Monotonic Functions

Let $y=f(x)$ be a differentiable function on an interval (a,b). If for any two points $x1,x2 \in (a,b)$ such that $x1<x2$, there holds the inequality $f(x1) \le f(x2)$, the function is called increasing (or non-decreasing) in this interval.

If this inequality is strict, i.e. $f(x1)<f(x2)$, then the function $y=f(x)$ is said to be strictly increasing on the interval (a,b).

Theorem

In order for the function $y=f(x)$ to be increasing on the interval (a,b), it is necessary and sufficient that the **first derivative of the function be non-negative everywhere in this interval:**

$f'(x) \ge 0 \; \forall x \in (a,b)$.

Define:

If a function $f(x)$ is differentiable on the interval (a,b) and belongs to one of the four considered types (i.e. it is increasing, strictly increasing, decreasing, or strictly decreasing), this function is called monotonic on this interval.