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FOURIER ANALYSIS OF CLIMATOLOGICAL DATA SERIES IN A TROPICAL ENVIRONMENT

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SUMMARY

Monthly-averaged 10-year climatological data series of global solar irradiation, average ambient temperature and mean relative humidity in Ibadan (lat. 7.43° N, long. 3.90° E), Nigeria, have been analysed using the Fourier series method. The pertinent amplitudes, phase angles, and harmonic angles have been obtained for each year of the series and for each variable. The study reveals that only the annual harmonic contributes in any significant way to explain the variance of the average temperature and the mean relative humitidy series, with percentages ranging between 57% and 91% for temperature and between 75% and 93% for the relative humidity. In the case of global solar irradiation, both the first and the second harmonics contribute almost equally to the total variance, with about 40% each. Typical annual time function parameters are provided for each of the three climatological variables.

KEY WORDS fourier; harmonic; time series; solar irradiation; ambient temperature; relative humidity; annual time function

INTRODUCTION

The use of harmonic or Fourier analysis to study solar irradiation has several advantages over other methods of solar data anlaysis and modelling. Firstly, the irradiation data itself is in a time series form *ab initio*, making it immediately amenable to the Fourier or harmonic series method. Secondly, it enables data consolidation, reducing data sets by orders of thousands without significant loss of information content in the observed phenomenon. Thirdly, especially in the case of Fourier transform methods, it enables the possibility of data rehabilitation, whereby incomplete data sets may be selectively filtered for noise to obtain very clean signals (Phillips, 1984). Finally, the harmonics and their phases contain detailed information on the observed phenomenon which give insight into its spatial and temporal distribution patterns.

The Fourier series analytic tool has been applied in hydrology and water resources for much longer than in solar climatology (Thomann, 1967; Adamonski, 1971; Kottegoda, 19880). Perhaps its earliest use in solar radiation analysis were by Balling (1983), Baldasano and Coronas (1983), Baldasano *et al.* (1988) and Salcedo and Baldasano (1984). Balling analysed the global monthly solar radiation from 221 stations in the United States and found that the average variance for the set explained by the first harmonic was 98.9%, varying between 92.9% and 99.9%. The second harmonic was explained between 0.01% and 6.56%.

The results of a Fourier analysis by Baldasano *et al.* (1988) on daily solar radiation data from 24 stations in Spain confirm that only the annual harmonic (i.e. the first harmonic) contributes in a determinant way to explaining the variance of the series, which ranges between 61% and 77%. The lower percentages compared to Balling's were attributed to the use of daily-averaged rather than monthly-averaged data.

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Salcedo and Balsadano (1984) also applied harmonic analysis to a 9-year annual series of daily global solar irradiation and a 10-year ambient temperature data (average, minimum and maximum) both for Barcelona city, Spain. The variance retained by the annual harmonic varied between 64% and 77% for the solar irradiation series and 61% and 70% for the temperature series. They used the analysis to deduce a 41-day phase angle between solar radiation and air temperature in Barcelona city.

Recently, Herrero (1993) used Fourier analysis on the monthly solar irradiation data from 41 stations in Spain and found that the contribution of the first harmonic ranged between 93% and 99.3%.

The purpose of the present study is to use the Fourier analytical tool to analyse 10-year time series of three climatological variables in a tropical environment, namely, the global solar irradiation, $G(t)$, the average ambient temperature, $T(t)$ and the average relative humidity $RH(t)$ in the city of Ibadan, Nigeria.

THE CLIMATOLOGICAL DATA

The data employed in this study have been obtained from the agroclimatological station of the International Institute of Tropical Agriculture (I.I.T.A.), Ibadan. For the global solar radiation and the average ambient temperature, the period of the series is the 10-year period 1980–89, while for the mean relative humidity, it is for the 10-year period 1983–92.

FOURIER SERIES (OR HARMONIC) ANALYSIS

The theory and application of Fourier series are well known and we only present the pertinent expressions here for application to the monthly-averaged daily climatological series, $H(t)$ of this study:

$$H(t) = H_{avy} + \sum_{m=1}^{N/2} R_m \cos\left(\frac{2\pi m}{\rho} t + \phi_m\right) \tag{1}$$

and also,

$$H(t) = H_{avy} + \sum_{m=1}^{N/2} a_m \sin\left(\frac{2\pi m}{\rho} t\right) + \sum_{m=1}^{N/2} b_m \cos\left(\frac{2\pi m}{\rho} t\right) \tag{2}$$

with the following least squares estimates for a_m and b_m:

$$a_m = \frac{2}{n} \sum_{i=1}^{N} H_i \sin\left(\frac{2\pi m}{\rho} i\right), \quad m = 1, 2, \ldots, (N/2) - 1 \tag{3}$$

$$a_{N/2} = 0 \tag{4}$$

$$b_m = \frac{2}{N} \sum_{i=1}^{N} H_i \cos\left(\frac{2\pi m}{\rho} i\right), \quad m = 1, 2, \ldots, (N/2) - 1 \tag{5}$$

$$b_{N/2} = \frac{1}{N} \sum_{i=1}^{N} H_i (-1)^i \tag{6}$$

where

$H(t)$ = value of climatological variable at time t
H_{avy} = year average of the N monthly-averaged daily climatological variable
H_i = monthly-averaged daily value of climatological variable for month 'i' in the series
n = period or number of terms in the series ($N = 12$ here)
R_m = amplitude of the mth harmonic = $\sqrt{a_m^2 + b_m^2}$
p = fundamental period (= 12 for annual cycle)

ϕ_m = phase angle of the mth harmonic, given by the following (Fagbenle and Karayiannis, 1994)

$$\phi_m = \begin{cases} \tan^{-1}(-a_m/b_m), & b_m > 0 \\ \tan^{-1}(-a_m/b_m) - \pi, & b_m < 0,\ a_m > 0 \\ \tan^{-1}(a_m/b_m) + \pi, & b_m < 0,\ a_m \le 0 \\ -\pi/2, & b_m = 0,\ a_m > 0 \\ \pi/2, & b_m = 0,\ a_m < 0 \\ \text{arbitrary}, & b_m = 0,\ a_m = 0 \end{cases} \tag{7}$$

The annual variation in a Fourier series dataset, determined by the first harmonic ($m = 1$), has one maximum value and one minimum value spaced apart by 6 months exactly. The semi-annual variation in the array dataset is determined by the second harmonic ($m = 2$) and has two equally spaced maxima and minima.

Furthermore, the phase angle ϕ_m for each harmonic locates the first crest relative to the origin and it is usually interpreted as the point in time at which maxima occur on the harmonic curve. To convert this time to data, it is necessary to adjust the time $t = 1$ to correspond to the Julian day nearest to the vernal equinox, 22 March, when the maximum irradiation obtains near the equator where Nigeria is located. Thus, we choose $t = 1$ to correspond to 15 March, $t = 2$ for 15 April, etc. It is seen that the situation is different from that in the northern latitudes where $t = 1$ corresponds nearly to the summer solstice, 22 June, when the global solar irradiation is highest in the northern hemisphere.

RESULTS AND DISCUSSION

Global solar irradiation series

Results of the harmonic analysis of the 10-year global solar radiation series are summarized in Tables 1 and 2. The first harmonic, which represents the annual tendencies in the data, appears not to be a dominant factor as might have been expected. In only two years, 1980 and 1989, is the contribution

Table 1. Total variance, contribution of the first two harmonics, and time first maximum occurs in the first two harmonics of the global solar irradiation series

Year	Total variance $(MJ/m^2\text{-day})^2$	Percentage harmonic contribution to variance		Units of time first maximum occurs in harmonics	
		1st	2nd	1st	2nd
1980	12·36	60·2	26·3	2·48	3·59
1981	6·72	25·7	49·2	2·14	4·22
1982	13·18	54·7	32·5	3·14	3·96
1983	6·27	25·6	55·8	2·36	4·16
1984	6·37	53·6	16·3	5·38	8·83
1985	5·24	22·2	73·2	4·83	4·64
1986	7·78	13·3	24·6	1·29	5·70
1987	4·19	34·4	47·6	3·25	4·16
1988	8·55	20·1	54·9	1·56	4·24
1989	7·30	70·2	8·7	2·49	5·00
DM*	4·39	44·0	39·4	2·86	4·22

*DM—results using data mean for the period from Table 5.

Table 2. Parameters of the first harmonic of the global solar irradiation series

Year	Irradiation G_{avy} (MJ/m²-day)	amplitude R_1 (MJ/m²-day)	Phase angle ϕ_1 (rad)	Coefficients a_1 (MJ/m²-day)	b_1 (MJ/m²-day)
1980	17·25	2·73	−1·2984	2·6266	0·7338
1981	17·35	1·32	−1·1221	1·1848	0·5705
1982	17·36	2·68	−1·6459	2·6767	−0·1981
1983	16·98	1·27	−1·2358	1·1955	0·4162
1984	17·65	1·85	−2·8205	0·5853	−1·7523
1985	16·57	1·08	−2·5301	0·6200	−0·8820
1986	16·67	1·02	−0·6746	0·6349	0·7938
1987	17·24	1·20	−1·7027	1·1914	−0·1565
1988	15·37	1·31	−0·8174	0·9558	0·8965
1989	18·09	2·26	−1·3021	2·1829	0·6012
DM*	17·06	1·39	−1·4970	1·3854	0·1023

*DM—results using data mean for the period from Table 5.

greater than 60%, while for six years out of ten, its contribution is about 30% or less. Indeed, the second harmonic appears equally as important in its contribution as the first harmonic.

This is in complete contrast to the results of harmonic studies of northern latitude irradiation. Balling (1983) found a remarkable coherent spatial pattern with first harmonic percentage variance levels ranging from 92.9% to 99.9% throughout the 221 US stations studied.

Similarly, Baldasano *et al.* (1988) found that only the first harmonic contributed in a determinant way to explain the variance, with percentages ranging between 61% and 77% throughout the 24 Spanish stations of the study. In his study of 41 stations also in Spain, Herrero (1993) also found that the first harmonic contribution ranged between 93% and 99.3%. If thus appears that the relative importance and dominance of the first irradiation harmonic diminishes near the equator, as suggested by studies of other stations throughout Nigeria (Fagbenle and Karayiannis, 1994).

The phase angle of maximum radiation of the first two harmonics also appears in Table 1 in time units, again with $t = 1$ for 15 March, $t = 2$ for 15 April, etc. It is seen that generally, the annual tendency in the data produces a maximum approximately between five and eight weeks after the 22 March vernal equinox date. Two years, 1984 and 1985, have much longer delay periods of 10 weeks and 16 weeks respectively. The first maximum in the second harmonic which indicates the semi-annual tendency is delayed generally by between two and three months.

With total variance ranging between 4 and 13 MJ/m²-day, a relatively narrow variation around the mean results with a maximum standard deviation of about 3·6 MJ/m²-day.

Parameters of the first harmonic given in Table 2 may be used to estimate the global solar irradiation in Ibadan within 10% of the data for any year, using only the first harmonic and the results for the data mean (DM) row. It is not possible to get any closer than this in the light of its low percentage contribution.

Average ambient termperature series

In the average ambient temperature series the annual tendency as exhibited by the first harmonic is clearly dominant, with values ranging between 57% and 91%, while the semi-annual tendency contributes relatively smaller percentages between 4% and 30%. For each year of the series, Table 3 shows the relevant parameters for estimation of the ambient temperature using only the first harmonic. Such estimates are seldom outside 1% of the data. Thus, in this case, a truly typical annual time function may be found using the values indicated on the DM row in Table 3, and the results are within 1·5% of the data for any year. Thus,

$$T(t) = 26{\cdot}7 + 1{\cdot}7275\cos\left(\frac{\pi}{6}t - 1{\cdot}2932\right)$$

or (8)

$$T(t) = 26{\cdot}7 + 1{\cdot}6614\sin\left(\frac{\pi}{6}t\right) + 0{\cdot}4734\cos\left(\frac{\pi}{6}t\right)$$

The first maxima in the first harmonic usually occurred, for the years under study, between the time units of 2·2 (21 April) and 2·8 (9 May), four to six weeks after the vernal equinox of 22 March. This is approximately the same range found for the global irradiation first harmonic. In the case of the second harmonic, its first maximum appeared about one month after that of the temperature first harmonic.

Average relative humidity series

The largest percentage contribution from the first harmonic to the total variance was found in the average relative humidity series in which, as shown in Table 4, the range was between 75% and 94%. With the exception of the two years 1984 and 1986, the second harmonic contribution to the total variance was always less than 10%.

Thus, the relative humidity may be estimated quite adequately from the values of the relevant parameters shown in Table 4, with deviations from data of less than 1% for any year. In this case, the typical annual time function is perhaps most adequately obtained from the values under DM in the last row of Table 4 as:

$$RH(t) = 74{\cdot}36 + 9{\cdot}64\cos\left(\frac{\pi}{6}t + 2{\cdot}4016\right)$$

or (9)

$$RH(t) = 74{\cdot}36 - 6{\cdot}5105\sin\left(\frac{\pi}{6}t\right) - 7{\cdot}1129\cos\left(\frac{\pi}{6}t\right)$$

For calculating the values of the parameters for the typical annual time function of each of the climatological variables shown in the last row of Tables 2 to 4, it was found sufficiently accurate to emply

Table 3. Relevant parameters of the first harmonic of the average ambient temperature series

Year	T_{avy} (°C)	Percentage contribution to total variance 1st	2nd	Phase angle (rad)	Coefficients a_1	b_1	Amplitude R_1 (°C)
1980	26·6	79·7	12·0	−1·3701	1·7920	0·3645	1·8287
1981	26·8	83·3	4·0	−0·9204	1·4588	1·1098	1·8330
1982	26·6	90·9	6·3	−1·1252	1·6934	0·8089	1·8767
1983	26·9	57·1	29·4	−1·3095	1·9203	0·5135	1·9877
1984	26·6	68·2	14·7	−1·4684	1·3765	0·1415	1·3838
1985	26·4	69·4	8·2	−1·2696	1·5673	0·4868	1·6412
1986	26·2	82·8	10·7	−1·3997	1·9233	0·3322	1·9517
1987	27·2	77·5	6·4	−1·5041	1·7907	0·1195	1·7947
1988	26·6	71·2	5·9	−0·8076	0·9019	0·8647	1·2481
1989	26·9	80·4	12·5	−1·5745	2·1898	−5·378*	2·1898
DM*	26·7	83·0	10·9	−1·2932	1·6614	0·4734	1·7275

*DM—results using data mean for the period from Table 5.

Table 4. Relevant parameters of the first harmonic of the average relative humidity series

Year	RH_{avy} (%)	Amplitude (%)	Percentage Contribution to total variance 1st	2nd	Phase angle (rad)	Coefficients a_1	b_1
1983	72·92	14·91	80·2	5·9	2·3326	−10·8006	−10·2769
1984	74·75	8·67	81·4	17·4	2·3751	−6·0217	−6·2379
1985	73·25	10·02	84·5	5·1	2·4291	−6·5585	−7·5721
1986	74·00	8·77	77·2	13·8	2·3040	−6·5221	−5·8593
1987	74·93	9·32	89·5	2·4	2·2910	−7·0139	−6·1389
1988	75·54	9·55	92·8	3·8	2·5224	−5·5520	−7·7691
1989	74·43	5·36	79·7	2·2	2·7931	−1·8377	−5·0383
1990	74·80	6·52	75·5	6·2	2·4897	−3·9622	−5·1784
1991	75·60	9·52	93·8	4·6	2·3823	−6·5630	−6·8970
1992	72·40	14·45	85·3	9·5	2·3519	−10·2735	−10·1610
DM*	74·36	9·64	94·2	4·9	2·4016	−6·5105	−7·1129

*DM—results using data mean for the period from Table 5.

Table 5. Mean monthly values of the climatological variables over the period of each series

Month	Irradiation G_{mean} (MJ/m^2-day) (1980–89)	Ambient temperature T_{mean} (°C) (1980–89)	Relative humidity RH_{mean} (%) (1983–92)
Jan.	16·18	26·91	61·59
Feb.	18·54	28·90	63·21
Mar.	18·62	28·83	68·97
Apr.	19·02	28·26	74·45
May	18·19	26·92	78·85
June	17·51	26·08	80·70
July	15·03	24·88	81·90
Aug.	14·12	24·88	82·28
Sept.	16·15	25·24	81·28
Oct.	17·56	26·13	76·26
Nov.	17·88	26·92	72·51
Dec.	15·85	26·09	68·34

the mean monthly values over the period of each series, as shown in Table 5. No higher accuracy was obtained from any more complicated least squares procedures.

CONCLUSIONS

The total variance in the climatological series studies ranged between 5 and 14 (MJ/m^2-day)^2 for global solar irradiation, 2·18 and 6·92 (°C)^2 for average ambient temperature, and 36 and 278 (%)^2 for relative humidity. Thus, both the irradiation and the ambient temperature display rather narrow variations about the mean of their annual series.

In contrast to findings from northern latitudes, the irradiation first harmonic at Ibadan has a very low percentage contribution to the total variance, much less than 50% in most of the years examined. In fact both the ambient temperature and the relative humidity series displayed dominant first harmonic characteristics and the concept of a typical annual time function fitted these two variables quite easily.

Phase angles of maximum radiation of the first harmonic indicate that the annual tendency in the data produced a maximum in both the irradiation and the ambient temperature generally between four and six weeks after the 22 March vernal equinox.

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