AE 5335 – Assignment 3

Collaboration Policy

This is an "open-book and open-notes" assignment, which means that consulting lecture notes/slides provided by the instructor and/or similar educational material from sources on the Internet is allowed. Using MATLAB® or similar software is allowed unless stated otherwise for specific problems. Discussions about this assignment with other students in the class are allowed and encouraged. However, the submitted assignment must reflect your own independent work. Plagiarism from any source is disallowed.

Statement of Academic Honesty: When submitting this assignment, you will be asked to state:

My signature below affirms that this submission is entirely my own work, and reflects my own understanding of the course content. This submission does not contain material copied or plagiarised from any other source including works of other students. I have read and I understand WPI's Academic Honesty Policy, and my conduct in preparing this submission has been in accordance with this Policy.

Instructions for Assignment Submission

- The submission format is a *single* PDF file, to be uploaded via *Canvas*. Multiple files will not be graded.
- Read WPI's Academic Honesty Policy, and the Student Guide to Academic Integrity.
- All deadlines provided are in Eastern Daylight Time. Late submissions will receive a grade penalty as follows.
  - Late by up to 48 hours after deadline: 10% penalty.
  - Late by 48 - 96 hours after deadline: 20% penalty.
  - Late by more than 96 hours after deadline: 30% penalty.
- The solution to each problem should be presented in the following format:
  - **Method:** e.g., "I did [insert basic outline of method] and obtained the following results." OR "I used the [insert name of command/tool/library] from [MATLAB or similar software]. This command/tool/library [insert brief explanation of what the tool does and/or how it works]."
  - **Results:** e.g. "The following plots/numbers/expressions indicate [what they indicate; insert plots / numbers, expressions etc]."
  - **Discussion:** e.g. "The [results above; details] match expected [behavior/values] because [reasons]." OR "The [results above; details] match expected [behavior/values], but may include minor numerical errors because [reasons]." OR "The [results above; details] are not sensible because [explain why you think the results are not correct]. My method appears to be correct, but I was not able to resolve [issue] despite trying [describe your attempts, including discussions, if any, with the instructor and/or TA]."
- Submissions in the form of software code (e.g., MATLAB® code) without an explanation of the method and discussion of results are not acceptable, and such submissions will be returned without grading. If and when code is included in the submission, it must be included as an appendix, not in the main submission.
- Illegible and/or untidy submissions will not be accepted or graded, and the students will forfeit points unless the work is entirely resubmitted. For these resubmissions, lateness penalties as above will apply.
- To scan handwritten work into a PDF file, use either a desktop scanner or a smartphone-based software application such as CamScanner, Microsoft Lens, or Adobe Scan. Watermarks, if any, left by free versions of these software applications are acceptable.
- Consult this page for examples of acceptable and unacceptable assignment submissions.

Problem 1. (25 points)

For the shortest Dubins path (previously assigned problem) between the following pair of waypoints with minimum radius of turn 150 m, implement a heading angle reference-based tracking algorithm.

(i) Initial: $\boldsymbol{p}_0 = (0,0)\ \mathrm{km},\ \chi_0 = 0°$, final: $\boldsymbol{p}_\mathrm{f} = (1,1)\ \mathrm{km},\ \chi_\mathrm{f} = 0°$.

Demonstrate the performance of this tracking algorithm for the initial condition $\boldsymbol{p}(0) = (0.1, 0.5)\mathrm{km},\ \chi(0) = 90°$. To do so, simulate the aircraft motion modeled by

$$\dot{\boldsymbol{p}} = \begin{bmatrix} V\cos\chi \\ V\sin\chi \end{bmatrix}, \qquad \dot{\chi} = a_\mathrm{h}(\chi_\mathrm{ref} - \chi).$$

where the speed $V = 30$ m/s and a_h is a positive constant.

Plot the trajectory followed by the aircraft for different values of a_h, namely, $a_\mathrm{h} = 0.1$, $a_\mathrm{h} = 1$, and $a_\mathrm{h} = 5$.

Problem 2. (30 points)

Consider a polynomial trajectory $\boldsymbol{p}(t) = (p_x(t), p_y(t), p_z(t))$ of the form

$$p_x(t) = \sum_{k=0}^{N} a_{xk} t^k, \qquad p_y(t) = \sum_{k=0}^{N} a_{yk} t^k, \qquad p_z(t) = \sum_{k=0}^{N} a_{zk} t^k.$$

Here, $N = 4$ and the coefficients are provided in the attached `.mat` file. The parameter t denotes time, and the trajectory is defined over the interval $t \in [0, 60]$ s. In the attached `.mat` file, the coefficients are provided in a 3×5 array, where the first, second, and third rows are coefficients a_{xk}, a_{yk}, a_{zk}, respectively. In each row, coefficients are arranged in descending powers, i.e., a_{x4} is the first row, first column etc.

We would like a quadrotor aircraft to follow this trajectory while always maintaining its yaw angle equal to the heading angle of the trajectory. Using the differential flatness property of the quadrotor dynamical model, find out and plot the following quantities over the entire time interval to enable trajectory tracking.

- Yaw, pitch, and roll angles ψ, θ, ϕ
- Body-axis coordinates $\boldsymbol{\omega}_\mathrm{tb}^\mathrm{b}$ of the angular velocity
- Magnitude F of the total thrust generated by the rotors
- Body-axis coordinates $\boldsymbol{M}^\mathrm{b}$ of the moments generated by the rotors
- Spin rates Ω_i of the four rotors

All relevant characteristics of the quadrotor aircraft ("Hummingbird") are provided in the following article on *Canvas*:

C. Powers, D. Mellinger, and V. Kumar, "Quadrotor Kinematics and Dynamics." In: K. Valavanis, G. Vachtsevanos (eds) *Handbook of Unmanned Aerial Vehicles,* 2015. Springer, Dordrecht.

Problem 3. (10 points)

A MATLAB® simulator of quadrotor aircraft dynamics is attached. Run this simulator by providing rotor spin rate inputs Ω_i, as found in the previous problem.

Demonstrate that the simulated states match those of the desired polynomial trajectory. The initial conditions may be assumed equal to those corresponding to the desired polynomial trajectory at time $t = 0$.

Problem 4. (15 points)

Now we would like a small fixed-wing aircraft to follow a polynomial trajectory similar to that described in Problem 2. The coefficients are provided in the attached `.mat` file. The order N and time interval are the same as in Problem 2. Using the differential flatness property of the fixed-wing particle dynamical model, find out and plot the following quantities over the entire time interval to enable trajectory tracking.

- Heading and climb angles χ, γ
- Magnitudes L, D, T of the lift, drag, and thrust
- Angle of attack α

All relevant characteristics of the fixed-wing aircraft are provided in the textbook by Beard & McLain (Aerosonde UAV in Appendix E).

Problem 5. (10 points)

Prepare a short ($\sim$ 500 words) summary of the following article provided on *Canvas*:

> M. Cutler and J. How, "Analysis and control of a variable-pitch quadrotor for agile flight." in *Journal of Dynamic Systems, Measurement, and Control* vol. 137, no. 10, pp. 101002, July 2015.

Explain what you consider as the main ideas, main findings, and/or weaknesses of the work reported in this article.

Problem 6. (10 points)

Prepare a short ($\sim$ 500 words) summary of the following article provided on *Canvas*:

> C. Richter, A. Bry, and N. Roy, "Polynomial trajectory planning for aggressive quadrotor flight in dense indoor environments." In: Inaba, M., Corke, P. (eds) *Robotics Research.* Springer Tracts in Advanced Robotics, vol 114, 2016. Springer, Cham.

Explain what you consider as the main ideas, main findings, and/or weaknesses of the work reported in this article.