

PIPG Module

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1 Template Optimal Control Problem

$$\text{minimize} \quad \sum_{t=1}^N x_t^\top Q_t x_t + q_t^\top x_t + u_t^\top R_t u_t + r_t^\top u_t, \quad (1a)$$

$$\text{subject to} \quad x_{t+1} = A_t x_t + B_t^- u_t + B_{t+1}^+ u_{t+1} + c_t, \quad t = 1, \dots, N-1, \quad (1b)$$

$$x_t \in \mathbb{D}_t^x, \quad u_t \in \mathbb{D}_t^u, \quad t = 1, \dots, N. \quad (1c)$$

To track known state reference x_t^{ref} and/or a control reference u_t^{ref} , choose $q_t = -2x_t^{\text{ref}}$ and $r_t = -2u_t^{\text{ref}}$. The boundary conditions on states and control are accounted in (1c).

2 Conic Optimization Problem

$$\text{minimize} \quad \frac{1}{2} z^\top P z + p^\top z \quad (2a)$$

$$\text{subject to} \quad H z - g \in \mathbb{K}, \quad (2b)$$

$$z \in \mathbb{D}. \quad (2c)$$

3 Template Extension to General SOCPs

$$\begin{aligned} &\text{minimize} \quad \sum_{t=1}^N x_t^\top Q_t x_t + q_t^\top x_t + u_t^\top R_t u_t + r_t^\top u_t, \\ &\text{subject to} \quad \left. \begin{aligned} x_{t+1} &= A_t x_t + B_t^- u_t + B_{t+1}^+ u_{t+1} + c_t, & t &= 1, \dots, N-1, \\ x_t &\in \mathbb{D}_t^x, \quad u_t \in \mathbb{D}_t^u, \\ F_t^0 x_t + G_t^0 u_t + h_t^0 &= 0, \\ F_t^1 x_t + G_t^1 u_t + h_t^1 &\leq 0, \\ F_t^2 x_t + G_t^2 u_t + h_t^2 &\preceq_2 0, \end{aligned} \right\} & t &= 1, \dots, N, \end{aligned}$$

where \preceq_2 is the generalized inequality representing a second-order cone.