# A new numerical algorithm for TV-based image inpainting with its applications for restoring ancient Thai painting images and removing subtitles from animes

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#### Introduction

Image inpainting is the process of reconstructing the missing or damaged regions of an image in order to make it more legible. In fine art museums, inpainting of damaged paintings are traditionally carried out by professional artists and usually very time consuming, not to mention the risk of completely destroying a precious ancient painting due to direct retouching.

Let  $\Omega \subset \mathbb{R}^2$  we search for the original image  $u:\Omega \to [0,\infty)$  from a damaged image  $z:\Omega \to [0,\infty)$  containing an inpainting domain  $D \subset \Omega$ 

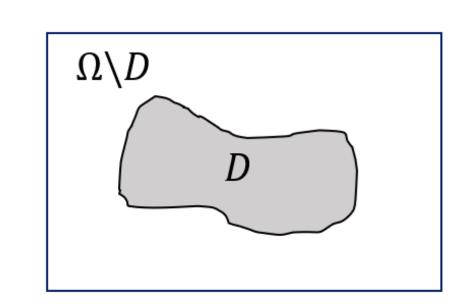


Figure 1: example of the image and its inpaint domain.

We compute the solution of the total variation (TV) based image inpainting model as given by [1]:

$$\min_{u \in BV(\Omega)} \left\{ \frac{\lambda}{2} \int_{\Omega \setminus D} (u - z)^2 d\Omega + \int_{\Omega} |\nabla u| d\Omega \right\} \tag{1}$$

$$\lambda = \begin{cases} \lambda, & x \in \Omega \setminus D \\ 0, & x \in D \end{cases}$$

 $\lambda$  is called the regularization parameter and  $\lambda>0$ ,  $BV(\Omega)$  represents the space of functions of bounded variation, and  $\int_{\Omega} |\nabla u| d\Omega$  is the TV regularization

However, these existing methods also suffer from difficulties related to the non-differentiability and non-linearity of the TV regularization. To develop efficient numerical methods for (1), it is still a challenging task.

# Objective

- 1. Study the numerical algorithms for the TV based image inpainting model.
- 2. Develop a new numerical algorithm for solving TV based image inpainting model
- 3. Apply a new numerical algorithm to painting restoration and removed subtitle from anime

## Methods

Following the split Bregman method introduced in [2] for TV based image denoising model, we reformulate the minimization problem (1) to the following unconstrained minimization problem.

$$\min_{u,\overrightarrow{w}} \left\{ \frac{\lambda}{2} \int_{\Omega} (u-z)^2 d\Omega + \int_{\Omega} |\overrightarrow{w}| d\Omega + \frac{\theta}{2} \int_{\Omega} (\overrightarrow{w} - \nabla u + \overrightarrow{b})^2 d\Omega \right\}$$
 (2)

We separate the minimization problem (2) into two subproblems and develop an alternating minimization procedure to approximate the solution.

$$u^{new} = \underset{u}{\operatorname{argmin}} \left\{ \frac{\lambda}{2} \int_{\Omega} (u - z)^2 d\Omega + \frac{\theta}{2} \int_{\Omega} \left( \overrightarrow{w}^{old} - \nabla u + \overrightarrow{b}^{old} \right)^2 d\Omega \right\}$$
(3)

$$\vec{w}^{new} = \operatorname{argmin} \left\{ \int_{\Omega} |\vec{w}| d\Omega + \frac{\theta}{2} \int_{\Omega} (\vec{w} - \nabla u^{new} + \vec{b}^{old})^2 d\Omega \right\}$$
(4)

We update the Bregman parameter by  $\vec{b}^{new} = \vec{b}^{old} + \nabla u^{new} - \vec{w}^{new}$ .

This process is repeated until the sequence of the approximate solution converges.

We hope the initial solution will affect to the rate of convergence. Our alternating minimization procedure is implemented in the multi-resolution framework or image pyramid [3]. We use split Bregman in small image and then scale up the result for using in large image initial solution.

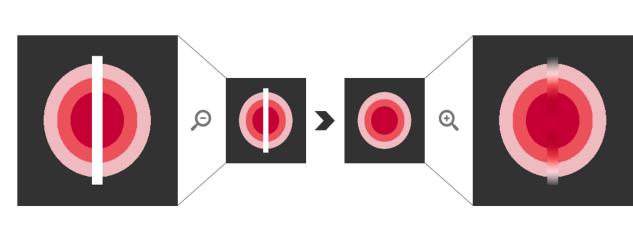


Figure 2: preparation of initial solution using image pyramid

In this work, we limit the depth of image pyramid to 4 levels.

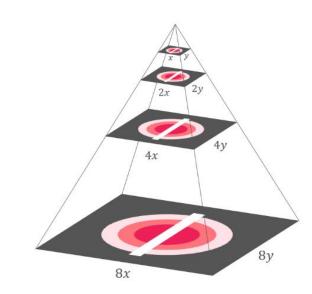
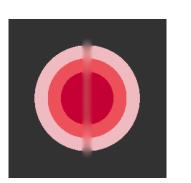
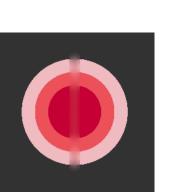
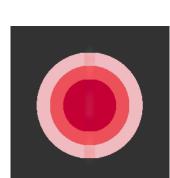


Figure 3: 4 levels of image pyramid.

We observe that the rate of convergence is fast at the beginning and then becomes slow. Thus, we hope iteration for a few times in original resolution level will be enough to make an inpainted image close to the original image. Hence, we limit the iteration in original resolution level to 10 times.







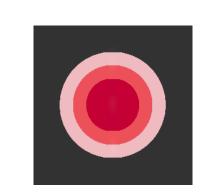


Figure 4: The inpainted images in step 5/10/50/100.

We select 5 Thai paintings and damaged them to compare restoration between split Bregman and our algorithm in both performance and quality.

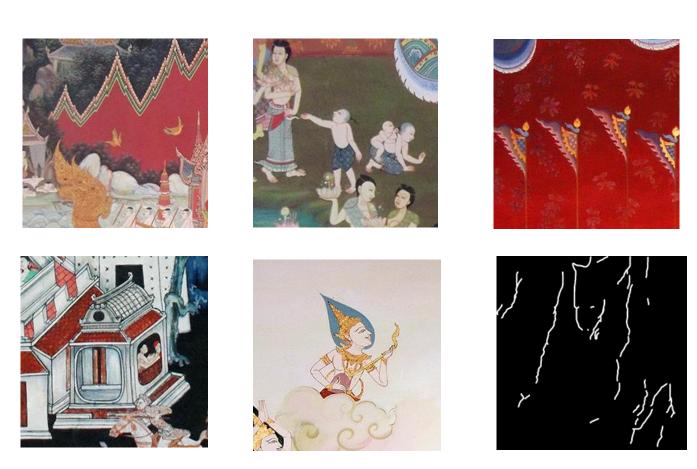


Figure 5: set of Thai paintings and their inpainting domains.

For removed anime subtitle, We use one minute of video which is 1280x720 pixels. This video has 20 lines of subtitle and we split video to 5 horizontal part for creating 5 examples of anime.



Figure 6: split video into 5 part.

Since the video is a sequence of images. We can use image inpainting to remove subtitles from the videos in each frame.

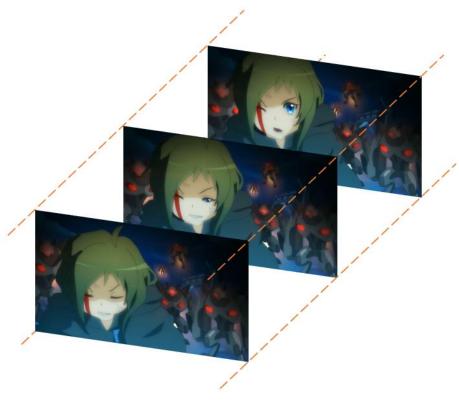


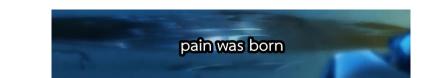
Figure 7: the video is a sequence of images.

Before we remove subtitles. We need to find the positions of them. Due to find these, we have developed a new algorithm to detect texts on subtitle by assuming that anime subtitles always have black border.

#### This is anime subtitle

Figure 8: anime subtitles have black border.

For more rapid time of processing, We suggest to use the result from the previous frame if SSIM outside the domain is more than 0.95 and use the previous frame as an initial solution if SSIM outside the domain is more than 0.90.



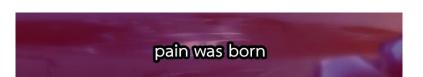


Figure 9: compute SSIM between both frames in red area.

#### Result

Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity (SSIM) are used to measure the quality of the restored images. The greater PSNR is, the better restored images are. In the same way, SSIM is close to one will indicate the images are better accordingly.









(a) Original

(b) damaged (c) Split Bregman (d) Our method

Figure 10: sample of Thai painting restoration.

Algorithm	Processing time	PSNR (dB)	SSIM
	(Seconds)		
Split Bregman	2.72	34.89	1.0000
Our algorithm	0.39	35.30	1.0000

Table 1: Average results of Thai painting restoration.

Algorithm	Processing time (Seconds)	PSNR (dB)	SSIM
Split Bregman	5,073.08	32.88	0.9654
Our algorithm	75.76	29.33	0.9454

Table 2: Average results of removing anime subtitles.

## Conclusion

In restoring ancient Thai painting image, our algorithm is 7 times faster than split Bregman and have more quality in both terms of PSNR and SSIM. For removing subtitles from animes, even split Bregman is better in quality but our algorithm is 67 times faster.

## Discussion

In this work, a new numerical algorithm for the TV based image inpainting model was proposed. As expected, our experimental results reveal that the new algorithm is significantly faster than the existing method.

## Reference

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- [2] T. Goldstein and S. Osher, "The Split Bregman Method for L1-Regularized Problems", SIAM Journal on Imaging Sciences. vol. 2, issue 2, pp. 323-343, 2009.
- [3] E.H. Andelson and C.H. Anderson and J.R. Bergen and P.J. Burt and J.M. Ogden. "Pyramid methods in image processing". 1984

