Combined and Composite Games in Evoluationary Game Theory*

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Building upon previous work in analyzing finite population size effects on evolutionary games, defining new game as a compositon of well known 2 and 3 strategy games and derive replicator dynamics for different microscopic interaction processes. Computing drift and drift reversal (loss of stability below a critical population size) both through extensive simulations and analytical derivations for each of the agent interaction processes. Developed visualization tools.

Usage: Secondary publications and information retrieval purposes.

Structure: You may use the `description` environment to structure your abstract; use the optional argument of the `\item` command to give the category of each item.

I. INTRODUCTION: RPS DYNAMICS

To understand the combined game, first must look at the single cases of rock paper scissors and Snowdrift.

A. Rock-Paper-Scissors Model

Previous work focusses on the famous cyclical rock-paper-scissors game [1]

The general rock paper scissors matrix is defined as:

$$\begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix} \tag{1}$$

For the standard symmetric case of the RPS game, $a = 0, c = -1, b = 1$ where rock crushes scissors, paper covers rock, and scissors cuts paper. Ties are awarded the payoff of 0 resulting in cyclical dynamics about the internal fixed point of $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

1. Replicator dynamics

Let x, y and z represent the fraction of players using rock, paper, and scissors respectively. z can be defined as $z = 1-x-y$, reducing the dimension of the replicator equations. Expected payoffs for each strategy can then be defined as follows:

$$\pi_R = ax + b\left(-x-y+1\right) + cy \tag{2}$$

$$\pi_P = ay + bx + c\left(-x-y+1\right) \tag{3}$$

$$\pi_S = a\left(-x-y+1\right) + by + cx \tag{4}$$

The replicator equations can be defined for (x, y) using the standard formula:

$$\dot{x}_i = x_i(\pi_i(x) - \langle\pi(x)\rangle) \tag{5}$$

where the average payoff (or mean fitness) in the population is:

$$\langle\pi(x)\rangle = x\pi_R + y\pi_P + z\pi_S$$

This leads to the following replicator equations for x and y:

$$\begin{aligned}\dot{x} = {} & xy\left(ax - ay - bx + b\left(-x-y+1\right) + cy - c\left(-x-y+1\right)\right) \\ & + x\left(-x-y+1\right)\left(ax - a\left(-x-y+1\right) - by + b\left(-x-y+1\right) - cx + cy\right)\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{y} = {} & xy\left(-ax + ay + bx - b\left(-x-y+1\right) - cy + c\left(-x-y+1\right)\right) \\ & + y\left(-x-y+1\right)\left(ay - a\left(-x-y+1\right) + bx - by - cx + c\left(-x-y+1\right)\right)\end{aligned} \tag{7}$$

The stability of this system of replicator equations can be analyzed by computing the eigenvalues of the Jacobian

* A footnote to the article title

matrix of functions $F = \dot{x}, G = \dot{y}$.

Should include a figure for RPS cyclic dynamics.

Add in the fixed points and stability for RPS.

B. Snowdrift game

II. COMBINED GAME DEFINITION

This work focusses on the composite game with the idea of combing snowdrift (SD) and rock-paper-scissors (RPS) into a single four strategy payoff matrix. Average payoffs, replicator equations and fixed points can subsequently be derived and the deterministic dynamics can be studied. The simulation tool can do all of this, plot trajectories both in 2d and and in the 3d pyramid simplex.

Define our combined game and explain the subgame etc. Link to previous work about RPS

III. MODEL AND STOCHASTIC DYNAMICS

We consider a finite population of size N evolving under stochastic evolutionary dynamics with S strategies. The population state is described by the vector $\mathbf{x} = (x_1, \ldots, x_{S-1})$, where $x_i = n_i/N$ and the final strategy is fixed by normalization. At each update step, individuals interact according to a payoff matrix $\mathbf{A}$ and reproduce according to a specified microscopic update rule. In the large–N limit, the stochastic dynamics can be approximated by a Fokker–Planck equation governing the probability density $P(\mathbf{x}, t)$.

IV. FOKKER-PLANCK EQUATION

Master equation can be derived and using the 2nd order taylor expansion of the transition probabilities for each strategy, a fokker-planck equation can be derived for each of the processes.

In the diffusion approximation, the master equation may be expanded to second order in $1/N$, yielding a Fokker-Planck equation of the form

$$\frac{d}{dt}P^\tau(x) = \frac{1}{N}\left[-\frac{d}{dx}(a(x)P^\tau(x)) + \frac{1}{2}\frac{d^2}{dx^2}\left(b^2(x)P^\tau(x)\right)\right] \quad (8)$$

Here $x \in (i, j, k)$ denotes the frequency of a given strategy in the RPS subsystem, and

$$a(i) = T^{PR} + T^{SR} + T^{+R} - T^{RP} - T^{RS} - T^{R+},$$

$$b(i) = \sqrt{\frac{1}{N}\left(T^{PR} + T^{SR} + T^{+R} + T^{RP} + T^{RS} + T^{R+}\right)}$$

for strategy i, rock and similarly $a(j), b(j)$ for paper and $a(k), b(k)$ for scissors can be defined in the same way with the pairs of transition probabilities into and out of the respective states and must be derived for each microscopic update process and set of payoff matrices. Full derivation of Fokker-Planck equation and expressions for $a(x), b(x)$ can be found in full report detailing 2nd order expansion of the master equation for the 4x4 game.

For each strategy, $a(x)$ corresponds to the deterministic replicator equation ($N \to \infty$), and $b(x)$ is the diffusion term corresponding to the stochastic drift. Code is capable of computing these values for each interaction process in order to plot the correct determinisic adjusted dynamics. For the local update process, this $a(x)$ is equal to the standard replicator payoff difference equation (with some scaling factor accounting for selection pressure w).

V. DRIFT MEASURES

To characterize finite-population effects beyond deterministic dynamics, we study the expected one–step change of suitable coexistence measures,

$$\langle \Delta H \rangle = \mathbb{E}[H(\mathbf{x}_{t+1}) - H(\mathbf{x}_t)].$$

The sign of $\langle \Delta H \rangle$ determines whether stochastic drift favors interior coexistence or boundary attraction. $\langle \Delta H \rangle$ can be written as a sum over the simplex ($1 \leq i, j \leq N - i, k \leq N - i - j$).

$$\langle \Delta H \rangle = \sum_{i=1}^{N}\sum_{j=1}^{N-i}\sum_{k=1}^{N-i-j}(H_s - H_{s'})\,T^{s \to s'} \quad (9)$$

for all the states $s, s' \in \{R, P, S, L\}$ and transition probabilities $T^{s \to s'}$.

A. Rock-Paper-Scissors Drift

For the three-strategy subsystem, the replicator equations have the constant of motion:

$$H_{\mathrm{RPS}} = -xyz \quad (10)$$

Also have the summation here.

$$\langle \Delta H_{\rm RPS}\rangle = \frac{2}{N}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \Big[z(y-x)(T^{RP}-T^{PR}) +y(z-x)(T^{RS}-T^{SR}) +x(y-z)(T^{SP}-T^{PS}) +xz(T^{P+}-T^{+P})+yz(T^{R+}-T^{+R}) +xy(T^{S+}-T^{+S})\Big] + \frac{2}{N^2}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \Big[z(T^{RP}+T^{PR})+y(T^{RS}+T^{SR}) +x(T^{SP}+T^{PS})\Big] \tag{11}$$

This expression shows that finite population drift is controlled by antisymmetric differences of transition probabilities between competing strategies.

B. Snowdrift Drift

The 4th strategy occurs with frequency $q=1-x-y-z$, we define the snowdrift coexistence measure

$$H_{\rm SD} = -q(1-q).$$

The expected value of H_{SD} over the entire simplex is then:

$$\langle \Delta H_{\rm SD}\rangle = \frac{12}{N}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \Big[q\big[(T^{R+}-T^{+R})+(T^{P+}-T^{+P}) +(T^{S+}-T^{+S})\big]\Big] + \frac{12}{N^2}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \Big[T^{+R}+T^{+P}+T^{+S}\Big] \tag{12}$$

The leading $1/N$ term governs directional drift between the RPS subspace and the loner strategy, while the $1/N^2$ term represents finite–population diffusion.

C. Four-Strategy Coexistence Drift

To capture full four-strategy coexistence, the constant of motion H_4 is defined

$$H_4 = -xyzq,$$

which vanishes on all faces of the simplex and is maximized in the interior.

$$\begin{aligned}\langle \Delta H_4\rangle = \frac{6}{N^7}\sum_{i=1}^{N}\sum_{j=1}^{N-i}\sum_{k=1}^{N-i-j} \Big[& ijk(N-i-j-k)(T^{RP}+T^{RS}+T^{R+}+T^{PR}+T^{PS}+T^{P+} \\ & +T^{SR}+T^{SP}+T^{S+}+T^{+R}+T^{+P}+T^{+S}) - (i-1)(j+1)k(N-i-j-k)T^{RP} \\ & -(i-1)j(k+1)(N-i-j-k)T^{RS} - (i-1)jk(N-i-j-k+1)T^{R+} \\ & -(i+1)(j-1)k(N-i-j-k)T^{PR} - i(j-1)(k+1)(N-i-j-k)T^{PS} \\ & -i(j-1)k(N-i-j-k+1)T^{P+} - (i+1)j(k-1)(N-i-j-k)T^{SR} \\ & -i(j+1)(k-1)(N-i-j-k)T^{SP} - ij(k-1)(N-i-j-k+1)T^{S+} \\ & -(i+1)jk(N-1-i-j-k)T^{+R} - i(j+1)k(N-1-i-j-k)T^{+P} - ij(k+1)(N-1-i-j-k)T^{+S}\Big]\end{aligned} \tag{13}$$

In the continuous limit

$$\begin{aligned}\langle \Delta H_4\rangle = \frac{6}{N}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \Big[& zq(y-x)(T^{RP}-T^{PR}) + yq(z-x)(T^{RS}-T^{SR}) + xq(y-z)(T^{SP}-T^{PS}) \\ & + yz(q-x)(T^{R+}-T^{+R}) + xz(q-y)(T^{P+}-T^{+P}) + xy(q-z)(T^{S+}-T^{+S})\Big] \\ + \frac{6}{N^2}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \Big[& zq(T^{RP}+T^{PR}) + yq(T^{RS}+T^{SR}) + xq(T^{SP}+T^{PS}) \\ & + yz(T^{R+}+T^{+R}) + xz(T^{P+}+T^{+P}) + xy(T^{S+}+T^{+S})\Big]\end{aligned} \tag{14}$$

This expression naturally decomposes into RPS–like drift terms, snowdrift–like drift terms, and diffusion corrections. The sign of $\langle \Delta H_4 \rangle$ determines whether stochastic dynamics favors four–strategy coexistence or boundary attraction.

D. Comparison of drift measures

Drift reversal, the loss of stability of an attracting fixed point (something about hopf bifurcation) below a critical population size, occurs at the point where the value of $\langle H \rangle$ changes sign. Depending on the initial parameters, this effect is observed in the RPS and SD subgames, and in some cases in both.

Diagram of the main cases - large population attraction to center - one case where drift in SD but strong RPS giving the rod shape. Other case where strong SD but drift in RPS, giving the spiral / plane in the center of the simplex.

VI. INTERPRETATION

While $\langle \Delta H_{\mathrm{RPS}} \rangle$ and $\langle \Delta H_{\mathrm{SD}} \rangle$ describe drift within lower–dimensional subspaces, $\langle \Delta H_4 \rangle$ captures stability of full interior coexistence. Sign reversals of $\langle \Delta H_4 \rangle$ generally occur at larger population sizes, reflecting the additional geometric suppression of interior states in the four–strategy simplex.

VII. FIRST-LEVEL HEADING: THE LINE BREAK WAS FORCED via \\

This sample document demonstrates proper use of REVTeX 4.2 (and LaTeX 2$_\varepsilon$) in mansucripts prepared for submission to APS journals. Further information can be found in the REVTeX 4.2 documentation included in the distribution or available at `http://journals.aps.org/revtex/`.

When commands are referred to in this example file, they are always shown with their required arguments, using normal TeX format. In this format, `#1`, `#2`, etc. stand for required author-supplied arguments to commands. For example, in `\section{#1}` the `#1` stands for the title text of the author's section heading, and in `\title{#1}` the `#1` stands for the title text of the paper.

Line breaks in section headings at all levels can be introduced using \\. A blank input line tells TeX that the paragraph has ended. Note that top-level section headings are automatically uppercased. If a specific letter or word should appear in lowercase instead, you must escape it using `\lowercase{#1}` as in the word "via" above.

A. Second-level heading: Formatting

This file may be formatted in either the `preprint` or `reprint` style. `reprint` format mimics final journal output. Either format may be used for submission purposes. `letter` sized paper should be used when submitting to APS journals.

1. *Wide text (A level-3 head)*

The `widetext` environment will make the text the width of the full page, as on page 7. (Note the use the `\pageref{#1}` command to refer to the page number.)

a. Note (Fourth-level head is run in) The width-changing commands only take effect in two-column formatting. There is no effect if text is in a single column.

B. Citations and References

A citation in text uses the command `\cite{#1}` or `\onlinecite{#1}` and refers to an entry in the bibliography. An entry in the bibliography is a reference to another document.

1. *Citations*

Because REVTeX uses the `natbib` package of Patrick Daly, the entire repertoire of commands in that package are available for your document; see the `natbib` documentation for further details. Please note that REVTeX requires version 8.31a or later of `natbib`.

a. Syntax The argument of `\cite` may be a single *key*, or may consist of a comma-separated list of keys. The citation *key* may contain letters, numbers, the dash (-) character, or the period (.) character. New with natbib 8.3 is an extension to the syntax that allows for a star (*) form and two optional arguments on the citation key itself. The syntax of the `\cite` command is thus (informally stated)

`\cite {` *key* `}`, or
`\cite {` *optarg+key* `}`, or
`\cite {` *optarg+key* `,` *optarg+key*... `}`,
where *optarg+key* signifies

key, or
`*`*key*, or
`[`*pre*`]` *key*, or
`[`*pre*`]` `[`*post*`]` *key*, or even
`*[`*pre*`]` `[`*post*`]` *key*.
where *pre* and *post* is whatever text you wish to place at the beginning and end, respectively, of the bibliographic reference (see Ref. [**?**] and the two under Ref. [**?**]). (Keep in mind that no automatic space or punctuation is applied.) It is highly recommended that you put the entire *pre* or *post* portion within its own set of braces, for example: `\cite { [ {`*text*`}]` *key*`}`. The extra set of braces will keep LaTeX out of trouble if your *text* contains the comma (,) character.

The star (*) modifier to the *key* signifies that the reference is to be merged with the previous reference into a single bibliographic entry, a common idiom in APS and AIP articles (see below, Ref. [**?**]). When references are merged in this way, they are separated by a semicolon instead of the period (full stop) that would otherwise appear.

b. Eliding repeated information When a reference is merged, some of its fields may be elided: for example, when the author matches that of the previous reference, it is omitted. If both author and journal match, both are omitted. If the journal matches, but the author does not, the journal is replaced by *ibid.*, as exemplified by Ref. [**?**]. These rules embody common editorial practice in APS and AIP journals and will only be in effect if the markup features of the APS and AIP BibTeX styles is employed.

c. The options of the cite command itself Please note that optional arguments to the *key* change the reference in the bibliography, not the citation in the body of the document. For the latter, use the optional arguments of the `\cite` command itself: `\cite *[`*pre-cite*`]` `[`*post-cite*`]{`*key-list*`}`.

2. *Example citations*

By default, citations are numerical[**?**]. Author-year citations are used when the journal is RMP. To give a textual citation, use `\onlinecite{#1}`: Refs. **? ?** . By default, the `natbib` package automatically sorts your citations into numerical order and "compresses" runs of three or more consecutive numerical citations. REVTeX provides the ability to automatically change the punctuation when switching between journal styles that provide citations in square brackets and those that use a superscript style instead. This is done through the `citeautoscript` option. For instance, the journal style `prb` automatically invokes this option because *Physical Review B* uses superscript-style citations. The effect is to move the punctuation, which normally comes after a citation in square brackets, to its proper position before the superscript. To illustrate, we cite several together [**? ? ? ? ? ?**], and once again in different order (Refs. [**? ? ? ? ? ?**]). Note that the citations were both compressed and sorted. Futhermore, running this sample file under the `prb` option will move the punctuation to the correct place.

When the `prb` class option is used, the `\cite{#1}` command displays the reference's number as a superscript rather than in square brackets. Note that the location of the `\cite{#1}` command should be adjusted for the reference style: the superscript references in `prb` style must appear after punctuation; otherwise the reference must appear before any punctuation. This sample was written for the regular (non-`prb`) citation style. The command `\onlinecite{#1}` in the `prb` style also displays the reference on the baseline.

3. References

A reference in the bibliography is specified by a `\bibitem{#1}` command with the same argument as the `\cite{#1}` command. `\bibitem{#1}` commands may be crafted by hand or, preferably, generated by BibTeX. REVTeX 4.2 includes BibTeX style files `apsrev4-2.bst`, `apsrmp4-2.bst` appropriate for *Physical Review* and *Reviews of Modern Physics*, respectively.

4. Example references

This sample file employs the `\bibliography` command, which formats the `report.bbl` file and specifies which bibliographic databases are to be used by BibTeX (one of these should be by arXiv convention `report.bib`). Running BibTeX (via `bibtex report`) after the first pass of LaTeX produces the file `report.bbl` which contains the automatically formatted `\bibitem` commands (including extra markup information via `\bibinfo` and `\bibfield` commands). If not using BibTeX, you will have to create the `thebibiliography` environment and its `\bibitem` commands by hand.

Numerous examples of the use of the APS bibliographic entry types appear in the bibliography of this sample document. You can refer to the `report.bib` file, and compare its information to the formatted bibliography itself.

C. Footnotes

Footnotes, produced using the `\footnote{#1}` command, usually integrated into the bibliography alongside the other entries. Numerical citation styles do this[2]; author-year citation styles place the footnote at the bottom of the text column. Note: due to the method used to place footnotes in the bibliography, *you must re-run BibTeX every time you change any of your document's footnotes.*

VIII. MATH AND EQUATIONS

Inline math may be typeset using the `$` delimiters. Bold math symbols may be achieved using the `bm` package and the `\bm{#1}` command it supplies. For instance, a bold α can be typeset as `$\bm{\alpha}$` giving $\boldsymbol{\alpha}$. Fraktur and Blackboard (or open face or double struck) characters should be typeset using the `\mathfrak{#1}` and `\mathbb{#1}` commands respectively. Both are supplied by the `amssymb` package. For example, `$\mathbb{R}$` gives $\mathbb{R}$ and `$\mathfrak{G}$` gives $\mathfrak{G}$

In LaTeX there are many different ways to display equations, and a few preferred ways are noted below. Displayed math will center by default. Use the class option `fleqn` to flush equations left.

Below we have numbered single-line equations; this is the most common type of equation in *Physical Review*:

$$\chi_+(p) \lesssim [2|\mathbf{p}|(|\mathbf{p}|+p_z)]^{-1/2} \begin{pmatrix} |\mathbf{p}|+p_z \\ px+ip_y \end{pmatrix}, \tag{15}$$

$$\left\{ \mathbb{1}234567890abc123\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum_b^a}{A^2} \right\}. \tag{16}$$

Note the open one in Eq. (16).

Not all numbered equations will fit within a narrow column this way. The equation number will move down automatically if it cannot fit on the same line with a one-line equation:

$$\left\{ ab12345678abc123456abcdef\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum_b^a}{A^2} \right\}. \tag{17}$$

When the `\label{#1}` command is used [cf. input for Eq. (16)], the equation can be referred to in text without knowing the equation number that TeX will assign to it. Just use `\ref{#1}`, where `#1` is the same name that used in the `\label{#1}` command.

Unnumbered single-line equations can be typeset using the `\[`, `\]` format:

$$g^+g^+ \rightarrow g^+g^+g^+g^+ \dots ~,~~q^+q^+\rightarrow q^+g^+g^+ \dots ~.$$

A. Multiline equations

Multiline equations are obtained by using the `eqnarray` environment. Use the `\nonumber` command at the end of each line to avoid assigning a number:

$$\begin{aligned}\mathcal{M} = \quad & ig_Z^2(4E_1E_2)^{1/2}(l_i^2)^{-1}\delta_{\sigma_1,-\sigma_2}(g_{\sigma_2}^e)^2\chi_{-\sigma_2}(p_2)\\ & \times[\epsilon_j l_i\epsilon_i]_{\sigma_1}\chi_{\sigma_1}(p_1),\end{aligned} \tag{18}$$

$$\begin{aligned}\sum |M^{\text{viol}}_g|^2 \;=\; & g^{2n-4}_S(Q^2)\;N^{n-2}(N^2-1)\\ & \times \left(\sum_{i<j}\right)\sum_{\text{perm}}\frac{1}{S_{12}}\frac{1}{S_{12}}\sum_\tau c^f_\tau\;.\end{aligned} \tag{19}$$

Note: Do not use `\label{#1}` on a line of a multiline equation if `\nonumber` is also used on that line. Incorrect cross-referencing will result. Notice the use `\text{#1}` for using a Roman font within a math environment.

To set a multiline equation without *any* equation numbers, use the `\begin{eqnarray*}`, `\end{eqnarray*}` format:

$$\begin{aligned}\sum |M^{\text{viol}}_g|^2 \;=\; & g^{2n-4}_S(Q^2)\;N^{n-2}(N^2-1)\\ & \times \left(\sum_{i<j}\right)\left(\sum_{\text{perm}}\frac{1}{S_{12}S_{23}S_{n1}}\right)\frac{1}{S_{12}}\;.\end{aligned}$$

To obtain numbers not normally produced by the automatic numbering, use the `\tag{#1}` command, where `#1` is the desired equation number. For example, to get an equation number of $(2.6')$,

$$g^+g^+ \rightarrow g^+g^+g^+g^+\dots~,~~q^+q^+\rightarrow q^+g^+g^+\dots~. \tag{2.6$'$}$$

a. A few notes on `tag`*s* `\tag{#1}` requires the `amsmath` package. Place the `\tag{#1}` command before the `\label{#1}`, if any. The numbering produced by `\tag{#1}` *does not affect* the automatic numbering in REVTEX; therefore, the number must be known ahead of time, and it must be manually adjusted if other equations are added. `\tag{#1}` works with both single-line and multiline equations. `\tag{#1}` should only be used in exceptional cases—do not use it to number many equations in your paper. Please note that this feature of the `amsmath` package is *not* compatible with the `hyperref` (6.77u) package.

Enclosing display math within `\begin{subequations}` and `\end{subequations}` will produce a set of equations that are labeled with letters, as shown in Eqs. (20b) and (20a) below. You may include any number of single-line and multiline equations, although it is probably not a good idea to follow one display math directly after another.

$$\begin{aligned}\mathcal{M} = \quad & ig_Z^2(4E_1E_2)^{1/2}(l_i^2)^{-1}(g_{\sigma_2}^e)^2\chi_{-\sigma_2}(p_2)\\ & \times[\epsilon_i]_{\sigma_1}\chi_{\sigma_1}(p_1).\end{aligned} \tag{20a}$$

$$\left\{ abc123456abcdef\alpha\beta\gamma\delta1234556\alpha\beta\frac{1\sum^a_b}{A^2}\right\}, \tag{20b}$$

Giving a `\label{#1}` command directly after the `\begin{subequations}`, allows you to reference all the equations in the `subequations` environment. For example, the equations in the preceding subequations environment were Eqs. (20).

1. Wide equations

The equation that follows is set in a wide format, i.e., it spans the full page. The wide format is reserved for long equations that cannot easily be set in a single column:

$$\mathcal{R}^{(\text{d})} = g_{\sigma_2}^e\left(\frac{[\Gamma^Z(3,21)]_{\sigma_1}}{Q_{12}^2-M_W^2}+\frac{[\Gamma^Z(13,2)]_{\sigma_1}}{Q_{13}^2-M_W^2}\right)+x_WQ_e\left(\frac{[\Gamma^\gamma(3,21)]_{\sigma_1}}{Q_{12}^2-M_W^2}+\frac{[\Gamma^\gamma(13,2)]_{\sigma_1}}{Q_{13}^2-M_W^2}\right)\;. \tag{21}$$

This is typed to show how the output appears in wide format. (Incidentally, since there is no blank line between the `equation` environment above and the start of this paragraph, this paragraph is not indented.)

IX. CROSS-REFERENCING

REVTEX will automatically number such things as sections, footnotes, equations, figure captions, and table captions. In order to reference them in text, use the `\label{#1}` and `\ref{#1}` commands. To reference a particular page, use the `\pageref{#1}` command.

The `\label{#1}` should appear within the section heading, within the footnote text, within the equation, or within the table or figure caption. The `\ref{#1}` command is used in text at the point where the reference is to be displayed. Some examples: Section I on page 1, Table I, and Fig. 1.

X. FLOATS: FIGURES, TABLES, VIDEOS, ETC.

Figures and tables are usually allowed to "float", which means that their placement is determined by LATEX, while the document is being typeset.

Use the `figure` environment for a figure, the `table` environment for a table. In each case, use the `\caption` command within to give the text of the figure or table caption along with the `\label` command to provide a key for referring to this figure or table. The typical content of a figure is an image of some kind; that of a table is an alignment.

Insert an image using either the `graphics` or `graphix` packages, which define the `\includegraphics{#1}` command. (The two packages differ in respect of the optional arguments used to specify the orientation, scaling, and translation of the image.) To create an alignment, use the `tabular` environment.

The best place to locate the `figure` or `table` environment is immediately following its first reference in text; this sample document illustrates this practice for Fig. 1, which shows a figure that is small enough to fit in a single column.

In exceptional cases, you will need to move the float earlier in the document, as was done with Table II: LaTeX's float placement algorithms need to know about a full-page-width float earlier.

Fig. 2 has content that is too wide for a single column, so the `figure*` environment has been used.

The content of a table is typically a `tabular` environment, giving rows of type in aligned columns. Column entries separated by `&`'s, and each row ends with \\. The required argument for the `tabular` environment specifies how data are aligned in the columns. For instance, entries may be centered, left-justified, right-justified, aligned on a decimal point. Extra column-spacing may be be specified as well, although REVTeX 4 sets this spacing so that the columns fill the width of the table. Horizontal rules are typeset using the `\hline` command. The doubled (or Scotch) rules that appear at the top and bottom of a table can be achieved enclosing the `tabular` environment within a `ruledtabular` environment. Rows whose columns span multiple columns can be typeset using the `\multicolumn{#1}{#2}{#3}` command (for example, see the first row of Table II).

Tables I, II, III, and IV show various effects. A table that fits in a single column employs the `table` environment. Table II is a wide table, set with the `table*` environment. Long tables may need to break across pages. The most straightforward way to accomplish this is to specify the `[H]` float placement on the `table` or `table*` environment. However, the LaTeX 2_ε package `longtable` allows headers and footers to be specified for each page of the table. A simple example of the use of `longtable` can be found in the file `summary.tex` that is included with the REVTeX 4 distribution.

There are two methods for setting footnotes within a table (these footnotes will be displayed directly below the table rather than at the bottom of the page or in the bibliography). The easiest and preferred method is just to use the `\footnote{#1}` command. This will automatically enumerate the footnotes with lowercase roman letters. However, it is sometimes necessary to have multiple entries in the table share the same footnote. In this case, there is no choice but to manually create the footnotes using `\footnotemark[#1]` and `\footnotetext[#1]{#2}`. `#1` is a numeric value. Each time the same value for `#1` is used, the same mark is produced in the table. The `\footnotetext[#1]{#2}` commands are placed after the `tabular` environment. Examine the LaTeX source and output for Tables I and IV for examples.

The `\setfloatlink` command causes the title of the video to be a hyperlink to the indicated URL; it may be used with any environment that takes the `\caption` command. The `\href` command has the same significance as it does in the context of the `hyperref` package: the second argument is a piece of text to be typeset in your document; the first is its hyperlink, a URL.

Physical Review style requires that the initial citation of figures or tables be in numerical order in text, so don't cite Fig. 2 until Fig. 1 has been cited.

TABLE I. A table that fits into a single column of a two-column layout. Note that REVTeX 4 adjusts the intercolumn spacing so that the table fills the entire width of the column. Table captions are numbered automatically. This table illustrates left-, center-, decimal- and right-aligned columns, along with the use of the `ruledtabular` environment which sets the Scotch (double) rules above and below the alignment, per APS style.

Left[a]	Centered[b]	Decimal	Right
1	2	3.001	4
10	20	30	40
100	200	300.0	400

[a] Note a.
[b] Note b.

ACKNOWLEDGMENTS

We wish to acknowledge the support of the author community in using REVTeX, offering suggestions and encouragement, testing new versions, ...

Appendix A: Appendixes

To start the appendixes, use the `\appendix` command. This signals that all following section commands refer to

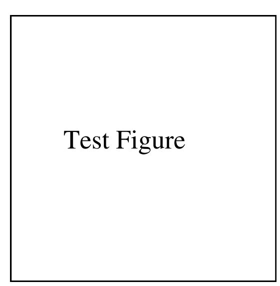

FIG. 1. A figure caption. The figure captions are automatically numbered.

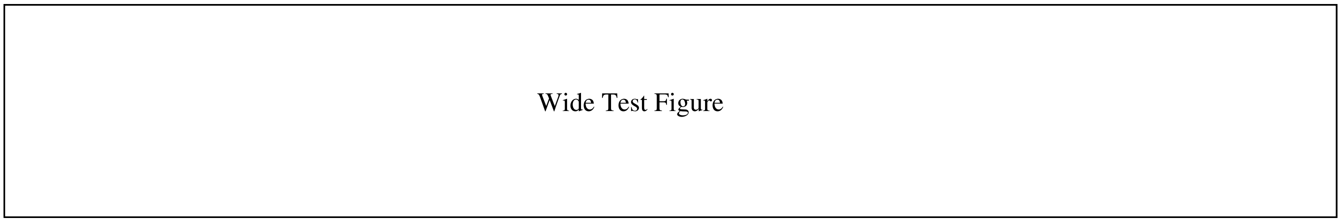

FIG. 2. Use the figure* environment to get a wide figure that spans the page in `twocolumn` formatting.

TABLE II. This is a wide table that spans the full page width in a two-column layout. It is formatted using the `table*` environment. It also demonstates the use of `\multicolumn` in rows with entries that span more than one column.

	D^1_{4h}		D^5_{4h}	
Ion	1st alternative	2nd alternative	lst alternative	2nd alternative
K	$(2e)+(2f)$	$(4i)$	$(2c)+(2d)$	$(4f)$
Mn	$(2g)$[a]	$(a)+(b)+(c)+(d)$	$(4e)$	$(2a)+(2b)$
Cl	$(a)+(b)+(c)+(d)$	$(2g)$[a]	$(4e)$[a]	
He	$(8r)$[a]	$(4j)$[a]	$(4g)$[a]	
Ag		$(4k)$[a]		$(4h)$[a]

[a] The z parameter of these positions is $z \sim \frac{1}{4}$.

appendixes instead of regular sections. Therefore, the `\appendix` command should be used only once—to setup the section commands to act as appendixes. Thereafter normal section commands are used. The heading for a section can be left empty. For example,

```
\appendix
\section{}
```

will produce an appendix heading that says "APPENDIX A" and

```
\appendix
\section{Background}
```

will produce an appendix heading that says "APPENDIX A: BACKGROUND" (note that the colon is set automatically).

If there is only one appendix, then the letter "A" should not appear. This is suppressed by using the star version of the appendix command (`\appendix*` in the place of `\appendix`).

TABLE III. Numbers in columns Three–Five are aligned with the "d" column specifier (requires the `dcolumn` package). Non-numeric entries (those entries without a ".") in a "d" column are aligned on the decimal point. Use the "D" specifier for more complex layouts.

One	Two	Three	Four	Five
one	two	three	four	five
He	2	2.77234	45672.	0.69
C[a]	C[b]	12537.64	37.66345	86.37

[a] Some tables require footnotes.
[b] Some tables need more than one footnote.

Appendix B: A little more on appendixes

Observe that this appendix was started by using

```
\section{A little more on appendixes}
```

Note the equation number in an appendix:

$$E = mc^2. \tag{B1}$$

TABLE IV. A table with numerous columns that still fits into a single column. Here, several entries share the same footnote. Inspect the LaTeX input for this table to see exactly how it is done.

	r_c (Å)	r_0 (Å)	κr_0		r_c (Å)	r_0 (Å)	κr_0
Cu	0.800	14.10	2.550	Sn[a]	0.680	1.870	3.700
Ag	0.990	15.90	2.710	Pb[b]	0.450	1.930	3.760
Au	1.150	15.90	2.710	Ca[c]	0.750	2.170	3.560
Mg	0.490	17.60	3.200	Sr[d]	0.900	2.370	3.720
Zn	0.300	15.20	2.970	Li[b]	0.380	1.730	2.830
Cd	0.530	17.10	3.160	Na[e]	0.760	2.110	3.120
Hg	0.550	17.80	3.220	K[e]	1.120	2.620	3.480
Al	0.230	15.80	3.240	Rb[c]	1.330	2.800	3.590
Ga	0.310	16.70	3.330	Cs[d]	1.420	3.030	3.740
In	0.460	18.40	3.500	Ba[e]	0.960	2.460	3.780
Tl	0.480	18.90	3.550				

[a] Here's the first, from Ref. **?** .
[b] Here's the second.
[c] Here's the third.
[d] Here's the fourth.
[e] And etc.

1. A subsection in an appendix

You can use a subsection or subsubsection in an appendix. Note the numbering: we are now in Appendix B 1.

Note the equation numbers in this appendix, produced with the subequations environment:

$$E = mc, \tag{B2a}$$

$$E = mc^2, \tag{B2b}$$

$$E \gtrsim mc^3. \tag{B2c}$$

They turn out to be Eqs. (B2a), (B2b), and (B2c).

[1] A. Traulsen, J. C. Claussen, and C. Hauert, Phys. Rev. Lett. **95**, 238701 (2005).

[2] Automatically placing footnotes into the bibliography requires using BibTeX to compile the bibliography.