High-fidelity Spacecraft Dynamics in Cislunar Space

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Equations of motion in the Mean Equator Mean Equinox (MEME) J2000 inertial frame with the origin at the instantaneous center of the Moon.

$$\dot{r}_{sc} = v_{sc} \tag{1a}$$

$$\begin{aligned}\dot{v}_{sc} = & -GM_M \frac{r_{sc}}{\|r_{sc}\|_2^3} + GM_E\left(\frac{r_E - r_{sc}}{\|r_E - r_{sc}\|_2^3} - \frac{r_E}{\|r_E\|_2^3}\right) + GM_S\left(\frac{r_S - r_{sc}}{\|r_S - r_{sc}\|_2^3} - \frac{r_S}{\|r_S\|_2^3}\right) \\ & - \frac{k_{sc}A_{sc}S_0r_0^2}{M_{sc}c}\left(\frac{r_S - r_{sc}}{\|r_S - r_{sc}\|_2^3}\right) \\ & + \frac{3}{2}GM_M M_{J2}R_M^2 \frac{r_{sc}}{\|r_{sc}\|_2^5}\left(3\sin^2\left(\arccos\left(\frac{r_E^\top \bar{r}_{sc}}{\|r_E\|_2\|\bar{r}_{sc}\|_2}\right) + \theta_{eq}\right) - 1\right),\end{aligned} \tag{1b}$$

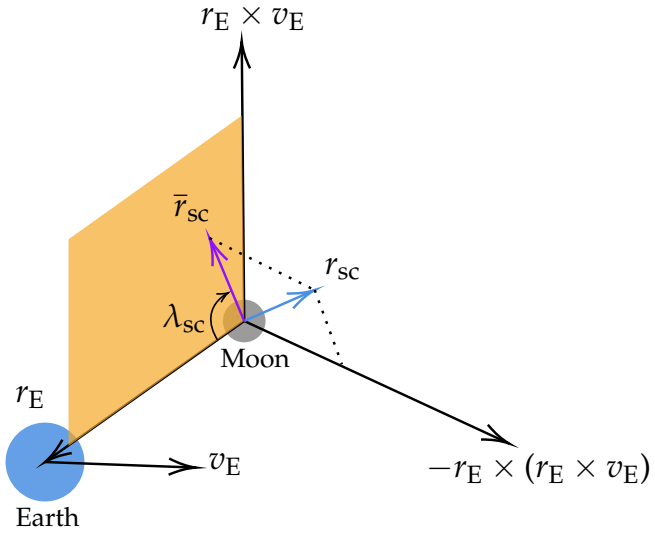
where

$$\bar{r}_{sc} = r_{sc} - \frac{r_{sc}^\top \bar{v}_E}{\|\bar{v}_E\|_2^2}\bar{v}_E, \tag{2}$$

$$\bar{v}_E = -r_E \times (r_E \times v_E) = -r_E^\top (v_E \times r_E). \tag{3}$$

Note that $r_E, \bar{v}_E$ and $r_E \times v_E$ form a right-handed set of orthogonal vectors. The projection of spacecraft position vector onto the plane formed by r_E and $r_E \times v_E$ is denoted by $\bar{r}_{sc}$. The angle between $\bar{r}_{sc}$ and r_E, denoted by λ_{sc}, quantifies the Moon latitude closest to the spacecraft.

The cannonball model of solar radiation pressure assumed here represents the spacecraft as a sphere. As a result, the cross-sectional area A_{sc} experiencing solar radiation is independent of spacecraft orientation.



r_{sc}	Position of spacecraft with respect to Moon
v_{sc}	Velocity of spacecraft with respect to Moon
r_E	Position of Earth with respect to Moon
v_E	Velocity of Earth with respect to Moon
r_S	Position of Sun with respect to Moon
k_{sc}	Reflectivity of spacecraft body
r_0	1 AU
A_{sc}	Cross-sectional area of spacecraft
S_0	Solar flux at distance r_0 from Sun
c	Speed of light in vacuum
G	Universal gravitational constant
M_{sc}	Mass of spacecraft
M_E	Mass of Earth
M_M	Mass of Moon
M_S	Mass of Sun
M_{J2}	J2 zonal harmonic coefficient for Moon, 2.024×10^{-4}
R_M	Radius of Moon, 1737.1 km
θ_{eq}	Equitorial inclination of Moon, 6.68°