

# MATRIX

A matrix is a collection of numbers arranged into a fixed number of rows and columns. The rows are called horizontal lines and columns are called vertical lines. Each number of a matrix is called element.

For example:

$A = (4)$

$B = \begin{pmatrix} 2 & 0 & 1 \\ 4 & 5 & 7 \end{pmatrix}$

$C = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$

## Order or Size of a Matrix

The number of rows and columns that a matrix has is called order of matrix. If matrix contains 'm' rows and 'n' columns, then it is an order of m by n and it is written as  $m \times n$ .

For example,

- 1.  $\begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$  is a matrix of order  $(2 \times 2)$
- 2.  $\begin{pmatrix} 4 & 3 & 8 \\ 2 & 6 & 9 \end{pmatrix}$  is a matrix of order  $(2 \times 3)$

Hence, the order or the size of a matrix is given by the number of rows followed by the number of columns.

## Notation of Matrix

The matrix which is generally denoted by the capital letter such as A, B, C, ..... etc. By the use of two suffixes, the elements of the matrix are denoted by the corresponding small letter along with two suffixes. 'i' is the first suffix and 'j' is the second suffix. The suffix 'i' represents the number of row and 'j' represent the number of a column.

Let  $A = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & 4 \\ 1 & 7 & 3 \end{pmatrix}$

In double suffix notation, the matrix A is denoted by  $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

So,

$a_{11} = 1, a_{12} = 2, a_{13} = 6$   
 $a_{21} = 4, a_{22} = 8, a_{23} = 4$   
 $a_{31} = 1, a_{32} = 7, a_{33} = 3$

## Equal Matrix

Two Matrix are equal, if all three of the following conditions are met:

- Each matrix has the same number of rows.
- Each matrix has the same number of columns
- Corresponding elements of each matrix are equal.

For example,  $A = \begin{pmatrix} 2 & 4 \\ 5 & 6 \end{pmatrix}$  and  $B = \begin{pmatrix} 2 & 6 \\ 5 & 4 \end{pmatrix}$

## Types of Matrix

TYPES OF MATRICES		
TYPE OF MATRIX	DEFINITION	EXAMPLE
ROW	That matrix has a single row with order is $(1 \times n)$ .	$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
COLUMN	That matrix has a single column, with its order is $(n \times 1)$ .	$B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
RECTANGULAR	That matrix that has different number of rows and columns and its order is $(m \times n)$ .	$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$
TRANSPOSE	Given a matrix $A$ , is called the transpose of the matrix $A$ is denoted by $A^T$ or $A'$ or $A^T$ or $A'$ . In symbols for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$	$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$
DIAGONAL	The square of a given matrix is the sum of square of each element in its square. The square of $A$ is $A^2$ .	$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
INVERSE	That matrix has an equal number of rows and columns, $n \times n$ , which has the inverse of $A$ is denoted by $A^{-1}$ . In symbols for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$	$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $A^{-1} = \frac{1}{4-6} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

## Types of Matrix

There are six types of matrices. They are as follow:

1. Row Matrix
2. Column Matrix
3. Square Matrix
4. Rectangular Matrix
5. Zero/Null Matrix
6. Identity/Unit Matrix

**1. Row Matrix:** If the matrix has only one row then it is called row matrix. For example,  $(2 \ 7 \ 9)$  is a row matrix of order  $(1 \times 3)$ .

**2. Column Matrix:** If the matrix has only one column then it is called column matrix. For example,  $\begin{pmatrix} 1 \\ 4 \\ 8 \end{pmatrix}$  is a column matrix of order  $(3 \times 1)$ .

**3. Square Matrix:** If the number of rows and the number of columns are same, then it is called the square matrix. For example,  $\begin{pmatrix} 3 & 7 \\ 4 & 8 \end{pmatrix}$  is a square matrix of order  $(2 \times 2)$ .

**4. Rectangular Matrix:** A matrix whose number of rows and columns are not same, which is not a square matrix is a rectangular matrix. For example,  $\begin{pmatrix} 3 & 7 & 9 \\ 2 & 0 & 8 \end{pmatrix}$  is a rectangular matrix of order  $(2 \times 3)$ .

**5. Zero or Null Matrix:** If the every elements are zero or null then it is called zero or null matrix. For example,  $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ ,  $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$  are all zero matrices of order  $(2 \times 2)$  and  $(2 \times 3)$  respectively.

**6. Identity or Unit Matrix:** If each of it diagonal elements is 1 and all other elements are zero is known as identity matrix. For example,  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  and  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  are identity matrices of order  $(2 \times 2)$  and  $(3 \times 3)$  respectively.

## Addition of Matrices

When A and B are two matrices of the same order, the sum of A and B by adding corresponding elements of A and B.

For example,

$$\text{If } A = \begin{pmatrix} 1 & 4 & 6 \\ 3 & 0 & 9 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 8 & 3 \\ 6 & 5 & 2 \end{pmatrix},$$

$$A + B = \begin{pmatrix} 1+2 & 4+8 & 6+3 \\ 3+6 & 0+5 & 9+2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 12 & 9 \\ 9 & 5 & 11 \end{pmatrix}$$

Subtraction of Matrices

When A and B are two matrices of the same order, the difference between A and B by  $A - B$  is the matrix by subtracting corresponding elements of A and B.

For example,

If  $A = \begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$  and  $B = \begin{pmatrix} 6 & 4 \\ 7 & 5 \end{pmatrix}$ , then

$$A - B = \begin{pmatrix} 4 - 6 & 2 - 4 \\ 5 - 7 & 3 - 5 \end{pmatrix}$$
$$= \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$$

$\begin{pmatrix} 4 & 6 \\ -2 & 0 \end{pmatrix} + \begin{pmatrix} -2 & 6 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 12 \\ -4 & 3 \end{pmatrix}$

To add two matrices, just add the corresponding numbers in each matrix together.

$(-2) + (-2) = -4$        $0 + 3 = 3$

It should now be obvious why the matrices have to be the same size (same order). The matrices above are all 2 by 2 matrices.

$\begin{pmatrix} 4 & 6 \\ -2 & 0 \end{pmatrix} - \begin{pmatrix} -2 & 6 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & -3 \end{pmatrix}$

$(-2) - (-2) = 0$        $0 - 3 = -3$

Subtracting is just the same, although you subtract the numbers in the second matrix from the corresponding numbers in the first.

Multiplication of a Matrix

Multiplication of a Matrix

If 'C' is any number and A is a matrix, then the matrix CA is obtained by multiplying each element of the matrix A by the number C.

For example,

If  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ , then  $4A = \begin{pmatrix} 1 \times 4 & 2 \times 4 \\ 3 \times 4 & 4 \times 4 \end{pmatrix}$

$$= \begin{pmatrix} 4 & 8 \\ 12 & 16 \end{pmatrix}$$

Transpose of a Matrix

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$       Original matrix

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T \Rightarrow \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

Transpose of a Matrix

The new matrix which is obtained from a given matrix A by interchanging its rows and columns is called the transpose of A or transpose matrix. It is denoted by  $A'$  or  $A^T$ . If the order of A is  $m \times n$ , then the order of A will be  $n \times m$ .

If  $A = \begin{pmatrix} 4 & 3 & 2 \\ 1 & 0 & 5 \end{pmatrix}$ , then  $A' = \begin{pmatrix} 4 & 1 \\ 3 & 0 \\ 2 & 5 \end{pmatrix}$

Here, A is of order  $(2 \times 3)$  and  $A'$  is of order  $(3 \times 2)$ .

## Determinant of a Matrix

Handwritten calculation of the determinant of a 3x3 matrix  $M$  on lined paper. The matrix is  $M = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$ . The calculation shows  $\det(M) = 1(0-24) - 2(0-20) + 3(0-5) = 1$ .

## Determinant of a Matrix

Suppose,  $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$  be a square matrix of order  $(2 \times 2)$ . The determinant of matrix  $A$  is denoted by  $|A|$  and is defined as,

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= a_{11}a_{22} - a_{21}a_{12}$$

For example,

$$\text{If } A = \begin{pmatrix} 5 & 4 \\ 8 & 7 \end{pmatrix}$$

$$= 5 \times 7 - 8 \times 4$$

$$= 35 - 32$$

$$= 3$$

## Singular and Non-Singular Matrix

### Ex 3: Classifying square matrices as singular or nonsingular

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 3 & -2 & 1 \\ 3 & 2 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 & -1 \\ 3 & -2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

Sol:

$$|A| = 0 \quad \Rightarrow \quad A \text{ has no inverse (it is singular)}$$

$$|B| = -12 \neq 0 \quad \Rightarrow \quad B \text{ has inverse (it is nonsingular)}$$

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## Singular and Non-Singular Matrix

If the determinant of  $A$  matrix is zero then it is called asingular matrix.

For example,

$$\text{If } A = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}, \text{ then } |A| = \begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix} = 6 - 6 = 0.$$

If the determinant of  $A$  matrix is not zero then it is called the Non-Singular matrix.

For example,

$$\text{If } A = \begin{pmatrix} 3 & 2 \\ 5 & 6 \end{pmatrix}$$

then,

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & 6 \end{vmatrix}$$

$$= 18 - 10$$

$$= 8$$