

ΔH

i - rock, j - paper, k - scissors, N - i - j - k + players

$H = -x(1-x)$

$\Delta H = \Delta H(t+1) - \Delta H(t)$,

$\Delta H = -x_{t+1}(1-x_{t+1}) - (-x_t(1-x_t))$

$\Delta H = -x_{t+1}(1-x_{t+1}) + x_t(1-x_t)$

$\Delta H = x_t(1-x_t) - x_{t+1}(1-x_{t+1})$

Rough equation:

$$\langle \Delta H \rangle = \sum_{i,j,k} \left(\Delta H_s - \Delta H_{s'} \right) T^{s \rightarrow s'}$$

$$\begin{aligned} \langle \Delta H \rangle = \frac{1}{N^6} \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N & \left[(N-i-j-k)(1-N+i+j+k)(T^{R+} + T^{P+} + T^{S+} + T^{+R} + T^{+P} + T^{+S}) \right. \\ & - (N-i-j-k+1)(-N+i+j+k)T^{R+} \\ & - (N-i-j-k+1)(-N+i+j+k)T^{P+} \\ & - (N-i-j-k-1)(2-N+i+j+k)T^{+R} \\ & - (N-i-j-k-1)(2-N+i+j+k)T^{+P} \\ & \left. - (N-i-j-k-1)(2-N+i+j+k)T^{+S} \right] \end{aligned} \quad (1)$$

$$q = 1 - x - y - z, p = N - i - j - k$$

$$\begin{aligned} \langle \Delta H \rangle = \frac{1}{N^6} \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N & \left[p(1-p)(T^{R+} + T^{P+} + T^{S+} + T^{+R} + T^{+P} + T^{+S}) \right. \\ & - (p+1)(-p)(T^{R+} + T^{P+} + T^{S+}) \\ & \left. - (p-1)(2-p)(T^{+R} + T^{+P} + T^{+S}) \right] \end{aligned} \quad (2)$$

$$\begin{aligned} \langle \Delta H \rangle = \text{scaling?} \int_0^1 dx \int_0^1 dy \int_0^1 dz & \left[q(1-q)(T^{R+} + T^{P+} + T^{S+} + T^{+R} + T^{+P} + T^{+S}) \right. \\ & - (q + \frac{1}{N})(1-q - \frac{1}{N})(T^{R+} + T^{P+} + T^{S+}) - (q - \frac{1}{N})(1-q + \frac{1}{N})(T^{+R} + T^{+P} + T^{+S}) \left. \right] \end{aligned} \quad (3)$$

Numerical integration in python code ./augRps.py, shows change of sign as expected.