High-fidelity Spacecraft Dynamics in Cislunar Space

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Equations of motion in the Mean Equator Mean Equinox (MEME) J2000 inertial frame with the origin at the instantaneous center of the Moon.

$$
\begin{aligned}
\dot{r}_{\mathrm{sc}} &= v_{\mathrm{sc}} \qquad (1a)\\
\dot{v}_{\mathrm{sc}} &= -GM_{\mathrm{M}}\frac{r_{\mathrm{sc}}}{\|r_{\mathrm{sc}}\|_2^3} + GM_{\mathrm{E}}\left(\frac{r_{\mathrm{E}}-r_{\mathrm{sc}}}{\|r_{\mathrm{E}}-r_{\mathrm{sc}}\|_2^3} - \frac{r_{\mathrm{E}}}{\|r_{\mathrm{E}}\|_2^3}\right) + GM_{\mathrm{S}}\left(\frac{r_{\mathrm{S}}-r_{\mathrm{sc}}}{\|r_{\mathrm{S}}-r_{\mathrm{sc}}\|_2^3} - \frac{r_{\mathrm{S}}}{\|r_{\mathrm{S}}\|_2^3}\right)\\
&\quad - \frac{k_{\mathrm{sc}}A_{\mathrm{sc}}S_0r_0^2}{M_{\mathrm{sc}}c}\left(\frac{r_{\mathrm{S}}-r_{\mathrm{sc}}}{\|r_{\mathrm{S}}-r_{\mathrm{sc}}\|_2^3}\right) \qquad (1b)\\
&\quad + \frac{3}{2}GM_{\mathrm{M}}M_{\mathrm{J2}}R_{\mathrm{M}}^2\frac{r_{\mathrm{sc}}}{\|r_{\mathrm{sc}}\|_2^5}\left(3\sin^2\left(\arccos\left(\frac{r_{\mathrm{sc}}^\top r_{\mathrm{E}} - \frac{r_{\mathrm{sc}}^\top v_{\mathrm{E}}}{\|v_{\mathrm{E}}\|_2^2}v_{\mathrm{E}}^\top r_{\mathrm{E}}}{\|r_{\mathrm{E}}\|_2\left\|r_{\mathrm{sc}} - \frac{r_{\mathrm{sc}}^\top v_{\mathrm{E}}}{\|v_{\mathrm{E}}\|_2^2}v_{\mathrm{E}}\right\|_2}\right) + \theta_{\mathrm{eq}}\right) - 1\right)
\end{aligned}
$$

r_{sc}	Position of spacecraft with respect to Moon
v_{sc}	Velocity of spacecraft with respect to Moon
r_{E}	Position of Earth with respect to Moon
v_{E}	Velocity of Earth with respect to Moon
r_{S}	Position of Sun with respect to Moon
k_{sc}	Reflectivity of spacecraft body
r_0	1 AU
A_{sc}	Cross-sectional area of spacecraft
S_0	Solar flux at distance r_0 from Sun
c	Speed of light in vacuum
G	Universal gravitational constant
M_{sc}	Mass of spacecraft
M_{E}	Mass of Earth
M_{M}	Mass of Moon
M_{S}	Mass of Sun
M_{J2}	J2 zonal harmonic coefficient for Moon, 2.024×10^{-4}
R_{M}	Radius of Moon, 1737.1 km
θ_{eq}	Equitorial inclination of Moon, 6.68°

The cannonball model of solar radiation pressure assumed here, represents the spacecraft as a sphere. As a result, the cross-sectional area A_{sc} experiencing solar radiation is independent of spacecraft orientation.