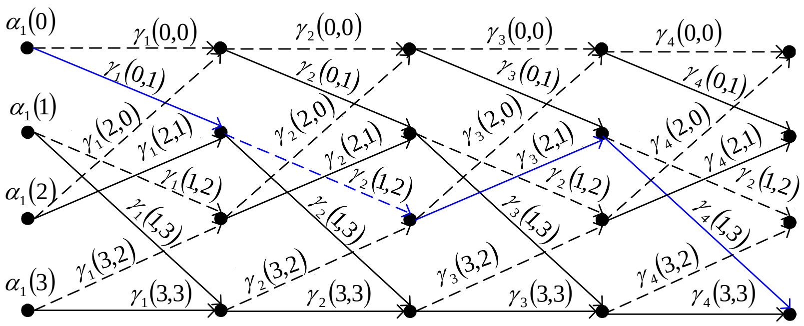
Turbo Equalization



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TuEqu example (p=1)

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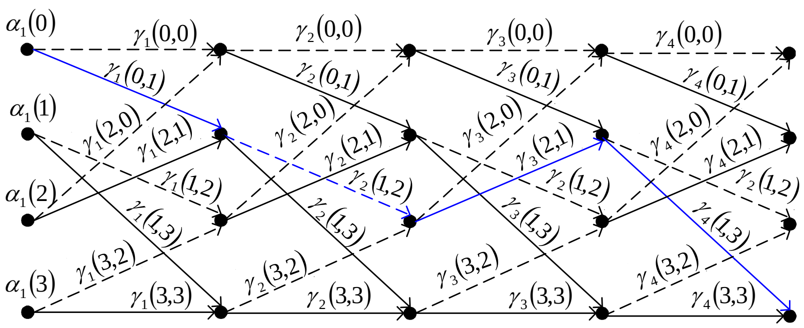
Turbo Equalization



$$\alpha_2(0) = \alpha_1(0).\gamma_1(0,0) + \alpha_1(2).\gamma_1(2,0) = 1.0{,}1955 + 0 = 0{,}1955$$
$$\alpha_2(1) = \alpha_1(0).\gamma_1(0,1) + \alpha_1(2).\gamma_1(2,1) = 1.0{,}1970 + 0 = 0{,}1970$$
$$\alpha_2(2) = \alpha_1(1).\gamma_1(1,2) + \alpha_1(3).\gamma_1(3,2) = 0$$
$$\alpha_2(3) = \alpha_1(1).\gamma_1(1,3) + \alpha_1(3).\gamma_1(3,3) = 0$$

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TuEqu example (p=1)

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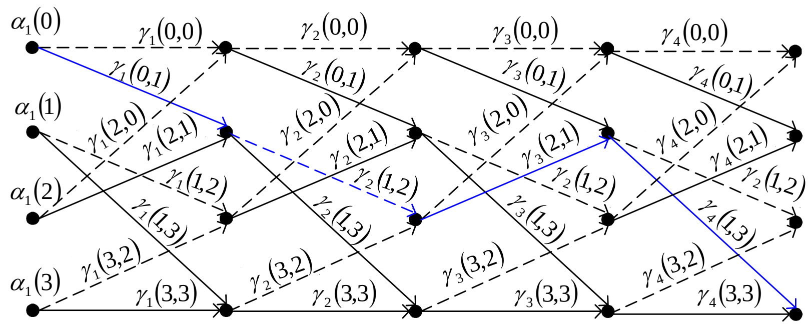
Turbo Equalization



$$\alpha_3(0) = \alpha_2(0).\gamma_2(0,0) + \alpha_2(2).\gamma_2(2,0) = 0{,}1955.0{,}2152 + 0 = 0{,}0421$$
$$\alpha_3(1) = \alpha_2(0).\gamma_2(0,1) + \alpha_2(2).\gamma_2(2,1) = 0{,}1970.0{,}1595 + 0 = 0{,}0312$$
$$\alpha_3(2) = \alpha_2(1).\gamma_2(1,2) + \alpha_2(3).\gamma_2(3,2) = 0{,}1970.0{,}0542 + 0 = 0{,}0106$$
$$\alpha_3(3) = \alpha_2(1).\gamma_2(1,3) + \alpha_2(3).\gamma_2(3,3) = 0{,}1970.0{,}0084 + 0 = 0{,}0017$$

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TuEqu example (p=1)

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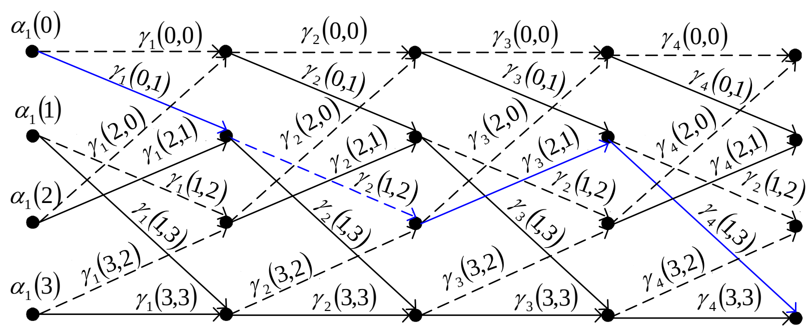
Turbo Equalization



$$\alpha_4(0) = \alpha_3(0).\gamma_3(0,0) + \alpha_3(2).\gamma_3(2,0) = 0{,}0029$$
$$\alpha_4(1) = \alpha_3(0).\gamma_3(0,1) + \alpha_3(2).\gamma_3(2,1) = 0{,}0078$$
$$\alpha_4(2) = \alpha_3(1).\gamma_3(1,2) + \alpha_3(3).\gamma_3(3,2) = 0{,}0070$$
$$\alpha_4(3) = \alpha_3(1).\gamma_3(1,3) + \alpha_3(3).\gamma_3(3,3) = 0{,}0051$$

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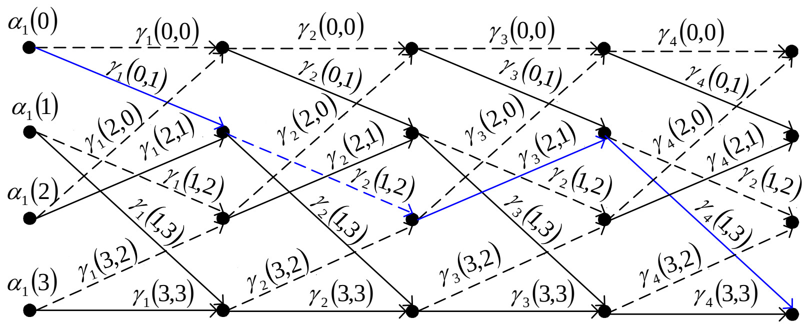
TuEqu example (p=1)



$$\alpha_5(0) = \alpha_4(0).\gamma_4(0,0) + \alpha_4(2).\gamma_4(2,0) = 0{,}0002$$
$$\alpha_5(1) = \alpha_4(0).\gamma_4(0,1) + \alpha_4(2).\gamma_4(2,1) = 0{,}0008$$
$$\alpha_5(2) = \alpha_4(1).\gamma_4(1,2) + \alpha_4(3).\gamma_4(3,2) = 0{,}0019$$
$$\alpha_5(3) = \alpha_4(1).\gamma_4(1,3) + \alpha_4(3).\gamma_4(3,3) = 0{,}0025$$

Turbo Equalization

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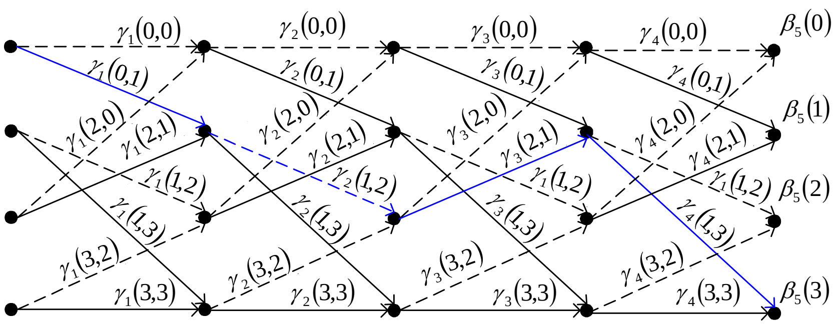
TuEqu example (p=1)



$$\beta_4(0) = \beta_5(0).\gamma_4(0,0) + \beta_5(1).\gamma_4(0,1) = 0{,}1.10^{-9}$$
$$\beta_4(1) = \beta_5(2).\gamma_4(1,2) + \beta_5(3).\gamma_4(1,3) = 0{,}5.10^{-9}$$
$$\beta_4(2) = \beta_5(0).\gamma_4(2,0) + \beta_5(1).\gamma_4(2,1) = 0{,}4.10^{-9}$$
$$\beta_4(3) = \beta_5(2).\gamma_4(3,2) + \beta_5(3).\gamma_4(3,3) = 0{,}5.10^{-9}$$

Turbo Equalization

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TuEqu example (p=1)

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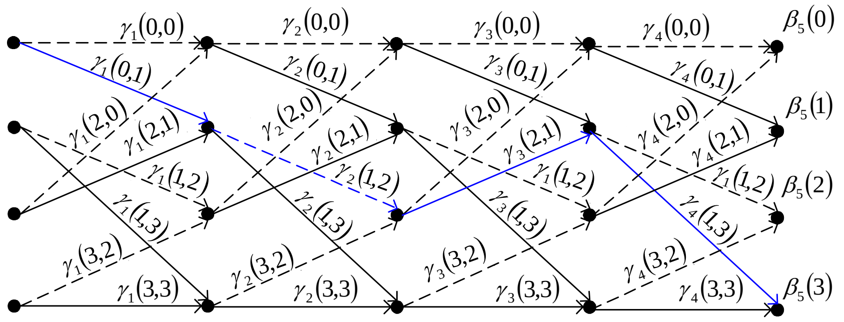
Turbo Equalization



$$\beta_3(0) = \beta_4(0).\gamma_3(0,0) + \beta_4(1).\gamma_3(0,1) = 0{,}0.10^{-9}$$
$$\beta_3(1) = \beta_4(2).\gamma_3(1,2) + \beta_4(3).\gamma_3(1,3) = 0{,}2.10^{-9}$$
$$\beta_3(2) = \beta_4(0).\gamma_3(2,0) + \beta_4(1).\gamma_3(2,1) = 0{,}1.10^{-9}$$
$$\beta_3(3) = \beta_4(2).\gamma_3(3,2) + \beta_4(3).\gamma_3(3,3) = 0{,}1.10^{-9}$$

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TuEqu example (p=1)

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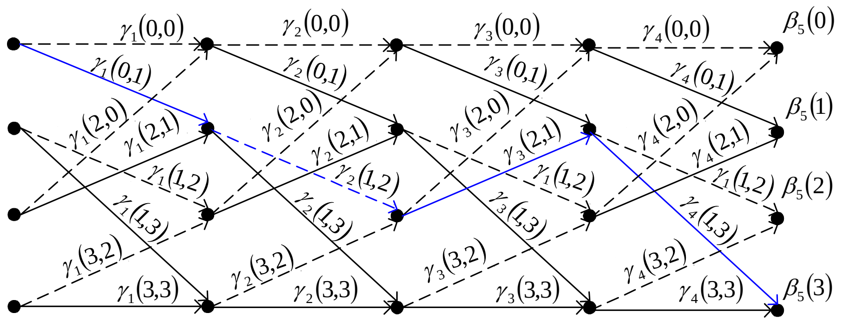
Turbo Equalization



$$\beta_2(0) = \beta_3(0).\gamma_2(0,0) + \beta_3(1).\gamma_2(0,1) = 0{,}4.10^{-10}$$
$$\beta_2(1) = \beta_3(2).\gamma_2(1,2) + \beta_3(3).\gamma_2(1,3) = 0{,}1.10^{-10}$$
$$\beta_2(2) = \beta_3(0).\gamma_2(2,0) + \beta_3(1).\gamma_2(2,1) = 0{,}2.10^{-10}$$
$$\beta_2(3) = \beta_3(2).\gamma_2(3,2) + \beta_3(3).\gamma_2(3,3) = 0{,}0.10^{-10}$$

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TuEqu example (p=1)

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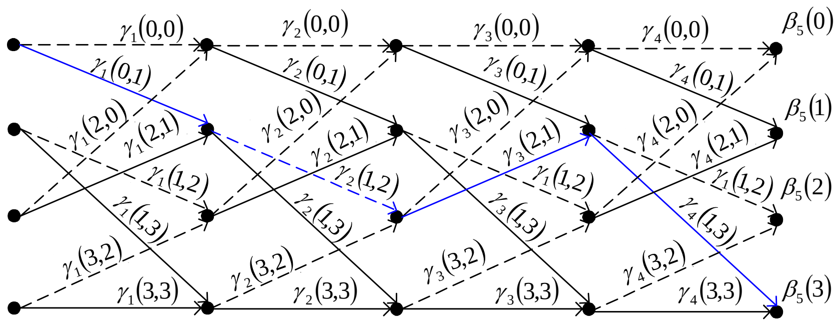
Turbo Equalization



$$\beta_1(0) = \beta_2(0).\gamma_1(0,0) + \beta_2(1).\gamma_1(0,1) = 0{,}1.10^{-10}$$
$$\beta_1(1) = \beta_2(2).\gamma_1(1,2) + \beta_2(3).\gamma_1(1,3) = 0{,}0.10^{-10}$$
$$\beta_1(2) = \beta_2(0).\gamma_1(2,0) + \beta_2(1).\gamma_1(2,1) = 0{,}1.10^{-10}$$
$$\beta_1(3) = \beta_2(2).\gamma_1(3,2) + \beta_2(3).\gamma_1(3,3) = 0{,}0.10^{-10}$$

TuEqu example (p=1)

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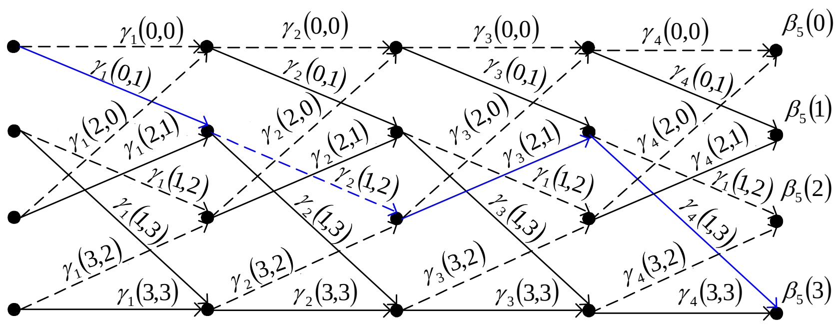
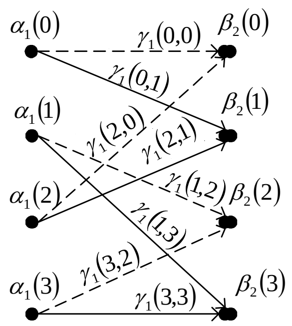
Turbo Equalization



$$L^E\left(c_n^\pi \mid \mathbf{r}\right) = \ln \frac{\left(\sum_{c_n^\pi=+1} p(S_{n-1}, S_n, \mathbf{r})\right)}{\left(\sum_{c_n^\pi=-1} p(S_{n-1}, S_n, \mathbf{r})\right)} = \ln \frac{\left(\sum_{c_n^\pi=+1} \alpha_{n-1}(s')\gamma_n(s',s)\beta_n(s)\right)}{\left(\sum_{c_n^\pi=-1} \alpha_{n-1}(s')\gamma_n(s',s)\beta_n(s)\right)}$$

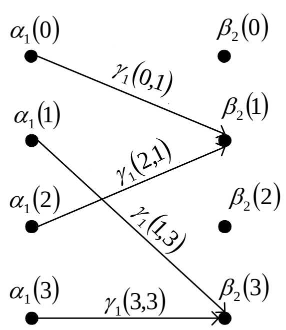
TuEqu example (p=1)



$$L^E(c_1 \mid \mathbf{r}) = \ln \frac{\left(\sum_{c_n^\pi=+1} p(S_{n-1}, S_n, \mathbf{r})\right)}{\left(\sum_{c_n^\pi=-1} p(S_{n-1}, S_n, \mathbf{r})\right)}$$

Turbo Equalization

TuEqu example (p=1)



$$\sum_{c_n^\pi=+1} p(S_{n-1}, S_n, \mathbf{r}) = \alpha_1(0).\gamma_1(0,1).\beta_2(1) + \alpha_1(1).\gamma_1(1,3).\beta_2(3)$$
$$+ \alpha_1(2).\gamma_1(2,1).\beta_2(1) + \alpha_1(3).\gamma_1(3,3).\beta_2(3)$$

Turbo Equalization

TuEqu example (p=1)

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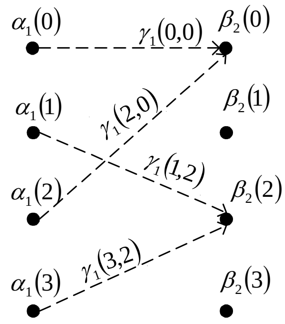
Turbo Equalization



$$\sum_{c_n^\pi=-1} p(S_{n-1},S_n,\mathbf{r}) = \alpha_1(0).\gamma_1(0,0).\beta_2(0) + \alpha_1(1).\gamma_1(1,2).\beta_2(2)$$
$$+\alpha_1(2).\gamma_1(2,0).\beta_2(0) + \alpha_1(3).\gamma_1(3,2).\beta_2(2)$$

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TuEqu example (p=1)

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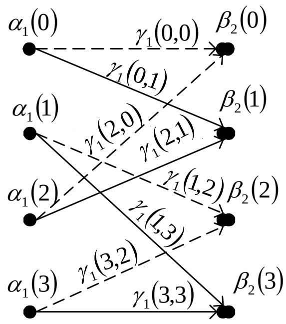
Turbo Equalization



$$L^E\left(c_1^\pi \mid \mathbf{r}\right) = \ln \frac{\left(\sum_{c_n^\pi=+1} p(S_{n-1},S_n,\mathbf{r})\right)}{\left(\sum_{c_n^\pi=-1} p(S_{n-1},S_n,\mathbf{r})\right)} = -1{,}7504$$

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Turbo Equalization

TuEqu example (p=1)

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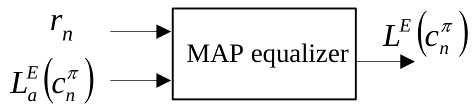

$$c_n = [1 \quad -1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1]$$

$$L^E\left(c_n^\pi\right) = [-1{,}7504 \quad 0{,}2874 \quad 0{,}7274 \quad 1{,}7291\ldots]$$

leads to wrong decisions

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Turbo Equalization

TuEqu example (p=1)

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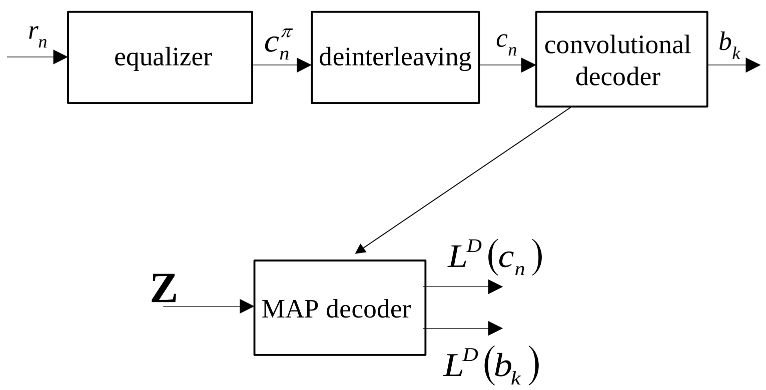

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TuEqu example (p=1)

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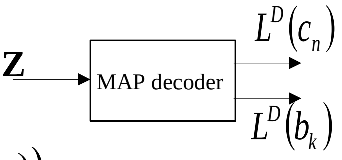
Turbo Equalization

$$\mathbf{Z} = [P(c_1 \mid \mathbf{r})\, P(c_2 \mid \mathbf{r}) \ldots P(c_N \mid \mathbf{r})]$$



$$P(c_n \mid \mathbf{r}) \cong \frac{\exp\left(c . L_{ext}^{E}(c_n \mid \mathbf{r})\right)}{1 + \exp\left(L_{ext}^{E}(c_n \mid \mathbf{r})\right)} \quad c \in \{0,1\}$$

$$L^E(c_n) = [-1{,}7504 \quad 4{,}4354 \quad -0{,}9099 \quad 3{,}5313\ldots]$$

$$\mathbf{Z}(c_n = 1) = [0{,}1480 \quad 0{,}9883 \quad 0{,}2870 \quad 0{,}9716\ldots]$$

$$\mathbf{Z}(c_n = 0) = [0{,}8520 \quad 0{,}0117 \quad 0{,}7130 \quad 0{,}0284\ldots]$$

TuEqu example (p=1)

Inatel
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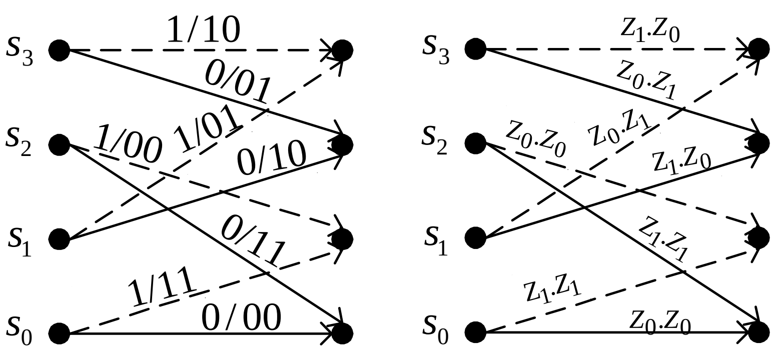
Turbo Equalization

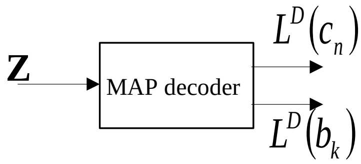


$$Z = [P(c_1 \mid \mathbf{r})\, P(c_2 \mid \mathbf{r}) \ldots P(c_N \mid \mathbf{r})]$$

$$\gamma(s_i, s_j) = P(b_k) . P(c_1 = c_{1,i,j} \mid \mathbf{r}) . P(c_2 = c_{2,i,j} \mid \mathbf{r})$$

there is no a priori information of the information bits

TuEqu example (p=1)



$$b_k = [1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$$

Hard decisions of the information bits:

$$\hat{b}_k = [1 \quad 1 \quad 0 \quad \boxed{0} \quad \boxed{1} \quad 0 \quad \boxed{0} \quad 1]$$

$$c_n = [1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad 1 \quad -1]$$

Hard decisions of the coded bits:

$$\hat{c}_n = [1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad \boxed{1} \quad 1 \quad \boxed{1} \quad 1 \quad -1 \quad 1 \quad 1 \quad 1 \quad \boxed{1}]$$

Turbo Equalization

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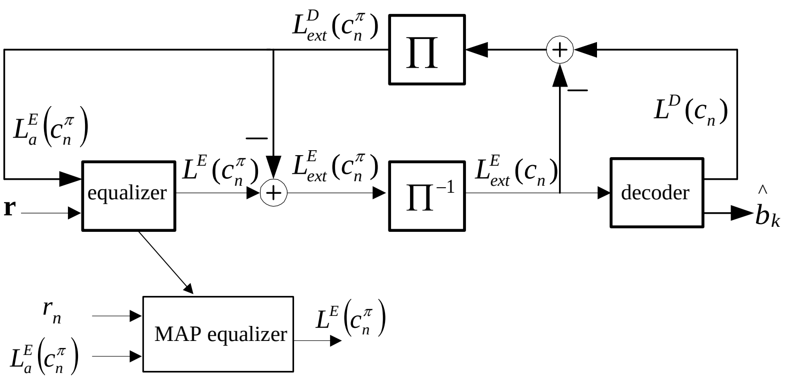
TuEqu example (p=2)



$$\gamma(s',s) = \frac{\exp\left(c.L_{ext}^D(c_n^\pi)\right)}{1+\exp\left(L_{ext}^D(c_n^\pi)\right)} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-(r_n - v_n)^2 / 2\sigma^2\right)$$

Turbo Equalization

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TuEqu example (p=2)

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Turbo Equalization

$p = 1$

$c_n^\pi = [1 \quad -1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1]$

$L^E(c_n^\pi) = [\boxed{-1{,}7504 \quad 0{,}2874} \quad 0{,}7274 \quad 1{,}7291 \ldots]$

leads to wrong decisions

$p = 2$

$c_n^\pi = [1 \quad -1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad -1 \quad 1 \quad -1]$

$L^E(c_n^\pi) = [\boxed{3{,}9617 \quad -4{,}7377} \quad 2{,}0979 \quad 1{,}8811 \ldots]$

errors corrected

TuEqu example (p=2)

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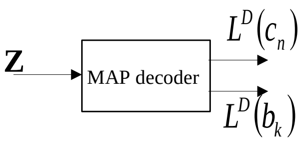
Turbo Equalization

$$\mathbf{Z} \equiv [P(c_1 \mid \mathbf{r})\, P(c_2 \mid \mathbf{r}) \ldots P(c_N \mid \mathbf{r})]$$



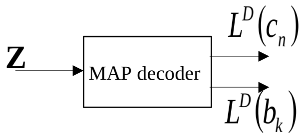
$$P(c_n \mid \mathbf{r}) \cong \frac{\exp\left(c.L_{ext}^E(c_n \mid \mathbf{r})\right)}{1 + \exp\left(L_{ext}^E(c_n \mid \mathbf{r})\right)} \quad c \in \{0,1\}$$

$$L^E(c_n) = [-0{,}9085 \quad 4{,}2859 \quad -1{,}3199 \quad 3{,}9053 \ldots]$$

$$Z(c_n = 1) = [0{,}2873 \quad 0{,}9864 \quad 0{,}2108 \quad 0{,}9803 \ldots]$$

$$Z(c_n = 0) = [0{,}7127 \quad 0{,}0136 \quad 0{,}7892 \quad 0{,}0197 \ldots]$$

TuEqu example (p=2)



$$b_k = [1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 0]$$

Hard decisions of the information bits:

$$\hat{b}_k = [1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad \boxed{0} \quad 1]$$

$$c_n = [1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad 1 \quad -1]$$

Hard decisions of the coded bits:

$$\hat{c}_n = [1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad 1 \quad \boxed{1}]$$

Turbo Equalization

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TuEqu example (p=2)

$p = 1$

Hard decisions of the information bits:

$$\hat{b}_k = [1 \quad 1 \quad 0 \quad \boxed{0} \quad \boxed{1} \quad 0 \quad \boxed{0} \quad 1]$$

Hard decisions of the coded bits:

$$\hat{c}_n = [1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad \boxed{1} \quad 1 \quad \boxed{1} \quad 1 \quad -1 \quad 1 \quad 1 \quad 1 \quad \boxed{1}]$$

$p = 2$

Hard decisions of the information bits:

$$\hat{b}_k = [1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad \boxed{0} \quad 1]$$

Hard decisions of the coded bits:

$$\hat{c}_n = [1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad 1 \quad \boxed{1}]$$

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Questions???

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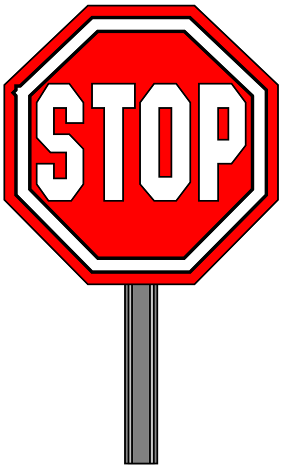

Turbo Equalization

Evolution of the LLR's

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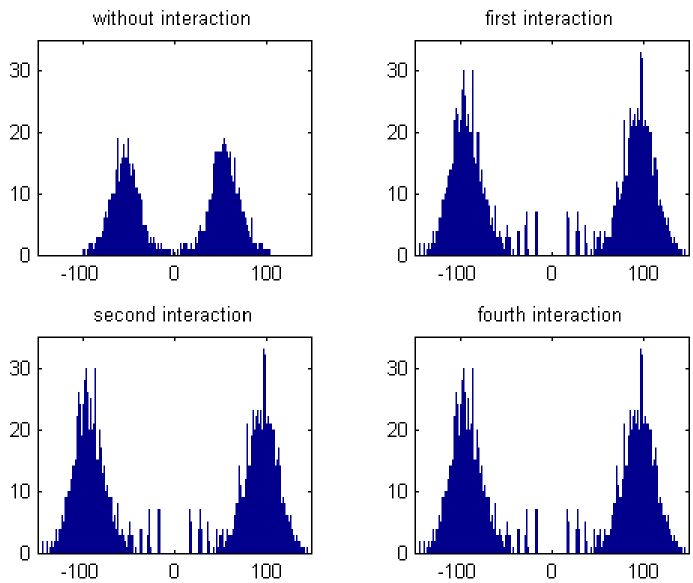

Results

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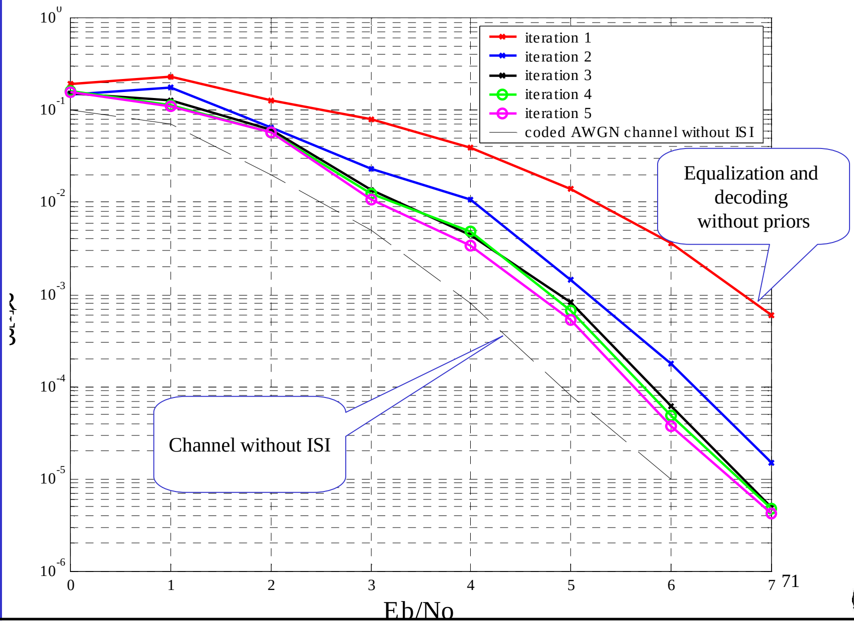
Turbo Equalization



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TuEqu using MAP

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Turbo Equalization

- The MAP algorithm applied in turbo equalization is adequate to low spectral efficiency modulations and channels exhibiting a low delay spread.
- In 1997, Glavieux proposed an *Interference Canceller* in the Turbo Equalizer for channels with strong delay spread and high order modulations.

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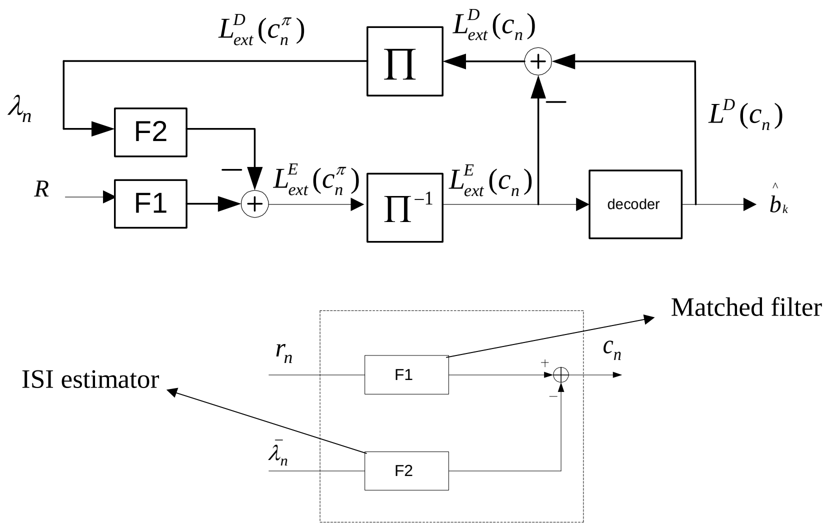
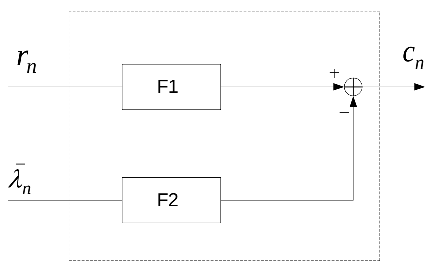
Turbo Equalization with IC

Turbo Equalization



Turbo Equalization with IC

Turbo Equalization



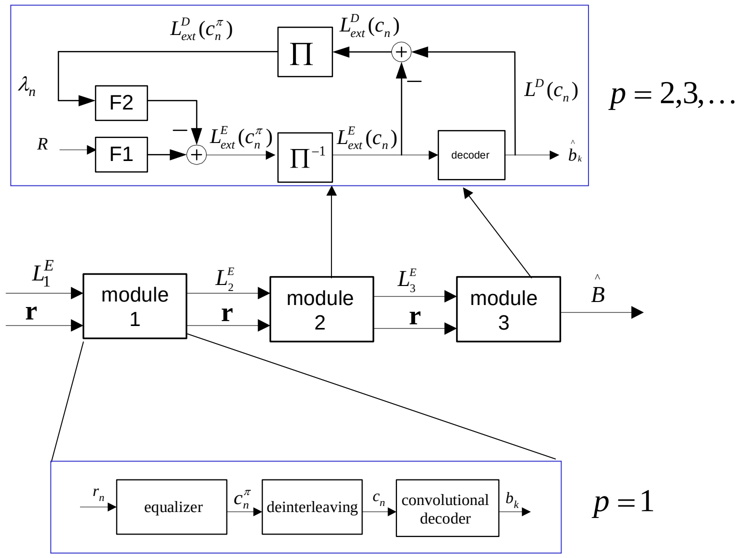
The input of the filter F2 is:

$$\bar{\lambda}_n = E\{c_k\} = P(c_n = +1).1 + P(c_n = -1).(-1)$$

$$= \frac{\exp(L(c_n))}{1+\exp(L(c_n))} + \frac{-1}{1+\exp(L(c_n))} = \tanh\left(\frac{L_{ext}^{D}(c_k)}{2}\right)$$

TuEqu example

Turbo Equalization



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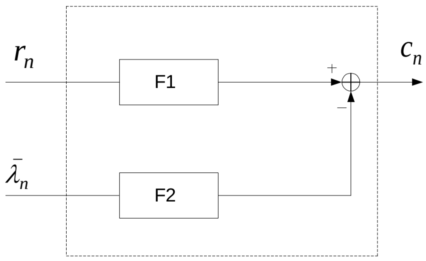
Adaptive Turbo Equalization

Turbo Equalization



For each stage, the equalizer is updated according to the mean - square - error (MSE) criterion [Laot]:

$$MSE = E\left\{\left|c_n - \lambda_n\right|^2\right\}$$

$$W_{k+1} = W_k - \mu.R(s_n - \hat{c}_n)$$

$$D_{k+1} = D_k + \mu.R(s_n - \hat{c}_n)$$

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Results [Laot]

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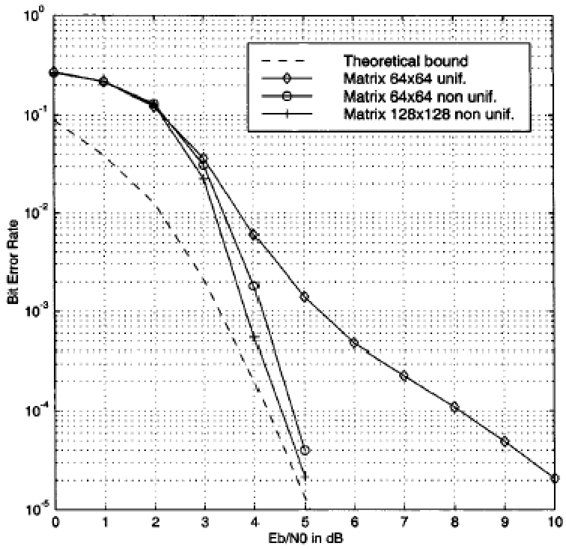
Turbo Equalization



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TuEq, other approaches

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Turbo Equalization

- Turbo equalization using block codes
- Turbo equalization using turbo codes
- Turbo equalization using joint channel estimation and MAP equalization (BCJR).
- Turbo equalization applied in multi-user detection of CDMA systems.

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- C. Laot, A. Glavieux and J. Labat. **Turbo Equalization: Adaptative Equalization and Channel Decoding Jointly Optimized.** IEE Journal on Selected Areas in Communications, vol.19, nº9, september,2001
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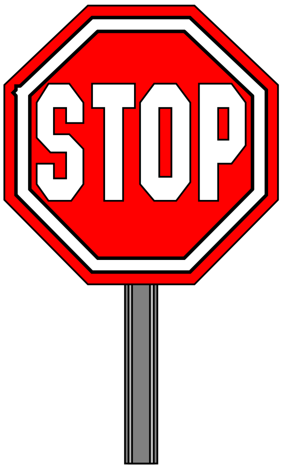
Questions???



Turbo Equalization