High-fidelity Spacecraft Dynamics in Cislunar Space

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The spacecraft equations of motion are represented in the Earth-centered inertial frame (ECI) defined by Earth's Mean Equator and Mean Equinox (MEME) at 12:00 Terrestrial Time on 1 January 2000, with the origin at the instantaneous Moon center. This frame is labelled as `J2000` in SPICE.

$$
\begin{aligned}
\dot{r}_{\mathrm{sc}} &= v_{\mathrm{sc}} && \text{(1a)}\\
\dot{v}_{\mathrm{sc}} &= -GM_{\mathrm{M}}\frac{r_{\mathrm{sc}}}{\|r_{\mathrm{sc}}\|_2^3} + GM_{\mathrm{E}}\left(\frac{r_{\mathrm{E}} - r_{\mathrm{sc}}}{\|r_{\mathrm{E}} - r_{\mathrm{sc}}\|_2^3} - \frac{r_{\mathrm{E}}}{\|r_{\mathrm{E}}\|_2^3}\right) + GM_{\mathrm{S}}\left(\frac{r_{\mathrm{S}} - r_{\mathrm{sc}}}{\|r_{\mathrm{S}} - r_{\mathrm{sc}}\|_2^3} - \frac{r_{\mathrm{S}}}{\|r_{\mathrm{S}}\|_2^3}\right)\\
&\quad - \frac{k_{\mathrm{sc}}A_{\mathrm{sc}}S_0r_0^2}{M_{\mathrm{sc}}c}\left(\frac{r_{\mathrm{S}} - r_{\mathrm{sc}}}{\|r_{\mathrm{S}} - r_{\mathrm{sc}}\|_2^3}\right) && \text{(1b)}\\
&\quad + \frac{3}{2}GM_{\mathrm{M}}M_{\mathrm{J2}}R_{\mathrm{M}}^2\frac{r_{\mathrm{sc}}}{\|r_{\mathrm{sc}}\|_2^5}\left(3\sin^2\left(\arccos\left(\frac{r_{\mathrm{E}}^\top \bar{r}_{\mathrm{sc}}}{\|r_{\mathrm{E}}\|_2\|\bar{r}_{\mathrm{sc}}\|_2}\right) + \theta_{\mathrm{eq}}\right) - 1\right),
\end{aligned}
$$

where

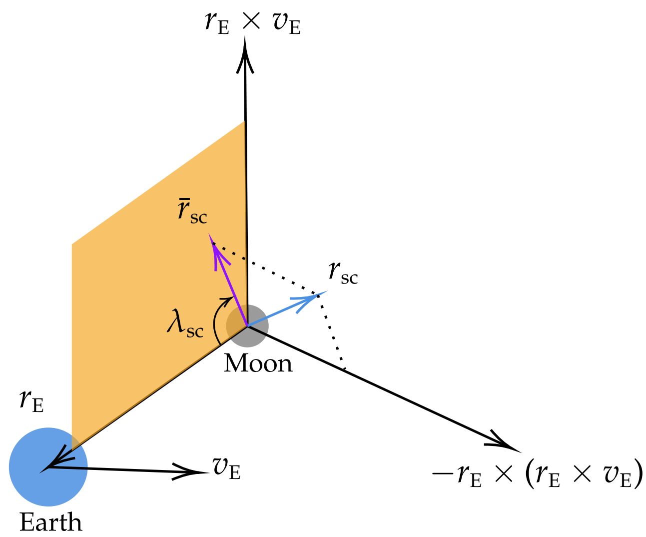
$$
\begin{aligned}
\bar{r}_{\mathrm{sc}} &= r_{\mathrm{sc}} - \frac{r_{\mathrm{sc}}^\top \bar{v}_{\mathrm{E}}}{\|\bar{v}_{\mathrm{E}}\|_2^2}\bar{v}_{\mathrm{E}}, && \text{(2)}\\
\bar{v}_{\mathrm{E}} &= -r_{\mathrm{E}} \times (r_{\mathrm{E}} \times v_{\mathrm{E}}) = -r_{\mathrm{E}}^\top(v_{\mathrm{E}} \times r_{\mathrm{E}}). && \text{(3)}
\end{aligned}
$$

Note that $r_{\mathrm{E}}, \bar{v}_{\mathrm{E}}$ and $r_{\mathrm{E}} \times v_{\mathrm{E}}$ form a right-handed set of orthogonal vectors. The projection of spacecraft position vector onto the plane formed by r_{E} and $r_{\mathrm{E}} \times v_{\mathrm{E}}$ is denoted by $\bar{r}_{\mathrm{sc}}$. The angle between $\bar{r}_{\mathrm{sc}}$ and r_{E}, denoted by λ_{sc}, quantifies the Moon latitude closest to the spacecraft.

The cannonball model of solar radiation pressure assumed here represents the spacecraft as a sphere. As a result, the cross-sectional area A_{sc} experiencing solar radiation is independent of spacecraft orientation.

Accessing Ephemeris via SPICE

NAIF SPICE toolkit for MATLAB, called `mice`, is necessary for querying ephemeris state of celestial bodies in specific coordinate frames. Download `mice` from here and include `mice\lib` and `mice\src\mice` to MATLAB path.



r_{sc}	Position of spacecraft with respect to Moon
v_{sc}	Velocity of spacecraft with respect to Moon
r_E	Position of Earth with respect to Moon
v_E	Velocity of Earth with respect to Moon
r_S	Position of Sun with respect to Moon
k_{sc}	Reflectivity of spacecraft body
r_0	1 AU
A_{sc}	Cross-sectional area of spacecraft
S_0	Solar flux at distance r_0 from Sun
c	Speed of light in vacuum
G	Universal gravitational constant
M_{sc}	Mass of spacecraft
M_E	Mass of Earth
M_M	Mass of Moon
M_S	Mass of Sun
M_{J2}	J2 zonal harmonic coefficient for Moon, 2.024×10^{-4}
R_M	Radius of Moon, 1737.1 km
θ_{eq}	Equitorial inclination of Moon, 6.68°

Data

- `pck00010.tpc` – Orientation and size/shape data for natural bodies.
- `naif0011.tls.pc` – Leap second kernel.
- `de421.bsp` – Position of planets and Moon between 1900 and 2050. See this document for more details.