Signal and image processing

Homework 3

1. Consider the following signal
   1. Write as a difference of two step signals.

**Ans**

* 1. Use the time shift property to find . Express your final answer as a ratio of two polynomials in .

**Ans**

* 1. Find the region of convergence of .

**Ans**

The region of convergence is .

1. Consider the following finite anti-causal signal where .
   1. Find the Z transform , and express it as a ratio of two polynomials in .

**Ans**

The given anti-casual signal is expressed as

The general finite transform of the finite anti-casual signal is expressed as

For the given signal, the expression of is following as

* 1. What is the region of convergence of ?

**Ans**

The region convergence is the entire of the complex plane because has no pole.

1. Consider the following causal signal
   1. Find the transform , and express it as a ratio of two polynomials in

**Ans**

From the transform table, . The transform of the given signal is expressed as

* 1. What is the region of convergence of ?

**Ans**

We arrange expressed as

By the convergence of the geometric series, the region of convergence is .

1. Consider the following Z transform
   1. Rewrite in terms of negative powers of

**Ans**

* 1. Find

**Ans**

* 1. Verify that is consistent with the initial value theorem.

**Ans**

For the initial value theorem,

We firstly evaluate

Then, we evaluate .

Therefore,

The initial value theorem is verified.

* 1. Verify that is consistent with the final value theorem.

**Ans**

For the final value theorem,

We firstly evaluate .

Then, we evaluate .

Therefore,

The final value theorem is verified.

1. Consider the following transform
   1. Find for using the synthetic division method.

**Ans**

We arrange the given signal,

For the synthetic division, we need to evaluate

For , we neglect term. The is expressed as

By the synthetic division method, x(k) is

* 1. Find using the partial fraction method.

**Ans**

We arrange the given signal,

We know that

By the delayed property of the transform, we obtain

Therefore,

By the scale property of the transform, we know that

Then,

By the delayed property of the transform, we obtain

Therefore,

Finally, we obtain expressed as

.

* 1. Find using the residue method

**Ans**

From the given signal,

By the residue method, is expressed as

For the given signal, there are two simple poles at

.

1. Consider the following transform. Find using the time shift property and the residue method.

**Ans**

We have to find . There is a multipole at .

By the delayed property and linearity,

Finally, we obtain,