

# Robot Control Programming Assignment 3

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## Part a:

- In order to generate cubic polynomial trajectory for the two joints of the robot, initially the initial and final conditions of the trajectory were defined as mentioned in the assignment.
- A symbolic variable  $t$  was defined.
- Then using the general form of the cubic polynomial trajectory and its derivative, matrices  $A$ ,  $a$  and  $B$  were defined such that  $A*a = B$ .
- $A$  is a  $4 \times 4$  matrix containing elements of trajectory substituted with  $t_0$  and  $t_f$ .
- $a$  is a  $4 \times 2$  matrix consisting all the coefficients of both the trajectories (column 1 consists coefficients for trajectory for joint 1 and column 2 for joint 2 respectively) which are to be found.
- $B$  is a  $4 \times 2$  matrix consisting initial and final parameters of both trajectories.
- Now by calculating  $a = (A^{-1}) B$ , value of all the coefficients are calculated.
- Following are the final equations of the trajectories obtained in the matlab.

$$th1 = (\pi * t^3) / 500 - (3 * \pi * t^2) / 100 + \pi$$

$$th2 = (\pi * t^3) / 1000 - (3 * \pi * t^2) / 200 + \pi / 2$$

- On differentiating these equations with respect to time, equations for angular velocities and angular accelerations can be found as follows.

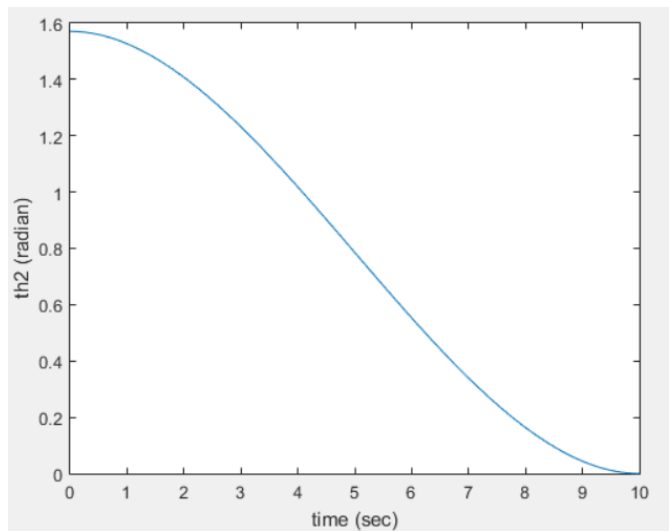
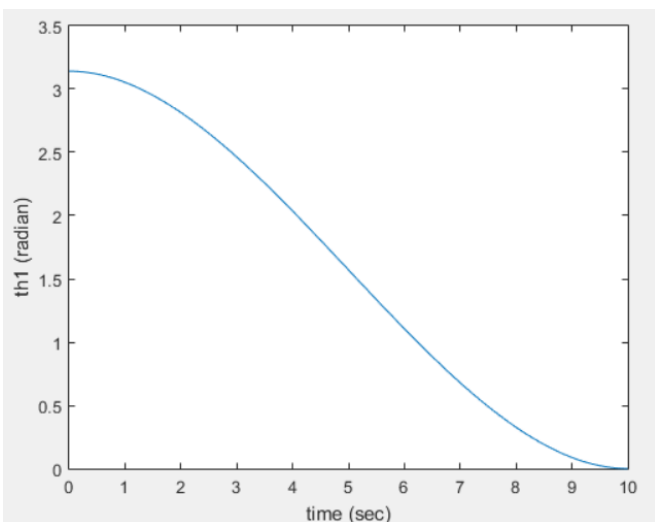
$$dth1 = (3 * \pi * t^2) / 500 - (3 * \pi * t) / 50$$

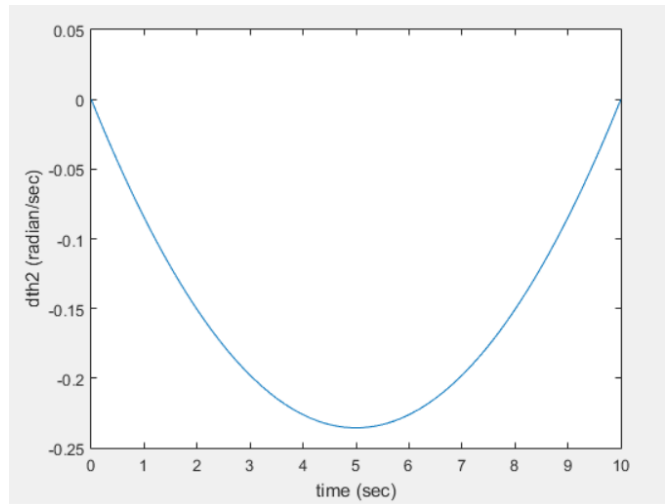
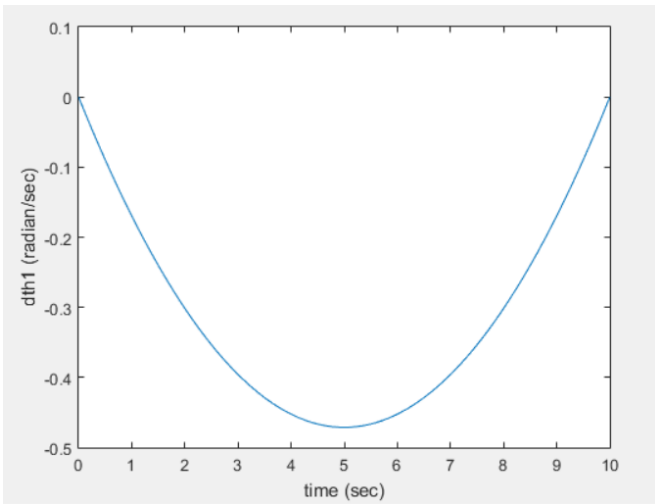
$$dth2 = (3 * \pi * t^2) / 1000 - (3 * \pi * t) / 100$$

$$ddth1 = (3 * \pi * t) / 250 - (3 * \pi) / 50$$

$$ddth2 = (3 * \pi * t) / 500 - (3 * \pi) / 100$$

- Following are the plots of the obtained trajectory.





### Part b:

- Using the dynamic equations derived in previous assignments, matrices  $M(q)$ ,  $C(q, \dot{q})$  and  $g(q)$  are obtained as follows.

$$M = \begin{bmatrix} (m1*d1^2 + m2*d2^2 + 2*m2*cos(th2)*d2*l1 + m2*l1^2 + I1 + I2), & (m2*d2^2 + l1*m2*cos(th2)*d2 + I2); \\ (m2*d2^2 + l1*m2*cos(th2)*d2 + I2), & (m2*d2^2 + I2) \end{bmatrix};$$

$$C = \begin{bmatrix} -(2*d2*dth2*l1*m2*sin(th2)), & -(d2*dth2*l1*m2*sin(th2)); \\ (d2*l1*m2*sin(th2)*dth1), & 0 \end{bmatrix};$$

$$G = \begin{bmatrix} (-sin(th1)*(d1*g*m1 + g*l1*m2) - d2*g*m2*sin(th1 + th2)); \\ (-d2*g*m2*sin(th1 + th2)) \end{bmatrix};$$

$$u = M*\ddot{dq} + C*\dot{dq} + G;$$

$$\text{Where } u = [u1; u2], \dot{dq} = [dth1; dth2], \ddot{dq} = [\ddot{dth1}, \ddot{dth2}]$$

### Part c:

- In order to design a stated feedback linearized controller, initially the feedback control input  $\tau$  was considered as  $\tau = M*v + C*\dot{dq} + G$
- Thus taking  $v = \ddot{dq}$
- Converting this equation into state space, we obtain A and B as follow.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0; \\ 0 & 0 & 0 & 1; \\ 0 & 0 & 0 & 0; \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$B = \begin{bmatrix} 0 & 0; \\ 0 & 0; \\ 1 & 0; \\ 0 & 1 \end{bmatrix};$$

- Now lamda were assigned and using place function, values of elements of matrix k were found.

- Now using defined symbolic representations of states (i.e. matrix of  $\theta_1$ ,  $\theta_2$ ,  $\dot{\theta}_1$  and  $\dot{\theta}_2$ ), desired states (i.e. matrix of  $\text{des\_}\theta_1$ ,  $\text{des\_}\theta_2$ ,  $\text{des\_}\dot{\theta}_1$  and  $\text{des\_}\dot{\theta}_2$ ) and desired control input  $v_d$ ,  $v$  can be calculated using following command.

$$v = -k*(\text{states}-\text{des\_states})+v_d;$$

- The desired states and  $v_d$  are obtained from the trajectory.
- The final control law obtained is

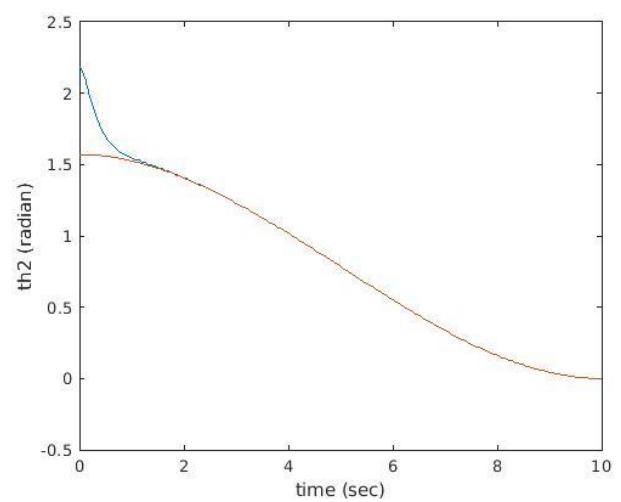
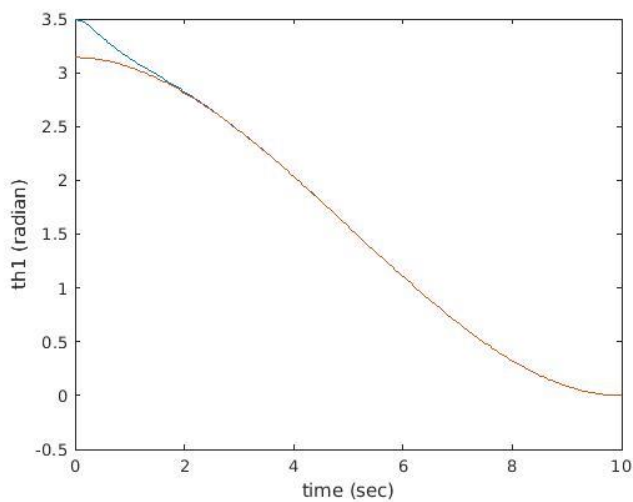
$$\tau = M*v+C*\dot{q}+G.$$

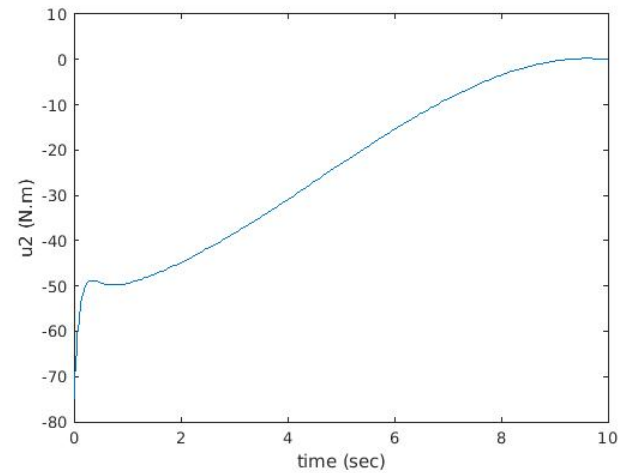
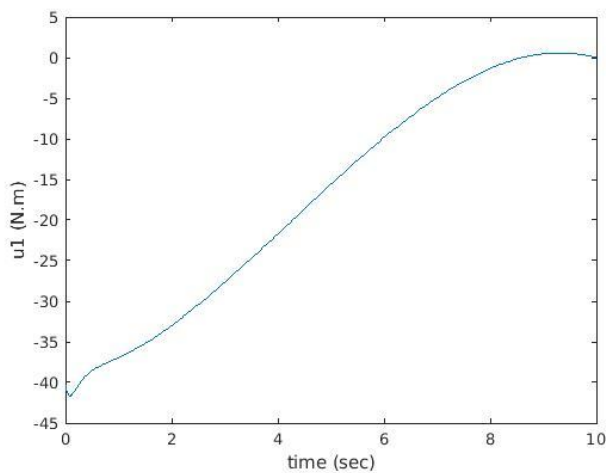
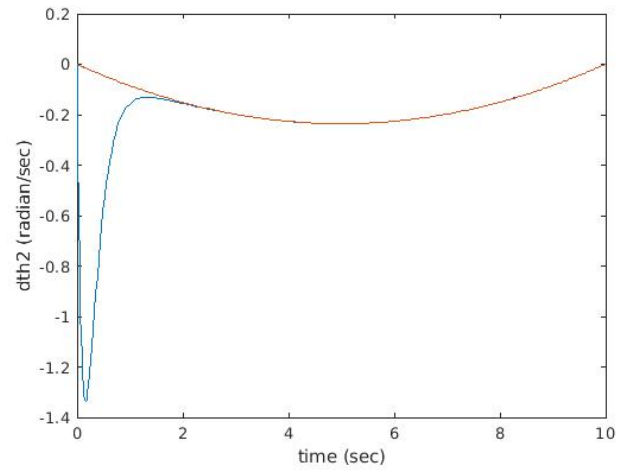
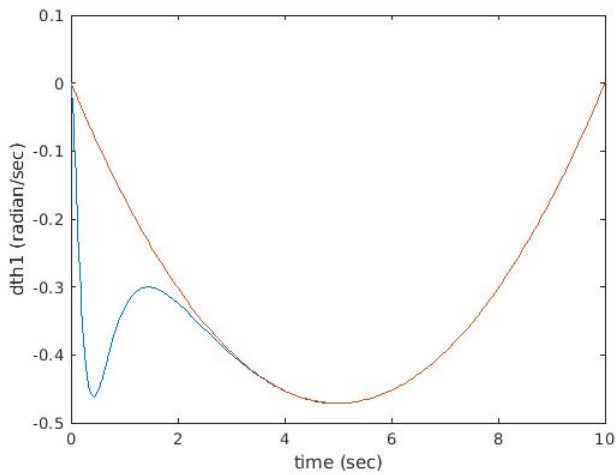
#### Part d:

- Now in order to make the system follow a trajectory, the equations of the trajectory are added in the ode function from which decided states are obtained.
- The control law designed in part c is substituted in the ode function and the final control input is updated accordingly.

#### Part e:

- Now using the ode function, simulation is run by providing the initial conditions  $\theta_1 = 200^\circ$ ,  $\theta_2 = 125^\circ$ ,  $\dot{\theta}_1 = 0$  and  $\dot{\theta}_2 = 0$
- Following plots were obtained from the simulation.

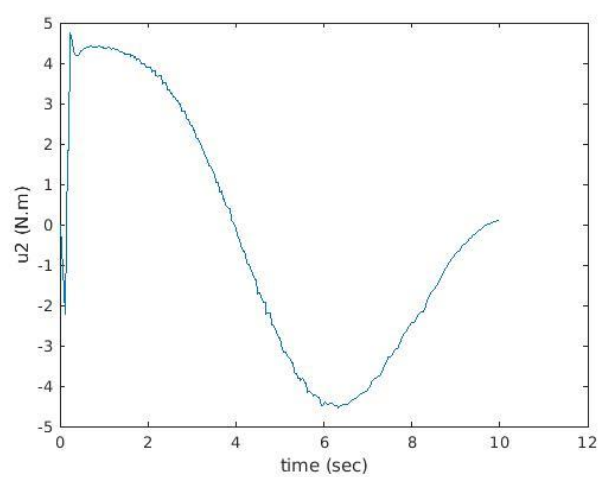
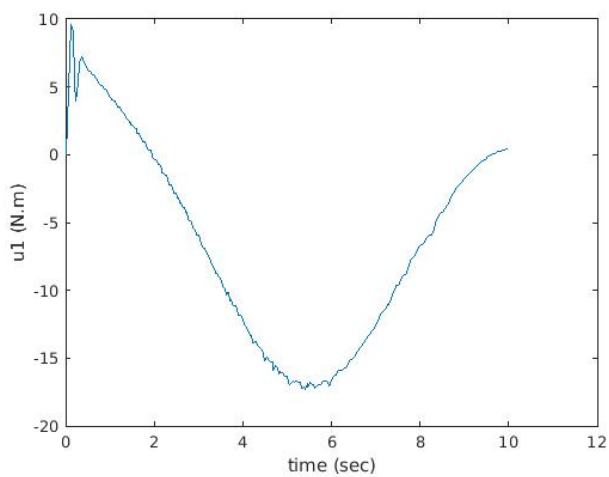
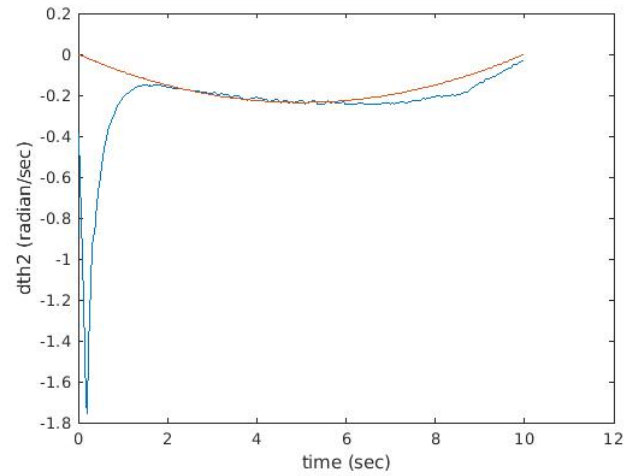
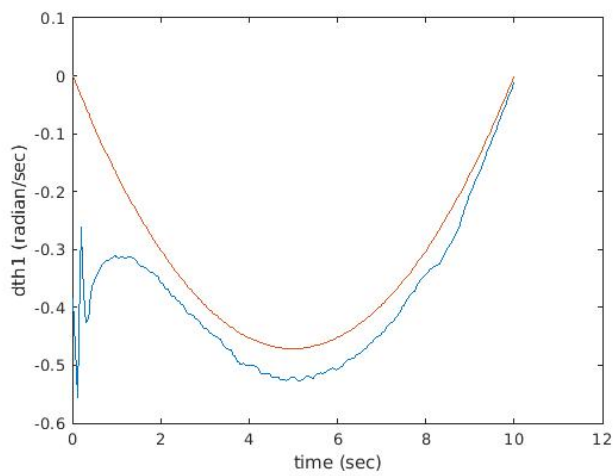
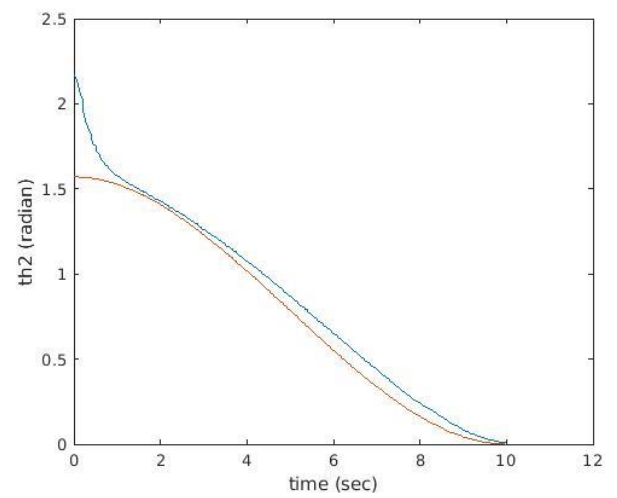
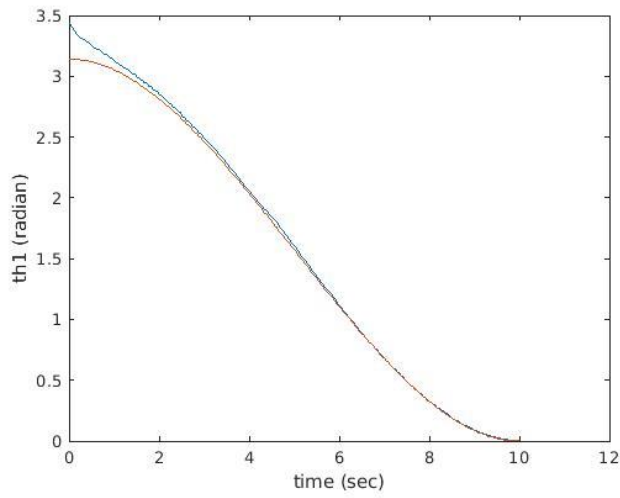




- Here the orange line represents the actual trajectory and the blue represents the actual output.
- It can be seen that initially the starting points of the system and the trajectory are different but as time proceeds, the system merges to the trajectory.

#### Part f:

- New rrobot\_traj\_control file was created in order to test the controller and trajectory in gazebo simulation.
- The equations of the trajectory were added in order to obtain desired states and the control law was implemented inside the while loop.
- The initial starting conditions were provided as  $\theta_1 = 200^\circ$ ,  $\theta_2 = 125^\circ$ ,  $d\theta_1 = 0$  and  $d\theta_2 = 0$ .
- The result obtained were stored and plotted in the end.
- The controller was successfully able to control the system through the trajectory.
- Following are the plots obtained.



- The orange curve represents the trajectory and blue curve represents the actual output of the system.
- It can be seen from the plots that the starting points of the trajectory and the system were different but as the time proceeds, the system converges to the trajectory.