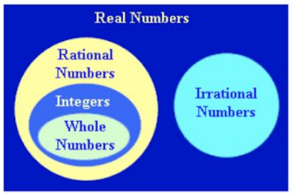
Rational and Irrational Number

Rational Numbers



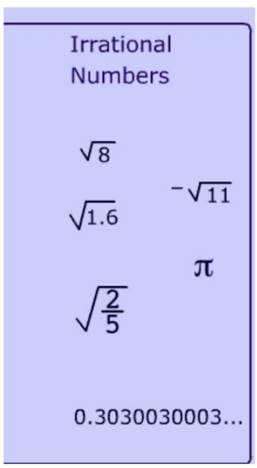
The number can be in different form. Some numbers can be in a form of fraction, ratio, root and with the decimal. If the number is in the form of $\frac{p}{q}$ (fraction) ,of two integer p and q where numerator p and q≠0 are called rational numbers.

$5\frac{2}{3}, \frac{7}{4}, \frac{3}{4}, \frac{3}{5}$ etc are the examples of rational numbers.

Rational number can be:

- All natural number
- All whole number
- All integer
- All fraction

Irrational Numbers



Example for Irrational Numbers

Numbers which cannot be expressed in a ratio (as a fraction of integer) or it can be expressed in decimal form is known as irrational numbers. It can neither be terminated nor repeated.

For example,

$\sqrt{7}$ = 2.64575131............

$\sqrt{5}$ = 2.23620679....... etc are irrational numbers.

$\sqrt{2}$,$\sqrt{3}$,$\sqrt{5}$,$\sqrt{6}$,$\sqrt{7}$, etc. are the examples of irrational number where the numbers are a non-terminating and a non-repeating number.

Some Results on Irrational Numbers

1. If we made an irrational number negative then it is always an irrational number.
   For example, $-\sqrt{5}$

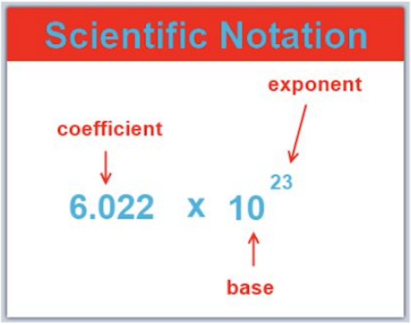
2. If we add a rational number and an irrational number then a result is always an irrational number.
   For example, $2 + \sqrt{3}$ is irrational.

3. If we multiply a non-zero rational number with an irrational number then it is always an irrational number.
   For example, $5\sqrt{3}$ is an irrational number.

4. The sum of two irrational number is not always an irrational number.
   For example, $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$, which is irrational.

5. The product of two irrational number is not always an irrational number.
   For example, $(2 + \sqrt{3}) \times (2 - \sqrt{3}) = 4 - 3 = 1$, which is rational.

Scientific Notation



Scientific Notation is used to handle very large or very small numbers. Scientists have developed a shorter method to express very large numbers. It is written as the product of a number (integer and decimal) and a power of 10.

Here are some examples of scientific notation.

$10{,}000 = 1\times10^{4}$	$34{,}567 = 3.4567\times 10^{4}$
$1000 = 1\times10^{3}$	$495 = 4.95 \times10^{2}$
$10 = 1\times10^{1}$	$98 = 9.8 \times10^{1}$(not usually done)
$\frac{1}{10}=1\times10^{-1}$	$0.23 = 2.3 \times10^{-1}$(not usually done)
$\frac{1}{100}= 1\times10^{-2}$	$0.026 = 2.6\times10^{-2}$
$\frac{1}{1000} = 1\times10^{-3}$	$0.0064 =6.4\times10^{-3}$
$\frac{1}{10{,}000} = 1\times10^{-4}$	$0.00088 = 8.8\times10^{-4}$

Example:

0.000457

$= \frac{0.000457\times1000000}{1000000}$

$=\frac{457}{10^{6}}$

$=457\times 10^{-6}$

$=4.57\times10^{2}\times10^{-6}$

$= 4.57\times10^{-4}$

Significant Figures

Numbers are often rounded to avoid reporting insignificant figures. Significant figures are often used in connecting with rounding.

Rounding 15.543 or 4.756 to 1 decimal place (d.p) seems sensible. The rounded figure is very close to an actual value.

15.543 = 15.5 (1 d.p)

4.756 = 4.8 (1 d.p)

But what happens if you round a very small number to 1 d.p?

0.00789 = 0.0 (1 d.p)

0.00456 = 0.0 (1 d.p)

This is not a useful answer. Another way to find an approximate answer with very small numbers is to use significant figures.

Counting significant figures

Rules for Counting Sig Figs

- Nonzero integers always count as significant figures
  - 3456 has 4 sig figs.
- Leading zeros are never significant
  - 0.000757 has 3 sig figs
- Captive zeros always count assignificant figures
  - 16.07 has 4 sig figs
- Trailing zeros are significant only if the number contains a decimal point.
  - 9.300 has 4 sig figs

Rules for significant counting

Significant figures start at the first non-zero number, so ignore the zeros at the front, but not the ones in between. Look at the following examples:

0.0067 (Here, 6 is the first significant figure and 7 is the second significant figure)

0.0508 From the first significant figure onwards, all zeros are included. It's only the zeros at the beginning that don't count. Here, 5 is the first significant figure, 0-second significant figure and 8 is the third significant figure.

Examples

1. Round 0.0724591 to 3 significant figures, look at the fourth significant figure. It's a 5, so round up.
   0.0724591
   Therefore, 0.0724591 = 0.0725 (3 s.f.)

2. Round 0.2300105 to four significant figures.
   Solution:
   To round to four significant figures, look at the fifth significant figures.It's a 1, so round down.
   0.2300105
   Therefore, 0.2300105 = 0.2300 (4 s.f)
   Even though 0.2300 is the same as 0.23, include the zeros to show that you have rounded to 4 significant figures.

Surds

a. $\sqrt{11}$ is a surd.
b. 4 is not a surd.
c. $\sqrt{9}$ is not a surd. $(\because \sqrt{9} = 3)$
d. $\sqrt{18}$ is a surd.
e. $\sqrt{81}$ is not a surd. $(\because \sqrt{81} = 9)$
f. $\sqrt{24}$ is a surd.

A surd is a square root which cannot be reduced to the whole number. If we can't simplify a number to remove a square root (or cube root) then it is a surd.

The numbers left in the square root form or cube root form etc. is called surds. The reason we leave them as surds is because in the decimal form. They would go on forever and this is a clumsy way of writing them.

Example: $\sqrt{3}$ (square root of 3) can't be simplified further.

Addition and Subtraction of Surds

Adding and subtracting surds are simple- however we need the numbers being square rooted (or cube rooted) to be the same.

$5\sqrt{8} + 2\sqrt{8} = 10\sqrt{8}$

$6\sqrt{7} - 3\sqrt{7} = 3\sqrt{7}$

However, if the number in the square root sign isn't prime, we might be able to split it up in order to simplify an expression. For example :

$\sqrt{12} + \sqrt{27} = \sqrt{4 \times 3} + \sqrt{9 \times 3}$

$= \sqrt{4 \times 3} + \sqrt{9 \times 3}$

$= 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

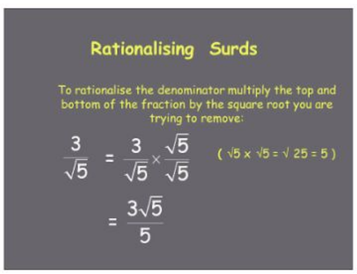
$= 5\sqrt{3}$

Rationalising the Denominator

It is untidy to have to have the fraction which has the surd denominator. This can be tidied up by multiplying the top and bottom of the fraction by the particular expression. This is known as rationalising the denominator. For example: $\frac{1}{\sqrt{2}}$ has an irrational denominator. We multiply the top and bottom by $\sqrt{2}$.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Now the denominator has the rational number.



Example for Rationalising the Denominator