A new numerical algorithm for TV-based image inpainting with its applications for restoring ancient Thai painting images and removing subtitles from animes

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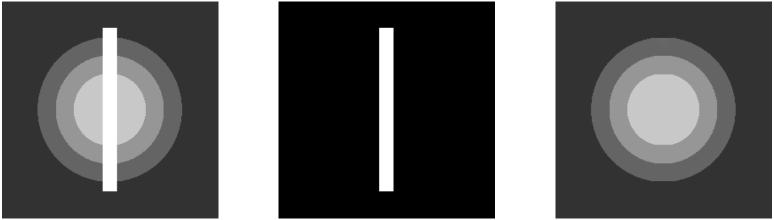
DPST student conference on Science and Technology 2019

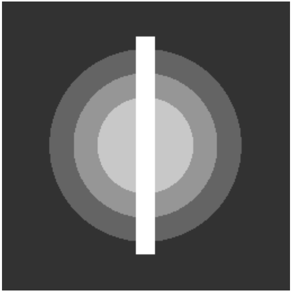
Image inpainting



(a) Damaged image (b) Inpainted domain (c) Restored image

Figure 1: Example of image inpainting

Image inpainting problem



grayscale image

color image

grayscale image

- image domain $\Omega \subset \mathbb{R}^2$
- inpainting domain $D \subset \mathbb{R}^2$
- physical position $\mathbf{x} = (x, y) \in \Omega$
- image intensity $V \subset [0, \infty)$
- grayscale image $u : \Omega \to V,\ z : \Omega \to V$
- without lose of generality $\Omega = [1, n]^2$ and $V = [0, 1]$ which $n > 0$ is positive integer

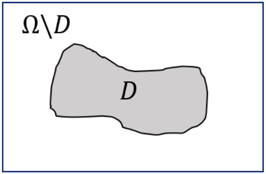


Figure 3: D is an inpainting domain

Total variation model for grayscale image inpainting

$$\min_u \{\mathcal{J}(u) = \frac{1}{2}\int_\Omega \lambda(u-z)^2 d\Omega + \int_\Omega |\nabla u| d\Omega\}$$

$$\lambda = \lambda(\mathbf{x}) = \begin{cases} \lambda_0, & x \in \Omega \setminus D \\ 0, & x \in D \end{cases}$$

T.F. Chan and J. Shen , "Mathematical models of local non-texture inpaintings", SIAM Journal on Applied Mathematics, vol. 62, no. 3, pp. 1019–1043, 2001.

Numerical algorithm

(1) Explicit time marching

(2) Fixed point iteration

(3) Split Bregman

Explicit time marching

$$\min_u \{\mathcal{J}(u) = \frac{1}{2}\int_\Omega \lambda(u-z)^2 d\Omega + \int_\Omega |\nabla u| d\Omega\}$$

$$\downarrow$$

$$\begin{cases} -\nabla\cdot\left(\dfrac{\nabla u}{|\nabla u|}\right) + \lambda(u-z) = 0, & \mathbf{x}\in(1,n)^2 \\ \dfrac{\partial u}{\partial n} = 0, & x\in\partial\Omega \end{cases}$$

L. I. Rudin, S. Osher, E. Fatemi, “Nonlinear total variation based noise removal algorithms”, Physica D: Nonlinear Phenomena, vol 60, issues 1–4, pp. 259-268, 1992.

Explicit time marching

$$u(\mathsf{x}, t_{k+1}) = u(\mathsf{x}, t_k) + \tau \left(\nabla \cdot \left(\frac{\nabla u(\mathsf{x}, t_k)}{|\nabla u(\mathsf{x}, t_k)|} \right) + \lambda(\mathsf{x})(u(\mathsf{x}, t_k) - z(\mathsf{x})) \right)$$

$$u(\mathsf{x}, t_0) = z \qquad t_k = t_0 + k\tau \ (\tau > 0) \qquad t_0 = 0$$

$$u(\mathsf{x}, t_0), u(\mathsf{x}, t_1), u(\mathsf{x}, t_2), u(\mathsf{x}, t_3), ..., \color{red}{u(\mathsf{x}, t^*)}$$

Limitation of explicit time marching

$$u(\mathbf{x}, t_{k+1}) = u(\mathbf{x}, t_k) + \tau \left(\nabla \cdot \left(\frac{\nabla u(\mathbf{x}, t_k)}{|\nabla u(\mathbf{x}, t_k)|} \right) + \lambda(\mathbf{x})(u(\mathbf{x}, t_k) - z(\mathbf{x})) \right)$$

$$\tau < 1$$

Fixed-point iteration

$$-\nabla \cdot \left(\frac{\nabla u^{[\nu+1]}}{|\nabla u|^{[\nu]}} \right) + \lambda(u^{[\nu+1]} - z) = 0,\ u^{[0]} = z$$

$$u^{[0]}, u^{[1]}, u^{[2]}, u^{[3]}, ..., u^{*}$$

C.R. Vogel and M.E. Oman,"Iterative methods for total variation denoising", SIAM Journal on Scientific Computing. vol. 17, pp. 227-238, 1996.

Numerical problem

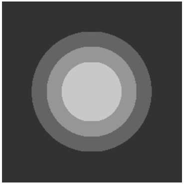


Figure 4: Example of image which has numerical problem

$$\frac{1}{|\nabla u|} = \frac{1}{\sqrt{u_x^2+u_y^2}} \to \infty$$

$$|\nabla u| \approx |\nabla u|_\beta = \sqrt{u_x^2 + u_y^2 + \beta},\ 0 < \beta \ll 1$$

Split Bregman

$$\min_{u}\{\mathcal{J}(u) = \frac{1}{2}\int_{\Omega}\lambda(u-z)^2 d\Omega + \int_{\Omega}|\nabla u| d\Omega\}$$

$$\downarrow$$

$$\min_{u,\boldsymbol{w}}\{\mathcal{J}(u,\boldsymbol{w}) = \frac{1}{2}\int_{\Omega}\lambda(u-z)^2 d\Omega + \int_{\Omega}|\boldsymbol{w}| d\Omega\} \qquad \text{which } w = \nabla u$$

$$\downarrow$$

$$\min_{u,\boldsymbol{w}}\{\mathcal{J}(u,\boldsymbol{w}) = \frac{1}{2}\int_{\Omega}\lambda(u-z)^2 d\Omega + \int_{\Omega}|\boldsymbol{w}| d\Omega + \frac{\theta}{2}\int_{\Omega}(\boldsymbol{w}-\nabla u+\boldsymbol{b})^2 d\Omega\}$$

T. Goldstein and S. Osher,"The Split Bregman Method for L1-Regularized Problems", SIAM Journal on Imaging Sciences. vol. 2, issue 2, pp. 323-343, 2009.

Split Bregman

$$\min_{u,w}\{\mathcal{J}(u,w)=\frac{1}{2}\int_\Omega \lambda(u-z)^2 d\Omega+\int_\Omega |\nabla w| d\Omega+\frac{\theta}{2}\int_\Omega (w-\nabla u+b)d\Omega\}$$

$$\downarrow$$

$$u^{\text{New}}=\arg\min_u\{\mathcal{J}_1(u)=\frac{1}{2}\int_\Omega \lambda(u-z)^2 d\Omega+\frac{\theta}{2}\int_\Omega (w^{\text{old}}-\nabla u+b^{\text{old}})d\Omega\}$$

$$w^{\text{New}}=\arg\min_w\{\mathcal{J}_2(w)=\int_\Omega |\nabla w| d\Omega+\frac{\theta}{2}\int_\Omega (w-\nabla u^{\text{New}}+b^{\text{old}})d\Omega\}$$

$$b^{\text{New}}=b^{\text{old}}+\nabla u^{\text{New}}-w^{\text{New}}$$

Color image

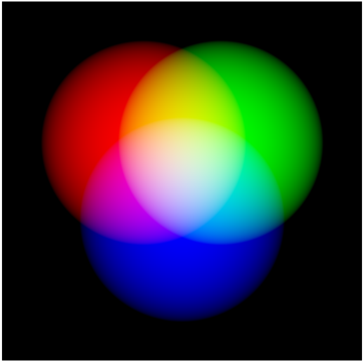


Figure 5: Color image create from red,green and blue color[1]

[1] ภาพจาก `https://commons.wikimedia.org/wiki/File:Additive_RGB_Circles-48bpp.png` สืบค้นเมื่อวันที่ 25 กันยายน 2561

Color image

$$u,\ z : \Omega \to V$$

$$\downarrow$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix},\ z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} : \Omega \to V^3$$

Total variation model for color image inpainting

$$\min_u \{\mathcal{J}(u) = \frac{1}{2}\int_\Omega \lambda(u-z)^2 d\Omega + \int_\Omega |\nabla u| d\Omega\}$$

$$\downarrow$$

$$\min_u \{\mathcal{J}(u) = \sum_{l=1}^{3} (\frac{1}{2}\int_\Omega \lambda(u_l - z_l)^2 d\Omega + \int_\Omega |\nabla u_l| d\Omega)\}$$

Split Bregman for Color image

$$\min_{u,\boldsymbol{w}}\{\mathcal{J}(u,\boldsymbol{w})=\frac{1}{2}\int_\Omega \lambda(u-z)^2 d\Omega+\int_\Omega |\nabla \boldsymbol{w}| d\Omega+\frac{\theta}{2}\int_\Omega (\boldsymbol{w}-\nabla u+\boldsymbol{b}) d\Omega\}$$

$$\downarrow$$

$$\min_{\boldsymbol{u},\boldsymbol{w_1},\boldsymbol{w_2},\boldsymbol{w_3}}\{\mathcal{J}(\boldsymbol{u},\boldsymbol{w_1},\boldsymbol{w_2},\boldsymbol{w_3})=\sum_{l=1}^{3}(\frac{1}{2}\int_\Omega \lambda(u_l-z_l)^2 d\Omega+\int_\Omega |\nabla \boldsymbol{w_l}| d\Omega + \frac{\theta}{2}\int_\Omega (\boldsymbol{w_l}-\nabla u_l+\boldsymbol{b_l}) d\Omega)\}$$

Quality measurement

(1) Peak Signal Noise Ratio (PSNR)

(2) Structural Similarity (SSIM)

Peak Signal Noise Ratio (PSNR)

$$\text{PSNR} = 10 \cdot log_{10}(\frac{1}{\sqrt{\text{MSE}}})$$

- MSE is mean square error which is MSE = $\frac{1}{nx \times ny} \sum (u - \tilde{u})^2$
- u is original image.
- $\tilde{u}$ is restored image from numerical method.
- The unit of PSNR is the decibel (dB).

Structural Similarity (SSIM)

$$\mathrm{SSIM}(u,\tilde{u}) = \frac{(2\mu_u\mu_{\tilde{u}} + 0.0001)(2\sigma_{u\tilde{u}} + 0.0009)}{(\mu_u^2 + \mu_{\tilde{u}}^2 + 0.0001)(\sigma_u^2 + \sigma_{\tilde{u}}^2 + 0.0009)}$$

- u is original image.
- $\tilde{u}$ is restored image from numerical method.
- μ_u is average of u
- $\mu_{\tilde{u}}$ is average of $\tilde{u}$
- σ_u is variation of u
- $\sigma_{\tilde{u}}$ is vriation of $\tilde{u}$
- SSIM has range between 0 to 1.

Synthetic image

Figure 6: Original image

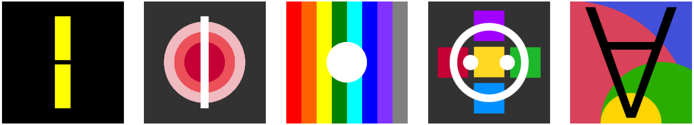


Figure 7: Damaged image

iteration $\leq 10,000$ round

$$\frac{||u_{new} - u_{old}||}{||u_{new}||} \geq 10^{-4}$$

Restoration result

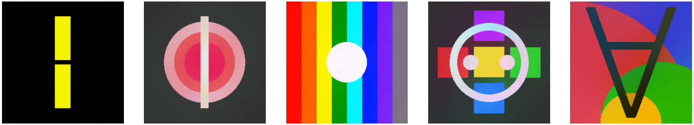


Figure 8: Explicit time marching

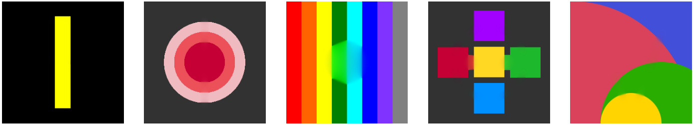


Figure 9: Fixed point iteration

Figure 10: Split Bregman

Performance of numerical method

Method	Processed time (Second)	PSNR (dB)	SSIM
Explicit time marching	120.68	16.72	0.9960
Fixed point iteration	74.81	38.67	0.9999
Split Bregman	14.06	39.42	0.9999

Table 1: Average image restoration result of numerical method
$\lambda = 250, \beta = 10^{-5}, \tau = 10^{-5}, \theta = 5$

Intial solution

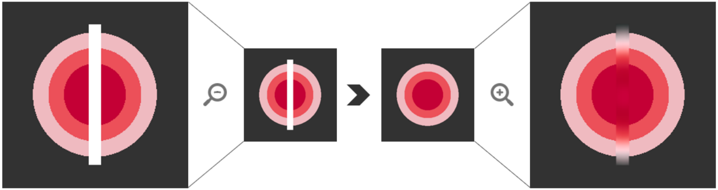


Figure 11: Find intial solution by using image pyramid

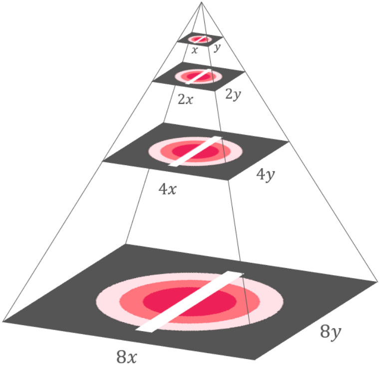


Figure 12: Image pyramid method

E.H. Andelson and C.H. Anderson and J.R. Bergen and P.J. Burt and J.M. Ogden. "Pyramid methods in image processing". 1984

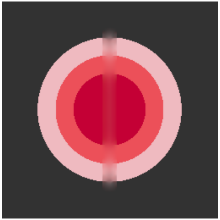
Restoration result with image pyramid

Iteration step	Processed time (Second)	PSNR (dB)	SSIM
Without Image pyramid	17.38	39.42	0.9999
10/1/1/10000	13.52	39.38	0.9999
10/3/3/10000	11.86	39.54	0.9999
10/10/10/10000	9.26	40.17	0.9999
100/1/1/10000	10.28	39.04	0.9999
100/3/3/10000	10.28	39.80	0.9999
100/10/10/10000	9.27	40.12	0.9999

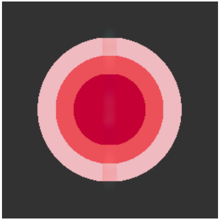
Table 2: Average result of Split Bregman method with Image pyramid

Iteration on Finest level

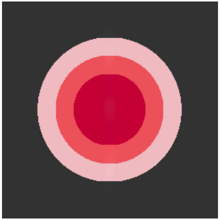
(a) 5 times



(b) 10 times



(c) 50 times



(d) 100 times

Figure 13: image pyramid with iteration 10/10/10 but have difference iteration on finest step

Restoration result with only 10 iteration on finest step

Iteration step	Processed time (Second)	PSNR (dB)	SSIM
Without Image pyramid	0.37	17.26	0.9963
10/1/1/10	0.40	28.54	0.9993
10/3/3/10	0.33	29.83	0.9994
10/10/10/10	0.38	32.56	0.9995
100/1/1/10	0.34	31.50	0.9999
100/3/3/10	0.36	31.99	0.9999
100/10/10/10	0.38	33.39	0.9998

Table 3: Average result of Resotration with only 10 itearation on finest step

Algorithm for Thai painting restoration

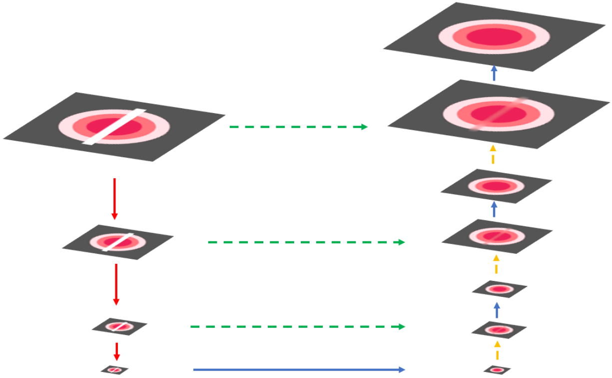


Figure 14: Our algorithm for Thai painting restoration

Algorithm for Thai painting restoration

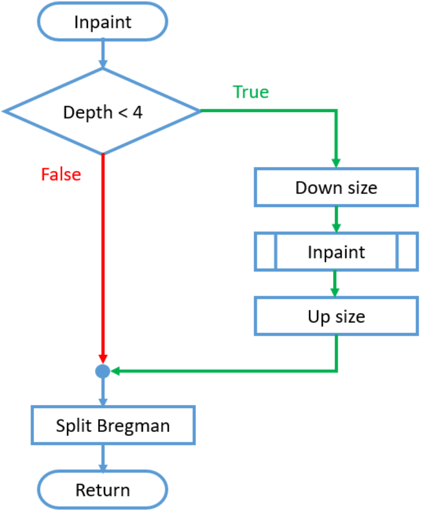


Figure 15: Our algorithm's flow chart

Thai painting restoration

Figure 16: Original Thai painting

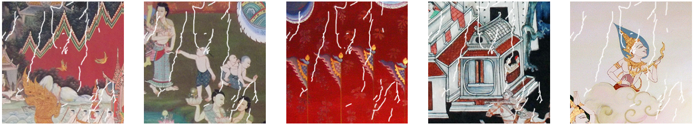


Figure 17: Damage Thai painting

Thai painting restoration result

Figure 18: Split Breman

Figure 19: Our algorithm

Comparison between 2 algorithms

(a) Original image

(b) Damaged image

(c) Split Bremgan

(d) Our algorithm

Figure 20: Comparison of images that have been expanded 4 times

Thai painting restoration result

Method	Processing time (Second)	PSNR (dB)	SSIM
Split Bregman	2.72	34.89	1.0000
Our algorithm	0.39	35.30	1.0000

Table 4: Average Thai painting restoration

Anime subtitle removal

Figure 21: Festival Asia Special Video - feat. Inori Aizawa

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat.

But I must explain to you how all this mistaken idea of denouncing of a pleasure and praising pain was born and I will not give you a complete account of the system, and expound the actual teachings of the great explorer of the truth, the master-builder of human happiness. No one rejects, dislikes, or avoids pleasure itself, because it is pleasure

しかし私は、喜びを非難して苦痛を賞賛するという誤ったこの考えがすべてどのようにして誕生したかをあなたに説明しなければならないから、私はあなたにその体系を完璧に説明し、真実を求める偉大な探究家、人間の喜びを築く建築家の実践的な教えを詳しく説明しよう

หากแต่ข้าพเจ้าต้องอธิบายต่อท่านในเหตุที่มโนคติอันความชิงชังอันหลงผิดทั้งหลาย ในการใฝ่หาและสดุดีบาดแผลว่าบ่มเพาะขึ้นได้เยี่ยงไรและ ข้าพเจ้าจักให้สาระแห่งระบบอันครบถ้วนแด่ท่าน แลประสิทธิ์คำสอนที่แท้แห่งการแสวงหาอันยิ่งใหญ่ในความจริง ซึ่งเป็นบูรพคณาจารย์ผู้สรรสร้างความผาสุขแห่งมวลมนุษย์ ฤจะมีผู้ใดมิรับ มิชอบ หรือเลี่ยงหลีกความสุข ด้วยเพราะมันเป็นสุขก็หาไม่

Figure 22: Lorem Ipsum in 4 languages [2]

[2] Lorem ipsum in difference language https://en.wikipedia.org/wiki/Lorem_ipsum, https://ja.wikipedia.org/wiki/Lorem_ipsum and https://th.wikipedia.org/wiki/ลอเร็มอิปซัม, 25 November 2018

Split video

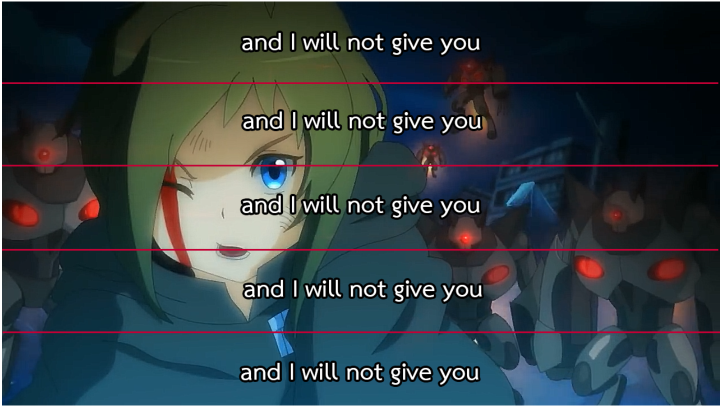


Figure 23: Split video into 5 parts

Video and image

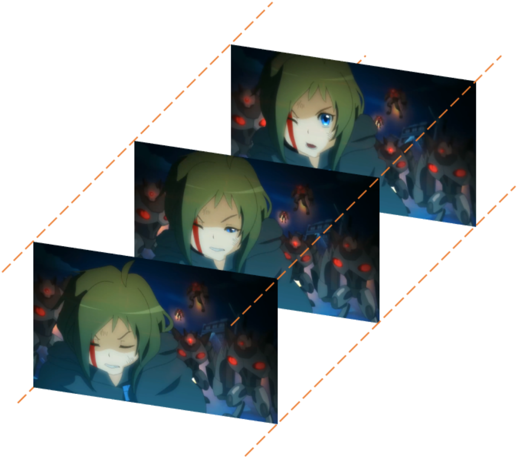
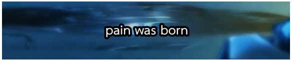


Figure 24: Video is sequences of image

Finding subtitle

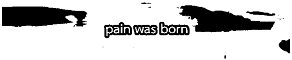
Figure 25: Anime subtitle always has black border

Finding subtitle



(a) Anime with subtitle

(b) mark black as white



(c) inverse color

(d) remove white which attach to edge

(e) remove smaller or larger object

(f) expand inpaint domain

Figure 26: our method to finding subtitle

Finding subtitle result

Language	Pixel in inpaint domain	Pixel which detected	Pixel which error	Error (percent)
Thai	23,222,220	24,083,125	2,141,201	9.22
English	27,278,745	28,598,424	3,714,321	13.62
Japanese	28,544,173	30,103,466	3,740,971	13.11

Table 5: Average error of finding inpainting domain in various language subtitles

Sequence of image and intial solution

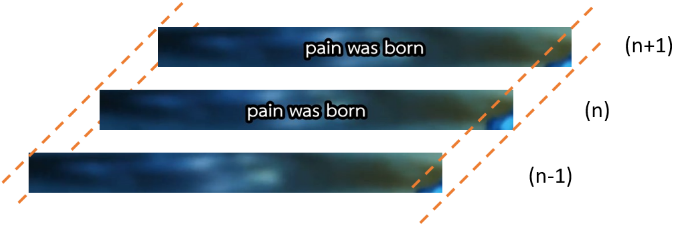


Figure 27: Image sequence in video which has subtitle

Consider similarities with SSIM

(a) Previous frame

(b) Current frame

(c) Area to find SSIM in previous frame

(d) Area to find SSIM in current frame

Figure 28: Area to calculate SSIM for skip frame and borrow frame method

Skip frame

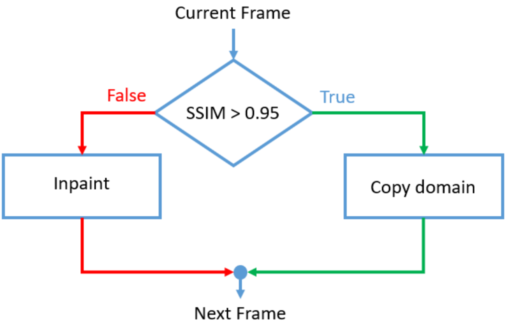


Figure 29: Skip frame method

Borrow frame

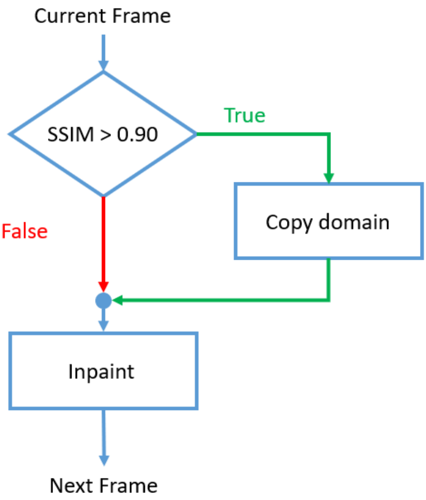


Figure 30: Borrow frame method

Skip and borrow frame

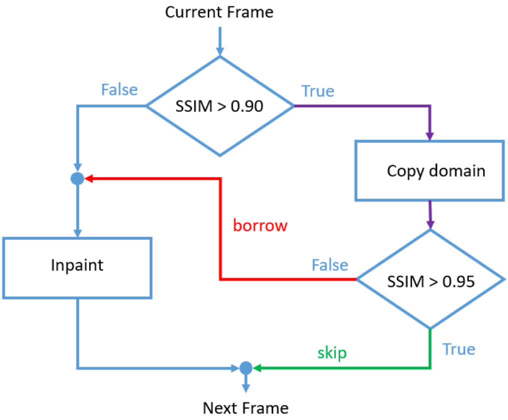


Figure 31: Skip and borrow frame method

Skip/Borrow frame result

Method	Processing time (Second)	PSNR (dB)	SSIM
Our first algorithm	141.29	31.39	0.9510
Skip frame	89.29	29.07	0.9408
Borrow frame	132.78	32.20	0.9655
Skip and borrow frame	75.76	29.33	0.9454

Table 6: Average result of removing subtitle from anime

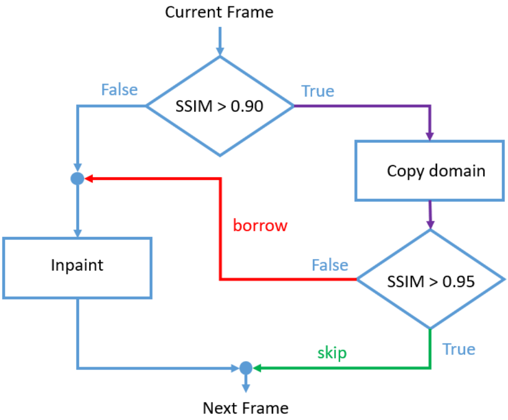


Figure 32: Skip and borrow frame method

Removing subtitle result

(a) Before

(b) After

Figure 33: sample frame of inpainting result

Removing subtitle result

Method	Processed time (Second)	PSNR (dB)	SSIM
Split Bregman	5073.08	32.88	0.9654
Our algorithm	75.76	29.33	0.9454

Table 7: Average result of removing subtitle with Split Bregman and our algorithm

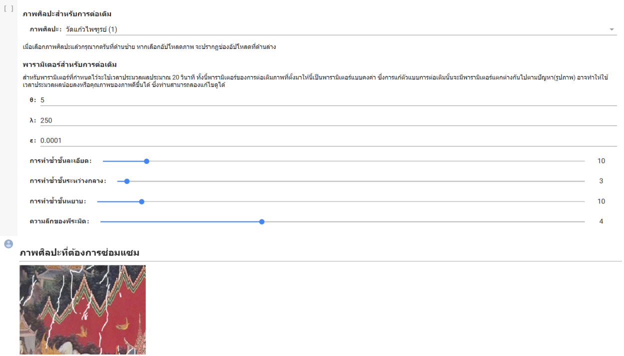


Figure 34: Try by yourself at https://bit.ly/demothai

Program for removing subtitle from anime

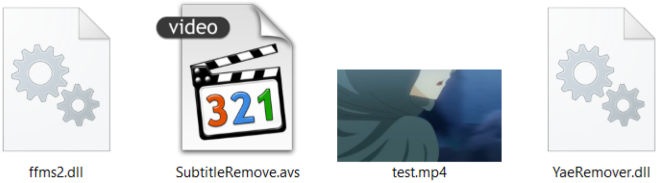


Figure 35: file for removing subtitle (https://bit.ly/demo-anime-inpaint)

```
LoadPlugin("ffms2.dll") #https://github.com/FFMS/ffms2
LoadPlugin("YaeRemover.dll") #our method
Video = FFMS2("test.mp4", ATrack=-1, fpsnum=24000, fpsden=1000, ColorSpace="RGB24", UTF8=True)

Result = Video.YaeRemover(Left = 280,Right = 1000,Top = 613,Bottom = 683,StokeWidth = 6)
return Result
```

Figure 36: source code in SubtitleRemove.avs

Thank you

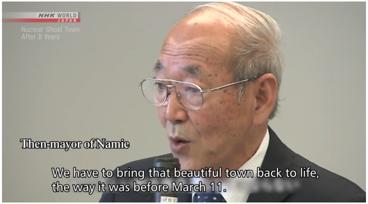
Why we need to remove subtitle

Figure 37: Ling Feng Zhe has Chinese subtitle because it had produced in China

If we don’t remove subtitle

(a) Justice Pao (Ch. 3)

(b) Wise and Foolish (Thai PBS)



(c) Nuclear ghost town (NHK World)

Figure 38: How TV channel work on forgien language subtitle