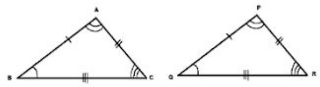
Congruence and Similarities

Congruent Triangles

The triangles having same size and shape are called congruent triangles. Two triangles are congruent when the three sides and three angles of one triangle have the measurements as three sides and three angles of another triangle. The symbol for congruent is ≅.

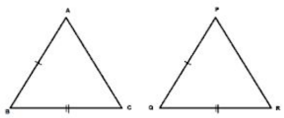
In the following figure, ΔABC and ΔPQR are congruent. We denote this as ΔABC ≅ ΔPQR.



Postulate and Theorems for Congruent Triangles

Postulate (SAS)

If two sides and the angle between them in one triangle are congruent to the corresponding parts in another triangle, then the triangles are congruent.



In the given figure,

AB ≅ PQ Sides (S)

∠B ≅ ∠Q Angle (A)

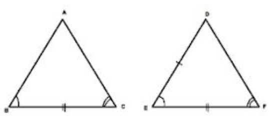
BC ≅ QR Side (S)

Therefore, ΔABC ≅ ΔPQR

Theorem (ASA)

A unique triangle is formed by two angles and the included side.

Therefore, if two angles and the included side of one triangle are congruent to two angles and the included side of the another triangle, then the triangles are congruent.



In the figure,

∠B ≅ ∠E Angle (A)

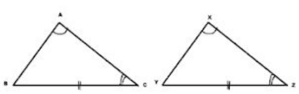
BC ≅ EF Side (S)

∠C ≅ ∠F Angle (A)

Therefore, ΔABC ≅ ΔDEF

Theorem (AAS)

A unique triangle is formed by two angles and non-included side. Therefore, if two angles and the side opposite to one of them in a triangle are congruent to the corresponding parts in another triangle, then the triangles are congruent.



In the figure,

∠A ≅ ∠X Angle (A)

∠C ≅ ∠Z Angle (A)

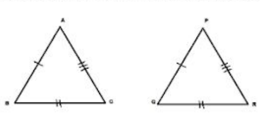
BC ≅ YZ Side (S)

Therefore, ΔABC ≅ ΔXYZ

Theorem (SSS)

A unique triangle is formed by specifying three sides of a triangle, where the longest side (if there is one) is less than the sum of the two shorter sides.

Therefore, if their sides of a triangle are congruent to three sides of another triangle, then the triangles are congruent.



In the figure

AB ≅ PQ Sides (S)

BC ≅ QR Sides (S)

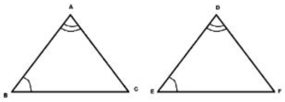
CA ≅ RP Sides (S)

Therefore, ΔABC ≅ ΔPQR

Similar Triangles

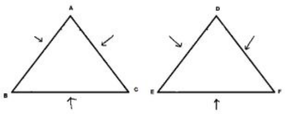
Methods of providing triangles similar

1. If the corresponding sides of a triangle are proportion to another triangle then the triangles are similar.
   Example

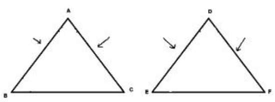
   If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, Then $\Delta ABC \sim \Delta DEF$

2. If the corresponding angle of a triangle is congruent to another triangle then, the triangles are similar.
   Example

   If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ then $\Delta ABC \sim \Delta DEF$

3. Conversing first and the second method we can prove triangle similar as their sides being proportional and angles congruent.
   Example

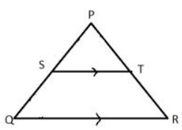
   

   $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$ then $\Delta ABC \sim \Delta DEF$

In case of overlapping triangles

When the lines are parallel in a triangle, then they intersect each other which divides the sides of a triangle proportionally.

Verification:

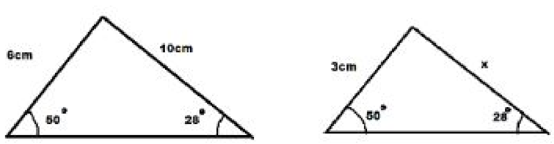


In ΔPQR and ΔSPT

Statements	Reasons
ST∕∕QR	Given
$\angle PST \cong \angle QSR$	Corresponding angles
$\Delta PQR \cong \Delta SPT$	Common Angle P
$\frac{PS}{SQ} = \frac{PT}{TR}$	ST∕∕QR

Example

Given the following triangles, find the length of x.



Solution:

The triangles are similar by AA rule.So, the ratio of lengths are equal.

$\frac{6}{3} = \frac{10}{x}$

or, 6x = 30

or, x = $\frac{30}{6}$

$\therefore$ x = 5 cm