1. LTI discrete-time system described with the difference equation

1.1. Find the characteristic polynomial and the input polynomial . Express in factored form.

**Ans**

The characteristic polynomial can be obtained in case of zero input, . The trial solution, , is substituted into the difference equation expressed as

By multiplying the equation with , the equation becomes

It appears that the polynomial on the left side is the characteristic polynomial given as

The can be written in the factored form expressed as

The input polynomial can be obtained by considering zero output, . By substitution, the equation is expressed as

By multiplying the equation with , the equation becomes

It appears that the polynomial on the left side is the input polynomial given as

1.2. Write down the general form of the zero-input response .

**Ans**

The general from of the zero-input response is expressed as

For the given system, the is

1.3. Find the zero-input response when the initial condition is and

**Ans**

By applying the initial conditions, the can be expressed as

The system of equations can be written in the matrix form given as

Therefore, the zero-input is response of the given initial conditions is

1.4. Write down the general form of the zero-state response when the input is .

**Ans**

The casual exponential input can be expressed as

It appears that the amplitude is and the exponential factor is . For the simple root of the characteristic polynomial, the general form of the zero-state response is given as

where ,

For the given system, . is expressed as

1.5. Find the zero-state response using the input in Problem 1.4.

**Ans**

The weighting coefficients () are as the following

We can write the as

1.6. Find the complete response using the initial condition in Problem 1.3 and the input in Problem 1.4.

**Ans**

The complete response system is the sum of and .

2. Compute and plot the impulse response of the LTI discrete-time system described the difference equation

**Ans**

The characteristic polynomial and the input polynomial of the system is expressed as the following equations.

It appears that . For the LTI system, the general impulse response is given as

where , and .

For the given system, the impulse response is expressed as

The coefficients are evaluated by

The impulse response is expressed as

We simplify the impulse response as the following

The impulse response of the given system is shown in the below figure.



\* In MATLAB, h(k) is computed by

h = 2\*(-1).^((k-1)/2).\*(0.5).^k.\*(k>=0).\*(mod(k,2)==1);

3. Echo detection: Let the input signal be the multifrequency chirp described by

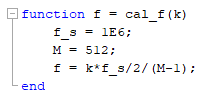
where k = 1, 2, …, M. Suppose the received signal consists of L = 2048 samples and is given by

where the transmitted signal which is zero-extended such that its length becomes samples, and is the atmospheric noise.

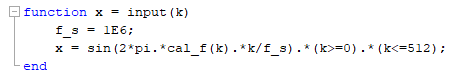
3.1. Generate and plot the M-point input signal using the above formulas.

**Ans**

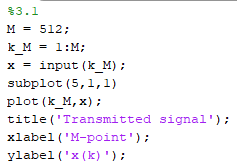
The frequency function, , is defined as a function in the MATLAB code.



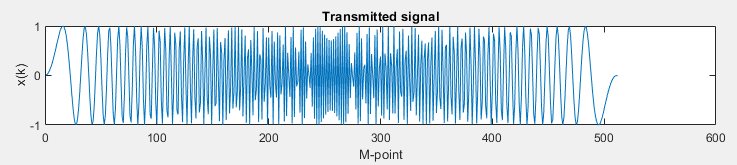
The input signal, is also defined as a function in the MATLAB code. The logical operations are used as a step function, , and the limitation for M-point ().



We generate the input signal by the following code.



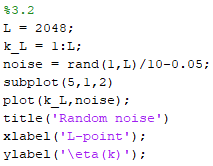
The plot of the transmitted signal is shown as



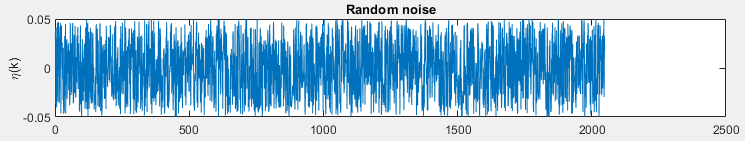
3.2. Generate the L-point noise using the MATLAB command rand such that its value is in the range [-0.05,0.05].

**Ans**

The noise in range [-0.05,0.05] is generated by the following code.



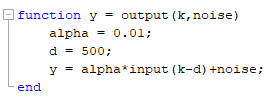
We plot the noise function shown as



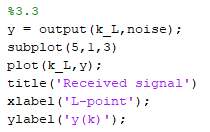
3.3. Generate and plot the received signal with attenuation factor , and delay.

**Ans**

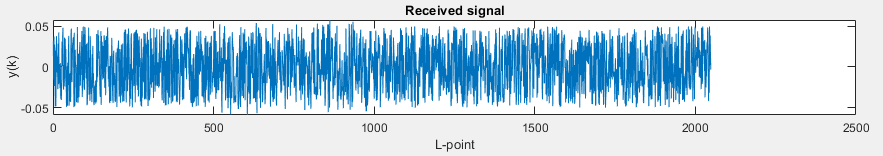
We define the output function as the following code.



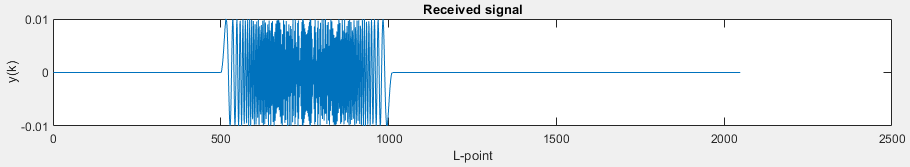
The received signal is generated by the given code.



The plot of the output signal is shown as



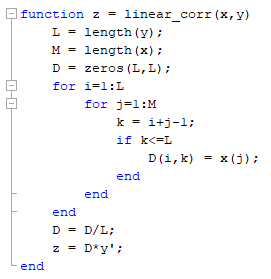
To confirm the delayed signal, we remove the noise shown in the below plot.



3.4. Perform linear cross-correlation of with the input signal and plot the result. Also determine the delay d from the graph.

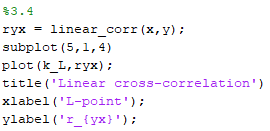
**Ans**

We define the linear cross-correlation as a function in the MATLAB code as the following

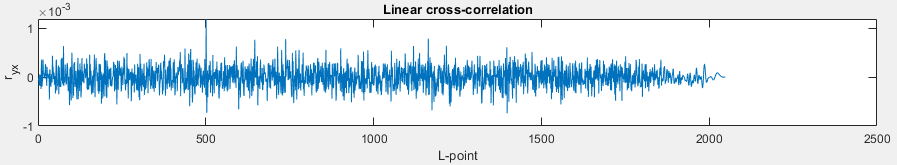


In the function, the cross-correlation matrix, , is constructed. The linear cross-correlation () is evaluated by

The linear cross-correlation of the system is calculated by



The result is shown in the following figure.

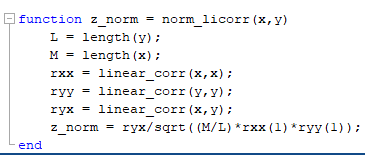


From the above graph, the delayed value () can be determined as the peak which is 501.

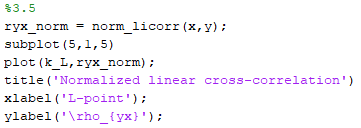
3.5. Perform normalized linear cross-correlation of with the input signal and plot the result.

**Ans**

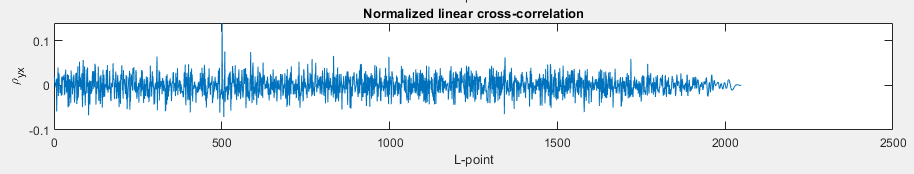
We define the normalized linear cross-correlation (NLCC) as a function in the MATLAB code.



LNCC is evaluated by the code.



The result of NLCC is shown in the following figure.



It appears that the peak of is about 0.1. The correlation between transmitted signal and received signal is not strong. The reason is that the amplitude of the received signal is 0.01 of the transmitted signal.