# ALGEBRA

ORDERED PAIR

Simply, an ordered pair is a pair of numbers which are used to locate the point on a coordinate plane. Ordered pairs are also used to show the position on a graph where the horizontal value (x') is the first and vertical value (y') is second. If the ordered pairs have two elements, it is written in the form (x, y) in which x is fixed as the second component. For example,

We can pair off the elements and member diagrammatically as follows: -

Elements	Numbers
x	20
y	30
z	40

We have drawn an arrow from the elements to the numbers. Such figure is called a balloon diagram or arrow diagram. The arrows are used to show the relationship between the ordered pairs.
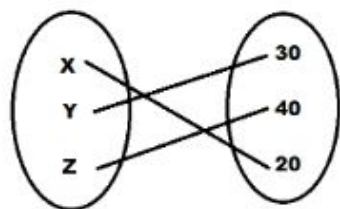
Equity of Ordered Pairs

When the first component and the second component of an ordered pair are correspondingly equal then it is called equality of ordered pairs. For examples,

(x, y) = (x, y)

(-2, -5) = (-2, -5)

(1, 2) = (1, 2)

$(4, 5) = (\frac{8}{2}, \frac{10}{2})$

Cartesian Product

Cartesian product is simply defined as the set of all possible ordered pairs with first element x and second element y. Mathematically, the cartesian product of two sets X and Y is written as,

$X \times Y = \{(x, y) ; x \in X \text{ and } y \in Y\}$

Cartesian Product

The cartesian product can be represented in 3 ways. They are as follows:
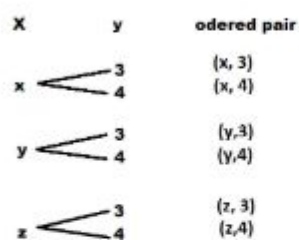
  Tree Diagram
  Mapping Diagram
  Graphical Representation

Tree Diagram

The tree diagram can be represented as follows:

Let, suppose two sets X = (x, y, z) and Y = (3, 4).

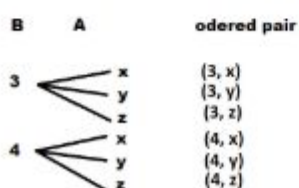Now, Taking x-component from set x and y-component from set y, then we can do all possible pairs as given below:

X	y	odered pair
x	3 / 4	(x, 3) / (x, 4)
y	3 / 4	(y,3) / (y,4)
z	3 / 4	(z, 3) / (z,4)

Tree diagram of order
pair

∴ X × Y = {(x, 3), (x, 4), (y, 3), (y, 4), (z, 3), (z, 4)}

Similarly, we can find the cartesian product B × A as follows:

B × A = {(3, x) (3, y) (3, z) (4, x) (4, y) (4, z)}

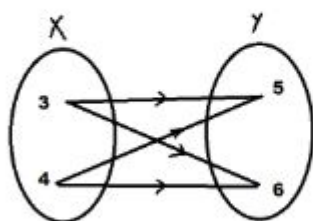| B	A	odered pair
3	x / y / z	(3, x) / (3, y) / (3, z)
4	x / y / z	(4, x) / (4, y) / (4, z)

Mapping Diagram

Mapping diagram can be represented as given below:

X= {3, 4} and Y = {5, 6}

X×Y = {3, 4} × {5,6} = {(3, 5) (3, 6) (4, 5) (4, 6)}
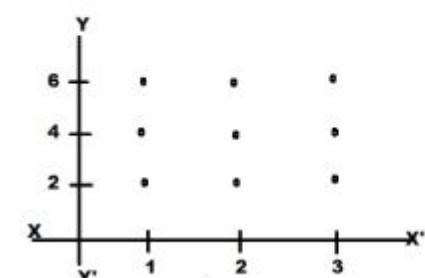

Here, Each arrow represented ordered pairs of A × B.

Graphical Representation

Graphical representation can be represented as below:

Suppose, X = {1, 2, 3} and Y = {2, 4, 6}

Then {X×Y} = {1, 2, 3} × {2, 4, 6} = {(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)}

RELATION

Generally, relation refers to the blood connection between the people. For example, brother and sister, father and son etc. But in mathematics, if A and B are two sets in which A × B is called the cartesian product of A and B, any subset of A × B is called a relation from A to B. The relation from A to B is denoted by R. The relation from A is called a relation on A.

For examples,

Let, A = {3, 5} and B = {4, 6}

Then, A × B = {3, 5} × {4, 6}

= {(3, 4), (3, 6), (5, 4), (5, 6)}

Let, R = {(3, 4), (5, 6)}

So, R is a subset of A × B.

So, R is a relation from A to B.

Representation of a relation

A relation can be represented in 5 ways as:

> By the set of ordered pairs
> By standard description using a rule of formula
> By table
> By graph
> By arrow diagram

By the set of ordered pairs

A relation can be represented in the set of ordered pairs in which there are two components. The first component is called x-component and the second component is called y-component.

For example,

> R = {(Sulav, Shree), (Siddartha, Bhagwat), (Asmita, Riya)}
> R = {(1, 1), (2, 2), (3, 3)}

By standard description using a rule or a formula

A relation can be represented by using the formulas and standard description.

For example,

> Let, A = {3, 4, 5} and B = {3, 4, 5}
> Then,
> A × B = {(3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5), (5, 3), (5, 4), (5, 5)}
> Here, R is a subset of A × B. So, R is a relation from A to B. The relation R can be shown as, R = {(x, y) : x=y}

> Let, A = (5, 6) and B = (7, 8)
> Then,
> A × B = (5, 6) × (7, 8)
> = {(5, 7), (5, 8), (6, 7), (6, 8)}
> Let, R = {(5, 7), (5, 8), (6, 8)}
> Here, R is a subset of A × B. So R is a relation from A to B. The relationship R can be represented as, R = {(x, y) : x<y}

By Table

A relation can also be represented in table which one as follows: -

Let, A = {2, 4} and B = {4, 6}
Then, A × B = {2, 4} × {4, 6}
= {(2, 4), (2, 6), (4, 2), (4, 6)}
Then, R is a relation from A to B. This relation can be represented as,

| x | 2 | 4 |
| y | 4 | 6 |

By Graph
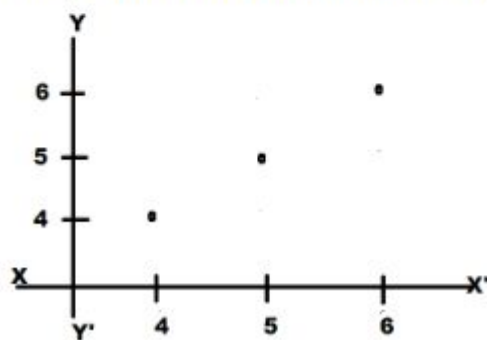
A relation can also be shown in graphs by the use of graph paper. For example,

Let, A = {4, 5, 6} and {4, 5, 6}
Then, A × B = {4, 5, 6} × {4, 5, 6}
= {(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}
So, R = {(4, 4), (5, 5), (6, 6)}
Here, R is a relation from A to B which is represented as,

Example for graph

By Arrows Diagram
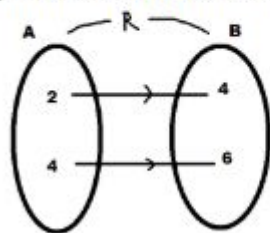
Let, A = {2, 4} and B = {4, 6}
Then A × B = {2, 4}×{4, 6}
= {(2, 4), (2, 6), (4, 4), (4, 6)}
Let, R = {(2, 4), (4, 6)}
So, R is a relation from A to B. This relation can be represented as,

Example for arrow diagram

Domain and Range of a Relation

Let R be a relation from A to B. Then, the set of all first elements of the ordered pairs of R are called the domain. For example,
Let, A = (1, 2) and B = (3, 4)
Domain of relation = (1, 3)
Similarly, the set of all the second elements of the ordered pair of R is called the range. For example,
Let, A = (1, 2) and B = (3, 4)
Range of a Relation = (2, 4)

Inverse relation

The interchanging of x-component and y-components of each pair of the relation of R is denoted by R^{-1}.
For example,
Let, A = {1, 2} and B = {3, 4}
Here, A × B = {1, 2} × {3, 4} = {(1, 3) (1, 4) (2, 3) (2, 4)}
Thus, R^{-1}= {(3, 1), (4, 1), (3, 2), (4, 2)}