Investigating Exponential random variables

Jimi Damon

11/18/2015

Overview

```
library(dplyr)
library(ggplot2)
set.seed(314159)
lambda=0.2
```

Simulations

I make use of the sapply function to return average of a 40 element exponential function for every element in a list of length 1000.

```
sims <- sapply(1:1000,function(x){ mean(rexp(40,lambda)) } )
```

This creates a 1000 element list in the variable sims.

Sample mean versus Theoretical mean

```
mean_sims <- mean(sims)
mean_sims
```

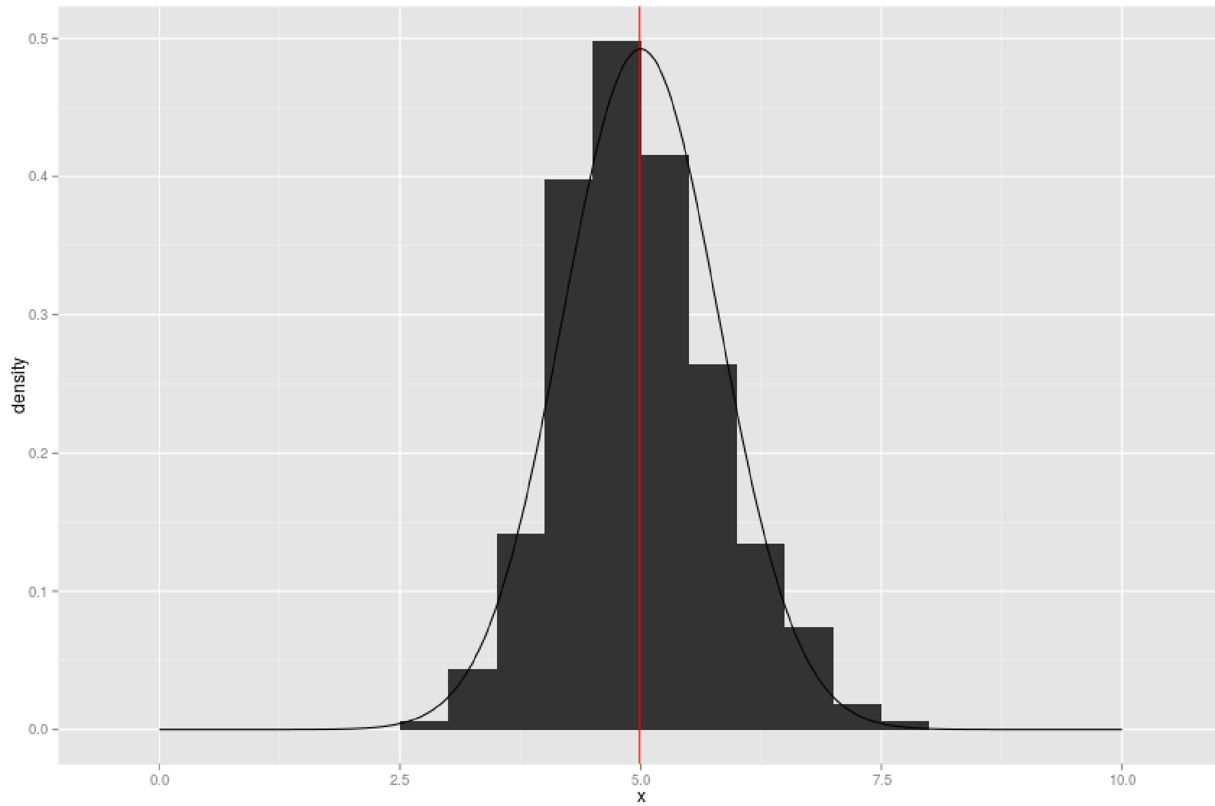
```
## [1] 4.98902
```

The expected mean for an exponential distribution would be $\frac{1}{\lambda}$ which equals

$$E[X] = \quad \frac{1}{\lambda} \quad = \frac{1}{0.2} \quad E[X] = \quad 5$$

t-Test of this mean value In order to determine how close this sample mean $\mu = \texttt{4.989}$ is to our ideal value of 5, I will run a t-test. I will be testing the null hypothesis that the average of these 1000 averages of sets of 40 exponential random variables is equivalent to 5.

```
ggplot(data.frame(x=sims),aes(x=x)) + geom_histogram(aes(y=..density..),binwidth=0.5) +
geom_vline(xintercept=mean(sims),colour="red") +
geom_line(data=(seq(0,10,0.001) %>% data.frame(x=.,y=dnorm(x=.,mean=5,sd=sqrt(var(sims))))),aes(x=x,y=y
```



Our plot clearly shows a normal shaped plot centered around the red line represnting the theoretical mean of the distribution.

I set up the t-test as $H_0 : \mu = 5$

P value for this is

```
tt <- t.test(sims,mu=5)
tt$p.value
```

```
## [1] 0.6684281
```

Since this value is greater than $\alpha(0.05)$, then we can accept our NULL hypothesis that our sample mean matches the theoretical.

Sample variance vs. Theoretical Variance

```
sd_sims <- sd(sims)
sd_sims
```

```
## [1] 0.8104647
```

The expected standard deviation for an exponential random variable would be

$$\frac{1}{\lambda}$$

We would get $\frac{1}{(0.2)^2}/sqrt(40)) = 0.79$.

As we can see , both of these values (variance and mean) are very close to their expected values and the graph shows a distribution that is very close to a normal distribution.

The skewness of this distribution tends to favor a fatter tail going to positive ∞. This makes sense since the exponential distribution is limited at x = 0 and cannot contribute anymore mass in range $x : (-\infty, 0]$.