PIPG Module

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1 Template Optimal Control Problem

$$\text{minimize} \quad \sum_{t=1}^{N} x_t^\top Q_t x_t + q_t^\top x_t + u_t^\top R_t u_t + r_t^\top u_t, \tag{1a}$$

$$\text{subject to} \quad x_{t+1} = A_t x_t + B_t^- u_t + B_{t+1}^+ u_{t+1} + c_t, \qquad t = 1, \dots, N-1, \tag{1b}$$

$$x_t \in \mathbb{D}_t^x, \quad u_t \in \mathbb{D}_t^u, \qquad t = 1, \dots, N. \tag{1c}$$

To track known state reference x_t^{ref} and/or a control reference u_t^{ref}, choose $q_t = -2x_t^{\text{ref}}$ and $r_t = -2u_t^{\text{ref}}$. The boundary conditions on states and control are accounted in (1c).

2 Template Extension to General SOCPs

$$\text{minimize} \quad \sum_{t=1}^{N} x_t^\top Q_t x_t + q_t^\top x_t + u_t^\top R_t u_t + r_t^\top u_t,$$

$$\text{subject to} \quad x_{t+1} = A_t x_t + B_t^- u_t + B_{t+1}^+ u_{t+1} + c_t, \qquad t = 1, \dots, N-1,$$

$$\left. \begin{array}{l} x_t \in \mathbb{D}_t^x, \quad u_t \in \mathbb{D}_t^u, \\ F_t^0 x_t + G_t^0 u_t + h_t^0 = 0, \\ F_t^1 x_t + G_t^1 u_t + h_t^1 \le 0, \\ F_t^2 x_t + G_t^2 u_t + h_t^2 \preceq_2 0, \end{array} \right\} \qquad t = 1, \dots, N,$$

where $\preceq_2$ is the generalized inequality representing a second-order cone.